

Group Preference-based Evolutionary Multi-objective Optimization with Non-Equally Important Decision Makers: Application to the Portfolio Selection Problem

Slim Bechikh, Lamjed Ben Said and Khaled Ghédira

SOIE Laboratory, High Institute of Management of Tunis, University of Tunis,
41, Rue de la Liberté, Cité Bouchoucha 2000 Le Bardo, Tunis, Tunisia
slim.bechikh@gmail.com, {lamjed.bensaid, khaled.ghedira}@isg.rnu.tn

Abstract: Recently, Evolutionary Multi-objective Optimization (EMO) researchers have addressed the task of incorporating Decision Maker's (DM's) preferences in EMO Algorithms (EMOAs) in order to guide the search towards the preferred region of the Pareto front which is called Region Of Interest (ROI). In fact, the DM is not interested in discovering the whole Pareto front especially with the increase of the number of objectives. Once the ROI is well-approximated, the DM can subsequently select the final solution to realize. Unfortunately, most of the proposed studies assume the uniqueness of the DM which is not the case for several decision making situations. Few preference-based EMOAs have addressed this task by guiding the search based on several reference points each corresponding to a particular DM then searching for an average ROI. However, this method does not resolve the problem and most DMs are still dissatisfied since the EMOA cannot achieve a consensus between the different negotiators. Additionally, DMs are not equally important from a hierarchical viewpoint. In this study, we address this problematic differently by providing the non-equally important DMs with a negotiation support system based on software agent paradigm to aggregate their conflicting preferences before the beginning of the evolutionary process. This negotiation system helps the DMs to confront and adjust their preferences through a number of negotiation rounds. The output of the system is a set of social preferences which will be injected subsequently in the EMOA in order to guide the search towards a satisfying social ROI. The proposed system is demonstrated to be helpful for such group decision making situation through a case study in addition to a practical instance of the Portfolio selection problem.

Keywords: group decision making, negotiation support system, evolutionary multi-objective optimization, DMs' hierarchy, Portfolio selection problem.

I. Introduction

Most real world problems involve the simultaneous optimization of several conflicting and incommensurable objectives under some constraints. The solution to a Multi-Objective Problem (MOP) corresponds to a set of trade-off solutions called the Pareto front which is

characterized by: (1) the ideal point and (2) the nadir one [10]. Over the two last decades, EMOAs have demonstrated their effectiveness and efficiency in approximating the whole Pareto front [5], [22]. However, in reality, the DM is not interested to the whole front rather than the portion of the front that matches at most his/her preferences, i.e., the ROI. Recently, several preference-based EMOAs were proposed to search for the optimal ROI [4], [14].

Nevertheless, most preference-based EMOAs assume the uniqueness of the DM. Very few of these algorithms consider the hypothesis that there exists more than one DM by injecting several reference points in the EMOA each corresponding to a particular DM than the algorithm provides an average ROI [12]. However, this mechanism does not resolve the problem since most DMs are still dissatisfied. In fact, the task of DMs' preference aggregation cannot be delegated to an EMOA. This latter cannot achieve a consensus between the different negotiators. Additionally, usually, DMs have different levels in terms of hierarchy. This aspect is also ignored in these algorithms. Motivated by these observations and inspired from the works [3], [6] and [9]; we propose in this paper a negotiation support system called W-NSS-GPA (Weighted Negotiation Support System for Group Preference Aggregation). W-NSS-GPA takes as inputs the DMs' preferences modelled as reference points and the DMs' hierarchy levels modelled by weights. Then, it provides as output a single Social Reference Point (SRP) that corresponds to an aggregation of all DM's preferences. By running the preference-based EMOA with this SRP, we obtain a social ROI corresponding to aggregated DMs' preferences. Each solution picked from this region is considered as a satisfying solution for each of the DMs. We aim by W-NSS-GPA to ensure the highest level of satisfaction for all DMs. Since, in real situation, the DMs' preferences are usually conflicting, NSS-GPA offers the DMs with a framework of negotiation to confront and update their preferences through a number of negotiation rounds while taking into account the importance level of each DM. In fact, each DM expresses his/her

