International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 17 (2025) pp. 1-21

https://doi.org/10.70917/ijcisim-2025-0273
Article

Mathematical Modeling and Optimization Methods for Characterizing the Properties of Recycled Asphalt Concrete Mixtures

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Abstract: A huge amount of waste asphalt mixture (RAP) is generated in the process of highway development, and in order to rationally utilize the resources, the recycling technology should be born. The study focuses on a series of mathematical modeling and optimization methods for characterizing the performance of recycled asphalt concrete mixtures. Through gray correlation analysis, the degree of influence of variables such as RAP admixture on the performance of the mix is identified. A BP neural network was introduced to construct a recycled asphalt mixture performance prediction model, and further combined with genetic algorithm to globally optimize the neural network parameters. Experiments on the performance of recycled asphalt in terms of needle penetration, softening point, ductility and rotational viscosity at different RAP dosages were carried out. The results showed that with the increase of recycled asphalt dosing, the needle penetration of aged asphalt increased from 28.18 to 64.89, the ductility increased from 14.10 cm to 58.55 cm, and the softening point decreased from 63.91°C to 48.00°C, which indicated that its low-temperature deformation capacity and construction and ease of use were significantly improved. The R^2 of the combination of BP neural network-genetic algorithm in four performance indexes, including dynamic stability, residual stability, etc., exceeded 0.9, and the predicted Pearson correlation coefficient of maximum bending and tensile strain reached 0.980, which is superior to the machine learning methods such as multivariate linear regression, vector machine, and random forest.

Keywords: RAP; recycled mix; gray correlation analysis; needle penetration; BP neural network; genetic algorithm

1. Introduction

In recent years, many highways constructed in the 1980s and 1990s have suffered from rutting, potholes, cracks, and other diseases under the pressure of overloading and traffic volume, and are in urgent need of maintenance or renovation to ensure their normal capacity [1]. Asphalt concrete is composed of asphalt, aggregate, stone dust cement and water and is generally used as the surface layer of asphalt road structures [2]. As we all know, the asphalt mixture used in the structural layer of the pavement is a kind of organic material with complex composition, and the asphalt mixture of the surface layer not only has to withstand the physical damage effects such as vehicle abrasion, external impact, but also due to the exposure to the external environment by wind, sun, freezing, flooding, temperature changes and other environmental effects of the combined effects of the role of the environment [3-5]. Therefore, the performance of asphalt changes with the growth of service life, asphalt slowly aging with time, adhesion and strength and other properties gradually decline, ultimately leading to asphalt concrete pavement rutting, congested package, waves, cracks, potholes, fly-away and other diseases [6-7].

When the performance of the road surface deteriorates to a certain extent and fails to meet the requirements of safe, comfortable, economical and fast driving for automobiles, it is necessary for specialized technicians to use specialized techniques to repair and maintain the diseased asphalt road surface to restore its service performance. During the maintenance, repair and reconstruction of severely damaged road surfaces, a large amount of waste pavement materials (RAP) are inevitably produced. These waste pavement materials contain a large amount of crushed stones, sand and aged asphalt, all of which are highly valuable materials that can be recycled and reused. The waste pavement materials are crushed, sieved and extracted to obtain these materials. It plays an immeasurable role in saving construction materials and reducing costs in road construction [8-10].

Asphalt pavement recycling technology is a technology that recycles, crushes, screens and extracts the waste pavement materials generated in the process of road renovation and reconstruction, and adds appropriate new aggregates (or not), new asphalt, and active fillers according to the recycled materials, and re-mixes, transports, paves, and mills them, to form asphalt mixtures whose road performance meets the relevant specifications [11-13]. Because the asphalt concrete recycling technology has the advantages of energy saving, environmental protection, low cost, cost saving, and short construction period, it is now more and more applied to road maintenance construction such as road repair and renovation [14]. The existing regeneration technology mainly includes hot regeneration technology, warm regeneration technology and cold regeneration technology [15]. Through the performance study of the regeneration mixture to explore whether its road performance meets the requirements of the pavement wear layer, it is possible to understand the strength formation process of cold regeneration mixture and the difference between the interior design and the actual project in a more in-depth manner, so as to more comprehensively assess the comprehensive performance of cold regeneration mixture, and to promote the development and application of regeneration technology [16-17].

The application of pavement regeneration technology in developed countries is relatively mature and popular, they have a long time tracking practice and research on asphalt mixture regeneration, forming a self-contained system of regeneration technology [18-19]. Jaawani et al. deeply analyzed the role of RAP in the production of concrete, combined with the experimental data provided by the existing research, and systematically analyzed the effect of RAP on the compressive strength, flexural strength and durability of concrete [20]. Tabaković et al. prepared asphalt mixtures containing different percentages of RAP with the aim of exploring the effect of RAP on the mechanical properties of binder-grade asphalt pavement mixtures, and it was found that RAP optimized the mechanical properties of binder mixtures, with mixtures containing 30% of RAP showing the best fatigue resistance [21]. Ma et al. similarly investigated the effect of different percentage contents of RAP on recycled asphalt concrete and designed a variety of test experiments to determine its properties such as dynamic modulus, stability, and moisture sensitivity, and found that the percentage content of RAP improves the dynamic modulus and rutting stability of the concrete, but decreases its resistance to thermal cracking and fatigue cracking [22].

In addition, Ding et al. formulated ordinary recycled asphalt mixtures and recycled asphalt mixtures based on rubberized asphalt stabilizing binder using 0%, 30%, and 50% RAP, and the rubberized asphalt and RAP percentages enhanced the low temperature resistance of recycled asphalt mixtures [23]. Santos et al. fabricated flexible mixtures with RAP contents of 20%, 30%, and 40%, and compared them to conventional mixtures were compared and their differences in stiffness, modulus, fatigue resistance and permanent deformation behavior were analyzed in the laboratory [24]. Zhu, J et al. designed tests such as dynamic modulus test and Marshall immersion to obtain the mechanical parameters of high modulus asphalt concrete containing high content of RAP, and found that a reasonable proportion of aggregate gradation and asphalt binder can improve the overall modulus asphalt concrete properties [25]. Laboratory test results by Sanchez-Cotte et al. showed that recycled concrete pavement-modified mixes had similar results to conventional hot-mix asphalt mixtures in terms of resilient modulus and indirect tensile strength, and that the source and content of the recycled concrete pavement affected the properties of the mixes [26]. Hoy et al. analyzed the effect of warm-mix and cold-mix re-testing techniques for RAP on the mechanical properties of asphalt concrete, where warm-mix asphalt concrete was made with asphalt cement AC60/70, while cold-mix asphalt concrete was made with asphalt emulsion CMS-2h, and the test results showed that CMS-2h is more suitable for pavement deterioration and rehabilitation, and it has better mechanical properties and is more environmentally friendly [27]. Oner et al. evaluated the performance of recycled mixtures with different percentages of reclaimed mixtures, based on three different warm-mix asphalt additives content, the mechanical properties and cost-effectiveness brought by the organic additives were best when the pavement reclaimed mix content was 30%, and the warm mix asphalt technology reduced the pavement construction cost [28].

At the same time, Mills-Beale et al. carried out a study on recycled concrete aggregate as a recycled mixture in hot mix asphalt and used mechanical properties to characterize the recycled mixture, the dynamic stiffness of the mixture in the test decreased with the increase of the recycled concrete aggregate, while the tensile strength ratio increased with the decrease of the recycled concrete aggregate [29]. Jiang, through electrochemical analysis method, the investigated the reaction mechanism of emulsified asphalt and aggregate before, when the contact between emulsified asphalt and aggregate occurs, the charged asphalt particles will be chemisorbed with the surface of the aggregate and rapidly distributed on the surface of the aggregate and through the series of reactions such as breaking the emulsion, distillation drying and so on to make the emulsified asphalt mixtures of the maximum adhesion [30]. Albayati et al. evaluated the performance of hot-mix hot-mix mixtures with the simultaneous use of RAP and recycled concrete aggregates. properties of recycled mixtures with both RAP and recycled concrete aggregates and tested the properties of recycled mixtures by scanning electron microscopy, Marshall stability, and indirect tensile strength tests, and the study showed that the recycled mixtures exhibited better performance [31].

