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Article

A Dual - One Dimensional Queuing Model for Cognitive Radio Network Transmission

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Abstract: This paper formulates the semi-passive backscatter transmission combined mode as a preemptive priority queueing system in two modes of hybrid transmission. In this mode, it is assumed that when there is no active primary user (PU) signal on the licensed channel, the secondary user (SU) signal can access this free channel in interweave mode. The PU has preemptive priority, i.e. when servicing the SU, the low-priority user is given low priority. If a primary user joins the system, the low-priority user's service is stopped and will resume when there are no high-priority users in the system. Conversely, when the licensed channel is busy with the PU's signal, the SU can access the channel in underlay mode at a lower transmission rate. Consequently, the performance of the SU, as well as the utilization of the spectrum, can be enhanced. Transmission between primary and secondary users is formulated as a dual one-dimensional queuing system. The system is governed by transient state equations and, using the generating function technique, the steady state probabilities, expected queue length and variance of both the primary and secondary users are obtained. Finally, some numerical calculations are carried out to investigate the relationship between the expected queue size and variance for various parameters.
Keywords: *Cognitive Radio Network, Queuing Models, Hybrid Transmission Preemptive Priority, Backscatter Transmission*.
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1. Introduction

Indeed, cognitive radio networks are a great and emerging technology. It is effectively used in spectrum analysis by introducing cognitive users to search for data transmission opportunities when the primary or licensed users (PU) disappear. Radio spectrum also enables services in many fields, including broadcasting, intelligent transport systems, communication networks for emergency services, and short-range internet for sensor-based devices. Over the past few years, wireless data traffic has increased significantly, making the available radio frequency (RF) spectrum below 10 GHz very limited (See [1], [4] and [12]). The wireless communication industry responded to this challenge by using RF above 10 GHz. However, using higher frequencies leads to increased path losses, as defined by the free space equation [10]. Blocking and shadowing effects also become more significant at higher frequencies in terrestrial radio communications. As demand for the use of the frequency spectrum increases, it is necessary to establish conditions or licenses for using this frequency band, or to find a way to enable licensed primary users (PUs) to make the most of their licenses, while also enabling unlicensed secondary users (USUs) to benefit when a PU license is unavailable. The cognitive radio (CR) technique aims to optimize spectrum usage by granting unlicensed SUs the opportunity to use unoccupied licensed bands of licensed PUs without affecting the transmission process. Clearly, the PU has pre-emptive priority, i.e. during service to a low-priority user, if a high-priority PU joins the system, the service to the low-priority user stops and will resume once there are no more high-priority users (See [9] and [13]). The efficiency of the spectrum sensing process depends on the sensing time and, consequently, the sensing energy. Although increasing the sensing time improves the performance of the sensing process, it reduces the remaining time for data transmission, thereby decreasing the throughput of the SUs [14]. In [7], the hybrid interweave/underlay transmission mode is modelled as an M/M/1 queuing model with a heterogeneous service rate. In this hybrid mode, it is assumed that, when the licensed channel is free of the primary user's (PU) signal, the secondary user (SU) can access this channel in interweave mode. Conversely, when the licensed channel is busy with the PU's signal, the SU can access the channel in underlay mode at a lower transmission rate. Based on this queuing model, expressions for the expected number of SUs in the network and the expected system time are derived in a similar form. The hybrid transmission mode is modelled as an M/M/1 queuing model where the arrival and service rates are subject to Poisson alternations [8]. For each arrival packet, a decision must be made as to whether to join or bypass the queue. In [10], a novel queueing framework is introduced for measuring the performance of hybrid interweave/underlay transmission modes in cognitive radio (CR) networks. The PU activity is modelled as a two-state continuous-time Markov chain. The matrix geometric method is used to calculate the average number of SUs in the network and the average delay for each SU. In [2], the PU activity is represented using three states: busy, concurrent and idle. In the busy state, SUs are not allowed to transmit. In the concurrent and idle states, however, the SU can transmit data using lower (underlay mode) or higher (interweave mode) transmission power, respectively. A partially observable Markov decision process (POMDP) framework is used to determine the optimal power transmission policy that maximizes the average throughput of the SU in a multi-channel cognitive radio (CR) network. As an extension to the models proposed in references [15], [5], [11] and [6], different service rates are considered for the underlay mode to guarantee that there is no harmful interference to the PU receiver. [3] Analyses a preemptive priority queuing system with two classes of single-server arrivals, where each class is assumed to follow a Poisson process with exponentially distributed service times. We are motivated to consider the model given in [11] in order to obtain the steady-state probabilities, the expected queue lengths, and the variance of both PU and SU users in explicit form. In this case, prioritized SU classes are considered and perfect channel sensing is assumed. The SUs are classified based on their delay requirements. A queueing model is used to formulate the system and describe the interaction between users. The service discipline between the PU and all SU classes is considered to be pre-emptive resume priority, as the PU has higher priority for accessing the channel upon arrival. Furthermore, the interaction between the two SU classes is modelled as preemptive resume priority, meaning that the PU with the higher priority can preempt the SU with the lower priority.

