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Article

Teachers' Roles and Teaching Behaviors in Mathematical Modeling Pedagogy

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Abstract: Mathematical modeling instruction, as a crucial approach to fostering students' mathematical application awareness and model-based thinking, necessitates a fundamental shift in teachers' teaching philosophies and behaviors. Adopting a "guidance-to-reconstruction" framework, this study examines the developmental path of teacher role transformation in this setting. The study first identifies the core objectives and behavioral characteristics of mathematical modeling instruction, arguing that the teacher's role evolves from a mere knowledge transmitter to a learning facilitator, context designer, and thinking facilitators. Through analysis of practical teaching cases and relevant literature, the paper reveals three stages of teacher role transformation: instructional guidance, modeling support, and instructional reconstruction. This progression demonstrates the teacher's crucial role in organizing problem situations, guiding students' modeling thinking, and promoting teamwork and reflection. The findings suggest that this behavioral shift constitutes not merely a methodological update but a fundamental reconceptualization of teachers' professional identity. Finally, the paper proposes targeted recommendations for teacher development based on practical reflection and interdisciplinary integration, offering theoretical and practical insights for the advancement of mathematical modeling instruction.

Keywords: Mathematical modeling teaching; Teacher role; Behavior teaching reconstruction

1. Introduction

Mathematical modeling instruction, as an open and exploratory pedagogical approach, requires students to engage with real-world problems through hypothesis formulation, model construction, and verification. In China, modeling-based exploratory courses are designed starting from early childhood [1-2]. Within this process, how teachers frame questions, provide support, and strategically intervene is pivotal for developing students' modeling thinking. Research indicates that distinct teacher roles and behaviors—such as guidance, facilitation, support, and reconstruction—significantly influence student performance across various modeling stages. This necessitates a fundamental shift in the teacher's role: from knowledge transmitter to learning guide, from textbook logic to authentic problem-solving, and from a focus on "standard answers" to an emphasis on process diversity and collaborative communication [3].

Existing research on mathematical modeling has primarily focused on four areas: (1) student competencies and learning processes; (2) instructional design and strategies [4-6]; (3) curriculum development and reform [7]; and (4) the development of teachers' modeling literacy [8]. However, significant gaps remain, particularly concerning the teacher's role in the classroom.

First, there is a lack of process-oriented, dynamic, and empirical research on teachers' specific behavioral patterns during modeling instruction. Most studies prescribe what teachers should do, but lack micro-level descriptions of their actual role transitions and interactive behaviors [6]. Second, few studies systematically analyze the mechanisms of teacher-student interaction within the modeling context, such as how teachers guide, support, and reconstruct students' thinking through dialogue and feedback [9]. Third, there is insufficient understanding of the real dilemmas and challenges teachers face, as well as the drivers behind their behavioral choices (e.g., teaching beliefs, contextual constraints) [10]. Fourth, research methods have been relatively simplistic, often relying on questionnaires and structured interviews rather than qualitative classroom observations, which limits the ability to reveal the processual mechanisms of role transformation [11].

To address these gaps, this study employs a qualitative case study approach to conduct a micro-level analysis of teacher-student interactions. It aims to delineate the dynamic trajectory of teacher role transformation—conceptualized as a path from guidance to reconstruction—and to identify the associated key teaching behaviors across different modeling stages [12].

2. Theoretical Research and Framework

2.1. Characteristics of mathematical modeling

See Table 1, as an important bridge connecting mathematical knowledge and the real world, mathematical modeling teaching has distinct teaching characteristics and a relatively systematic teaching process [13]. First, mathematical modeling instruction is realistic. Modeling tasks often originate from real-world problems (such as the environment, transportation, the economy, and epidemics). It emphasizes the connection between mathematics and reality, encouraging students to apply mathematical knowledge to explain phenomena, solve problems, and make decisions [14]. Second, mathematical modeling instruction is open-ended. Problems often lack single, standard answers; solutions vary, and results are subject to uncertainty. This encourages students to think from multiple perspectives and express themselves creatively. Third, it is exploratory, emphasizing problem awareness and the inquiry process. Students are required to continuously formulate hypotheses, revise models, and reflect on their conclusions. The teacher's role is to guide rather than directly impart knowledge. Fourth, it is collaborative, often conducted in small groups, emphasizing collaborative communication and fostering students' communication skills, division of labor, and collective decision-making abilities [15]. Fifth, it is comprehensive, encompassing knowledge from multiple disciplines, including mathematics, computer science, physics, statistics, engineering, and social sciences, integrating modeling concepts, mathematical tools, and technical techniques [16]. Sixth, it features process-based evaluation, which emphasizes recording and reflecting on process outcomes (such as modeling logs and midterm reports), focusing on model rationality, completeness of expression, and suggestions for improvement. The mathematical modeling process can be summarized into six stages: problem raising, model construction, model solving, model analysis, model application and correction, and expression and reflection [17].

