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Article

# A Novel Stochastic–Deep Learning Hybrid: GRU with GBM for Financial Forecasting

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**Abstract:** Financial time series are nonlinear, stochastic, and volatile which makes it a complicated task to accurately predict such data. The paper is a comparative study of the Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) models in predicting complex financial time-series data, which has been augmented with the Geometric Brownian Motion (GBM) model. To determine the predictive efficiency and the strength of the new GBM-based LSTM and GRU frameworks, they are tested on several large-scale stock market datasets of India. Standard error and goodness-of-fit is used to test the performance of the models to prevent unreliable validation of models across datasets. The experimental findings indicate that the addition of GBM helps the LSTM and GRU models to work much better in terms of forecasting because of their ability to effectively learn stochastic price dynamics and volatility patterns that are inherent to financial markets. The GRU with GBM architecture also achieves the best performance across all datasets compared to the isolated LSTM and GRU architectures, making it more accurate and stable in its predictions. The fact that the performance of various stocks varies also indicates the flexibility of the hybrid approach in varying market conditions. The results overall demonstrate that the GRU-GBM hybrid model is a highly dependable and effective model of short-term financial forecasting, especially in highly unstable and uncertain market conditions. The given strategy can be considered a potential alternative to informed decision-making support in the use of investment analysis and risk management.
**Keywords:** GBM, GRU, LSTM, Stock Market Prediction, Metrics.

## 1. Introduction

The analysis of predictive modeling in financial markets is a very complex process because of volatile, time-based, and multifaceted financial data. To illustrate, stock price forecasting is a very difficult endeavor since this process is affected by a lot of unpredictable internal and external influences like market mood, macroeconomic performance, and geopolitical occurrences. Older forecasting methods based on fundamental and technical analysis are typically inadequate when they are based on the premise that the assumptions are mostly fixed, and cannot keep up with the dynamism of the market. This difficulty is further compounded by the abundance of financial information that is most of the time misled by exogenous factors, such as economic cycles and political events that result in abrupt and pronounced adjustments of market trends. Recent studies in this field have tried to overcome these weaknesses in forecasts by incorporating more and more machine learning-based procedures, which can be used to model complex and non-linear relationships in financial data. As the stock markets are very dynamic environments for investment, predictive models that are accurate and adaptive must be created in order to make sound predictions.

In this paper, the authors explore the applicability of hybrid deep learning models to predict stock prices on various performance benchmark datasets. The four models considered are GRU, LSTM, and hybrid ones with Geometric Brownian Motion (GBM) added, i.e., GRU+GBM and LSTM+GBM. These hybrid models, in particular, aim to integrate stochastic market behavior into deep learning models.

- It is proposed to use a combination of recurrent neural networks with probabilistic trajectory modelling of GBM to achieve better predictive performance in the proposed GRU+GBM and LSTM+GBM models. GRU and LSTM add good time-based learning capacity, and GBM adds volatility-based trend representation to the model.

The principal contributions of the work are:

- The proposal of testing a hybrid GBM-based model with deep learning integrated into it to predict financial time series.

  Showing the superiority of GRU+GBM with five real stock datasets.

- A thorough statistical comparison of the results with the Tukey HSD test to determine the existence of significant differences in performance.
- To give a full performance interpretation by providing R2, MAE, and RMSE-based evaluation.
- Verifying that GRU + GBM can be statistically supported with consistent and stable improvements compared to GRU, LSTM, and LSTM + GBM.

The paper is structured as follows: Section 2. Literature Review 3. Materials and methods. In this section, we discuss the proposed framework GRU + GBM. Section 4. Experimental 5. Results and Discussion. Finally, conclusion and future work.