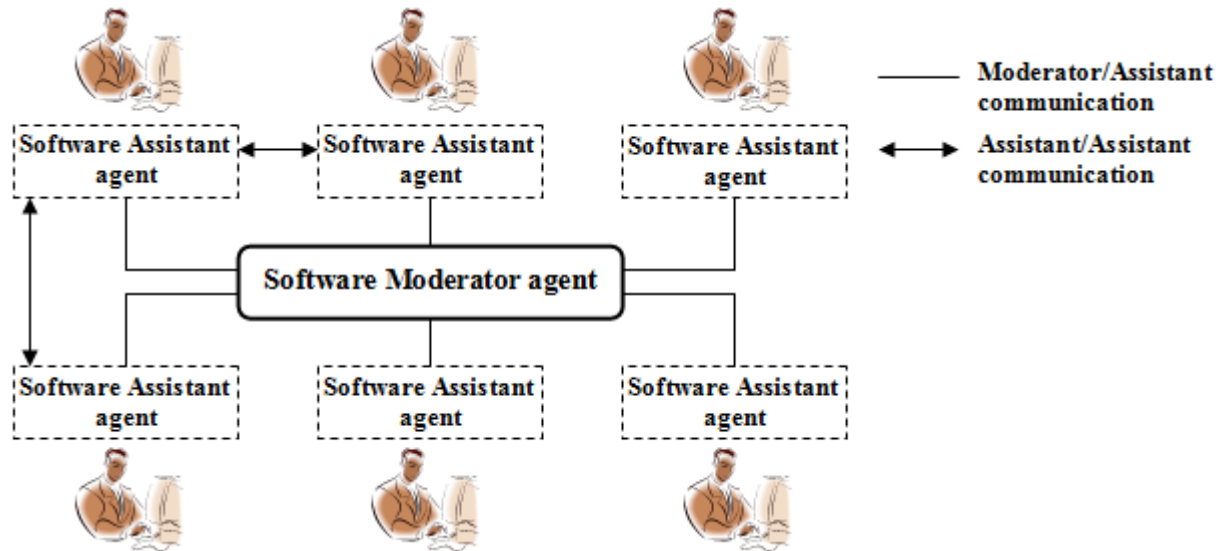


Figure 1. W-NSS-GPA architecture.

preferences in the form of an aspiration level vector (i.e., a reference point) where each aspiration level value lies between the corresponding nadir value and the ideal one. After minimizing the gap between DMs' preferences, the SRP supplied by W-NSS-GPA corresponds to the average vector of all aspiration level vectors. Consequently, the basic principle of W-NSS-GPA is to provide the mean of the DMs' reference points after minimizing the variance (the conflicts) between them.

In this work, we assume that there are no coalitions between the DMs. The remainder of this paper is structured as follows. Section II describes W-NSS-GPA. Section III demonstrates the usefulness of such system through a case study. Section IV valorizes the proposed system through its application to a practical instance of the Portfolio selection problem. Section V concludes the paper and provides some avenues for future research.

II. W-NSS-GPA

A. Overview

Since our goal consists in providing a framework for different non-equally important DMs to negotiate their preferences, we choose to exploit the software agent paradigm [11]. In this work, we propose an agent-based system for group preference negotiation (W-NSS-GPA) to bring closer DMs' reference points through a certain number of negotiation rounds. In fact, each DM is represented and assisted by a software agent called Assistant agent. The overall process is supervised and controlled by a software Moderator agent. Fig. 1 shows the architecture of the proposed system. We note that each Assistant agent can communicate with any of the other Assistant agents in addition to the Moderator agent.

Initially, each human DM agent E_j ($j=1,\dots,q$) expresses his/her preferences as an aspiration level vector $AV_j = [a_{j1}, \dots, a_{jM}]$ where M is the number of objectives. Additionally, he/she provides for each objective f_i ($i=1,\dots,M$) the acceptable deviation quantity σ_{ji} from his/her

specified aspiration level a_{ji} so that the agent is still satisfied with these specified deviations. Once all DM agents have expressed their preferences, the negotiation process begins. At each negotiation round of W-NSS-GPA, a DM agent may have one of the following two states: (1) *satisfied* or (2) *dissatisfied*. These two states are defined as follows:

Definition 1: A DM agent E_j ($j=1,\dots,q$) is said to be satisfied if all its aspiration levels a_{ji} ($i=1,\dots,M$) σ -coincide with the social aspiration levels. An aspiration level a_{ji} is said to σ -coincide with the social aspiration level a_i^{avg} if and only if: $a_i^{avg} - \sigma_{ji} \leq a_{ji} \leq a_i^{avg} + \sigma_{ji}$. We note that $a_i^{avg} = (\sum_{j=1}^q a_{ji})/q$ and $SRP = [a_1^{avg}, \dots, a_M^{avg}]$.

Definition 2: A DM agent E_j ($j=1,\dots,q$) is said to be dissatisfied if at least one of his/her aspiration levels does not σ -coincide with the relative social aspiration level.

