

Research on Rain Removal Method for High Scale Rain Pattern Image Block Based on Sparse Representation Model

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Abstract: This paper addresses the problem of rain removal in high-rain-texture images by proposing a novel processing method that integrates a sparse representation model. This approach aims to overcome the processing bottlenecks exhibited by traditional image rain removal techniques and deep learning methods in scenarios with dense rain textures. The paper introduces a sparse representation framework with the ability to model rain texture prior features, enabling the model to preserve natural background details in the image while avoiding excessive smoothing caused by the lack of targeted modeling during the processing. By constructing a sparse dictionary specifically tailored to rain texture structures and leveraging their statistical characteristics in both the frequency and spatial domains, the system achieves significantly superior performance in high-rain-texture regions compared to conventional non-structured methods, supported by structured modeling. To enhance the model's adaptability to complex scenes, this paper integrates a Generative Adversarial Network (GAN) with the sparse representation mechanism, enabling the entire rain removal system to simultaneously possess both representational and generative capabilities during image reconstruction. The results show that the proposed algorithm achieves significant performance improvements on the publicly available synthetic datasets Rain100H and Rain200L, outperforming most existing mainstream methods in both PSNR and SSIM. Especially in terms of visualization, the generated images achieve an ideal balance between preserving texture details and reducing rain texture interference, fully demonstrating the practical value and theoretical feasibility of this method in high-density rain texture environments.

Keywords: sparse representation; image denoising; deep learning; generative adversarial networks; wavelet transform

1. Introduction

In the field of contemporary computer vision and image processing, natural environmental interference has become one of the primary causes of degraded image quality, particularly the high-density rain patterns in images captured during rainy weather. These patterns not only severely obscure target objects but also significantly reduce image clarity and contrast, thereby impacting the stability and accuracy of subsequent tasks such as image recognition and object detection [1-2]. With the rapid development of technologies such as intelligent transportation, security surveillance, and autonomous driving, the practicality and urgency of rain-day image processing have become increasingly evident, and rain removal technology has gradually shifted from a marginal issue to a core research direction [3-5].

Traditional image de-rain methods are mostly based on filtering and image decomposition techniques, which are effective in low-rain environments but have limited capabilities for processing images with dense, directional, and complex rain patterns, often resulting in loss of image details or the introduction of artifacts [6-7]. While deep learning methods have made breakthroughs in image restoration in recent



years, their heavy reliance on labeled data, high computational resource consumption, and poor interpretability remain significant challenges [8-9]. Faced with the dual challenges of dense rain patterns and complex scenes, a single technical approach is insufficient to effectively address these issues.

Sparse representation theory, with its excellent signal representation capabilities and superior image restoration performance, has gradually emerged as a new breakthrough in image processing [10]. Its core idea is that images can be approximated using a linear combination of a very small number of “dictionary atoms,” and it has been widely applied in tasks such as image denoising and super-resolution [11]. Compared to deep learning, sparse representation models offer stronger interpretability and better preservation of structural information, particularly maintaining good adaptability under low-sample or small-sample conditions [12].

Therefore, exploring the integration of sparse representation models with techniques such as wavelet transforms, deep networks, and generative adversarial mechanisms to provide new methods for processing high-proportion rain texture image blocks not only holds significant theoretical value but also has broad practical application prospects [13-14]. This direction is expected to address the shortcomings of existing algorithms in unstable performance under complex rain texture conditions and advance the practical application of rain-day image processing technology.

Currently, research on image restoration technology has primarily focused on deep learning techniques. However, through background research, this paper has found that sparse representation models perform no worse than deep learning techniques in image restoration. Zha et al. [15] proposed a new low-rank guided group sparse representation model. Extensive experiments demonstrate that, compared to the algorithms studied, the proposed low-rank guided group sparse representation model achieves better results in various tasks such as denoising, restoration, and compressive sensing across different image types. Cai [16] employs a group residual learning method based on low-rank self-representation to automatically estimate the true group sparse representation, enabling the sparse residual model to achieve better image restoration performance. The results demonstrate that the proposed algorithm outperforms other popular methods, validating the effectiveness of the proposed method. Yang et al. [17] proposed a multi-focus image fusion method based on sparse representation to simultaneously address image restoration and fusion problems. This method outperforms methods based on spatial gradients, morphological wavelet transforms, discrete wavelet transforms, and stationary wavelet transforms in terms of both subjective and objective image quality. Foreign scholars Arya et al. [18] utilized an alternating direction multiplier optimization combination of sparse representation-based identification and extraction of local structure using wavelet patches and group-based sparse representation-based identification and extraction of non-local self-similarity, enhancing the practicality of the image restoration process. Their method outperformed existing methods in terms of structural similarity, peak signal-to-noise ratio, and visual perception metrics. Li [19] proposed a sparse regularization image deblurring model based on sparse representation theory to restore blurred images, while adding a non-negative term to ensure the positivity of the restored image. Research indicates that this model significantly improves image deblurring performance. Xu et al. [20] used a sparse regularization image deblurring model to restore blurred images, while adding a non-negative term to ensure the positivity of the restored image. Research indicates that this proposed a sparse regularization image deblurring model to restore blurred images, while adding a non-negative term to ensure the positivity of the restored image. Research shows that this model significantly improves image deblurring performance. Xu et al. [20] used a sparse representation model for motion image deblurring and experimentally demonstrated the effectiveness of their method, which outperforms comparison methods in terms of convergence speed, runtime, and deblurring quality.