## 2. LITERATURE REVIEW

The theory of non-random walk and the effectual market hypothesis suggest that past stock prices have high commercial importance, implying that the study of historical prices could help predict future trends [1-3]. The machine learning models have shown more potential than the classic statistical methods, such as Autoregressive Integrated Moving Average (ARIMA), a time-series model of prediction which uses differencing [4]. Time-series financial forecasting is significant in decision-making by investors, financial analysts, and policymakers [5]. Geometric Brownian Motion is a model that is used to forecast stock prices based on the predictor variables through the estimation of the return, volatility, and drift, which has a high prediction accuracy with a Mean Absolute Percentage error of 20 percent and less [6]. The LSTM-based models are also evaluated by comparing them with the traditional machine learning techniques and testing a variety of models and performance indicators, and sentiment influence on the stock market dynamics [7]. The current deep learning techniques provide a dynamic methodology through the use of vast amounts of historical information, something that conventional models fail to do with the assistance of statistical and econometric methods. Past predictions of the stock market using LSTM networks have demonstrated good performance of the network in identifying market trends. When machine learning methods are applied alongside net growth analysis, which determines the growth and shrinkage rates of financial variables [8], [9], the prediction outcomes are enhanced. The methodology is also superior in short-term market prediction as compared to the traditional statistical models based on the results of the performance evaluation [10].

The standard assessment measures that are employed in the evaluation process include RMSE, MAPE, and correlation coefficients. As it is shown in this paper, the complexity of models and prediction accuracy can be determined by the choice of network architecture [11]. The prediction of financial data will be more transparent and reproducible with the use of readily available stock market data, including open, high, low, and closing prices [12]. This recursive approach makes the model continue perfecting its predictions as it considers the varying market trends. The strength of this method is also confirmed by checks that are performed with the help of different sizes of inputs, which prove that it is more effective in improving the quality of forecasts than the traditional models [13]. The results of the experiment indicate the capability of LSTM models to learn complex stock price dynamics and provide them with accurate short-term forecasts, which is an additional indicator of their application in the analysis of financial markets [14].

Discusses the use of deep learning models to predict stocks, compares the deep recurrent neural networks, which are based on LSTM and GRU, a bidirectional stacked design, and a unidirectional stacked design, to the shallow neural networks, using the S&P 500 data, and it finds that the deep stacked LSTM was most effective in both short and long-term prediction [15]. The integration of both technical indicators, investor sentiment, and financial indicators forms another new model of stock predictions, and the dimension reduction methods of LASSO and PCA are used. The LSTM and GRU models comparison shows that both can forecast stock price, and LASSO is higher performance in comparison to PCA [16]. Also, financial news sentiment is proven to play a significant role in predicting the upcoming trends, and both LSTM-News and GRU-News perform as well as possible. An interactive deep-learning framework that can provide the most appropriate prediction model [17].

The success of deep learning models in financial time-series prediction has been demonstrated in many works. LSTM models are also found to be better or equal to GRU, ANN and SVM methods in stock and index prediction tasks with high predictive accuracy and high trading returns on datasets which include S&P 500, CNX-Nifty, IBM, NIFTY-50, BANKNIFTY and Moroccan stock markets [18], [19], [21], [22], [24], [26], [27]. Hybrid and stacked models, such as LSTM-GRU, Bi-LSTM-GRU, and multi-model integrations, even improve the accuracy of forecasting, robustness, and long-term prediction performance of standalone deep learning and traditional models [22], [23], [27], [28], [29]. Another notable feature presented in several studies is the use of advanced preprocessing and feature engineering methods, including ICA-based normalization, sentiment analysis, and dimensionality reduction, where the LASSO-based feature selection method has shown to be higher than PCA in enhancing the accuracy of the model [18], [20]. On the whole, these results support the idea that hybrid deep-learning models with streamlined structures and enhanced input features offer high-performance in complicated financial and economic time-series prediction problems [18] - [29].

The existing literature suggests that even though deep learning models, like LSTM and GRU, can be more effective in predictive accuracy of stock prices, stochastic models built on Geometric Brownian Motion (GBM), can still be very useful in predicting in the short term, modeling risks, and forecasting uncertainty in a variety of financial markets [30], [34], [37], [38]. A variety of works improve classical GBM with the use of memory effects, stochastic volatility, crisis regimes, multidimensional dependencies, and Monte Carlo simulations, which results in better predictive analysis of major indexes and sectoral data such as S&P 500, DAX, SHANGHAI Composite, and emerging markets [31], [32], [35], [41]. Comparative studies also indicate that ARIMA, as a traditional statistical model, can perform better than GBM and neural networks in certain limited duration contexts, which brings out the comparative merits of the statistical and stochastic models [33], [36]. Even more recent hybrid and ensemble models that combine GBM with Kalman filters or deep learning models and machine learning frameworks can show a considerable decrease in forecasting error and a greater robustness, even in big-data and high-frequency conditions [39], [40]. The findings are a great impetus to build hybrid GBM -deep learning modules that integrate probabilistic price processes with nonlinear learning of features in financial time series to provide stronger and more valid predictive features of the financial time series [30] - [41].