Table 1. Mathematical modeling teaching process diagram

Stage	Specific Tasks	Teaching Key Behaviors
Problem-raising (understanding the problem)	Identify real-world problems and key variables, and context	Teachers create real situations to stimulate students' interest
Model building	Build mathematical models (define variables, set relationships, list equations, etc.)	Teachers guide students to summarize and abstract, and put forward reasonable hypotheses
Model solution	Use mathematical tools (algebra, geometry, calculus, statistics, programming, etc.) to solve models	Teachers provide appropriate technical and methodological support
Model Analysis	Analyze the rationality of the results and test the interpretability and limitations of the model	Teachers guide students in critical thinking and model evaluation
Model application and modification	Apply the model to new situations and optimize the model based on actual deviations	Teachers help students optimize their expressions and ways of expression
Expression and reflection	Write a modeling report, present and communicate, and reflect on the modeling process and collaborative experience	Teachers guide students to summarize the process and summarize their experiences

2.2. Key teaching behaviors

In mathematical modeling instruction, "key teaching behaviors" refer to the specific actions that teachers implement to guide students through the entire modeling process, which are highly valuable and instructive. These behaviors not only reflect the multiple roles teachers play in teaching (such as guide, facilitator, reconstructor, and reflector) but also significantly influence students' modeling abilities, thinking quality, and learning attitudes [18]. First, classroom guidance refers to the teacher's actions during the teaching process to promote student participation in modeling activities through organization, regulation, and guidance. Key features include creating authentic problem situations to stimulate student interest in modeling; guiding students to discuss, analyze, and integrate modeling ideas; implementing classroom Q&A sessions and brainstorming to foster deeper thinking; and managing the classroom rhythm and teamwork dynamics to ensure modeling progress [19].

Second, problem design involves teachers selecting or creating modeling-worthy problems based on teaching objectives and student learning situations to stimulate students' modeling exploration. Key teaching behaviors include selecting realistic, open-ended, and explorable problems; appropriately decontextualizing and recontextualizing the problems to make them suitable for classroom use; grading the structure and difficulty of the problems; and stimulating students' problem awareness and their ability to raise new questions [20].

Third, modeling guidance refers to the key actions teachers take during the student modeling process, providing appropriate support and intervention to help students complete the mathematization, model construction, solution, and verification steps. This includes providing mathematical tools and knowledge integration suggestions; guiding students to analyze variable relationships and make reasonable hypotheses; encouraging students to use diagrams, algorithms, and other methods to build models; and finally, providing feedback and guidance during model verification and revision [21].

The fourth type of assessment guidance refers to the teacher's diagnostic, formative, or summative evaluation of students' modeling processes and outcomes during modeling instruction, while also guiding students in self- and peer assessment. This includes developing evaluation criteria (both process- and outcome-oriented); guiding students to reflect on key aspects of the modeling process; providing feedback to promote the development of modeling skills and teamwork; and encouraging students to engage in peer assessment to develop their evaluation literacy [22].

3. Study Design

Teachers' in-depth development of teaching cases is of great significance to teaching itself and cultivating students' abilities. In other words, the process of teaching mathematical modeling is actually the process of cultivating mathematical modeling ability. How to guide students in mathematical modeling is an important soul-searching question in cultivating modeling ability.