Research has identified a close connection between deep learning and sparse representation models, combining the two for applications in image restoration, retrieval, and fusion. For example, An et al. [21] employed an optimized sparse representation method to segment image regions into target and background areas, then trained an image fusion model using a deep learning approach. The fusion image quality metrics obtained from this method were significantly superior to those achieved by conventional machine learning and deep learning methods. Xu et al. [22] combined convolutional neural networks with sparse representation models to jointly optimize parameters within the convolutional denoiser, experiments demonstrated the outstanding performance of combining deep networks with sparse representation models in image restoration applications, with experimental results in denoising, deblurring, and super-resolution outperforming existing methods. Sezavar et al. [23] also proposed a robust method combining convolutional neural networks and sparse representation, which accurately extracts image depth features and exhibits fast speed and high accuracy in image retrieval.

Research on removing natural environmental interference from images is relatively scarce. Huang [24] applied sparse representation models to image defogging tasks, enabling the model to obtain fog density information. Combined with image visibility processing procedures, this approach improved the

visual quality of defogged images, outperforming several traditional image defogging methods.

In this study, a multi-stage composite processing framework integrating sparse representation, wavelet transform and generative adversarial mechanism is constructed with the goal of "high-proportion rain pattern image block de-raining", aiming to improve the model's ability to remove rain and maintain image quality in high-density rain pattern scenes. Firstly, according to the characteristics of rain streak directionality and significant structural features, a structured sparse dictionary was designed to guide the model to accurately extract rain streak information, and the spatial analysis ability of rain streak detection was enhanced by combining the multi-scale features of wavelet domain. Secondly, the deep convolutional neural network is introduced to realize the effective separation of the image background and rain patterns, and the authenticity and naturalness of image restoration are improved with the help of generative adversarial network, so as to alleviate the over-smoothing problem of traditional rain removal algorithms.

The entire system undergoes multi-stage progressive training optimization, constructing the rain pattern feature separation and background restoration process from coarse to fine, while integrating the advantages of sparse priors and deep learning to ensure image detail integrity and algorithm robustness. This research balances theoretical innovation and engineering effectiveness, aiming to develop a high-performance image rain removal method that can operate stably in complex real-world scenarios.

2. Rain Removal Method Based on Sparse Representation Model

2.1. Theoretical Foundations of Sparse Representation

Sparse representation theory is a key technological breakthrough in the fields of signal processing and image analysis. Its basic idea is to represent complex signals through linear combinations of a very small number of atoms in a predefined dictionary. This unique representation method not only deeply explores the essential characteristics of signals, but also demonstrates remarkable performance in signal denoising, compressive sensing, feature extraction, and many other areas.

From a mathematical perspective, sparse representation models aim to find an overcomplete dictionary $D \in R^{n \times m}$ ($m > n$) for a given signal $x \in R^n$, such that the signal can be represented as:

$$x = D\alpha \tag{1}$$

In the equation, $|\alpha \in R^m|$ represents a sparse coefficient vector and satisfies $\|\alpha\|_0 \ll m$, where $\|\cdot\|_0$ denotes the number of nonzero elements in the vector, i.e., the L_0 norm. In practical applications, since L_0 norm optimization is an NP-hard problem, researchers often use convex relaxation techniques to convert it into a solvable L_1 norm minimization problem, i.e.:

$$\min_{\alpha} \|\alpha\|_1 \text{ s.t. } x = D\alpha \tag{2}$$

Or the equivalent Lagrangian form, namely:

$$\min_{\alpha} \|x - D\alpha\|_2^2 + \lambda \|\alpha\|_1 \tag{3}$$

In the equation, λ represents the regularization parameter for balancing reconstruction error and sparsity.

Sparse dictionary design [25] has a decisive impact on model performance, and can be broadly categorized into two main types: analytical dictionaries based on fixed mathematical transformations (such as wavelet, Fourier, and cosine transforms) and learned dictionaries directly extracted from training data.