Depending on its priority class, the SU can access the channel in either interweave or underlay mode. If no PU is using the channel, the SU can access it in interweave mode. Otherwise, if a higher-priority PU is occupying the channel, the SU can access the channel in underlay mode. This paper is organized as follows. Section 2 describes the model, along with the notation and assumptions used throughout the paper. Also presented is a formulation of the equations governing the system in the transient case. Furthermore, explicit expressions are obtained for the generating functions for the queue size in each PU and SU class. Using the technique of partial fractions, we derive explicit forms for the steady-state probabilities. The efficiency of the given system is analyzed by finding explicit expressions for the expected queue lengths in both classes, E_q (the expected queue length of the secondary users) and E_F (the contribution to the mean queue size in the free situation), and the variance and mean queue length of the entire system are given in Section 3. Section 4, presents some numerical results to illustrate the behavior and efficiency of the model for different values of its parameters, and Section 5 lists the conclusions.

2. Formulation and solution

In this section, we formulate the two modes hybrid transmission in which the semi-rear transmission is combined with a preemptive priority queuing system.

Consider B and F states express cases in which PU is active (Busy state) and inactive (Free state) respectively. The holding times in the states B and F are exponentially distributed with mean $\frac{1}{\mu_p}$ and $\frac{1}{\lambda_p}$, respectively. Let P_F and P_B are the probabilities that the licensed channel is free and busy from the PU in steady state case, respectively. Hence

$$P_F = \frac{\mu_p}{\mu_p+\lambda_p} \text{ and } P_B = \frac{\lambda_p}{\mu_p+\lambda_p} \tag{2.1}$$

We assume a single queue of SU's signal with infinite capacity with a Poisson arrival process with rate λ_s. If the licensed channel is in state case F, the SU's signal at the head of the queue starts to transmit his data. If the channel is in state case B the SU signal use backscattering transmission.

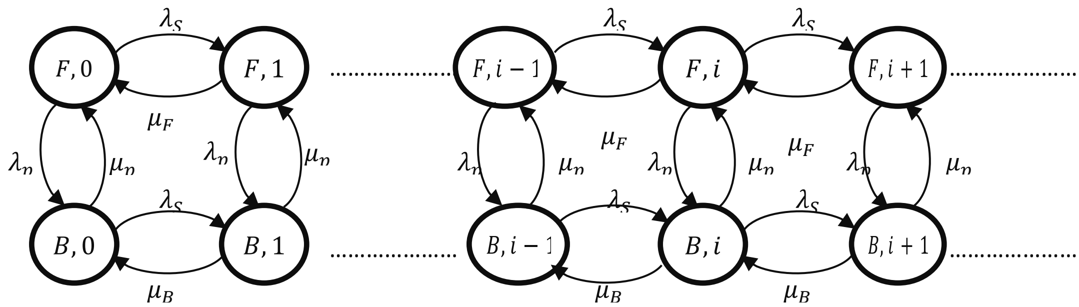

Fig. 1: Dual- one dimensional transmissions for CR

From the SU's point of view, we can consider the licensed channel to be a server with two different rates. The first rate is the interweave transmission rate on state F, and the second rate is the backscatter transmission rate on state B. The serving times for the SUs in interweave and backscatter transmissions are exponentially distributed with rates μ_F and μ_B, respectively.