3.1. Research subjects

This study involved 28 students, 15 boys and 13 girls, selected from Teacher Z's class, Grade 9, at a public high school in Shiyan City. The purpose of this study was to provide a model for literacy-oriented teacher role-playing instructional practices to cultivate students' mathematical modeling literacy. Based on the research objectives, the sample selection required a teacher with extensive teaching experience. Teacher Z has 22 years of teaching experience and holds a master's degree in mathematics and science education. She has seven years of experience in incorporating mathematical modeling activities into her teaching. Teacher Z divided the class into seven groups of four and conducted mathematical modeling instruction according to the seven stages of understanding, simplification, mathematization, mathematical solution, interpretation, verification, and revelation. To foster a vibrant and thoughtful discussion, the mathematics classroom was transformed into a student discussion community, where conclusions were reached independently through mutual debate, discussion, and reasoning, rather than the teacher's authority. Overall, Teacher Z was a suitable sample for this study.

3.2. Research Methods

This study employed a case study method using non-participant observation, conducting on-site classroom observations. Within the context of a complete situation, observation and analysis yielded data that provided a deeper understanding of the teacher's role in mathematical modeling instruction. Before data collection, influencing factors were carefully considered. When analyzing and describing the data, an understanding of the teacher's behavior and context was prioritized. Data collection was conducted using classroom observations. After data collection was complete, multivariate data analysis and summarization were conducted to facilitate source correction. This study quantified qualitative data to reveal key behavioral characteristics of the teacher's role.

4. Data Collection and Analysis

The data collected for this study covered Chapter 2, Quadratic Functions, Equations, and Inequalities; Chapter 3, Concepts and Properties of Functions; and Chapter 4, Exponential and Logarithmic Functions, all covered in the first semester of the first year of high school. These units encompass 15 classes in total. The first 12 classes covered the definitions, concepts, and properties of basic high school functions. Data for this study were collected from classes 12 through 15, with mathematical modeling activities taking place over three classes, totaling 138 minutes. During each data collection session, a camera and recorder were set up in the classroom. This ensured that both the research subjects and the class were observed in the most comfortable and natural environment possible, thereby obtaining the most natural and authentic data. Math classes were videotaped throughout, and notes were collected and recorded of teacher and student discussions and physical interactions. During group discussions or individual presentations, the interaction between the teacher and students was recorded, capturing detailed conversational content.

Based on the research framework, a coding system was developed to analyze the teaching case. The mathematical modeling stages, teaching behaviors, and roles were represented by symbols to form a coding table: the first code represents the stages, which include understanding C1, simplification C2, mathematization C3, mathematical solution C4, interpretation C5, verification C6, and disclosure C7; the second code represents the teacher's role: creator C, promoter P, problem solver A, epiphany E, reconstruction Q, and decision maker D.

This study analyzed three teaching scenarios (138 minutes total), accumulating 648 utterances. To enhance the reliability of the data analysts, this study employed triangulation. After converting the video recordings into transcripts, each transcript was analyzed by two researchers with backgrounds in mathematics and science education. The two raters were provided with the analysis criteria and verbally briefed to ensure their understanding of the analysis framework and criteria. Only then were the raters' coding results compared. After the analysis was complete, the two raters' coding results were cross-compared. Through this cross-comparison, the 10% discrepancies in the coding were discussed repeatedly until consensus was reached.

5. Study Results

5.1 The proportion of each teaching behavior of the teacher's role in mathematical modeling teaching

See Table 2, during mathematical modeling instruction, teachers and students engaged in 648 conversations. The roles played, from lowest to highest frequency, were creator (12 (1.9%)), insight maker (30 (4.7%)), decision maker (48 (7.4%)), puzzle solver (75 (11.6%)), facilitator (165 (25.6%)), and reconstructor (318 (49.3%). Therefore, the most common roles played by teachers were facilitator and reconstructor.

The mathematization and verification stages are relatively complex in mathematical modeling instruction, requiring students to understand the mathematical conditions and properties inherent in real-world problems while also optimizing the mathematical model. Consequently, during these stages, teachers were more likely to act as facilitators (165/648 (25.6%)) and reconstructors (318/648 (49.3%)). Teacher intervention was least frequent during the understanding stage, with 12 (1.9%) of the teachers acting as creators, primarily proposing hypotheses. The table below shows the proportion of teacher behaviors in each stage of mathematical modeling instruction, representing the ratio of the number of individual behaviors to the total number of teaching behaviors performed during that role.