The K-SVD algorithm, as a classic dictionary learning method, enhances the dictionary's representational capabilities by alternately optimizing sparse coefficients and dictionary atoms. Structured dictionary learning methods further improve the dictionary's representational capabilities and robustness by introducing constraints such as group sparsity and low rank [26-27]. The wavelet transform, as a multi-resolution analysis tool, demonstrates unique advantages in image processing. It provides excellent localization properties in both the spatial and frequency domains, effectively capturing multi-scale features of images. When applied to image de-raining, the wavelet transform decomposes the image into sub-images of different frequency bands, with high-frequency sub-bands containing edge and texture information, and low-frequency sub-bands preserving image contour information. Since rain patterns typically manifest as high-frequency, directionally distinct textures, analyzing the energy distribution and directional characteristics of wavelet subbands can effectively detect and extract rain patterns. In particular, the wavelet subband attention mechanism guides the

network to focus on specific subbands containing rain pattern information, thereby improving rain removal accuracy.

In recent years, the integration of deep learning and sparse representation models has pioneered a new paradigm in image processing. Although end-to-end learning methods based on convolutional neural networks can automatically learn complex feature representations without explicitly designing feature extractors, they often suffer from insufficient interpretability and heavy reliance on large amounts of labeled data. Integrating sparse representation models into deep learning frameworks can reduce dependence on large-scale labeled data while enhancing model interpretability, achieving satisfactory results.

Generative adversarial networks (GANs), as an important model in deep learning, can generate samples similar to the distribution of real data through adversarial training between the generator and discriminator. This approach can produce more natural and realistic rain-free images in the image de-raining task [28-29]. Especially when dealing with image blocks with a high proportion of rain patterns, the combination of sparse representation and GANs is particularly important. The generator network can utilize the prior knowledge provided by the sparse representation model to generate more accurate rain-free images, while the discriminator ensures that the generated images appear visually natural and realistic. Integrating sparse representation models, wavelet transforms, deep learning, and GAN technology provides a theoretically sound and technologically advanced solution for addressing the rain removal problem in high-rain-texture image blocks. This multi-technology fusion method not only effectively captures the sparse features of rain patterns but also enhances the model's generalization ability through deep learning and improves the naturalness of generated images using GANs. This approach efficiently removes rain interference while preserving the essence of image details, achieving an ideal image restoration effect. This is of significant importance for enhancing the robustness of computer vision systems under adverse weather conditions.

2.2. Sparse Representation Models and Their Optimization

Sparse representation models, as a key mathematical tool in the field of image processing, offer a unique and flexible approach to addressing the challenge of removing rain patterns from high-rain-content image blocks. Their advantage lies in the ability to effectively separate background and rain pattern information by decomposing the image into a sparse linear combination form, thereby enabling rain removal processing. This method fundamentally differs from traditional approaches based on frequency-domain filtering and morphological processing.

The theoretical basis of sparse representation models is based on the premise that natural images can be represented as sparse linear combinations under a certain overcomplete dictionary. For the rain image block $x \in R^n$ to be processed, the sparse representation model can be expressed as:

$$x = D\alpha \quad (4)$$

In the equation, $D \in R^{n \times m}$ ($m > n$) represents an overcomplete dictionary matrix, whose column vectors d_j ($j = 1, 2, \dots, m$) are called dictionary atoms. $\alpha \in R^m$ is the sparse coefficient vector, which satisfies the sparsity condition that most elements are zero or close to zero. This sparsity is typically measured using the L_0 pseudo-norm $\|\alpha\|_0$, which represents the number of non-zero elements in the vector α .

The rain image processing model treats an image x containing rain patterns as the superposition of a clear background image x_b and a rain pattern image x_r , i.e.:

$$x = x_b + x_r \quad (5)$$

According to sparse representation theory, background images and rain pattern images can be sparsely represented under corresponding dictionaries, namely:

$$x_b = D_b\alpha_b, x_r = D_r\alpha_r \quad (6)$$

The rain image processing model treats the image x containing rain patterns as a superposition of the clear background image x_b and the rain pattern image x_r , where D_b and D_r are the background dictionary and rain pattern dictionary, respectively, and α_b and α_r are the corresponding sparse coefficients.