Mathematical models can quantitatively correlate the performance indicators of asphalt mixtures (e.g., water damage resistance, high temperature stability, etc.) with parameters such as asphalt type, aggregate gradation, and mineral admixture, and make clear the weight of each parameter's influence on the performance [32-33]. Zhu, Y et al. used molecular dynamics simulation to study the mixing process of reclaimed asphalt both on the surface and within the layer, and they analyzed the new asphalt, aged asphalt, and reclaimer interactions and mutual diffusion between them and found that when the reclaimed asphalt content is too high, a large amount of reclaimed asphalt will diffuse into the original asphalt, which is not conducive to the recovery of the properties of aged asphalt [34]. Rondón-Quintana et al. investigated the phenomenon of fatigue in asphalt mixtures and developed mathematical models for this purpose, and they concluded that fatigue in asphalt mixtures is related to the loading pattern, the type of loading and the laboratory resting period to which the samples were subjected had a strong correlation [35]. Wang et al. constructed a mathematical prediction model in order to optimize the thermal conductivity of asphalt mixtures, which related the number of phases of the asphalt mixtures to the alignment structure while integrating the effects of asphalt, aggregates, pore space, and water on the thermal conductivity, and obtained the order of the influence of the different factors on the thermal conductivity to be the number of compaction> asphalt aggregate ratio> asphalt aggregate ratio> water content> gradation [36]. Water content > gradation [36].Ren et al. established a mathematical model of effective thermal conductivity of asphalt mixtures based on the principle of minimum thermal resistance and investigated the effect of morphology factor on the thermal conductivity of asphalt mixtures, and found that the effective thermal conductivity was positively correlated with the agglomerate content, the thermal conductivity of the aggregates, and the matrix of the fine aggregates, and negatively correlated with the morphology factor and the porosity [37]. Provatorova et al. established a mathematical model for the characterization of modified asphalt concrete, through which the maximum permissible deviation of the binder dosage and the main indexes in the asphalt concrete mixture were accurately calculated to provide data references for asphalt concrete improvement [38].

The article investigates how to efficiently and environmentally utilize recycled asphalt pavement material (RAP). The study combines grey correlation analysis, BP neural network and genetic algorithm to carry out the performance modeling and optimization of recycled asphalt mixtures. Firstly, various types of raw materials and their performance test results are introduced. The performance of basalt, limestone aggregates and RAP with different densities and sieve sizes were analyzed by the percentage of passing the square hole sieve, and the recycled mix ratios with different HMM and RAP dosages were designed. The analysis of asphalt properties was also carried out using three measurement methods, i.e., needle penetration measurement, softening point measurement and ductility measurement. On this basis, the gray linkage coefficient was introduced to clarify the correlation between different variables (RAP dosage, asphalt type, gradation, etc.) and the mix performance. Finally, a recycled asphalt mixture performance prediction and optimization system is constructed based on BP neural network and genetic algorithm.BP neural network can simulate the learning mechanism of the human brain, and through the training of a large amount of data, it establishes a nonlinear mapping relationship from the raw material parameters to the performance of the mixture, and realizes intelligent prediction of the performance. The genetic algorithm draws on the natural evolutionary principle of survival of the fittest to globally optimize the parameters in the neural network model and search for the best mix design scheme.

2. Analysis of raw material properties and identification of key influencing factors for recycled mixes

2.1. Analysis of technical properties of raw materials and design of mixing ratio

In this chapter, the raw materials and RAPs selected for the study were tested for their technical properties, and the mix proportions were designed for HMM-13, RHMM-13 with different RAP dosages, and thermally regenerated Sup-20 with 30% RAP dosage.

2.1.1. Aggregates and fillers

In this paper, two types of aggregates, basalt and limestone, are selected. The filler selected is the mineral powder made from limestone after grinding finely. According to the “highway engineering aggregate test procedures” (JTG3432-2024) for aggregate performance tests and density test, aggregate density and filler indicators are shown in Table 1 and Table 2, the results show that all aggregates meet the specification requirements. The sieving results of limestone, basalt and mineral powder are shown in

Table 3, Table 4 and Table 5 respectively.

Table 1. Test results of aggregate density.

Aggregate type		Apparent relative density	Bulk relative density	Asphalt mixture
Limestone	1#	2.719	2.710	Sup-20
	2#	2.715	2.657	
	3#	2.708	2.654	
	4#	2.699	2.642	Sup-20, HMM-13
Basalt	1#	2.947	2.864	HMM-13
	2#	2.934	2.857	
	3#	2.936	2.846	
	4#	2.929	2.839	

Table 2. Filler performance indicators and test results.

Test indicators		Test results	Standard requirements	Test methods
Water content/%		0.806	≤1	JTG3430,T0103
Relative density		2.719	≥2.5	JTGE42,T0352
Hydrophilic coefficient		0.843	<1	JTGE42,T0353
Particle size range	<0.6mm	100	100	JTGE42,T0351
	<0.15mm	96.291	90~100	
	<0.075mm	81.303	75~100	

Table 3. The screening results of basalt.

Screen hole size	Percentage passing through the square-hole sieve / %									
	16.0	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
1#	100	84.24	40.29	2.73	0.62	0.36	0.36	0.36	0.36	0.36
2#	100	100	96.17	10.82	3.57	1.92	1.92	1.92	1.92	1.92
3#	100	100	100	84.59	10.92	5.24	3.18	1.28	1.28	1.28
4#	100	100	100	96.23	66.08	22.14	5.78	2.94	2.41	2.41

Table 4. Limestone screening results.

Screen hole size	Percentage passing through the square-hole sieve / %									
	16.0	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
1#	100	91.28	68.10	10.23	1.08	1.08	1.08	1.08	1.08	1.08

2#	100	100	97.59	23.47	6.01	3.15	3.15	3.15	3.15	3.15
3#	100	100	100	91.72	45.23	14.64	5.13	2.38	2.38	2.38
4#	100	100	100	99.92	84.20	40.93	8.24	3.74	3.74	3.74

Table 5. Screening results of the mineral powder.

Screen hole size	Percentage passing through the square-hole sieve / %									
	16.0	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
Mine powder	100	100	100	100	100	100	100	100	96.10	85.82

2.1.2. Asphalt

This paper adopts the "highway engineering asphalt and asphalt mixture test procedures" (JTG E20-2011) on the SBS modified asphalt indicators for testing, test results are shown in Table 6, SBS asphalt performance indicators can meet the specifications.

Table 6. Performance indicators and test results of SBS modified asphalt.

Testing items	Test method	Standard requirements	Test results
Penetration (25℃)/0.1mm	T0604	40~70	59.17
Penetration (25℃)/0.Softening point/℃	T0606	≮70	73.19
1mm	T0605	≮25	52.08
Softening point/℃	T0662	≮80	84.2
Extension (5cm/min, 5℃)/cm	T0610	±0.5	-0.063
Extension (5cm/min, 5℃)/cm	T0604	≮65	83.29
Elastic recovery (25℃)/%	T0605	≮15	34.16

2.1.3. RAP

The source of RAP was a local pavement which had been crushed and by screening it was obtained under 15mm.After extracting, centrifuging and distilling the RAP according to the specifications, the old asphalt was obtained along with the old aggregate. The extracted and dried aggregates were sieved in water to get the gradation composition of the old aggregates and the oil to rock ratio of RAP was calculated to be 4.27%. The sieving results of the old aggregates are shown in Table 7.

Table 7. Result of screening of old mine materials.

Screen hole size	Percentage passing through the square-hole sieve / %									
	16.0	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
RAP	100	99.12	87.28	40.12	21.37	13.18	11.02	7.43	5.13	3.51

According to the test methods specified in the specification "Test Procedure for Asphalt and Asphalt Mixture in Highway Engineering" (JTGE20-2019), the recycled old asphalt is subjected to 25℃ needle penetration test (T0606), 5℃ ductility test (T0605), softening point test (T0606) and 60℃, 135℃, 175℃ viscosity test, etc., to test the relevant technical indicators, and to assess its physical properties and viscosity were evaluated to analyze the aging grade of old asphalt, and the test results are shown in Table

8.

Table 8. Old asphalt-related technical indicators.

Technical indicators	Old asphalt	Testing specification
Penetration (25℃) / 0.1 mm	28.18	T0604
Softening point / °C	63.91	T0606
Extension (5℃) / cm	14.10	T0605
Viscosity at 60℃ / mPa·s	2783	T0624
Viscosity at 135℃ / mPa·s	1851	T0625
Viscosity at 175℃ / mPa·s	726	T0626

Will be able to 25 ℃ penetration and 135 ℃ viscosity can be divided into the old asphalt aging grade, divided based on the old asphalt aging grade table for judgment, the old asphalt used in this paper belongs to the old SBS class of asphalt pavement binder, and its aging grade for the third level.