Table 2. Proportion of each teaching behavior of the teacher's role in the seven stages of mathematical modeling teaching

Role Behavior process		**Understand → Simplify → Mathematical → Mathematical solution → Interpretation → Verify → Reveal**					
Founder 12 (1.9%)	Information Processing	12 (100%)	/	/	/	/	/
	Propose a hypothesis	12 (100%)	/	/	/	/	/
Promoter 165 (25.6%)	Motivation	3 (25%)	3 (21.8%)	3 (4%)	3 (10%)	10 (32.1%)	15 (31.3%)
	Problem representation	6 (50%)	6 (38.2%)	18 (24%)	6 (20%)	/	/
	Pattern Recognition	/	75 (45.5%)	12 (16%)	/	114 (35.8%)	9 (18.8%)
	Path analysis	/	45 (27.3%)	9 (12%)	/	129 (40.6%)	21 43.8%)
Puzzle Solver 75 (11.6%)	Relationship Building	/	15 (9.1%)	12 (16%)	/	/	/
	Process Solution	/	/	24 (32%)	/	18 (5.7%)	3 (6.3%)
	Causal Interpretation	/	/	30 (40%)	15 (50%)	63 (19.8%)	15 (31.3%)
Enlightened One 30 (4.7%)	Model Evaluation	/	/	/	12 (40%)	180 (56.6%)	33 (68.8%)
	Reflection	/	/	/	24 (80%)	129 (40.6%)	18 (37.5%)
	Method Summary	/	/	/	12 (40%)	/	6 (12.5%)
Reconstructor 318 (49.3%)	Inspection and Evaluation	/	/	/	/	129 (40.6%)	12 (25%)
	Idea expansion	/	/	/	/	108 (34%)	24(50%)
	Model reconstruction	/	/	/	/	21 (6.6%)	12(25%)
Decision makers 48 (7.4%)	Diverse solutions	/	/	/	/	12 (3.8%)	18 (37.5%)
	Choose the best	/	/	/	/	/	6 (12.5%)

According to the data in the table above, teachers do not play a single role in mathematical modeling instruction, but rather the result of the interaction of multiple roles. The teaching process involves repeated reasoning and argumentation, and therefore, the proportion of teaching behaviors for each role varies. As shown in the table above, the top three teacher roles are re-constructor, facilitator, and problem solver. From understanding to simplification, teachers act as both creators and facilitators. Information processing and hypothesis formulation account for 100% of the creator's teaching behaviors. From simplification to mathematization, teachers are not only facilitators but also assist by acting as problem solvers. In their facilitator role, teachers engage in motivational stimulation, problem representation, and path analysis. Students, under the teacher's facilitation, engage in pattern recognition and relationship construction. Among all teaching behaviors, pattern recognition accounts for the highest proportion, at 75/165 (45.5%), followed by problem representation at 63/165 (38.2%). From mathematization to mathematical solution, teachers play the dual role of problem solver and facilitator. Teachers, in the role of problem solver, explain causal relationships, solve process problems, and construct relationships. They also play the role of facilitator, stimulating motivation and representing problems. Causal explanation accounts for the highest proportion of all teaching behaviors, at 30/75 (40%), followed by process solution, at 24/75 (32%). From the mathematical problem-solving to the explanation stage, teachers act as epiphanies, supplemented by problem solvers and facilitators. In the epiphanic role, teachers evaluate models, reflect on problems, and summarize methods. For more difficult problems, they also play the role of problem solver, explaining causal relationships. They also play the role of facilitator, stimulating motivation and representing problems. Problem reflection accounts for the highest proportion of all teaching behaviors, at 24/30 (80%). From the explanation to the verification stage, teachers' roles are complex and diverse, primarily as reconstructors, encompassing epiphanies, problem solvers, facilitators, and decision-makers. In the role of reconstructor, teachers rebuild models, expand ideas, and conduct verification and evaluation. In the role of epiphanies, teachers evaluate models, reflect on problems, and summarize methods. Model evaluation accounted for the highest proportion of 180/318 (56.6%), followed by inspection and evaluation of 129/318 (40.6%). From the interpretation to the revelation stage, decision makers were sublimated on the basis of the previous stage, which was the result of comprehensive effects, among which model evaluation accounted for the highest proportion of 33/48 (68.8%).