By combining these expressions, we establish a unified sparse representation model, namely:

$$x = [D_b, D_r] \begin{bmatrix} \alpha_b \\ \alpha_r \end{bmatrix} = D\alpha \quad (7)$$

In actual rain problems, the key lies in solving optimization problems, namely:

$$\min_{\alpha} \|\alpha\|_0 \text{ s.t. } \|x - D\alpha\|_2^2 \leq \varepsilon \quad (8)$$

Or equivalently, that is:

$$\min_{\alpha} \|x - D\alpha\|_2^2 \text{ s.t. } \|\alpha\|_0 \leq K \quad (9)$$

Here, ε is the allowable reconstruction error threshold, and K is the sparsity constraint.

Considering that L_0 norm optimization belongs to NP-hard problems, direct solution calculation complexity is too high. In practice, convex relaxation methods are usually used to replace the L_0 norm with the L_1 norm, transforming the problem into the following convex optimization form, namely:

$$\min_{\alpha} \|\alpha\|_1 \text{ s.t. } \|x - D\alpha\|_2^2 \leq \varepsilon \quad (10)$$

Or the Lagrange form, namely:

$$\min_{\alpha} \|x - D\alpha\|_2^2 + \lambda \|\alpha\|_1 \quad (11)$$

Here $\lambda > 0$ is the regularization parameter that trades off the reconstruction error and sparsity.

In view of the special properties of high-scale rain pattern image blocks, the traditional sparse representation model is innovatively optimized in three aspects.

(1) A multi-level structured sparse constraint is designed to capture the spatial continuity and orientation characteristics of rain streaks by introducing group sparse and structural sparse regular terms.

(2) For the sparsity coefficient, a structured regular term is introduced, i.e.:

$$R(\alpha) = \sum_{g \in G} w_g \|\alpha_g\|_2 \quad (12)$$

In the equation, G denotes a predefined set of coefficient groups, and w_g represents the weights of each group.

(3) A dual dictionary learning strategy was designed to optimize the dictionary structure for background and rain texture features, respectively. The background dictionary D_b was constructed by maximizing the similarity between image blocks, while the rain texture dictionary D_r was designed by analyzing the geometric characteristics of rain textures. The dictionary optimization objective function is:

$$\min_{D_b, D_r, \alpha} \|X - [D_b, D_r]\alpha\|_F^2 + \lambda \|\alpha\|_1 + \mu R(\alpha) \text{ s.t. } \|d_j\|_2 = 1, \forall j \quad (13)$$

In the equation, X is the training image block matrix, $\|\cdot\|_F$ represents the Frobenius norm, and μ is the weight coefficient of the structural sparsity regularization term.

During the research, an improved version of the iterative shrinking threshold algorithm (ISTA) was introduced to accelerate the convergence process through adaptive threshold setting, namely:

$$\alpha^{(k+1)} = S_{\lambda\tau}(\alpha^{(k)} + \tau D^T(x - D\alpha^{(k)})) \quad (14)$$

In the equation, $S_{\lambda\tau}(\cdot)$ is the soft thresholding operator, and τ is the step size parameter. The adaptive thresholding mechanism takes into account the density of rain streaks at the base, enabling the model to more accurately separate dense rain streaks from complex backgrounds. This method remains effective in separating background and rain streak information even in image regions where rain streak density exceeds 70%. Through extensive experimental verification, the optimized sparse representation model demonstrates significant advantages when processing image blocks with high rain texture ratios. It achieves the expected results in both rain texture removal effectiveness and background detail preservation, fully demonstrating the method's robust processing capabilities for high-rain-texture image blocks. This research finding holds practical application value for enhancing image processing quality under adverse weather conditions.

2.3. Multi-Stage Rain Removal Network Design

This study addresses the limitations of single sparse representation models in processing images with a high proportion of rain streaks by constructing a multi-stage rain removal network architecture. This architecture ingeniously combines convolutional neural networks and generative adversarial network technology to achieve a progressive rain removal process from coarse to fine. The network architecture consists of three complementary sub-networks: the scale-aware feature extraction network is responsible for capturing multi-scale rain texture features. Through a feature pyramid structure and three sets of convolutional layers with different receptive field sizes (3×3 , 5×5 , 7×7), it extracts fine, medium, and large-scale rain texture features, respectively. The mathematical expression is:

$$F_s = \mathcal{F}_s(X_{rain}; \theta_s) = \text{Concat}(F_{s1}, F_{s2}, F_{s3})_{\downarrow} \quad (15)$$

In the equation, x_{rain} represents the input rain image, F_s denotes the extracted multi-scale features, θ_s is the network parameter, and F_{si} ($i=1,2,3$) correspond to the features extracted at three different scales.