In short, predictive capabilities in the stock market have been improved with the use of deep learning algorithms in the shape of LSTM networks. Conventional financial models, such as GBM, as revolutionary as they are, are not able to offer the level of flexibility required to draw out sophisticated market behavior. The transition to machine learning has enabled researchers to build more precise and versatile forecasting frameworks that have the potential to extract the market mood, macro economy, and technical indicators. We compare four models, namely, GRU, LSTM, GRU+GBM, and LSTM+GBM, in this work. We evaluate their predictive capabilities through important evaluation metrics by subjecting them to rigorous testing and performance testing. The results of our study will provide meaningful information on the effectiveness of hybrid methods in financial predictions, giving relevant implications to the investment and financial practitioners.

## 3. Methodology

This section presents the models used in this paper, specifically LSTM + GBM and GRU + GBM, which combine deep learning methods with Geometric Brownian Motion to predict time series more effectively. Additionally, LSTM and GRU models are used independently to compare the performance of both models.

### *A. LSTM*

LSTM is an RNN model that is efficient in learning long-term dependencies and sequential relationships within the data. As opposed to vanilla RNNs, the LSTM networks address the vanishing gradient problem with the presence of explicit elements, that is, the forget gate, input gate, and output gate. These gates are what regulate the retention, updating, and deletion of values in the memory cell and allow the model to retain helpful context over time. LSTMs, therefore, lend themselves well to tasks that are dependent in time, such as time-series forecasting and natural language processing.

Forget gate $F_t = \sigma(G_f \ . [h_{t-1}, x_t] + B_f)$ (1)

Input gate $I_t = \sigma(G_i \ . [h_{t-1}, x_t] + B_i)$ (2)

Cell state update gate $C_t = tanh(G_c \ . [h_{t-1}, x_t] + B_c)$ (3)

Cell state $C_t = F_t \odot C_{t-1} + I_t \odot C_t$ (4)

Output gate $O_t = \sigma(G_o \ . [h_{t-1}, x_t] + B_o)$ (5)

Hidden state $C_t = O_t \odot \tanh(C_t)$ (6)

In this context, tanh refers to the hyperbolic tangent activation function; $G_f$, $G_i$, $G_c$, and Go are the respective weight matrices; σ denotes the sigmoid activation function; and $B_f$, $B_i$, $B_c$, and $B_o$ represent the corresponding bias terms. The operator ⊙ signifies element-wise (Hadamard) multiplication.

*Table 1.* Statistical Overview of Stock Data

| Stock | Samples | Min Price | Mean Price | Max Price | Std Dev | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| HCL. | 5030 | 14.8625 | 327.4341 | 1358.2 | 326.4203 | 1.197954 | 0.57916 |
| INFY | 6595 | 1.543164 | 398.2334 | 1939.5 | 423.6622 | 1.748046 | 2.619475 |
| TCS. | 5030 | 38.3875 | 1096.802 | 4019.15 | 1039.981 | 1.075968 | 0.143145 |
| TECHM | 4010 | 52.325 | 515.8746 | 1806.1 | 349.7294 | 1.123734 | 1.024487 |
| WIPRO | 6595 | 0.7675 | 158.9235 | 721.5 | 131.6246 | 1.707514 | 3.505724 |

## *B. GRU*

GRUs are engineered to manage sequential data by identifying dependencies across time while using fewer parameters.