5.2. The proportion of teaching behaviors that act as promoters at different stages

Facilitators play an important role in different stages of mathematical modeling. Table 3 shows that in mathematical modeling instruction, Teacher Z most needed to intervene as a facilitator, with the highest proportion of interventions occurring in the two stages: the simplification to mathematization stage, where she intervened 219 times out of 669 times, accounting for 32.7%. The second highest proportion was the interpretation to verification stage, where she intervened 345 times out of 669 times, accounting for 51.6%. In this proportion, the first number represents the sum of the number of teaching behaviors in that stage, while the second number represents the sum of all teaching behaviors across all stages. Each proportion in the table represents the ratio of the number of teaching behaviors in that stage to the total number of teaching behaviors across all stages. The following describes the main teaching behaviors in which Teacher Z intervened as a facilitator during the mathematical modeling stage.

Table 3. The number of times teachers act as facilitators in different stages of mathematical modeling

Teaching stage → Teaching behavior	**Motivation**	**Problem representation**	**Pattern Recognition**	**Path analysis**	**Total**
Understand → Simplify	3 (0.45%)	6 (0.9%)	0	0	1.35%
Simplify → Mathematical	36 (5.4%)	63 (9.4%)	75 (11.2%)	45 (6.73%)	32.7%
Mathematical → Mathematical solution	3 (0.45%)	18 (2.7%)	12 (1.8%)	9 (1.35%)	6.3%
Mathematical solution → Interpretation	3 (0.45%)	6 (0.9%)	0	0	1.35%
Interpretation → Verify	102 (15.2%)	0	114 (17.08%)	129 (19.3%)	51.6%
Verify → Reveal	15 (2.2%)	0	9 (1.35%)	21 (3.14%)	6.7%
Total	24.2%	13.9%	31.4%	30.5%	100%

(1) From simplification to mathematization

The goal of the simplification to the mathematization stage is to construct a mathematical model. Whether a mathematical model can become a cognitive model for the entire class requires the class's shared understanding. Teacher Z intervened in this stage, employing a facilitator role. The three most important teaching behaviors demonstrated in this stage were pattern recognition (75/669 students (11.2%)), problem representation (63/669 students (9.4%)), and path analysis (45/669 students (6.73%)). Under these teaching behaviors, students were required to conduct path analysis and pattern recognition based on mathematical properties. The teacher's facilitator role of inspiration and guidance helped students clarify their thinking and deepen their understanding. When Teacher Z asked students to conduct path analysis, her primary goal was to help them develop their thinking. She often intervened through problem representation, encouraging students to actively question, criticize, refute, and support their ideas. Expressing a position requires sufficient understanding and critical thinking before proposing a modeling idea. For example, when a student argued that an employee's final salary should be related to their monthly performance, Teacher Z used questioning and problem representation to inspire the student to identify a model.

(2) From interpretation to verification

This stage is the result of the teacher's dual role as both facilitator and reconstructor. While the re-constructor primarily engages in testing, evaluation, and developing ideas, the facilitator engages in motivational stimulation and path analysis, helping students deeply develop their constructive thinking and shaping their logical thinking. Therefore, this dual role of teaching behavior in this stage is crucial for teaching implementation. Path analysis and pattern recognition accounted for the highest proportion of all teaching behaviors, at 129/669 (19.3%) and 114/669 (17.08%), respectively.

5.3. The proportion of teaching behaviors of re-constructors at different stages

The above case studies and data analysis demonstrate the importance of the facilitator and reconstructor roles for mathematical modeling teachers. Further qualitative role analysis reveals the impact of reconstructor and facilitator behaviors on students. Teachers' teaching behaviors vary depending on their role. Table 3 shows four teaching behaviors in which facilitators intervene at different stages of mathematical modeling. Table 4 shows three teaching behaviors in which reconstructors intervene at different stages of mathematical modeling.

Table 4. The number of times teachers act as re-constructors in different stages of mathematical modeling

Teaching stages Teaching process behavior	**Inspection and Evaluation**	**Idea expansion**	**Model reconstruction**	**Total**
Interpretation → Verify	129(42.2%)	108(35.3%)	21(6.9%)	84.4%
Verify → Reveal	12 (3.9%)	24 (7.8%)	12 (3.9%)	15.6%
Total	46.1%	43.1%	10.8%	100%

Table 4 shows that Teacher Z intervened in the teaching stage as a reconstructor, and the most obvious performance was in the stage from explanation to verification, where he intervened in 258/306 (84.4%). The following will describe the main teaching behaviors of Teacher Z's intervention as a reconstructor in the mathematical modeling teaching stage.