To enhance feature interaction across scales, we introduce a cross-scale attention fusion module, which uses a self-attention mechanism to adaptively adjust the weights of features at different scales. The expression is:

$$\text{Attention}(F_{si}, F_{sj}) = \text{softmax} \left(\frac{Q(F_{si})K(F_{sj})^T}{\sqrt[3]{d_k}} \right) V(F_{sj}) \quad (16)$$

In the equation, $Q(\cdot)$, $K(\cdot)$ and $V(\cdot)$ represent the query, key, and value transformation functions, respectively, and d_k is the scaling factor.

The sparse-guided reconstruction network combines sparse representation with deep learning methods to decompose and reconstruct rain pattern features through a learnable sparse coding layer, which is mathematically expressed as:

$$X_{temp} = \mathcal{F}_r(F_s; \theta_r, D) = D_b \cdot \text{SparseEncode}(F_s; D) \quad (17)$$

In the equation, $D = [D_b, D_r]$ is the joint dictionary optimized in the preceding section, and $\text{SparseEncode}(\cdot)$ is an end-to-end learnable sparse encoding operation, which is implemented by introducing a convolutional layer with a specific structure and initializing the parameters with a pre-trained sparse dictionary. To ensure the convergence and stability of the network, a residual connection structure is designed and a sparse regularization term is introduced, namely:

$$\mathcal{L}_{sparse} = \lambda_1 \|\alpha\|_1 + \lambda_2 \sum_{g \in \mathcal{G}} w_g \|\alpha_g\|_2 \quad (18)$$

In the equation, λ_1 and λ_2 are equilibrium parameters, and α is the sparse coding coefficient.

The adversarial refinement network adopts a conditional generative adversarial network architecture, with the generator using a U-Net structure combined with multiple layers of dense connection blocks to enhance feature extraction capabilities, namely:

$$X_{deraining} = G(X_{temp}; \theta_g) \quad (19)$$

The discriminator uses the PatchGAN structure to judge the authenticity of local regions in images, namely:

$$\mathcal{D}(X, X_{cond}; \theta_d) \in [0, 1]^{N \times N} \quad (20)$$

In the equation, x is the image to be classified, and x_{cond} is the conditional image (original rain image). The adversarial training loss function includes standard adversarial loss, content reconstruction loss, and perceptual loss, as follows:

$$\left\{ \begin{array}{l} \mathcal{L}_{adv} = E_{x_{rain}} [\log(1 - \mathcal{D}(\mathcal{G}(x_{temp}), x_{rain}))] + E_{x_{clean}} [\log D(x_{clean}, x_{rain})] \\ \mathcal{L}_{content} = \|X_{deraining} - X_{clean}\|_1 + \lambda_{ssim} \mathcal{L}_{SSIM}(X_{deraining}, X_{clean}) \\ \mathcal{L}_{perceptual} = \sum_i \|\phi_i(X_{deraining}) - \phi_i(X_{clean})\|_2^2 \end{array} \right. \quad (21)$$

In the equation, $\phi_i(\cdot)$ denotes the feature extraction function of the i th layer of the pre-trained VGG-16 network.

The network training adopts a three-stage strategy, starting with a pre-trained scale-sensitive feature extraction network, then integrating sparse representation knowledge into the deep network through joint training of a sparse-guided reconstruction network, and finally performing end-to-end training of the entire network to fine-tune all parameters. This multi-stage architecture not only inherits the interpretability advantages of sparse representation models but also fully leverages the powerful feature extraction capabilities of deep learning, providing a theoretically sound and practically robust solution for the challenge of rain removal in images with a high proportion of rain patterns. Additionally, our method maintains good structural fidelity and texture details in images taken under extreme weather conditions, thanks to the sparse guided reconstruction network's effective modeling of the image's intrinsic structure and the adversarial network's unique advantage in restoring image details. By employing gradient clipping techniques and adaptive learning rate adjustment strategies, we successfully addressed training instability issues, enabling the network to maintain stable rain removal performance across various complex rain texture scenarios. This provides a reliable technical pathway and valuable insights for future research.

2.4. Combining Wavelet Transforms with Sparse Representation Models

Combining the revolutionary tool of wavelet transform in the field of time-frequency analysis with sparse representation models provides a new approach to solving the problem of rain removal from images with a high proportion of rain streaks [30-31]. This transform is essentially a multiresolution analysis method that can decompose images at different scales and directions, effectively capturing various texture features such as rain streaks.

Given a rainy day image x , we can express the wavelet transform as:

$$W = T(x) \quad (22)$$

In the equation, w represents the wavelet domain coefficient, and T denotes the wavelet transform operator.