Update gate $z_t = \sigma(G_z \ .[h_{t-1}, x_t] + B_z)$ (7)

Reset gate$R_t = \sigma(G_r \ .[h_{t-1}, x_t] + B_r)$ (8)

Candidate activation

$\hat{h}_t = \tanh(G_h \ .[R_t \odot h_{t-1}, x_t] + B_h)$ (9)

Hidden state $h_t = (1 - Z_t) \odot h_{t-1} + Z_t \odot h_t$ (10)

## *C. LSTM+GBM*

GBM is a typical stochastic process based on the modeling of the development of the prices of stocks and other financial series, as it can arise from stochastic and deterministic transformations. GBM is characterized by a diffusion (volatility) and a stochastic differential of the drift (the expected return). Mathematically, the GBM is the model that describes the development of the price as (see eqn. 11)

where $S(t)$ is the asset price at time $t$, $\mu$ is the drift rate, $\sigma$ is the volatility, and $W(t)$ is a Wiener process or Brownian motion. In this paper, GBM is employed to create a simulated feature that captures the stochastic nature of stock prices and hence enriches the input space for the deep learning model. By combining this simulated GBM feature with LSTM-based models, the model leverages both historical patterns and theoretically grounded stochastic behavior in order to enhance its capability of generalizing and predicting real financial time series more precisely.

## *D. GRU+GBM*

GBM is a strictly defined stochastic process that probabilistically models the dynamics of the prices of assets by replicating deterministic trends and pure randomness using a stochastic differential equation. It is rigorously defined as

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right) \qquad (11)$$

where S(t) represents the asset price at time t, μ is the drift rate, σ is the volatility, and W(t) is a Wiener process (Brownian motion). In this work, GBM is used to create a synthetic feature that preserves the random nature of stock price movements, thereby improving input representation for deep learning algorithms. The feature generated by the GBM is then mixed with GRU networks that can capture high-order temporal relationships in sequential financial data. GRUs apply gating mechanisms to keep track of important information over time steps and provide a computationally efficient method compared to standard RNNs and LSTMs. By inputting both historical stock prices (Open, High, Low, Close) and the generated synthetic GBM feature into a stacked GRU model, the developed approach takes advantage of both observed price movement and stochastic theoretical movement. The resulting hybrid technique maximizes predictive precision and generalizability by harvesting both data-driven patterns and stochastic randomness inherent in financial markets.

## *E. GBM Simulation*

Simulate a synthetic stochastic feature using GBM: Refer eqn. 11.

Define this as a function:

$$g(t;\mu,\sigma,S_0) = S_{GBM}(t) \tag{12}$$

Thus, for each time step t, the GBM feature becomes:

$$x_t^{GBM} = g(t) \tag{13}$$

Augmented Input Vector, Let the original input at time t be:

$$x_t \in \mathbb{R}^n \tag{14}$$

Then the augmented input vector becomes:

$$\tilde{x_t} = \begin{bmatrix} x_t \\ x_t^{GBM} \end{bmatrix} \in \mathbb{R}^{n+1} \tag{15}$$

GRU Cell Equations (Augmented). Refer to the eqns. 7-10.

Output Prediction Layer.

Let $\hat{y}_t$ denote the output prediction (e.g., the next closing price): $\hat{y}_t = Vh_t + c$ (16)

where V and c are training parameters. Final Combined Model Given:

$$\tilde{x_t} = \begin{bmatrix} x_t \\ g(t;\mu,\sigma,S_0) \end{bmatrix} \tag{17}$$

Then the GRU update is:

$$h_t = GRU(\tilde{x_t}, h_{t-1}) \tag{18}$$

$$\hat{y}_t = Vh_t + c \tag{19}$$

This model incorporates fully the GBM-derived stochastic information into the deep sequential learning process using GRU.

## 4. EXPERIMENTAL

### A. Model Development

The dataset might include erroneous or absent data points, which can negatively impact the model's performance. To address this, these issues must be resolved before the dataset is utilized. One typical approach for managing missing values is to impute them with the mean of the respective feature or remove the row if the dataset is sufficiently large. Additionally, the data undergoes normalization to ensure all features are on a comparable scale. It then outlines the training and prediction phases, with the complete model development process illustrated in Figure 1.