After each negotiation round, the Moderator agent constructs a set of direction rules from the observed DM agents' preferences (cf. section II.B.1). These direction rules guide the DMs when updating their preferences so that the consensus rate increases and hence the negotiation process converges towards a SRP more quickly. A satisfied DM agent can follow the direction rules in order to stop the negotiation processes as soon as possible so that he/she finishes the negotiation with a satisfied state. However, a dissatisfied DM agent can have two attitudes: (1) passive who will pursue the Moderator's direction rules or (2) active who has one of the two following behaviors:

- *manipulator*: this kind of DMs will lie about his/her true preferences in order to direct the SRP towards his/her preferences. For example, an agent E_j aspiration level a_{ji} is set to 0.4 (with $\sigma_{ji}=0.1$). However, the corresponding current social aspiration level a_i^{avg} is found to be equal to 0.7. E_j will lie in the next negotiation round by putting a_{ji} to 1.0

in order to try to decrease a_i^{avg} towards his/her true preferences about the i^{th} objective (i.e., 0.4).

- *non-manipulator*: this kind of DMs will search for the dissatisfied DMs and will invite them to update their preferences with the aim of modifying the SRP towards his/her preferences. For example, for two dissatisfied DMs E_1 and E_2 , we suppose that, for the objective f_1 , we have: $a_{11} = 0.5$, $a_{21} = 0.3$ and $a_1^{avg} = 0.8$. E_1 will send a request to E_2 in order to invite him/her to decrease a_{21} (eventually, while respecting his/her specified acceptable deviation σ_{21}). In fact, by decreasing a_{11} and a_{21} , a_1^{avg} value will decrease and will become closer to a_{11} and a_{21} which decreases the dissatisfaction level for both DMs.

Manipulation in voting systems is seen to be a dishonest behavior which should be avoided [6], [15]. For this reason, the Moderator agent which has a global overview about the overall negotiation system, may perceive that there exist manipulations during the negotiation rounds and hence detects the manipulator agents. In fact, in NSS-GPA, a manipulation is seen as an abrupt change in the DM's preferences which aims to increase the DM's individual welfare. Manipulation seems to be a selfish and dictatorial behavior because if all agents are manipulators, the consensus will never be reached and hence the negotiation process will never end. For this reason, the Moderator agent has the role to detect manipulators and to punish them by retrieving them the right to update their preferences without following the global direction rules. In fact, manipulators will be punished by forcing them to pursue the global direction rules issued from the Moderator software agent.

B. Conceptual details

1) Production of direction rules and process control

(a) Determination of the set of preferences to be updated

For each aspiration level a_{ji} of a DM agent E_j , the Moderator computes the mean gap separating a_{ji} from the aspiration levels a_{ki} of the other agents while taking into account the importance degree w_j of each DM agent E_j as follows:

$$MG_aspiration(a_{ji}, w) = \sum_{k=1, k \neq j}^q |w_j a_{ji} - w_k a_{ki}| / (q-1) \quad (1)$$

where $w = (w_1, \dots, w_q)$ is the vector expressing the DMs' hierarchy such that the sum of all weights equals 1. After that, the Moderator agent calculates the average mean gap for each aspiration component a_i as follows:

$$Average_MG(a_i, w) = \sum_{j=1}^q MG_aspiration(a_{ji}, w) / q \quad (2)$$

The Moderator agent can now determine the preferences to be updated in order to increase the consensus level. In fact, the Moderator agent aims to minimize the mean gap of each aspiration level by using the following rule named **R1**:

If ($MG_aspiration(a_{ji}, w) > Average_MG(a_i, w)$) **Then**

Update (a_{ji});

End If

(b) *Determination of the set of DM agents invited to update their preferences*

At the beginning of the negotiation process, all DM agents can be invited to update their preferences. In fact, if an aspiration level evaluation a_{ji} is to be changed, then automatically the DM agent E_j is invited to modify his/her preferences. This mechanism allows evading the problem of group tyranny [13]. After some negotiation rounds, the consensus level will increase. In order to preserve this increase and encourage the consensus improvement, we minimize the number of aspiration level evaluations to be updated. This is achieved by minimizing the number of DM agents invited to modify their preferences. These agents are identified as follows:

For each DM agent E_j , the Moderator agent computes the mean gap separating the agent's preferences from all other agents' preferences while taking into account the DMs' hierarchy as stated by equation (3):

$$MG_agent(E_j, w) = \sum_{k=1, k \neq j}^q Gap(E_j, E_k) / (q-1) \quad (3)$$

where $Gap(E_j, E_k, w) = \sum_{i=1}^M |w_j a_{ji} - w_k a_{ki}| / M \quad (4)$

Then, the Moderator calculates the average agent mean gap:

$$Average_AMG = \sum_{j=1}^q MG_agent(E_j, w) / q \quad (5)$$

Now, the Moderator agent can identify the agents that should update their reference points in order to augment the consensus level. This is achieved by minimizing the mean gap of each agent by the following rule named **R2**:

If ($MG_agent(E_j, w) > Average_AMG$) **Then**

Invite_for_update (E_j);