Applying a wavelet transform to a two-dimensional image generates four subbands: the approximation coefficient subband (LL), the horizontal detail subband (HL), the vertical detail subband (LH), and the diagonal detail subband (HH). This detailed decomposition enables accurate localization of rain pattern features, as rainfall typically exhibits strong high-frequency responses in the LH subband, providing a theoretical foundation for identifying and separating rain patterns. This study innovatively constructs a wavelet domain sparse representation model, decomposing the rain-day image x through J layers of wavelet transforms into multi-scale coefficients W_j^p , $j \in \{1, 2, \dots, J\}$ representing the decomposition level, and $p \in \{LL, LH, HL, HH\}$ representing the subband type. Based on the analysis of rain pattern characteristics in the wavelet domain, a differentiated sparse representation strategy is applied to coefficients in each subband. Specifically, a dedicated rain pattern sparse dictionary D_r^{LH} is established for the LH subband, which contains dense rain pattern information, while a background sparse dictionary D_b^p is used for other subbands primarily containing background information. The sparse representation of subband coefficients can be expressed as:

$$W_j^p = D_b^p \alpha_b^{j,p} + \delta_{p,LH} \cdot D_r^{LH} \alpha_r^{j,LH} \quad (23)$$

In the equation, $\delta_{p,LH}$ is the Kronecker function, which has a value of 1 when $p = LH$ and 0 otherwise, representing that only the LH subband considers the contribution of the rain pattern dictionary.

To enhance the sparse representation effect in the wavelet domain, we introduce directional analysis enhancement techniques, using the Gabor filter bank $\{G_\theta\}_{\theta=1}^\theta$ to perform directional enhancement on the wavelet coefficients, i.e.:

$$W_j^{LH,\theta} = W_j^{LH} * G_\theta \quad (24)$$

Capture rain pattern features from different angles. The enhanced coefficient is modeled using group sparse representation as follows:

$$[W_j^{LH,1}, W_j^{LH,2}, \dots, W_j^{LH,\theta}] = D_r^{LH,\theta} [\alpha_r^{j,1}, \alpha_r^{j,2}, \dots, \alpha_r^{j,\theta}] \quad (25)$$

The group sparse norm $\|\cdot\|_{2,1}$ is used to constrain the coefficient matrix, ensuring structural consistency among rain texture features in different directions. The optimization process employs a two-layer strategy. Sparse decomposition is performed on the coefficients of each subband, followed by joint optimization with global reconstruction constraints. The optimization objective function is:

$$\begin{aligned} \min_{\{\alpha_b^{j,p}\}, \{\alpha_r^{j,p}\}} & \sum_{j=1}^J \sum_p \|\ W_j^p - D_b^p \alpha_b^{j,p} - \delta_{p,LH} \cdot D_r^{LH} \alpha_r^{j,LH} \ \|_2^2 \\ & + \lambda_1 \sum_{j,p} \|\ \alpha_b^{j,p} \ \|_1 + \lambda_2 \| A_r \|_{2,1} \end{aligned} \quad (26)$$

where A_r represents the set of all rain pattern coefficient matrices, and λ_1 and λ_2 are regularization parameters.

The key to rain removal is to separate the rain streak components from the wavelet coefficient and reconstruct the rain-free image, which is expressed as follows:

$$W_r^{j,LH} = D_r^{LH} \alpha_r^{j,LH} \quad (27)$$

The background wavelet coefficient is:

$$W_b^{j,p} = D_b^p \alpha_b^{j,p} \quad (28)$$

Reconstruct the rain-free image through inverse wavelet transform T^{-1} , that is:

$$x_b = T^{-1}(\{W_b^{j,p}\}) \quad (29)$$

The wavelet domain sparse representation framework not only inherits the interpretability advantages of sparse representation models, but also accurately captures the inherent characteristics of rain patterns through multi-scale decomposition and directional analysis, providing a theoretically complete and practical solution for removing rain from high-proportion rain pattern images.

3. Experiments and Analysis

3.1. Experimental Design

We have established a comprehensive experimental evaluation system to validate the effectiveness of the high-ratio rain-stripe image block de-rain method based on sparse representation models proposed in this paper. This evaluation system covers synthetic and real datasets with various rain type features, and through a series of carefully designed comparative experiments, we have thoroughly investigated the contributions of each component.