Table 9. The criteria for classifying the aging level of old asphalt.

	Old road petroleum asphalt pavement binder		Old SBS type asphalt pavement binder					
Viscosity (Pa·s)	η≤1.6		η≤1.6	1.6<η≤3			η>3	
Penetration (0.1mm)	P>30	10<P≤30	P>30	P>30	20<P≤30	10<P≤20	20<P≤30	10<P≤20
Aging grade	I	II	I	II	III	IV	V	VI

2.2. Analysis of asphalt properties

The performance of asphalt itself directly determines the mix RAP quality. In this section, the three classical experiments of needle penetration, softening point and ductility will be used to observe the softness and hardness of asphalt at different temperatures, heat resistance and tensile deformation instincts. The overall test program of the study for RAP extraction and sieving and asphalt property examination is shown in Figure 1.

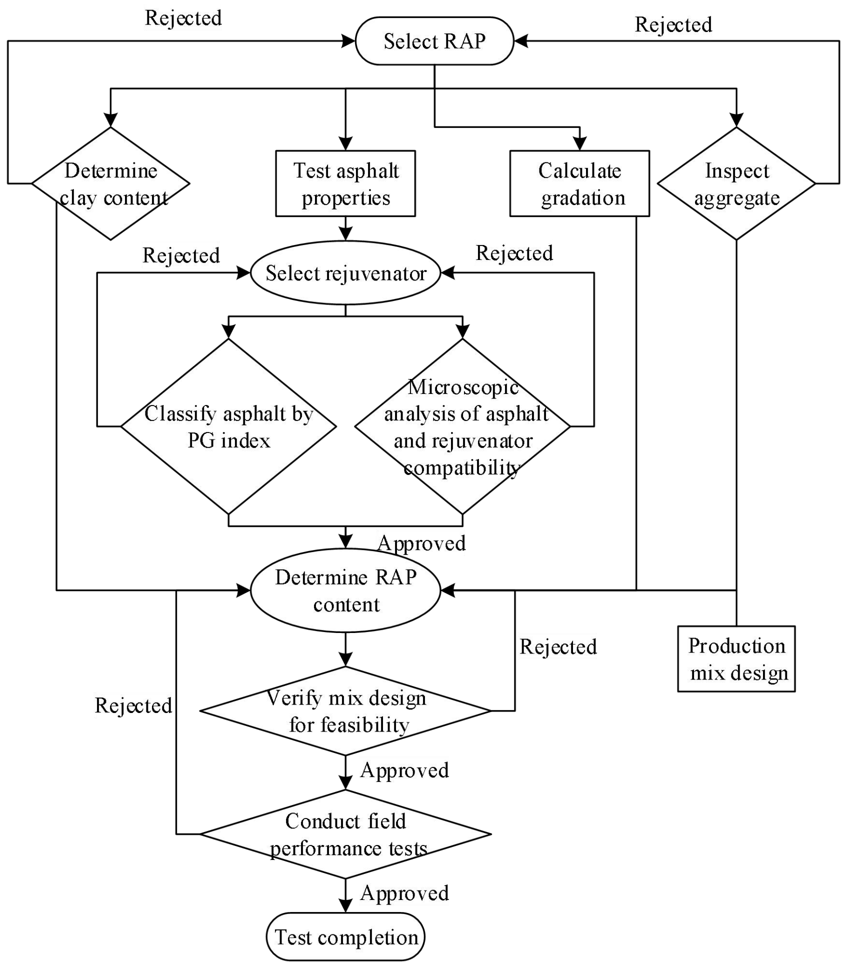

Figure 1. Overall process of the test plan.

2.2.1. Needle penetration determination method

Needle penetration determination method steps mainly include the following steps:

(1) Preparation: first prepare the specimen, generally cylindrical or square, the size should fully represent the actual engineering situation. Clean up the test site to ensure that the ground is flat and free of obvious impurities.

(2) Installation of pin-in device: Install the pin-in device on the specimen as required. The specific installation method and type of device are related to the geological conditions and need to be selected according to the actual situation.

(3) Conduct the test: gradually apply the load as required, observe the deformation of the pin-in device while applying the load, and record the corresponding load and deformation data.

(4) Stop the test: When the test data are sufficient, stop applying the load and record the final needle depth and load data.

2.2.2. Softening point determination method

Softening point determination method steps mainly include the following steps:

(1) Preparation: Prepare the specimen, make sure the surface of the specimen is flat and without obvious impurities. At the same time, prepare the softening point test instrument, check whether the instrument is intact, to ensure the smooth progress of the test.

(2) Install the specimen: put the specimen into the center of the heating plate of the softening point test instrument, and fix the specimen with the fixture.

(3) Start heating: Turn on the heating power of the test instrument and gradually increase the temperature of the heating plate. At the same time, observe the changes in the specimen, when the specimen softens the small iron ball falls, record the temperature at this time.

(4) End the test: When the specimen is completely softened and falls off, stop heating and turn off the test instrument. Record the final softening point temperature and analyze and evaluate.

2.2.3. Methods for determining ductility

The steps of the ductility test mainly include the following steps:

(1) Preparation: Prepare the specimen, make sure the surface of the specimen is flat and without obvious impurities. At the same time, prepare the ductility test instrument, check whether the instrument is intact, to ensure the smooth progress of the test.

(2) Installation of specimen: Put the specimen into the fixture of the ductility test instrument, and make sure the specimen is firmly fixed. According to the requirements of the test, adjust the distance between the fixtures and the test speed and other parameters.

(3) Start the test: Start the test instrument and begin to stretch the specimen. During the stretching process, observe the deformation of the specimen and record the tensile length and the tensile length when the specimen is damaged.

(4) End test: When the specimen is damaged or reaches the preset tensile length, stop the test and turn off the test instrument. Record the final tensile length and form of destruction, and analyze and evaluate.

2.3. Gray correlation analysis process of core influences on asphalt mixture performance

There are many factors affecting the performance of recycled asphalt mixtures, such as dynamic modulus, void ratio, oil-rock ratio, etc. In order to investigate which factor has the greatest influence, the key factor with the greatest influence on the performance of mixtures is identified with the help of gray correlation analysis among many factors.

The steps of gray correlation analysis for the core factors affecting the performance of asphalt mixture are as follows: (1) data processing and selection; (2) solving the gray correlation coefficient value between the subsequence and the parent sequence; (3) solving the gray correlation value; (4) ranking the gray correlation value, and drawing conclusions. The gray correlation analysis process is shown in Figure 2.

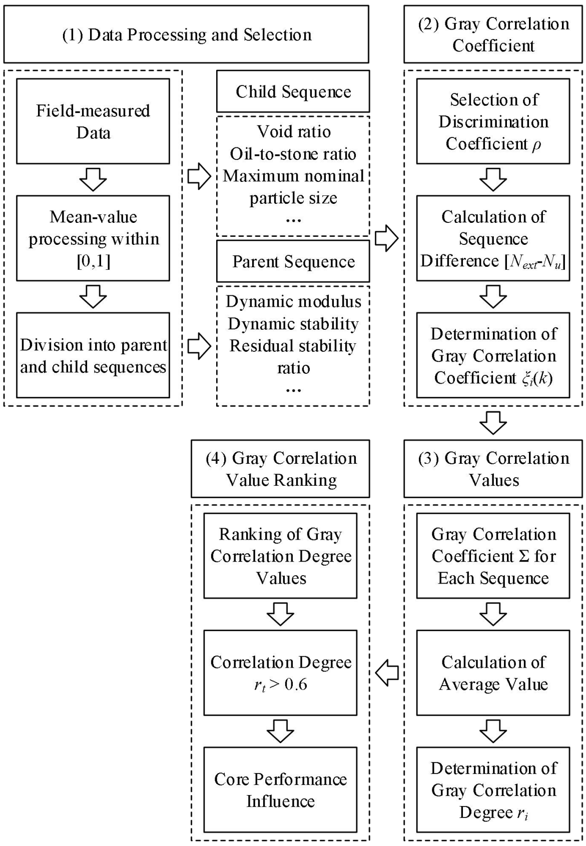

Figure 2. Gray correlation analysis process.