If $P_{F,i}(t)$ and $P_{B,i}(t)$ are the transient state probabilities that the licensed channel is free and busy from PU, respectively and there are i of the SU's in this moment waiting to be served at time t (transmitted using the licensed channel)

$$P'_{F,0}(t) = -(\lambda_p + \lambda_s)P_{F,0}(t) + \mu_F P_{F,1}(t) + \mu_p P_{B,0}(t)$$

$$P'_{B,0}(t) = -(\mu_p + \lambda_s)P_{B,0}(t) + \mu_B P_{B,1}(t) + \lambda_p P_{F,0}(t)$$

$$P'_{F,i}(t) = -(\lambda_s + \lambda_p + \mu_F)P_{F,i}(t) + \lambda_s P_{F,i-1}(t) + \mu_F P_{F,i+1}(t) + \mu_p P_{B,i}(t) \qquad i \geq 1$$

$$P'_{B,i}(t) = -(\lambda_s + \mu_p + \mu_B)P_{B,i}(t) + \lambda_s P_{B,i-1}(t) + \mu_B P_{B,i+1}(t) + \lambda_p P_{F,i}(t) \qquad i \geq 1$$

Hence in the steady state case we have $P_{F,i} = \lim_{t\to\infty} P_{F,i}(t)$ and $P_{B,i} = \lim_{t\to\infty} P_{B,i}(t)$, which yield to:

$$(\lambda_p + \lambda_s)P_{F,0} = \mu_F P_{F,1} + \mu_p P_{B,0} \tag{2.2}$$

$$(\mu_p + \lambda_s)P_{B,0} = \mu_B P_{B,1} + \lambda_p P_{F,0} \tag{2.3}$$

$$(\lambda_s + \lambda_p + \mu_F)P_{F,i} = \lambda_s P_{F,i-1} + \mu_F P_{F,i+1} + \mu_p P_{B,i} \qquad i \geq 1 \tag{2.4}$$

$$(\lambda_s + \mu_p + \mu_B)P_{B,i} = \lambda_s P_{B,i-1} + \mu_B P_{B,i+1} + \lambda_p P_{F,i} \qquad i \geq 1 \tag{2.5}$$

Let us define the generating probability functions of $P_{F,i}$'s and $P_{B,i}$'s as:

$$G_F(z) = \sum_{i=0}^{\infty} z^i P_{F,i} \text{ and } G_B(z) = \sum_{i=0}^{\infty} z^i P_{B,i} \tag{2.6}$$

Multiplying Eq.(2.4) by z^i , summing over i and adding to Eq. (2.2), we get

$$(\lambda_s + \mu_F + \lambda_p)G_F(z) = \lambda_s z G_F(z) + \mu_p G_B(z) + \frac{\mu_F}{z}\left(G_F(z) - P_{F,0}\right) + \mu_F P_{F,0} \tag{2.7}$$

Similarly, using Eq. (2.5) and Eq. (2.3), we get

$$(\lambda_s + \mu_B + \mu_p)G_B(z) = \lambda_s z G_B(z) + \lambda_p G_F(z) + \frac{\mu_B}{z}\left(G_B(z) - P_{B,0}\right) + \mu_B P_{B,0} . \tag{2.8}$$

Substituting from Eq.(2.8) into Eq.(2.7), we obtain

$$g(z)G_F(z) = P_{B,0}\mu_B \mu_p z + P_{F,0}\mu_F\left(\lambda_s z(1-z) + \mu_p z - \mu_B(1-z)\right)$$

where

$$\begin{aligned} g(z) = {} & \lambda_s^2 z^3 - \left(\lambda_s^2 + \mu_F\lambda_s + \mu_B\lambda_s + \lambda_s\lambda_p + \lambda_s\mu_p\right)z^2 \\ & + \left(\lambda_s\mu_F + \lambda_s\mu_B + \mu_F\mu_B + \lambda_p\mu_B + \mu_p\mu_F\right)z - \mu_F\mu_B \end{aligned} \tag{2.9}$$