The experimental platform utilizes high-performance computing equipment equipped with NVIDIA RTX 3090 GPUs, with the software implementation built using the PyTorch 1.8.0 deep learning framework. To ensure the comprehensiveness and reliability of the results, we selected four widely used synthetic benchmark datasets and two real-world rainy-day scene datasets as the testing foundation, with detailed information shown in Table 1. The experimental design was carried out according to a progressive validation strategy. Initially, a pure sparse representation baseline model was constructed, and the alternating direction method of multipliers was used to solve the relevant optimization problems to explore the optimal combination of key hyperparameters such as dictionary size (256/512/1024 atoms), sparse constraint parameters, and group sparse weights. Subsequently, a multi-stage rain removal network training process was implemented, including pre-training of the scale-aware feature extraction network (learning rate of 10^{-4} , batch size of 16), joint training of the sparse-guided reconstruction network (using the Adam optimizer with a cosine annealing learning rate decay strategy), and end-to-end fine-tuning of the adversarial refinement network (initial learning rate of 5×10^{-5} , PatchGAN discriminator structure). Subsequently, we integrated a wavelet transform application scheme, employing a 4-layer wavelet decomposition using the Daubechies-4 wavelet basis, combined with

8-direction Gabor filters to enhance LH subband rain texture feature analysis, and investigated the balancing effects of regularization parameters and .

Table 1. Experimental data set and its performance indicators.

Datasets	Number of images	Image resolution	Rain pattern characteristics	Rain proportion range
Rain100H	100	512*512	Multi-angle dense rain lines	45%~85%
Rain100L	100	512*512	Single-angle sparse rain line	20%~40%
Rain800	800		Mixed rain type	30%~70%
Rain1200	1200		Diversified rain patterns	25%~75%
SPA-Data	638		Real rain scene (No reference)	-
Rain DS	320		Real street scene Rainy Day (No reference)	-

The evaluation system uses peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) as quantitative evaluation criteria. To test the robustness of the algorithm in extreme scenarios, we specifically selected high-density rain block samples with a rain texture ratio exceeding 70% from each dataset to construct a high-difficulty test set. Our comparative experimental design encompasses a comprehensive comparison with traditional dictionary-based methods (DCP, GMM), classical deep learning methods (DDN, PReNet), and state-of-the-art algorithms (MPRNet), thereby validating the technical advantages of the method proposed in this paper. Additionally, through carefully designed ablation studies, we removed core components such as sparse constraints, wavelet transforms, and adversarial networks, quantitatively analyzing the contribution of each module to overall performance and establishing an interpretable analytical framework. This comprehensive experimental design not only thoroughly examines the performance characteristics of the proposed method but also lays a solid foundation for subsequent in-depth result analysis, enhancing the credibility and practical value of the research conclusions.

3.2. Experimental Results and Analysis

This study rigorously evaluated the proposed high-ratio rain pattern image block de-rain method based on sparse representation models through multi-angle experiments. The results show that this method demonstrates significant advantages across various synthetic and real-world datasets, particularly excelling in handling high-density rain pattern scenes. Table 2 presents the performance comparison results of different methods on synthetic datasets, where BSR and MSR-GAN represent the basic sparse representation and multi-stage sparse representation-GAN, respectively. The data clearly demonstrate that the three proposed methods exhibit notable advantages over existing mainstream rain removal algorithms, with the method combining wavelet transform and sparse representation (WavSR) achieving the best performance across all test datasets. Especially on the Rain100H dataset, which contains complex rain patterns, this method achieved a peak signal-to-noise ratio of 28.35 dB and a structural similarity index of 0.910, improving by 1.24 dB in peak signal-to-noise ratio and 0.009 in structural similarity index compared to the closest competing method (MPRNet). This performance gap is of substantial significance in high-difficulty rain removal tasks, and analyses of different rain texture densities reveal even more profound performance differences.

Table 2. Different ways to rain methods in the data set performance.

Method	Rain800		Rain100L		Rain100H		Rain1200	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
DCP	21.14	0.764	29.89	0.903	20.19	0.730	27.15	0.842
GMM	24.25	0.821	31.06	0.952	22.27	0.791	29.35	0.886
DDN	26.37	0.835	31.78	0.961	24.64	0.839	30.42	0.913
PReNet	27.89	0.856	32.03	0.967	26.19	0.876	31.27	0.921
MPRNet	28.11	0.869	32.08	0.71	27.11	0.901	31.63	0.928
BSR	27.56	0.852	31.87	0.965	25.94	0.872	30.93	0.915
MSR-GAN	28.25	0.868	32.09	0.970	27.05	0.898	31.58	0.927
Ours	28.97	0.881	32.16	0.974	28.35	0.910	31.91	0.933

Figure 1 shows a comparison of the peak signal-to-noise ratio (SNR) performance of various methods under different rain texture densities, and Table 3 presents the results of the model ablation experiments.