2.3.1. Data processing and selection

Five macroscopic performance characteristics such as dynamic modulus, dynamic stability, residual stability, split tensile strength ratio, ultimate bending and tensile strain were selected as the parent sequence, and 12 material composition characteristics such as void ratio, nominal maximum particle size, and oil-to-rock ratio were selected as the sub-sequences, respectively, for the convenience of data characterization, in which asphalt types of 70# asphalt, 90# asphalt, 110# asphalt, and SBS modified asphalt were represented by the numbers 1, 2, and 3, respectively, 4 to represent, and all other data are expressed in the form of raw data. At the same time, in order to avoid the evaluation model due to the different data outline and affect the evaluation model, choose to homogenize the dimensionless processing, that is, each class of data to the average value as a unit, all the data are divided by the average value of the data class.

2.3.2. Solving for the gray correlation coefficient

Each asphalt mixture property is defined as a parent sequence N_{0x} and each material composition characteristic is defined as a subsequence N_i, then the gray correlation coefficient is

$$\xi_i(k) = \frac{\min\limits_i \min\limits_k \left|N_{0xk} - N_{ik}\right| + \rho \max\limits_i \max\limits_k \left|N_{0xk} - N_{ik}\right|}{\left|N_{0xk} - N_{ik}\right| + \rho \max\limits_i \max\limits_k \left|N_{0xk} - N_{ik}\right|} \tag{1}$$

$$i = 1, 2, \cdots, 12; x = 1, 2, 3, 4, 5$$

In Eq. (1): $N_{0xk} - N_{ik}$ denotes the difference between the x parent sequence and the i subsequence at the k th point; ρ is the correlation resolution coefficient, which is generally 0~1. The smaller ρ is, the larger the resolution is, and it is taken as 0.5 in the calculation.

2.3.3. Solving for gray correlation values

The gray correlation value r_i can be directly used to judge the degree of influence of the material composition characteristics on the performance of asphalt mixtures, the higher the value of correlation, i.e., the greater the degree of influence. The formula is

$$r_i = \frac{1}{m} \sum_{k=1}^{m} \xi_i(k) \tag{2}$$

In equation (2): m is the total number of samples.

2.3.4. Gray correlation ranking

The gray correlation values of the five asphalt mixture properties were ranked separately to compare the magnitude of influence of each material compositional characteristic.

2.4. BP Neural Networks and Genetic Algorithms

In this section, two major methods, BP neural network and genetic algorithm, are introduced to constitute a system to study the performance of recycled asphalt mixtures from prediction to optimization. Through data training, the final performance of the mix is predicted from the raw material parameters, and among the huge number of possibilities, the search is made to find out that mix solution with optimal performance.

2.4.1. BP Neural Networks

BP neural network is a multilayer feed-forward neural network trained according to the error back propagation algorithm.The computational process of BP neural network mainly includes forward computational process and backward computational process.With the popularization of the application of matlab in the research of various disciplines, BP neural network has become the most widely used neural network nowadays.BP neural network is the flexible network structure, which has a strong nonlinear mapping ability. The number of intermediate layers of the network and the number of neurons in each layer can be set according to the specific situation of the research object, and at the same time, the difference in the design structure will make its performance differentiated. However, the training process of BP neural network can lead to slow convergence of the algorithm, which is easy to fall into the local minima.

2.4.2. Genetic algorithms

Genetic Algorithms (GA) originated from computer simulations of biological systems, and are essentially an efficient, parallel, global search method that automatically acquires and accumulates knowledge about the search space during the search process, and adaptively controls the search process in order to find the optimal solution. The implementation of genetic algorithms requires first determining a scheme for "digitally" encoding potential solutions to the problem, and establishing a mapping relationship between phenotypes and genotypes. A population is then initialized with random numbers, and after an appropriate decoding process, each genetic individual is evaluated for fitness using a fitness function, and is selected using a selection function according to some rules.

2.4.3. Network design and training

After combining the BP neural network and genetic algorithm to jointly construct a recycled asphalt mixture performance prediction network, its structure is shown in Figure 3. The network structure is selected as 8-10-5 single hidden layer neural network model, the hidden layer transfer function selects hyperbolic tangent S-shaped function, and the output layer transfer function selects linear function. The number of neurons in the input layer is 8, and the number of neurons in the output layer is 5. For the hidden layer, the number of neurons is not fixed, and it needs to be corrected according to the actual training situation, and the number of neurons in the hidden layer of the network is set to be 10 through the analysis of the number of steps of training and the result of error.

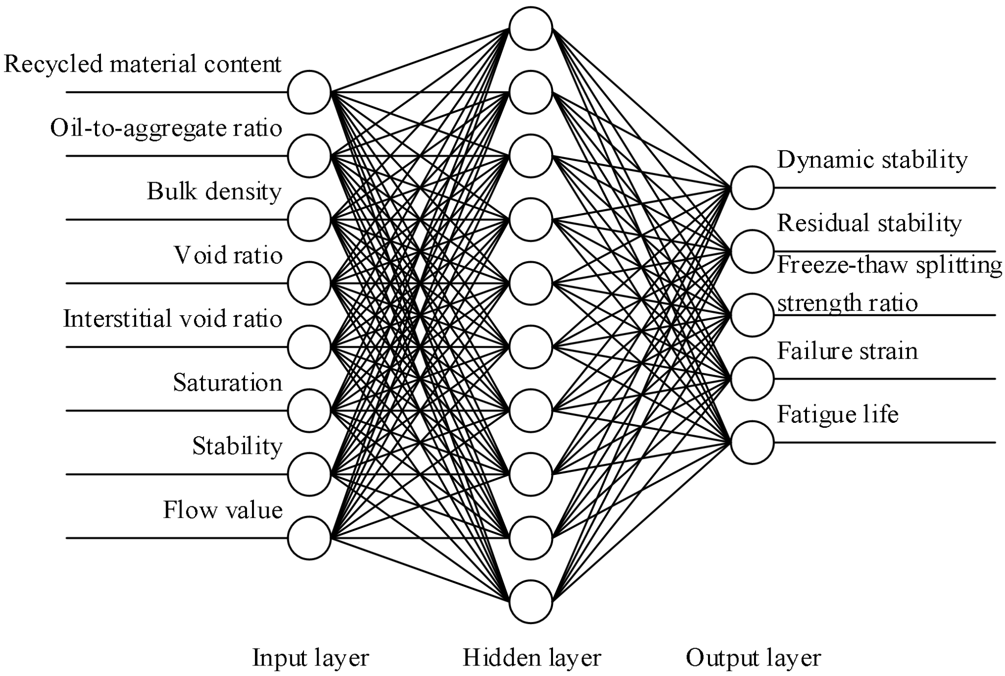

Figure 3. Network structure of performance prediction of recycled asphalt mixture.

3. Neural network-based performance prediction of recycled asphalt mixtures

After understanding the asphalt performance analysis methods and their core influencing factors in Chapter 2, Chapter 3 focuses on analyzing the performance of recycled asphalt binders and the prediction ability based on BP neural network-genetic algorithm.

Firstly, reclaimed asphalt at different dosage is set up to study its conventional performance and viscosity characteristics, and reveal its law of change with the proportion of reclaimed asphalt. Subsequently, a combination of BP neural network and genetic algorithm is introduced to analyze its training data to achieve accurate prediction of important indexes such as dynamic stability, residual stability, maximum bending and tensile strains, and fatigue life, and to compare and analyze with other models.

3.1. Properties of recycled asphalt binders

3.1.1. General Properties of Recycled Asphalt

In order to study the effect of recycled asphalt on the performance of recycled asphalt binding material, first of all, using the three conventional indicators of needle penetration, ductility, softening point, in the old asphalt mixed with different proportions of recycled asphalt binding material performance analysis, the test results are shown in Figure 4.

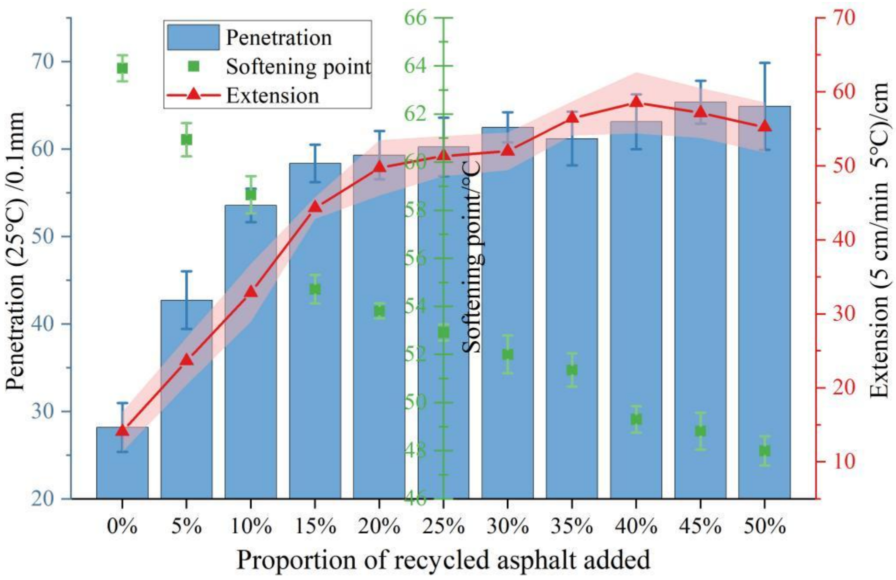

Figure 4. Asphalt performance under different proportions of recycled asphalt.