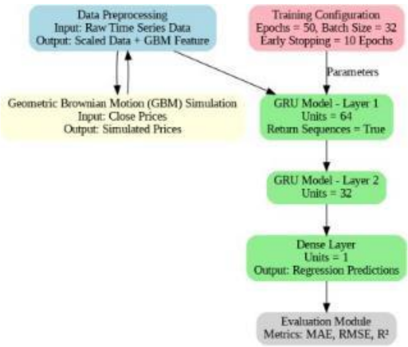


**Figure 1:** Complete model development process

The experiment is executed in Python, with the data being pulled from the Kaggle competition (https://www.kaggle.com/datasets/soumendraprasad/nifty50-stocks). The study is done using historical stock prices of different industries, including technology, services, and entertainment, to analyze and confirm the applicability of stock forecasting in different industries. The research utilizes five sets of stock data. The time periods of these stocks are: 01-01-2003 to 23-03- 2023 of HCL and TCS, 01-01-1997 to 23-03-2023 of Infosys and Wipro, and 02-01- 2007 to 23-03-2023 of Tech Mahindra. Figure 2 shows the historical plot of these five stocks, whereas Table 1 shows their descriptive statistical analysis.

*Table 1.* Statistical Overview of Stock Data

**Figure 2**. The original series of five stock daily closing prices.

The trends of financial stocks are characterized by complex and fluctuating schemes due to market forces, external economics, and inner randomness. These trendy movements add noise to the data, and it is hard to forecast correctly. To overcome such challenges, a powerful forecasting model is required that can be able to draw up meaningful trends out of the raw stock data without any noise and junk data. In the current paper, we used GRU-based and LSTM-based deep learning models, which are optimized by GBM, to enhance the predictive accuracy. Data preprocessing, noise removal, hyperparameter tuning, and training are some of the major steps involved in the methodology in order to render the financial time series data to be effectively processed.

## *B. Data Preprocessing and Noise Handling*

Financial trends on the stock are complex and volatile as they are influenced by changes in the market, external economic forces, and internal randomness. The dynamic trends add noise to the data set and it is difficult to foresee them. To overcome this, an effective forecasting model that has the strength to effectively extract informative patterns in raw stock data and filter irrelevant noise must be used. GRU-based and LSTM-based deep learning models were used in this study, and both are optimized GBM to enhance the performance of prediction. The main steps involved in the architecture include preprocessing of data, noise removal, optimization of hyperparameters, and training of models to achieve effective manipulation of financial time series data. Figure 3. shows the Structure optimization Stream plan.

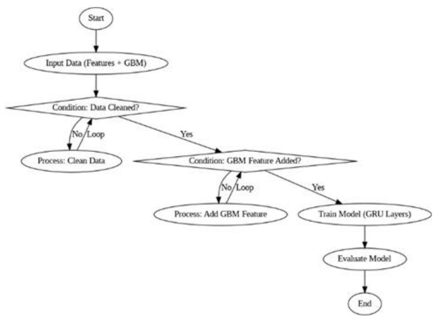


**Figure 3.** Stream plan of structure optimization.

## *C. GBM+GRU*

In this research work, a deep learning model founded on the GRU has been constructed to forecast the stock prices of five of the leading IT industry companies in the NIFTY50 index, i.e., HCLTECH, INFY, TCS, TECHM, and WIPRO. Historical daily stock prices with Open, High, Low, and Close values were utilized as features of each stock in the dataset. Preprocessing first involved converting the date column to the datetime type, setting it as the index, and deleting missing values to ensure data integrity. Open, High, Low, and Close features and the target variable, Close price, were normalized with MinMaxScaler to [0, 1] to facilitate quick convergence as well as efficient learning for neural networks. The data was divided into training and test sets in an 80:20 ratio. The input had to be reshaped into a 3-dimensional form [samples, time steps, features], where the time step was chosen as 1 in order to utilize one previous observation at a time for GRU to process the data. A stacked GRU architecture was used with two layers: the first had 64 units returning sequences, and the second had 32 units followed by a Dense layer for output. The model was trained and compiled with the Adam optimizer and the mean squared error loss function. Early stopping was used to avoid overfitting by tracking validation loss with a patience of 10 epochs. Upon being trained, the model made predictions on the test set that was then inverse transformed back to the original scale to facilitate proper evaluation. The performance of the model was measured based on the MAE, RMSE, and the $R^2$. Actual vs predicted price visualization was conducted to determine how the model estimated the trends in the market. Generally, the GRU model performed extremely well on time series financial data to predict stock price movement successfully over temporal relationships.