$$G_F(z)=\frac{P_{B,0}\mu_B\,\mu_p z+P_{F,0}\mu_F\left(\lambda_s z(1-z)+\mu_p z-\mu_B(1-z)\right)}{g(z)},$$

since $\tilde{\mu}-\lambda_s=\mu_F P_{F,0}+\mu_B P_{B,0}$, and $\tilde{\mu}$ is the average service rate of SU's, $\tilde{\mu}=\mu_F P_F+\mu_B P_B$, then we can rewrite $G_F(z)$ as following:

$$G_F(z)=\frac{\mu_p(\tilde{\mu}-\lambda_s)z+\mu_F P_{F,0}(1-z)(\lambda_s z-\mu_B)}{g(z)}. \tag{2.10}$$

Similarly,

$$G_B(z)=\frac{\lambda_p(\tilde{\mu}-\lambda_s)z+\mu_B P_{B,0}(1-z)(\lambda_s z-\mu_F)}{g(z)}. \tag{2.11}$$

The denominator $g(z)$ has three roots, $\frac{z_1}{\lambda_s^2}$, z_2, z_3, where $z_1 z_2 z_3=\mu_F\mu_B$, then Eq. (2.10) can be written as

$$G_F(z)=\frac{\mu_p(\tilde{\mu}-\lambda_s)z+P_{F,0}\mu_F(1-z)(\lambda_s z-\mu_B)}{(\lambda_s^2 z-z_1)(z-z_2)(z-z_3)}$$

$$=\frac{\mu_p(\tilde{\mu}-\lambda_s)z+P_{F,0}\mu_F(1-z)(\lambda_s z-\mu_B)}{-z_1 z_2 z_3\left(1-\lambda_s^2\frac{z}{z_1}\right)\left(1-\frac{z}{z_2}\right)\left(1-\frac{z}{z_3}\right)}, \qquad \left|\lambda_s^2\frac{z}{z_1}\right|,\left|\frac{z}{z_2}\right|,\left|\frac{z}{z_3}\right|<1$$

$$=-\frac{1}{z_1 z_2 z_3}\left(\mu_p(\tilde{\mu}-\lambda_s)z+P_{F,0}\mu_F(1-z)(\lambda_s z-\mu_B)\right)\left(1-\lambda_s^2\frac{z}{z_1}\right)^{-1}\left(1-\frac{z}{z_2}\right)^{-1}\left(1-\frac{z}{z_3}\right)^{-1}$$

$$=-\frac{1}{z_1 z_2 z_3}\left(\mu_p(\tilde{\mu}-\lambda_s)z+P_{F,0}\mu_F(1-z)(\lambda_s z-\mu_B)\right)$$

$$\times\left(1+\lambda_s^2\frac{z}{z_1}+\left(\lambda_s^2\frac{z}{z_1}\right)^2+\left(\lambda_s^2\frac{z}{z_1}\right)^3+\dots\right)\times\left(1+\frac{z}{z_2}+\left(\frac{z}{z_2}\right)^2+\left(\frac{z}{z_2}\right)^3+\dots\right)$$ The coefficients of

$$\times\left(1+\frac{z}{z_3}+\left(\frac{z}{z_3}\right)^2+\left(\frac{z}{z_3}\right)^3+\dots\right)$$

the parameter z are the transition probability. After expanding the pervious formula into terms of z , we get

$$P_{F,1}=\frac{-1}{z_1 z_2 z_3}\left(\mu_p(\tilde{\mu}-\lambda_s)+P_{F,0}\mu_F\left((\lambda_s+\mu_B)-\mu_B\left(\frac{\lambda_s^2}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\right)\right)$$

$$P_{F,2}=\frac{-1}{z_1 z_2 z_3}\left(\mu_p(\tilde{\mu}-\lambda_s)\left(\frac{\lambda_s^2}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)+P_{F,0}\mu_F\left(-\lambda_s+(\lambda_s+\mu_B)-\mu_B\left(\frac{\lambda_s^2}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\right.\right.$$