The enhancement effect of recycled asphalt on the performance of aged asphalt can be clearly seen in Figure 4. From section 2.1.3, it can be seen that the initial aging grade of the old asphalt of the study is class III, and its needle penetration is 2.818mm, softening point is 63.91°C, and the ductility is 14.10cm at 5°C under the test environment at 25°C. With the increase in the amount of reclaimed asphalt blending, the needle penetration and ductility of the blended asphalt gradually increase, and the softening point gradually decreases. In the ratio of recycled asphalt and aged asphalt 1:1, the needle penetration of the mixed asphalt is enhanced to 64.89, and the softening point decreases to 48°C, which indicates that the degree of asphalt softening has been improved, and the material becomes softer. The ductility was improved to 58.55 cm, indicating that the deformation ability of mixed recycled material at low temperature was greatly enhanced and the crack resistance was improved. Overall, the addition of recycled asphalt successfully awakened the performance of aged asphalt, especially when the blending amount was 5% to 30%, the improvement was most rapid.

3.1.2. Viscosity characteristics of recycled asphalt

In order to further explore the performance of recycled asphalt in terms of viscosity, recycled asphalt will continue to be blended into aged asphalt at 0-50% in 5% increments to prepare the resulting recycled asphalt mixtures. Comparative experiments were conducted with rotational viscosities at 135°C and 175°C, respectively. The rotational viscosities of the mixtures at each recycled asphalt percentage are shown in Fig. 5.

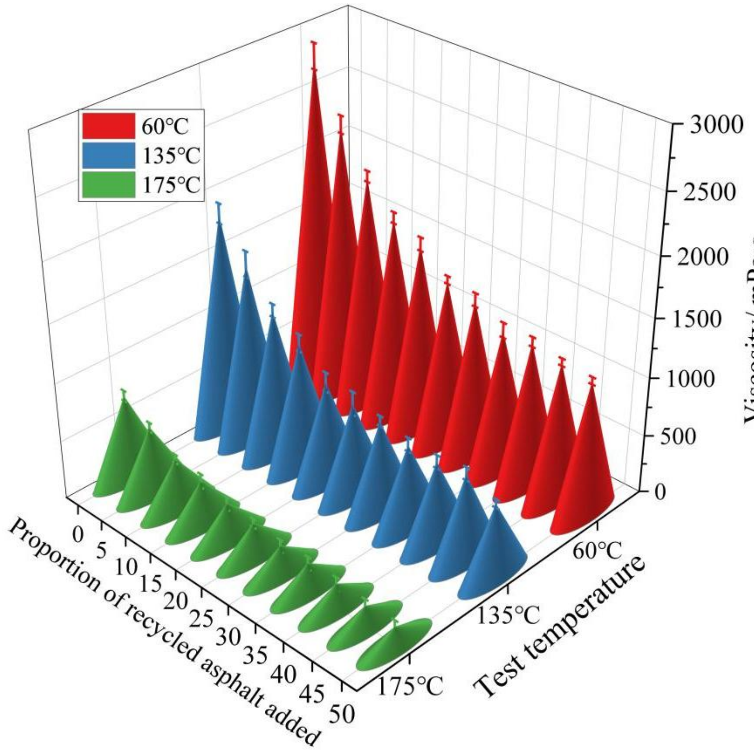

Figure 5. Rotational viscosity under different proportions of recycled asphalt.

As the proportion of recycled asphalt increased, the rotational viscosity of the mixes at 60°C, 135°C, and 175°C was significantly reduced. The viscosity decreased from 2783 mPa·s to 1152 mPa·s for aged asphalt at 60°C, 625 mPa·s at 135°C, and even more so to 152 mPa·s at 175°C.This change suggests that the incorporation of recycled asphalt helped to improve the fluidity properties of aged asphalt, making it easier to mix and pave open during construction. Especially in high temperature conditions, the viscosity decreased more. However, when the dosage of more than 30%, the viscosity decline tends to level off, which suggests that in the actual project, the recycled asphalt dosage is not the higher the better, but need to be combined with the conventional performance and specific construction requirements to determine the optimal ratio.

3.2 Research on mix performance prediction based on BP neural network-genetic algorithm

On the basis of clarifying the basic properties of recycled asphalt, this section further explores how to construct an efficient mix performance prediction system using BP neural network with genetic algorithm. The network is trained through a large amount of experimental data, and the advantages of BP neural network in terms of prediction accuracy and stability are verified by comparison with models such as multiple linear regression, SVR and random forest.

3.2.1. Network Sample Set Selection

A network sample set was formed from the test data of aggregate, asphalt and recycled asphalt mixes, and the MATLAB tool was used to train and learn the network, in which there were eight input parameters: the RAP admixture (RC), fineness modulus (FM), effective asphalt content (A_e), rutting factor at 60°C ($G^*/\sin\delta$), work of adhesion (W_{as}), creep rate (m, -10°C), creep strength (S, -10°C), and fatigue life of asphalt (N_{fa}, 25°C-2.5%); and four output parameters: dynamic stability (DS), residual stability (MS_0), maximum bending and tensile strain (ε_B), and fatigue life of mix (N_{fm}, 15°C-0.2).

Some of the parameter input and output statistics used in this paper for modeling the network for predicting the performance of recycled asphalt mixtures based on BP neural network-genetic algorithm are shown in Tables 10 and 11, respectively.

Table 10. Statistics of Modeling Input Parameters.

Serial Number	RC (%)	FM	A_e (%)	$G^*/\sin\delta$ (Pa)	W_{as} (mJ/m^2)	m	S (%)	N_{fa}
1	10	6.357	4.31	6150	82.69	0.425	124	14193
2	10	5.824	4.42	5432	91.15	0.447	115	17728
3	20	6.180	4.27	7254	79.49	0.418	131	9822
4	20	5.903	4.35	6614	85.79	0.431	126	13662
5	30	6.422	4.23	8150	74.23	0.406	142	5804
6	30	5.875	4.46	7400	81.61	0.422	135	9252
7	40	6.711	4.18	9394	69.85	0.395	151	3478
8	40	5.907	4.52	8638	76.39	0.413	146	5111
9	50	6.836	4.12	10430	64.27	0.387	162	1826
10	50	6.059	4.58	9782	71.87	0.404	157	4126
11	10	6.286	4.33	5911	86.45	0.436	121	16265
12	10	5.787	4.39	5484	93.76	0.452	118	18781
13	20	6.225	4.29	6904	80.37	0.427	134	11202
14	20	5.856	4.44	6512	87.75	0.439	128	14868
15	30	6.519	4.25	7759	75.05	0.416	144	6737
16	30	5.929	4.48	7424	82.80	0.428	138	10584
17	40	6.641	4.21	9018	70.41	0.401	154	3867
18	40	5.972	4.54	8410	77.71	0.419	149	5900
19	50	6.763	4.15	10102	65.74	0.392	165	2247
20	50	6.112	4.61	9539	73.22	0.408	159	4624
…	…	…	…	…	…	…	…	…

Table 11. Model output parameter statistics.