## 5. RESULTS AND DISCUSSION

### A. *Model Performance Comparison*

This research work evaluates the performance of four deep learning models, namely LSTM, GRU, LSTM+GBM, and GRU+GBM, over five different datasets: TCS, INFOSYS, HCLTECH, WIPRO, and TECHM. Key performance metrics, including MAE, RMSE, and the $R^2$ coefficient, have been used to analyze the training and testing errors. It possesses the lowest values of MAE and RMSE but the highest $R^2$ values, which indicate improved accuracy and reliability for financial time-series forecasting, whereas Table 2 indicates the respective parameter settings by algorithm.

**Table 2.** Configuration details of the algorithms implemented in this study

| MODEL | Component | Parameter | Value / Description |
|---|---|---|---|
| GRU+GBM | Target Variable | Close | Stock closing price prediction |
| | Feature Scaling | MinMax Scaler | Feature range (0,1) |
| | Train-Test Split | Train: 80%, Test: 20% | Split applied to the scaled dataset |
| | Additional Feature | GBM Simulated | Geometric Brownian Motion-generated data added as a feature |
| | GBM Parameters | μ (Drift) | 0.01 |
| | | σ (Volatility) | 0.02 |
| | | T (Time Horizon) | 1 year |
| | | dt (Time Step) | 1/252 (daily interval for trading days) |
| | GRU Model Architecture | Units (GRU Layer 1) | 64 |
| | | Units (GRU Layer 2) | 32 |
| | Batch Size | 32 | Number of samples per batch |
| | Loss Function | MSE | Loss function aimed at training |
| | Optimizer | Adam | Adaptive optimization algorithm |
| | Epochs | 50 | Maximum training iterations |
| | Early Stopping | Patience 10 epochs | Ends training if no development at all |
| LSTM+GBM | Feature Scaling | MinMaxScaler | Feature range: (0,1) |
| | Additional Feature | GBM Simulated | Geometric Brownian Motion-generated data added as a feature |
| | GBM Parameters | μ (Drift) | 0.01 |
| | | σ (Volatility) | 0.02 |
| | | T (Time Horizon) | 1 year |

| | | | |
|---|---|---|---|
| | | dt (Time Step) | 1/252 (daily interval for trading days) |
| | Train-Test Split | Train: 80%, Test: 20% | Split applied to scaled dataset |
| | LSTM Model Architecture | Units (LSTM - Layer 1) | 64 |
| | | Units (LSTM - Layer 2) | 32 |
| | Batch Size | 32 | Number of samples per batch |
| | Loss Function | MSE | Loss function aimed at training |

### *B. Evaluation Indicator*

This study evaluates the performance of the model using two significant evaluation factors, goodness of fit and predictive validity. The measures of evaluation used in stock forecasting are RMSE, eqn. 21, MAE eqn. 22, and R² eqn. 23. Of these, RMSE and MAE directly test the quality of fit of the model.

$$\textbf{Mean Absolute Error} = \frac{1}{N}\sum_{i=1}^{N}|y_{i}-\hat{y}_{i}| \qquad (20)$$

$$\textbf{Mean Squared Error} = \frac{1}{N}\sum_{i=1}^{N}(y_{i}-\hat{y}_{i})^{2} \qquad (21)$$

$$RMSE = \sqrt{MSE} \qquad (22)$$

Table 3 The obtained experimental results of five datasets of Indian stocks have shown that the hybrid model of GRU+GBM always does better with the metrics of MAE, RMSE, and $R^2$ in comparison to the standalone LSTM and GRU models. Specifically, GRU+GBM can provide very small prediction error in cases of Infosys and Wipro, MAE equals 0.00019 and 3.30 10, respectively, and $R^2$ equals 0.999, which mean that the predictive accuracy and stability are very good. In spite of the fact that LSTM+GBM is also better than traditional deep learning models, GRU+GBM is better in terms of its generalization on all datasets. The findings confirm that Geometric Brownian Motion combined with recurrent neural networks is effective in capturing the stochastic characteristics of market dynamics and retaining the time-dependent characteristics, which is highly effective in predicting the price of stocks. Figure 4. Visualize the actual and Predicted Stock Prices on the HClTECH Test Data using varios models.