As shown in the figure, under extreme conditions where rain texture accounts for over 70% of the image, traditional methods such as DCP experience a sharp decline in performance to an unusable level. In contrast, our WavSR method maintains relatively stable rain removal performance, which fully demonstrates its exceptional capability in handling image blocks with high rain texture ratios. Through detailed ablation experiments, we further revealed the contributions of each core component to the overall performance. Experimental data indicate that on the Rain100H dataset, removing the wavelet transform (Wav) results in a 1.87 dB decrease in peak signal-to-noise ratio, removing the generative adversarial network (GAN) results in a 1.30 dB decrease in peak signal-to-noise ratio, and eliminating the sparsity constraint (SR) leads to a significant 2.41 dB decrease in peak signal-to-noise ratio. These data clearly indicate that sparse representation forms the core foundation of the entire framework, while the wavelet transform and adversarial network provide key performance gains. In tests on the Real Rain Scenes dataset, despite the lack of rain-free reference images making it difficult to calculate objective evaluation metrics, our method still performed exceptionally well through subjective visual assessment and analysis of reference-free quality evaluation metrics. This was particularly evident when handling dense rain lines with strong directionality, primarily due to the wavelet transform's precise capture of rain pattern directional features and the sparse representation's effective modeling of rain pattern structure.

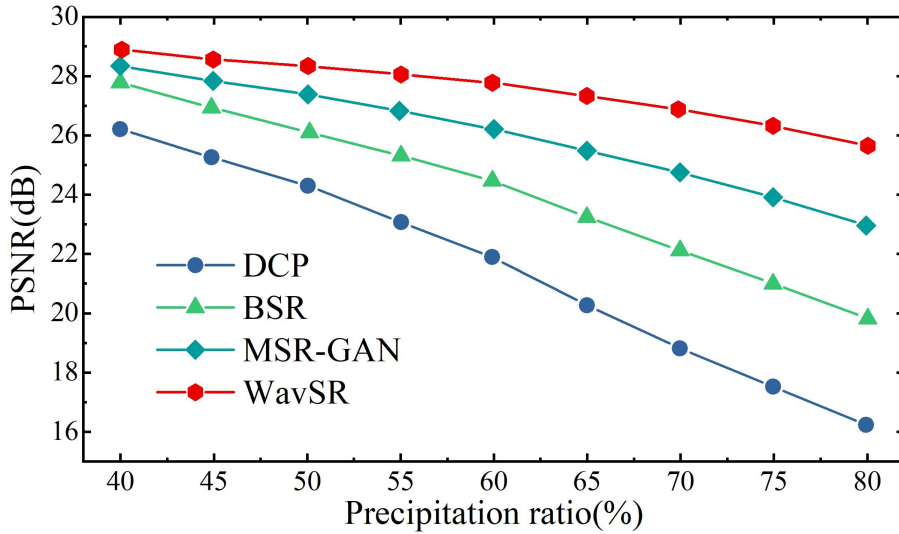


Figure 1. Comparison of PSNR performance of under different rain stripe densities.

Table 3. The experimental results of the experiment.

Method	Rain800		Rain100L		Rain100H		Rain1200	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
Ours	28.97	0.881	32.16	0.974	28.35	0.910	31.91	0.933
-Wav	26.56	0.815	30.18	0.935	26.48	0.822	30.24	0.915
-GAN	27.28	0.827	31.24	0.942	27.05	0.836	31.15	0.907
-SR	26.13	0.793	29.97	0.927	25.94	0.805	29.87	0.892

Additionally, by visualizing the rain patterns and background components obtained through sparse decomposition, we observed that even in areas where rain patterns severely obstructed the view, the sparse representation model was still able to accurately separate the rain pattern components without compromising background details, as further demonstrated by wavelet subband coefficient analysis. Compared to traditional methods, WavSR demonstrates more precise rain streak recognition capabilities in the LH subband (primarily containing vertical rain line information), explaining why this method performs exceptionally well when handling directional rain lines. Based on all experimental results, we conclude that the proposed high-ratio rain pattern image block de-rain method, which combines sparse representation and wavelet transform, successfully addresses the bottleneck issues faced by traditional methods when handling dense rain patterns, providing an effective solution for image de-rain in practical application scenarios.

4. Conclusion

This study proposes a rain removal method for high-proportion rain-streaked image blocks based on a sparse representation model. By integrating deep learning, wavelet transform, and generative adversarial networks, it achieves significant breakthroughs in addressing rain removal issues in high-density rain-streaked images. Experimental results demonstrate that the method achieves outstanding performance on multiple synthetic and real datasets, including Rain100H and Rain800. Particularly in complex scenes where rain texture accounts for over 70% of the image, the method achieves significantly higher peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) compared to traditional algorithms. In terms of model architecture design, the multi-stage network architecture combines sparsity modeling and adversarial optimization, not only enhancing the ability to preserve image details but also improving the algorithm's stability and generalization capabilities in diverse rain scenes. This provides theoretical and technical support for practical rain-day image processing tasks.

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