Serial Number	DS	MS_0 (%)	ε_B (με)	N_{fm}
1	3671	85.53	2249	7859
2	3254	90.09	2487	10192
3	3939	82.69	1713	5973
4	3508	84.32	1968	7240
5	4289	77.55	1241	3854
6	3863	79.61	1527	5130
7	4916	72.14	721	1972
8	4402	74.91	1057	3427
9	5462	67.29	612	1081
10	4984	70.15	1114	2141
11	3578	86.26	2301	8126
12	3312	91.55	2560	10594

13	4024	81.89	1780	6338
14	3685	83.9	2040	7801
15	4169	78.77	1330	4274
16	3782	79.94	1587	5673
17	4835	72.96	784	2408
18	4282	75.62	1179	3983
19	5389	68.28	682	1425
20	4866	70.03	1257	2771
…	…	…	…	…

3.2.2. Prediction results of BP neural network-genetic algorithm

The predicted values of dynamic stability, residual stability, maximum bending and tensile strains and fatigue life were obtained from the above BP neural network-genetic algorithm training data, and regression analysis was performed with the measured values. Pearson's correlation coefficient, correlation coefficient (R^2) and root mean square error (RMSE) were selected to evaluate the prediction accuracy of the neural network in these four aspects, in which the closer the correlation coefficient is to 1, the smaller the RMSE is, and the smaller the error is.

Figures 6-9 show the model predicted-actual values of dynamic stability, residual stability, maximum bending and tensile strains and fatigue life for 50 sets of data, respectively. The left side of the picture shows the distribution of model predicted and actual values and their fitting, and the right side shows the distribution of residuals for the 50 sets of predicted values.

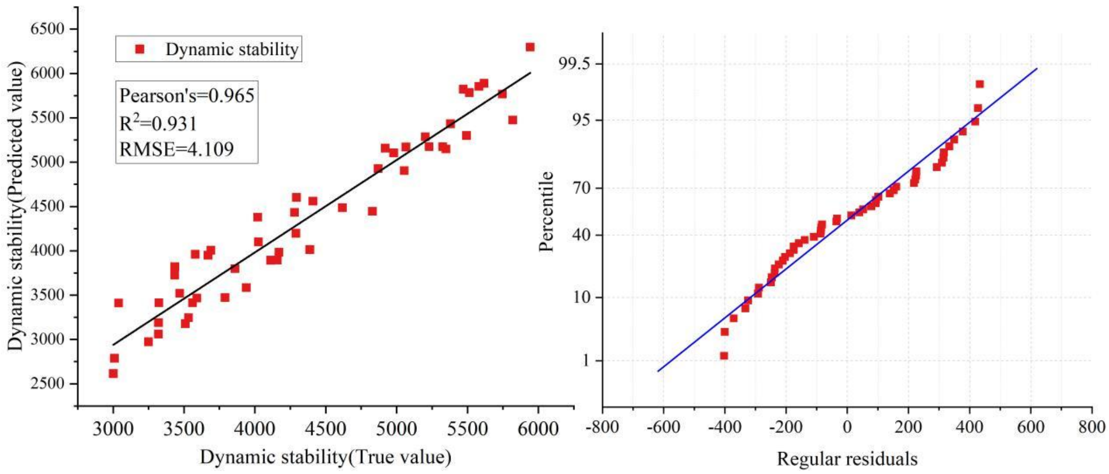

Figure 6. Model-predicted values versus actual values of dynamic stability.

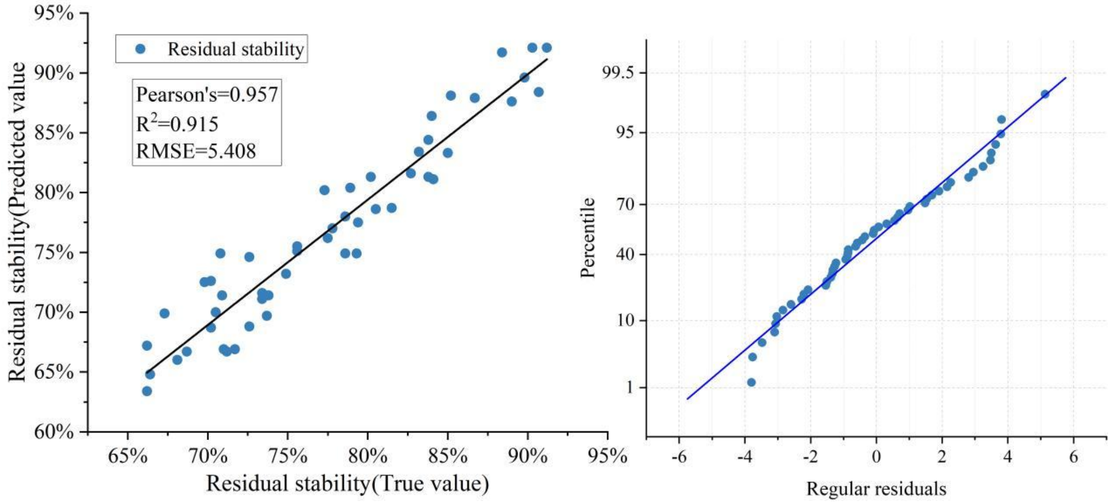

Figure 7. Model-predicted values versus actual values of residual stability.

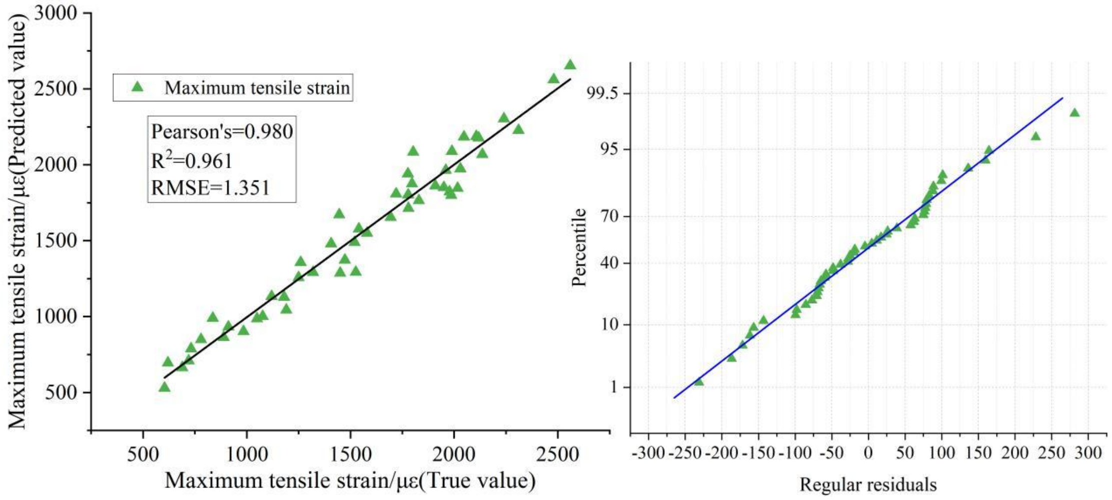

Figure 8. Model-prediction values and actual values of the maximum tensile strain.

bw

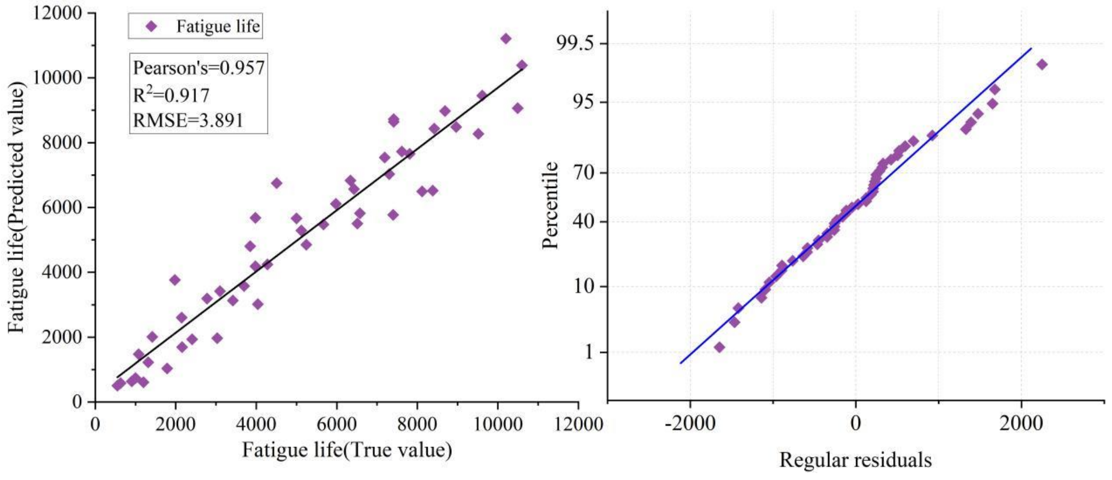

Figure 9. Model-predicted values versus actual values of fatigue life.