## 6. Discussion

Tables 4 and 5 summarize the Tukey HSD analysis of MAE and RMSE to compare the four forecasting models, namely GRU, LSTM, GRU+GBM, and LSTM+GBM, and determine statistically significant performance variations. The table presents results of mean differences, confidence, and adjusted p-values, where a significant difference is considered p-adj < 0.05 in the reject column. The findings indicate that both hybrid models (GRU+GBM and LSTM+GBM) are much better than the standalone LSTM model in terms of both metrics, though non-significant distinctions between GRU and LSTM alone are observed. In general, the analysis proves the fact that hybrid models are more accurate in terms of forecasting in any situation where the reject value is labeled with the word True. Figure 5. Shows evaluating the fit and predictive MAE and RMSE of various models.

Table 3: Prediction comparison results across different models

| DATASET | MODEL | MAE | RMSE | R2 |
|---|---|---|---|---|
| | LSTM+GBM | 0.014087933 | 0.01624857 | 0.996489 |
| | **GRU+GBM** | **0.00857146** | **0.01094335** | **0.998683** |
| | LSTM | 23.95700974 | 31.7499554 | 0.997494 |
| TCS | GRU | 18.68099043 | 23.9069207 | 0.998579 |

| | | | | |
|---|---|---|---|---|
| | LSTM+GBM | 0.000272741 | 0.00033059 | 0.997994 |
| | **GRU+GBM** | **0.000191668** | **0.00025622** | **0.999539** |
| | LSTM | 13.80633365 | 21.6582411 | 0.99751 |
| INFOSYS | GRU | 8.063531994 | 11.7049781 | 0.999273 |
| | LSTM+GBM | 0.003055939 | 0.00433378 | 0.998787 |
| | **GRU+GBM** | **0.002662833** | **0.00358276** | **0.999119** |
| | LSTM | 5.855481743 | 7.89597088 | 0.999032 |
| HCLTECH | GRU | 6.301308637 | 8.5488637 | 0.998865 |
| | LSTM+GBM | 0.000101836 | 0.00016773 | 0.998427 |
| | **GRU+GBM** | **3.30E-05** | **4.87E-05** | **0.99954** |
| | LSTM | 3.697748856 | 4.63923409 | 0.998971 |
| WIPRO | GRU | 2.802372459 | 3.72527702 | 0.999336 |
| | LSTM+GBM | 0.055119646 | 0.06999321 | 0.998453 |
| | **GRU+GBM** | **0.034242286** | **0.04502175** | **0.998523** |
| | LSTM | 7.629033813 | 10.5786407 | 0.998786 |
| TECHM | GRU | 8.369775757 | 10.8292352 | 0.998727 |

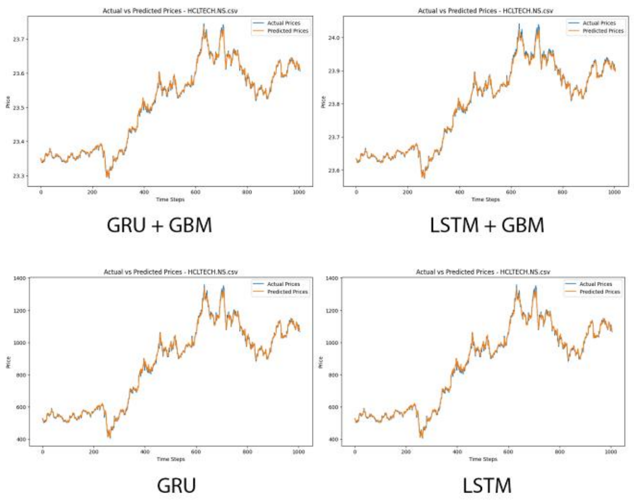


**Figure 4.** Comparison of Actual and Predicted Stock Prices on Test Data using Models

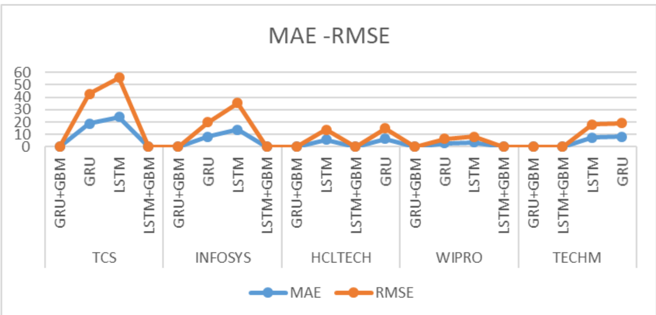


**Figure 5.** Evaluating the fit and predictive MAE and RMSE of various models

Table 4 Tukey HSD Test for MAE Multiple Comparison of Means - Tukey HSD, FWER=0.05