(1) From interpretation to verification

The core of the stage from interpretation to verification lies in the students' process of constructing different models. Teachers need to reflect on, expand upon, and reconstruct the models initially constructed by students. Teachers need to break through the existing modeling framework and expand and optimize it. During this stage, Teacher Z intervenes in student learning through the role of a re-constructor. Two prominent teaching behaviors during this stage are testing and evaluation (129/306 students (42.2%)) and idea development (108/306 students (35.3%)). Students engage in divergent thinking and construct ideas based on mathematical properties. Inspiring and comparing these behaviors through the role of a re-constructor helps students see richer mathematical relationships. In the following teaching dialogue, Teacher Z asks students to compare the practical effectiveness of their models, asking, "Is it realistic? So, if we solve the model and it doesn't fit the reality, what should we do?" This helps students focus on the similarities and differences between the original function model's unrealistic and realistic approaches. Students begin to notice these differences, gradually identifying questions and formulating hypotheses.

The aforementioned teaching activities primarily involve verification and evaluation. When teachers continuously focus on constructing different models, they ask, "If you were a member of this company—perhaps a salesperson or a manager—are you satisfied with this linear function model? If so, please provide your reasons. If not, please build a suitable function model and save the results." To help students see more model possibilities, teachers invite students to observe and propose hypotheses. When students lack new ideas or their logic is unclear, teachers intervene as re-constructors, allowing students to explore different modeling approaches within the framework provided by the teacher. This stage also directly impacts the overall quality of the modeling activity. Therefore, expanding ideas and verification, and evaluation are the primary teaching activities during this stage.

(2) From verification to disclosure

The revealing phase, the final stage of mathematical modeling instruction, aims to empower students to apply known mathematical models to decision-making. Verifying the model's plausibility can be challenging for students, requiring them to contextualize the scenario before making a decision. Within the context of mathematical modeling, the model presented in the reconstructed data often undergoes some changes. Teacher Z often intervenes before verifying the model's plausibility, encouraging students to conduct preliminary analysis and enhance the comparative advantage between different models. This phase provides students with room to expand their thinking towards optimal decision-making.

5.4. Analysis and Discussion

Compared to traditional mathematics classes, the teacher's role in the classroom has gradually changed. Mathematical modeling requires handling complex modeling processes, and the influence of the facilitator or re-constructor role can be seen in the details of teaching. Having students engage in problem representation, path analysis, and pattern recognition facilitates mathematical modeling activities. Problem representation requires students to understand real-life mathematical problems, organize the characteristic attributes of mathematical problems, and clarify mathematical thinking. Pattern recognition is a higher-level cognitive activity, requiring students to identify consistent pattern structures across multiple conditions. Path analysis involves identifying cognitive biases through precise verbal descriptions and correcting them. Verification and evaluation validate the rationality of model construction and determine the optimal solution. These high-level cognitive activities demonstrate valuable skills.

The case teacher uses Blum's modeling cycle process as the teaching stage. In this teaching stage, the six roles of teachers are constructed based on the teacher's teaching behavior, student learning behavior, and teacher-student interaction relationship, namely creator, promoter, problem solver, epiphany, re-constructor, and decision maker. This study found three points worth discussing from the research results: (1) The role analysis framework applies to the field of mathematics education and can also be extended to the field of science to analyze the classroom teacher role and teaching behavior of student-student interaction. (2) The teacher role analysis framework and key teaching behaviors in this article apply to case teacher research and can also be verified with more case data using structural equation models. (3) The teacher's role in the analysis framework is generated relative to the teacher-student interaction modeling process. The contribution of this study to this analysis framework is to clarify the key teaching behavior attributes behind each role.

6. Conclusion and Recommendations

6.1. Conclusion

This study explored the implications of individual teachers' teaching behaviors as they engaged in different roles at different stages of mathematical modeling instruction. Teachers adjusted their primary teaching behaviors based on their roles at different stages to maximize the effectiveness of mathematical modeling instruction. Facilitators and re-constructors contributed the most to the overall mathematical modeling instructional process, exceeding 74.9%. Based on these findings, this study reached the following two main conclusions:

The case teacher's intervention as a facilitator in mathematical modeling teaching is mainly in the verification stage, followed by the mathematization stage. The intervention as a re-constructor is mainly concentrated in the verification stage. The main teaching behaviors of the facilitator in the verification stage include path analysis and pattern recognition; the main teaching behaviors in the mathematization stage are problem representation and pattern recognition. The main teaching behaviors of the re-constructor in the verification stage include idea development and verification and evaluation. Among them, pattern recognition is the most common teaching behavior, followed by path analysis and problem representation.