From the above figure, it can be clearly seen that the data points of the four dimensional aspects are densely distributed around the fitting line, and the R^2 is above 0.9 with 0.931, 0.915, 0.961 and 0.917, respectively, which indicates that the dynamic stability, residual stability, maximum bending and tensile strain and fatigue life predicted by the BP neural network-genetic algorithm have good correlation with the measured values, which confirms the accuracy of the BP neural network-genetic algorithm modeling accuracy. The model has the best prediction performance for the maximum bending strain, with Pearson's correlation coefficient of 0.980 and root mean square error of 1.351. From the distribution of residuals, it can be seen that the conventional residuals are mainly concentrated in ±100, which indicates that the prediction error of the model can be controlled at about 5%. In terms of fatigue life, for the sample data with fatigue life greater than 10000 times, the prediction line still passes through the center of the data points stably, indicating that the model has grasped the loading capacity of the material very well.

3.2.3. Model comparison experiments

In order to further verify the advantages of BP neural algorithm selected in this paper for predicting the performance of mixes, multiple linear regression, SVR and random forest regression models were used as the comparison models, and the two properties of stability and freeze-thaw splitting residual strength ratio were selected for the prediction model building and testing. Pearson's correlation coefficient (Person's), correlation coefficient (R^2) and root mean square error (RMSE) are still used as evaluation indexes.

(1) Stability

Randomly selected 20 sets of data, Figure 10 shows the predicted results of the stability of the four models in the above data sets and their fitting analysis, and Figure 11 shows the residual plots of the four models.

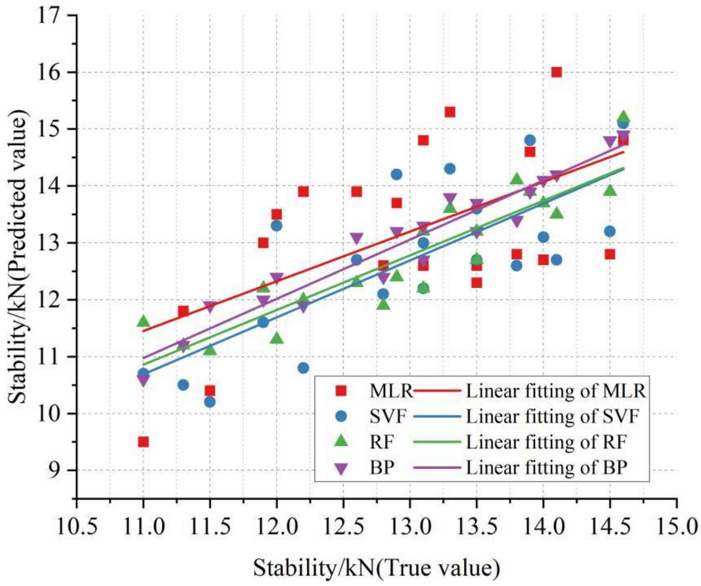

Figure 10. The stability prediction results of models and their fitting analysis.

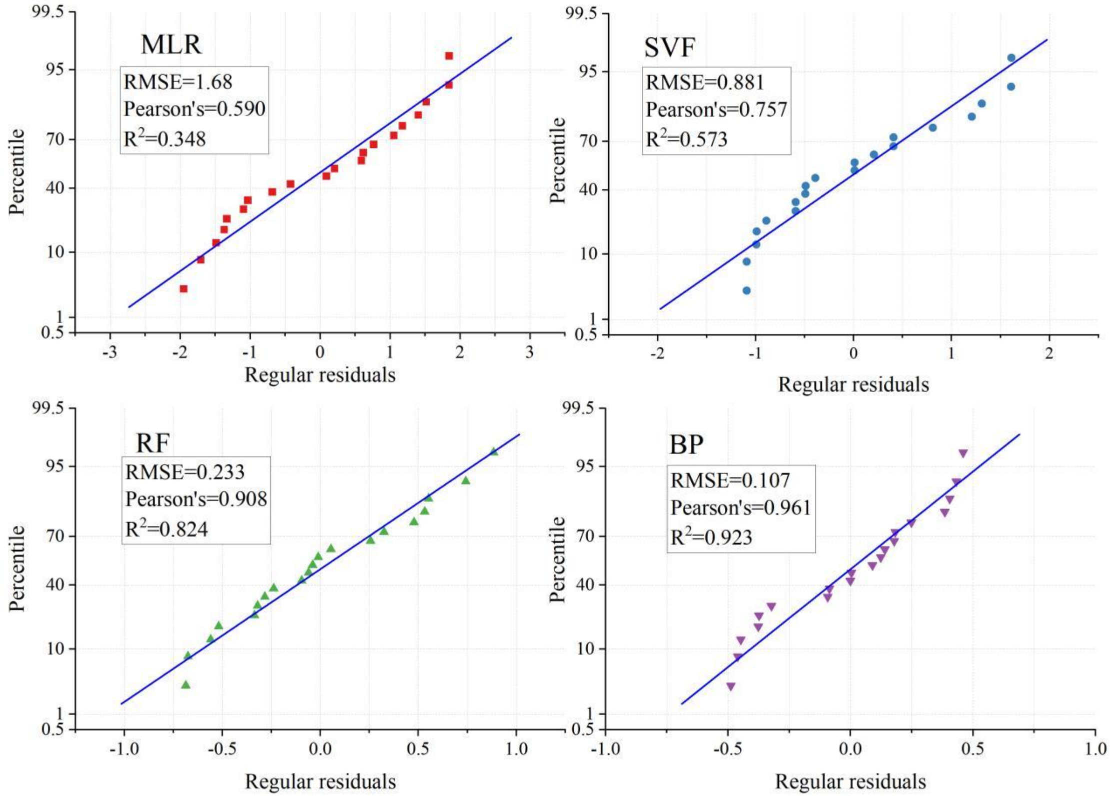

Figure 11. Residual analysis of the stability of 4 models.

In the stability actual-prediction scatterplot, the training results of multiple linear regression and SVR have more discrete points off the diagonal, and the scatter is more dispersed, while the training results of random forest are slightly more concentrated; the training results of BP neural network are the most concentrated, with the scatter distributed near the diagonal. The multivariate linear regression model has an RMSE of 1.68 and an R^2 of only 0.348, which is equivalent to saying that it fails to capture nearly two-thirds of the variations in the 20 sets of data. The support vector regression model is slightly better, with the RMSE dropping to 0.881, but the R^2 of 0.573 suggests that the prediction accuracy is still less than ideal. The Random Forest model has an RMSE of 0.233, the data points on the scatterplot have begun to tighten around the fitted line, its residuals are concentrated between -0.5 and 0.5, and its R^2 has improved to 0.824. The BP neural network model performs the best, with a root-mean-square error of only 0.107 and a high R^2 of 0.923, meaning that the model is able to explain more than 92% of data variation. From the residual distribution, the prediction error of BP neural network is basically controlled within ±0.5kN, and there is no obvious systematic bias, which indicates that the model is not only highly accurate, but also very stable.

(2) Freeze-thaw splitting residual strength ratio

The scatter plots of the predicted and true values of the four prediction methods on the data set regarding the freeze-thaw splitting residual strength ratio are shown in Fig. 12, and the model residual plots are shown in Fig. 13.

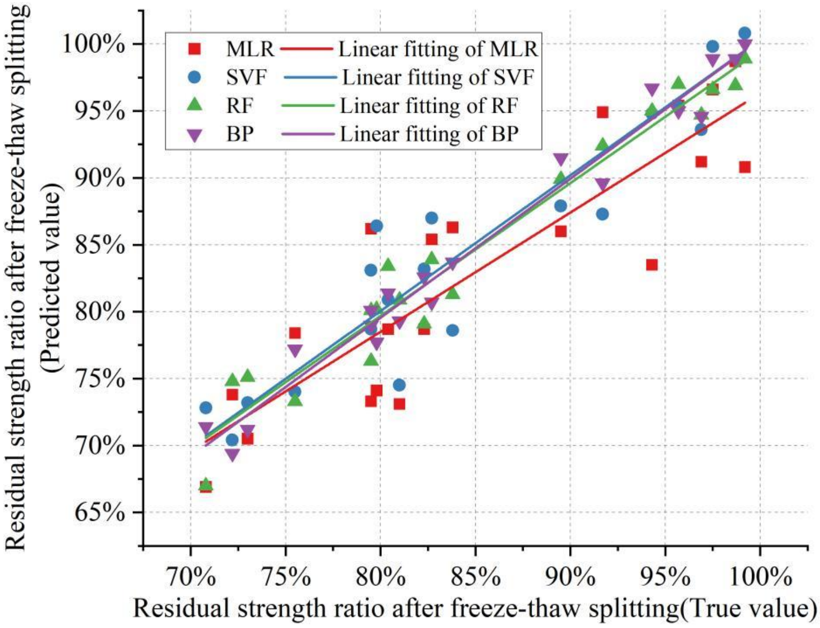

Figure 12. The results of the residual strength ratio after freeze-thaw splitting.