| Group1 | Group2 | Meandiff | P-adj | Lower | Upper | Reject |
|---|---|---|---|---|---|---|
| GRU | GRU+GBM | -8.8345 | 0.0597 | -17.966 | 0.2971 | FALSE |
| GRU | LSTM | 2.1455 | 0.906 | -6.986 | 11.2771 | FALSE |
| GRU | LSTM+GBM | -8.8291 | 0.0599 | -17.9606 | 0.3025 | FALSE |
| GRU+GBM | LSTM | 10.98 | 0.016 | 1.8484 | 20.1115 | **TRUE** |
| GRU+GBM | LSTM+GBM | 0.0054 | 1 | -9.1262 | 9.1369 | FALSE |
| LSTM | LSTM+GBM | -10.9746 | 0.016 | -20.1061 | -1.843 | **TRUE** |

Table 5 Tukey HSD Test for RMSE Multiple Comparison of Means - Tukey HSD, FWER=0.05

| Group1 | Group2 | Meandiff | P-adj | Lower | Upper | Reject |
|---|---|---|---|---|---|---|
| GRU | GRU+GBM | -11.7311 | 0.0611 | -23.912 | 0.4499 | FALSE |
| GRU | LSTM | 3.5614 | 0.8364 | -8.6197 | 15.7424 | FALSE |
| GRU | LSTM+GBM | -11.7248 | 0.0613 | -23.906 | 0.4562 | FALSE |
| GRU+GBM | LSTM | 15.2924 | 0.0117 | 3.1114 | 27.4735 | **TRUE** |
| GRU+GBM | LSTM+GBM | 0.0062 | 1 | -12.175 | 12.1873 | FALSE |
| LSTM | LSTM+GBM | -15.2862 | 0.0118 | -27.467 | -3.1052 | **TRUE** |

## 7. Conclusions

This work proposes a hybrid forecasting framework, GRU-GBM, which integrates GRU with GBM for modeling and predicting stock prices more accurately and robustly. The performance of the GRU-GBM model is rigorously tested by a systematic comparison with shallow deep learning models like standalone GRU, LSTM, and their ensembling with GBM on several financial time series datasets. Experimental results confirm the following significant findings:

• The interaction between GBM as an artificial stochastic factor and GRU significantly reinforces the predictive power of the model by engaging both historical patterns and theoretical price behavior. This exchange preserves both the deterministic structure and stochasticity typical of financial time series.

• The GRU+GBM model performs healthier than the GRU model unaccompanied and the LSTM model alone in MAE, RMSE, and the $R^2$ in all types of scenarios, and it even outperforms all other types of scenarios.

Compared to LSTM+GBM, the hybrid GRU+GBM generates lower error and higher accuracy, highlighting the advantage of using GRU's potent sequence learning in conjunction with stochastic modeling for improved generalization and stability. It displays very low error percentages and high R2 values, reflecting accurate and robust predictions.

• GBM stochastic model components included in deep learning models provide better adaptability and consistency in forecasting compared to conventionally upgraded ones in unstable markets.

Even with these encouraging findings, the method has some shortcomings:

• The performance of the model is hyperparameter-sensitive, e.g., number of GRU layers, learning rate, and number of epochs, which need to be fully tuned for best performance.

• Although GBM introduces stochastic realism, it involves constant volatility and drift, which can fail to represent the nonlinear, regime-switching dynamics of actual financial markets.

• Future studies might include investigation of other stochastic processes or the integration of the GRU-GBM model with bio-inspired optimization methods (e.g., ACO, PSO) for further boosting predictive performance and computational effectiveness.

In conclusion, the GRU-GBM hybrid model is demonstrated to possess robust and stable performance in financial prediction and has an important enhancement on regular deep learning models with good ability in capturing the stochastic dynamics of stock price changes.

### Author Contributions

M. Poornima wrote the main manuscript text, prepared figures, and tables. N. Nithyapriya reviewed the manuscript. All authors have read and agreed to the published version of the manuscript.


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### Conflict of Interest Statement

There is no conflict of interest.


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