The purpose of the case teacher's teaching behavior intervention at different stages is different. In the mathematization stage of problem representation teaching behavior, the case teacher's intervention is to ask students to express their problem thinking and to summarize or generalize the characteristics of mathematical problems in real situations; in the verification stage, the intervention is to develop students' comparative advantages in constructing various mathematical models. In the mathematization stage of path analysis teaching behavior, the intervention is to help students observe the rich mathematical relationships in real situations; in the verification stage, the intervention is to help students clarify the internal logic of various function models. In the mathematization stage of pattern recognition teaching behavior, the case teacher intervenes with inductive teaching behavior, and will ask students to select or integrate the most representative mathematical feature from multiple similar mathematical features.

6.2. Discussion

As facilitators, teachers' primary teaching activities involve problem representation and pattern recognition. The best way to establish a connection between these two is through problem schematization. Schematization involves interpreting the underlying structural prototype of a mathematical modeling problem. Students need to filter and process this information into meta-cognition within their minds. Schematized problem representation fosters effective mental experience and patterns, further enhancing students' mathematical modeling abilities. Schematized instruction in mathematical modeling fosters the formation of structured mathematical modeling knowledge and optimizes cognitive structure. This "optimization" transforms stored mathematical knowledge into "production formulas," facilitating the construction and activation, transfer, and application of knowledge.

As re-constructors, teachers' most important teaching behavior is to expand their thinking, and the best teaching content for expanding thinking is strategic knowledge. Strategic knowledge is methodological knowledge about how to learn and think. It includes general thinking strategies (how to grasp the overall direction of problem solving); modeling strategies (thinking about mathematical representation, hypothesis, construction, solution, verification, and adjustment); and modeling methods (mathematical techniques such as algebra, geometry, trigonometry, and anti-trigonometry). The student's mathematical modeling process is an active process of thinking. The greatest difference between students lies in their problem-solving strategies. Strategic knowledge can help students improve their mathematical modeling skills. By enabling students to achieve a state of experiential, conditional, and automated learning through path analysis, teachers can subtly change their modeling skills and shift the process from imitative learning and application transfer to spontaneous generation.

It is important to acknowledge the limitations of this study. As a qualitative case study, its findings are derived from an in-depth analysis of a specific classroom context and a single, experienced teacher. While this approach provides rich insights into the micro-processes of role transformation, the generalizability of the proposed framework needs to be further tested with larger and more diverse samples. Future research could employ mixed methods to validate and refine these role categories across different educational settings and teacher populations.

From a contemporary perspective, the inclusion of mathematical modeling in regular middle school instruction holds practical significance and value for national talent development, economic revitalization, and the development of key sectors. The urgent need for mathematical modeling instruction in middle schools is driving fundamental changes in current teacher education philosophies, teaching methods, curriculum design, content development, and academic assessment. Mathematical modeling instruction is an inevitable choice for curriculum reform in this new era and a crucial opportunity to promote the development of innovative teaching content.

Author Contributions

Conceptualization, Y.Z. and N.N.Z.; methodology, Y.Z., N.N.Z. and A.F.M.A.; software, A.F.M.A.; validation, Y.Z. and N.N.Z.; formal analysis, Y.Z.; investigation, Y.Z.; resources, Y.Z.; data curation, Y.Z.; writing—original draft preparation, Y.Z.; writing—review and editing, N.N.Z. and A.F.M.A.; visualization, Y.Z.; supervision, A.F.M.A.; project administration, N.N.Z. and A.F.M.A.; funding acquisition, Y.Z. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The data generated during this study are not publicly available due to privacy reasons. The data contain identifiable information of minor students and their teacher within a specific classroom context. Informed consent for participation was obtained, but it did not extend to public archiving of the records.

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Conflicts of Interest

The authors report there are no competing interests to declare.

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