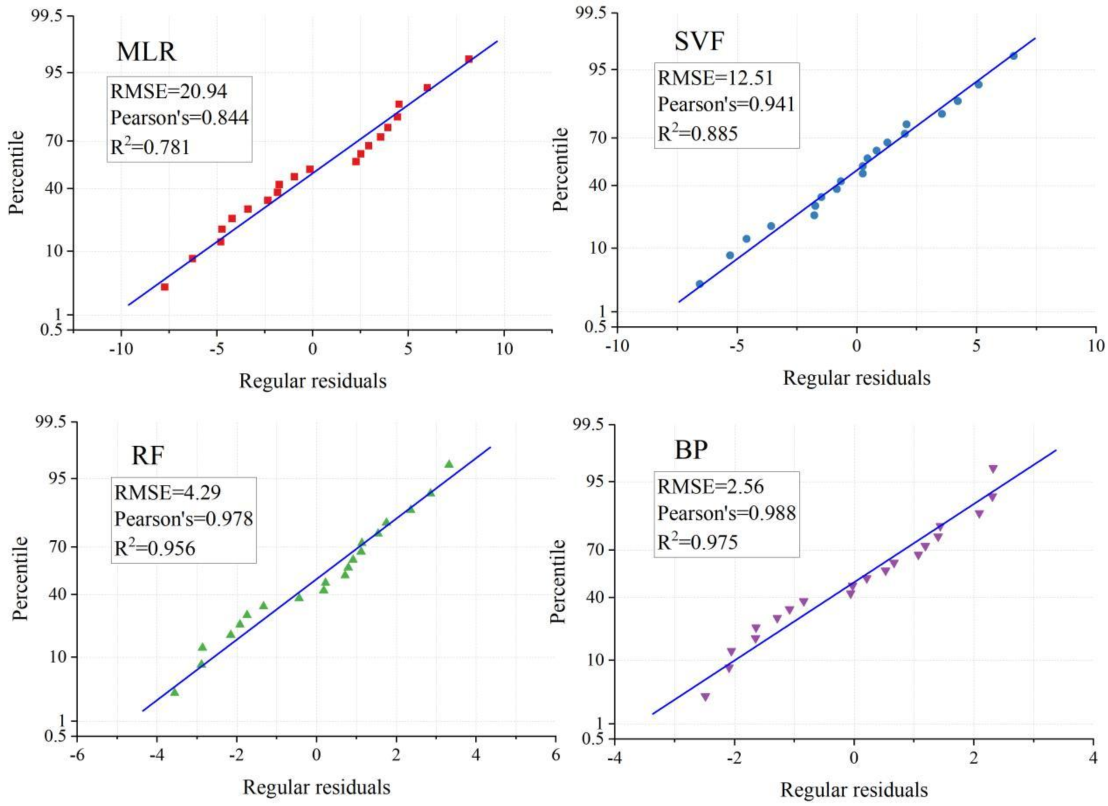

Figure 13. Residual analysis of the residual strength ratio after freeze-thaw splitting.

The predictive performance of the four models in terms of freeze-thaw splitting residual strength ratio is significantly better than in terms of stability, and it is still the BP neural network model that achieves the highest prediction accuracy. The Pearson's correlation coefficient of multiple linear regression Pearson's r = 0.884, RMSE is 20.94, through the prediction-actual graph in Figure 13 can be found that the data of multiple linear regression generally lies under the fitting curve, indicating that its prediction value is generally low. The R^2 of SVF and Random Forest reached 0.885 and 0.956, indicating that the prediction trends of these two models are more accurate.BP neural network showed its super learning ability again, and the RMSE of the prediction of the residual strength ratio of freeze-thaw splitting of the mixture was only 2.56, and the Pearson's correlation coefficient was 0.988, with the R^2=0.975. Even in the region of high residual strength ratio over 80%, the prediction points of the BP neural network are still close to the diagonal line, which confirms that its prediction of high-performance mixes is also reliable.

4. Conclusion

In this study, a set of research methods around recycled asphalt concrete mixes, from characterization analysis to performance prediction, was constructed by combining gray correlation analysis, BP neural network and genetic algorithm.

The experimental data clearly show that, with the addition of recycled asphalt into the aging grade III asphalt, when the ratio is increased to 1:1, at this time, the needle penetration of the mix recovers from 28.18 to 64.89, and the ductility jumps even more dramatically to 58.55 cm, the softening point of the mix decreases to 48.00 °C, and the viscosity characteristics of the asphalt at 175 °C even decreases to 152 mPa-s. This series of changes implies that with the addition of recycled asphalt, the hardness and low-temperature brittleness of aging asphalt were significantly improved, and it regained the flexibility and crack resistance.

The prediction model based on BP neural network and genetic algorithm performs excellently in the prediction of four core indexes, namely, dynamic stability, residual stability, maximum bending and tensile strain, and fatigue life, and the coefficients of determination between the predicted values and the actual values, R^2, are 0.931, 0.915, 0.961, and 0.917 respectively. When compared with the models of multivariate linear regression, support vector machine, etc., the BP model is able to achieve the best performance in terms of lowest root mean square error and highest goodness of fit, and its RMSEs for stability and freeze-thaw splitting residual strength ratio were 0.107 and 2.56, respectively.

5. Outlook

This chapter looks at the application of the model to expand the development of recycled asphalt mixtures in terms of new material design, multi-scale evaluation, full life cycle analysis and intelligent quality control.

5.1. New material design and process optimization based on predictive models

The BP neural network-genetic algorithm model constructed in this study has been shown to be able to accurately predict a number of properties of recycled asphalt mixtures. In the future, this model can be used to reverse-guide the design of new recycling agents and modified materials. For example, by adjusting the asphalt rheological properties in the input parameters of the model, the effects of different bio-based recycling agents or nanomaterials on the long-term performance of mixtures can be simulated, and the optimal formulations can be screened out at the laboratory stage. At the same time, by combining the key influencing factors identified in the ash correlation analysis, the parameters of the recycling process can be optimized, e.g., by accurately controlling the RAP dosage, the type of new asphalt and the mixing process, so as to maximize the use of RAP under the premise of guaranteeing the performance.

5.2. Establishment of a multi-scale performance evaluation system

The performance prediction model established in this paper is mainly based on macro test data. Future research should, on this basis, correlate the macro performance with the micro- and meso-scale features to construct a multi-scale performance evaluation system. For example, image recognition technology can be used to analyze the microstructure of the aggregate-asphalt interface, and the interface feature parameters can be used as inputs to the neural network, so as to reveal the intrinsic mechanism between material composition-interface structure-macroscopic performance in the model.

5.3. Full life cycle performance and environmental impact assessment

This research model focuses on the mechanical and road performance of mixes. The next step can be from the perspective of the whole life cycle, the genetic algorithm is no longer only looking for the optimal solution of performance, but seeking the Pareto optimal solution under the multi-objective of performance, cost and environmental benefits. Realize the sustainable development of road engineering. By comparing and analyzing the environmental impact and resource consumption of recycled asphalt mixtures and traditional asphalt mixtures in various stages of raw material extraction, production and processing, construction, use and maintenance, as well as waste disposal, we can optimize the production process and use of recycled asphalt mixtures, and reduce the environmental footprints of the whole life cycle of recycled asphalt mixtures.

5.4. Application of intelligent monitoring and quality control technology

The prediction model in this study is static. In the future, with the help of advanced intelligent monitoring technology and quality control means, real-time monitoring and dynamic control of the process of production, construction and use of recycled asphalt mixtures will be realized. These real-time data can be used as online inputs to the model for dynamic prediction and correction of the long-term performance of the mix. For example, using IoT technology, sensor technology and big data analysis technology, the quality of raw materials, production process parameters, construction quality indicators and pavement use performance of recycled asphalt mixtures are collected and analyzed in real time, so that potential quality problems can be detected in a timely manner and corresponding measures can be taken to adjust and optimize. The application of intelligent monitoring and quality control technology will help to improve the quality stability and reliability of recycled asphalt mixtures and reduce engineering risks.

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