$$\left.\left.-\mu_B\left(\left(\frac{\lambda_s^2}{z_1}\right)^2+\left(\frac{1}{z_2}\right)^2+\left(\frac{1}{z_3}\right)^2+\frac{\lambda_s^2}{z_1 z_3}+\frac{1}{z_1 z_2}+\frac{1}{z_2 z_3}\right)\right)\right)$$

$$P_{F,3} = \frac{-1}{z_1 z_2 z_3}\left(\mu_p\left(2\mu_0-\lambda_s\right)\left(\left(\frac{\lambda_s^2}{z_1}\right)^2+\left(\frac{1}{z_2}\right)^2+\left(\frac{1}{z_3}\right)^2+\frac{\lambda_s^2}{z_1 z_3}+\frac{1}{z_1 z_2}+\frac{1}{z_2 z_3}\right)\right.$$

$$+P_{F,0}\mu_F\left(-\lambda_s\left(\frac{\lambda_s^2}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)+\left(\lambda_s+\mu_B\right)\left(\left(\frac{\lambda_s^2}{z_1}\right)^2+\left(\frac{1}{z_2}\right)^2+\left(\frac{1}{z_3}\right)^2+\frac{\lambda_s^2}{z_1 z_3}+\frac{1}{z_1 z_2}+\frac{1}{z_2 z_3}\right)\right.$$

$$\left.\left.-\mu_B\left(\left(\frac{\lambda_s^2}{z_1}\right)^3+\left(\frac{1}{z_2}\right)^3+\left(\frac{1}{z_3}\right)^3+\frac{\lambda_s^4}{z_1^2 z_3}+\frac{\lambda_s^2}{z_1 z_3^2}+\frac{\lambda_s^4}{z_1^2 z_2}+\frac{\lambda_s^2}{z_1 z_2^2}+\frac{1}{z_2 z_3^2}+\frac{1}{z_2^2 z_3}+\frac{\lambda_s^2}{z_1 z_2 z_3}\right)\right)\right)$$

In general,

$$P_{F,i} = \frac{x_1 x_2 x_3}{\lambda_s^2}\left(P_{F,0}\mu_F\left(\lambda_s \Psi_{i-2}+\mu_B \Psi_i\right)-\left(\mu_p\left(2\mu_0-\lambda_s\right)+P_{F,0}\mu_F\left(\lambda_s+\mu_B\right)\right)\Psi_i\right) \qquad (2.12)$$

By the same way, we can calculate $P_{B,i}$, where the generating function for the transition busy states is as Eq. (2.11):

$$P_{B,i} = \frac{x_1 x_2 x_3}{\lambda_s^2}\left(P_{B,0}\mu_B\left(\lambda_s \Psi_{i-2}+\mu_F \Psi_i\right)-\left(\lambda_p\left(2\mu_0-\lambda_s\right)+P_{B,0}\mu_B\left(\lambda_s+\mu_F\right)\right)\Psi_i\right) \qquad (2.13)$$

where

$$\Psi_i = \sum_{\substack{k_1,k_2,k_3\geq 0\\ k_1+k_2+k_3=i}}^{i} \prod_{t=1}^{3} x_t^{k_t} \quad \text{and} \quad x_1=\frac{\lambda_s^2}{z_1},\quad x_2=\frac{1}{z_2},\quad x_3=\frac{1}{z_3}$$

Eqs. (2.12) - (2.13) are formulated in terms of $P_{F,0}$ and $P_{B,0}$, we can deduce them from the numerators of Eqs. (2.10)-(2.11) as

$$P_{F,0} = \frac{\mu_P\left(\lambda_s-2\mu_0\right)z_0}{\mu_F\left(1-z_0\right)\left(\lambda_s z_0-\mu_B\right)} \quad \text{and} \quad P_{B,0} = \frac{\lambda_P\left(\lambda_s-2\mu_0\right)z_0}{\mu_B\left(1-z_0\right)\left(\lambda_s z_0-\mu_F\right)}$$

where z_0 is the smallest root for the denominator.

3. Efficiency Analysis

The expected queue size in the system defined as

$$E_q = E_F + E_B ,$$

where E_F considered the contribution in the free situation to the mean queue size and E_B considered the contribution in the busy case to the mean queue size [16].

According to (2.6) the expected queue size can be obtained from the relation

$$E_q = E[i] = \left.\frac{dG(z)}{dz}\right|_{z=1} = \left.\frac{dG_F(z)}{dz}\right|_{z=1} + \left.\frac{dG_B(z)}{dz}\right|_{z=1},$$

where

$$E_F = \left.\frac{dG_F(z)}{dz}\right|_{z=1} = \frac{\mu_P(\mu_0 - \lambda_s)\left(-\lambda_s^2 + \mu_F\lambda_s + \mu_B\lambda_s + \lambda_p\lambda_s + \mu_P\lambda_s - \mu_F\mu_B\right)}{\left(\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)\right)^2} - \frac{\mu_F(\lambda_s - \mu_B)P_{F,0}}{\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)}$$

and

$$E_B = \left.\frac{dG_B(z)}{dz}\right|_{z=1} = \frac{\lambda_P(\mu_0 - \lambda_s)\left(-\lambda_s^2 + \mu_F\lambda_s + \mu_B\lambda_s + \lambda_p\lambda_s + \mu_P\lambda_s - \mu_F\mu_B\right)}{\left(\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)\right)^2} - \frac{\mu_B(\lambda_s - \mu_F)P_{B,0}}{\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)}$$

then the expected queue size is

$$E_q = \frac{\left(\mu_P + \lambda_p\right)(\mu_0 - \lambda_s)\left(-\lambda_s^2 + \mu_F\lambda_s + \mu_B\lambda_s + \lambda_p\lambda_s + \mu_P\lambda_s - \mu_F\mu_B\right)}{\left(\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)\right)^2} - \frac{\mu_F(\lambda_s - \mu_B)P_{F,0} + \mu_B(\lambda_s - \mu_F)p_{B,0}}{\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)}$$

after simplifying

$$E_q = \frac{\lambda_s}{(\mu_0 - \lambda_s)} + \frac{\mu_F(\mu_B - \lambda_s)P_{F,0} + \mu_B(\mu_F - \lambda_s)p_{B,0} - (\lambda_s - \mu_B)(\lambda_s - \mu_F)}{\left(\lambda_p + \mu_p\right)(\mu_0 - \lambda_s)}$$

$$E\left[i(i-1)\right] = \left.\frac{d^2G(z)}{dz^2}\right|_{z=1} = \frac{1}{\left(\lambda_s^2 - b + c - d\right)^2}\Big[\left(P_{F,0}\mu_F(\lambda_s - \mu_B) + P_{B,0}\mu_B(\lambda_s - \mu_F)\right)$$

$$.\left(6\lambda_s^2 - 4b + 2c\right) + \left(P_{F,0}\mu_F\lambda_s + P_{B,0}\mu_B\lambda_s\right)\left(-2\lambda_s^2 + 2b - 2c + 2d\right)\Big]$$

$$+\frac{\left(\mu_p + \lambda_p\right)(\mu_0 - \lambda_s)}{\left(\lambda_s^2 - b + c - d\right)^3}\left[\left(\lambda_s^2 - b + c - d\right)\left(-6\lambda_s^2 + 2b\right) - \left(6\lambda_s^2 - 4b + 2c\right)\left(-2\lambda_s^2 + b - d\right)\right]$$

then the variance can be easily calculated as

$$Var(i) = \frac{d^2G}{dz^2} + \frac{dG}{dz} - \left(\frac{dG}{dz}\right)^2$$

4. Numerical Results and Discussions

We present in this section some illustrated numerical examples to investigate the behavior and the effect of some variables against the expected queue size. In the illustrated figure 2, we can see the increase of the expected queue size as λ_s increases in some cases ($\mu_p = 0.4, 0.7$ and 0.9)

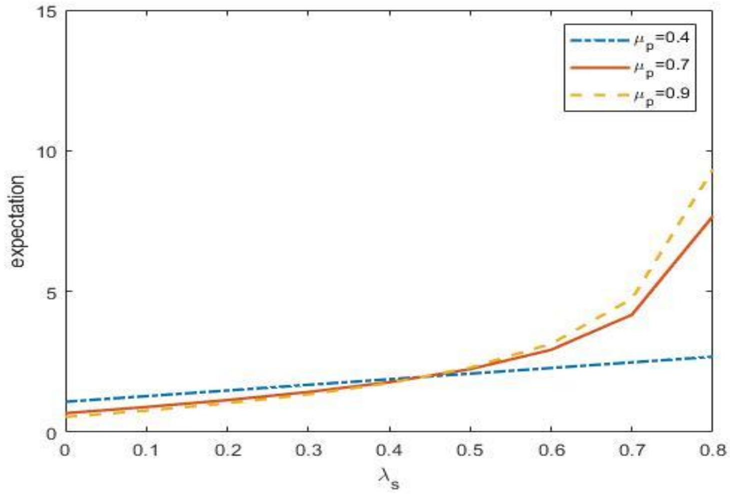

Fig. 2 : The expected queue size against λ_s with various values of μ_p

In Table 1 we calculate the expected queue size with various values of $0.2 \le \mu_B \le 0.8$ with fixed values of $\lambda_s = 0.3,\ \lambda_p = 0.2,\ \mu_p = 0.1,\ \mu_F = 0.6$, z_0 is the only root of the Eq.(2.9) that contained in the unite circle, we notice that the value of the expected queue size decrease as μ_B increases.

Table 2 shows the relation between the expected queue size and λ_s , $0.1 \le \lambda_s \le 0.7$, it is clear that the expected queue size increase as λ_s increases.

Table 1: The relation between expected queue sizes and the variance against various values of μ_B

$\lambda_s = 0.3,\ \lambda_p = 0.2,\ \mu_p = 0.1,\ \mu_F = 0.6$					
μ_B	z_0	$P_{F,0}$	$P_{B,0}$	E_q	Var
0.2	0.285104	0.041192	0.082384	12.2472	159.220
0.3	0.226875	0.021087	0.042174	3.12651	17.3514
0.4	0.438447	0.080785	0.161571	1.68471	6.56310
0.5	0.565741	0.153396	0.306793	1.34895	2.71620
0.6	0.585786	0.166665	0.333333	0.99981	2.00001
0.7	0.599433	0.175809	0.351617	0.78213	1.71620
0.8	0.609247	0.182953	0.365906	0.63618	1.60620
0.9	0.616612	0.187446	0.374892	0.52468	1.57090

Table 2: The relation between expected queue sizes and the variance against various values of λ_s

$\lambda_p = 0.2,\ \mu_p = 0.1,\ \mu_F = 0.6,\ \mu_B = 0.9$					
λ_s	z_0	$P_{F,0}$	$P_{B,0}$	E_q	Var
0.1	0.668815	0.282797	0.500000	0.096779	0.431700
0.2	0.643497	0.234024	0.468047	0.259927	0.900100
0.3	0.616612	0.187446	0.374892	0.524680	1.570900
0.4	0.588562	0.143500	0.287000	0.955918	2.819600
0.5	0.559860	0.102560	0.205139	1.700900	5.717500
0.6	0.531097	0.064944	0.129888	3.194830	14.64990
0.7	0.502811	0.030756	0.061511	7.605160	66.39079

5. Conclusion

Here, we modelled cognitive radio as a two-dimensional queue model. The first dimension represents the state of the primary user (wave), and the second dimension represents the number of secondary users (waves) waiting to be transferred via the primary wave's lane when it is free. The transient state equations governing the system were deduced and the steady state equations were then derived. The method of probability generating functions was used to obtain and compute the explicit formulas for $P_{F,i}$ and $P_{B,i}$, the probabilities that the licensed channel is free or occupied by the primary user (PU) respectively, and that i secondary users (SU) are waiting to be served at time t (i.e. sent using the licensed channel). The expected number and variance of the queue length were deduced and computed for some numerical values (see Tables 1 and 2).

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