End If

(c) *Process control*

Here, the Moderator agent firstly computes the SRP which corresponds to the weighted sum of all DMs reference points, i.e., $SRP = [a_1^{avg}, \dots, a_M^{avg}]^T$ such that:

$$a_i^{avg} = \sum_{j=1}^q w_j a_{ji} \quad \forall i = 1, \dots, M \quad (6)$$

After that, the Moderator agent calculates the gap separating each DM agent's reference point from the SRP:

$$Gap_from_SRP(E_j, w_j) = \sum_{i=1}^M |w_j a_{ji} - a_i^{avg}| / M \quad (7)$$

Then, the Moderator agent computes the average of all gaps separating the agents from the collective opinion:

$$Average_Gap_from_SRP = \sum_{j=1}^q Gap_from_SRP(E_j, w_j) / q \quad (8)$$

The overall negotiation process is controlled by the Moderator agent based on: (i) the *Consensus Rate* (*CR*) and (ii) the parameter *MaxIter*. The *CR* is expressed as follows:

$$CR = 1 - Average_Gap_from_SRP \quad (9)$$

MaxIter corresponds to the maximum allowed number of negotiation rounds. This parameter is important since it ensures that the process ends after a set of negotiation rounds. Based on *CR* and *MaxIter*, the Moderator agent controls the negotiation process by the following rule called **R3**:

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If (numIter ≤ MaxIter) Then
  If (CR < α) and (NOT All_Satisfied) Then
    Execute_update (R1); /*All DM agents can be
      invited to the update operation*/
  Else If (CR < β) and (NOT All_Satisfied) Then
    Execute_update (R1, R2); /*Some DM agents
      can be invited to the update operation*/
  Else
    Stop_negotiation ();
  End If
End If
Else
  Stop_negotiation ();
End If

```

where *numIter* is the current negotiation round index; α and β are two control parameters which are specified before the beginning of the negotiation process such that α ∈ [0,1], β ∈ [0,1] and (α < β). *All_Satisfied* is a Boolean variable indicating whether all DMs are satisfied (cf. Definition 1).

(d) Direction rule production

Once the preferences to be updated and the agents invited for the update operation are identified, the Moderator furnishes the advice rules to the DM agents as follows:

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Advice rule A1: If (wjaji < aiavg) Then
  Invite the agent Ej to increase aji;
End If
Advice rule A2: If (wjaji > aiavg) Then
  Invite the agent Ej to decrease aji;
End If

```

The objective of these rules is to bring closer DMs' preferences in order to reach a high level of consensus. According to these rules, least important DMs will be probably more frequently invited to update their aspiration levels than most important DM agents and vice versa. This fact emphasizes further the consideration of DMs' hierarchy in our system.

2) Manipulator isolation

Manipulation is a bad and undesirable behaviour in group decision making situations and especially in social choice theory [15]. For this reason, we offer the Moderator agent the ability to detect such behaviour and to penalize manipulator

DMs. In NSS-GPA, a manipulation is an abrupt change in one DM's aspiration level, which does not respect the relative accepted deviation of the last negotiation round, in such a way the preference update modifies the SRP in the direction of the manipulator preferences. We assume in our system that each DM has the right to make *M* manipulations during the negotiation since, in real world negotiation situations, a negotiator may give up one of his/her aspiration levels without aiming to manipulate the negotiation. For example, if a DM agent *E_j* is satisfied with all social aspiration levels except one aspiration level *a_{ji}* where the *ith* objective is not so important for him/her, then he/she prefers updating his/her preferences so that to increase the consensus rate which augments the chance of ending the negotiation process with an almost satisfied state. Thus, the Moderator agent considers a DM agent to be a manipulator if he/she performs (*M* + 1) manipulations. When, a DM agent is detected as a manipulator, the Moderator deprives him/her not only of manipulations but also of sending/receiving messages to/from other agents. Hence, the manipulators are isolated and are obliged to update their preferences according to the global direction rules; thereby increasing the consensus rate and making the negotiation process further converging. Manipulator isolation is an important mechanism to avoid selfishness, untrustworthiness and dictatorship behaviors.

3) Dissatisfied non-manipulator DMs' communication

A dissatisfied non-manipulator DM agent would like to decrease his/her dissatisfaction degree by negotiating with other dissatisfied DMs. As mentioned above, a DM is said to be dissatisfied if at least one of his/her aspiration levels does not σ-coincide with the social aspiration levels (cf. Definition 2). For example, consider the case of 5 objectives and 10 DMs, and suppose that there are three DM agents *E₁*, *E₂* and *E₃* which are dissatisfied with the third social aspiration level *a₃^{avg}* such that: *a₃^{avg}* = 0.6, *a₁₃* = 0.4, *a₂₃* = 0.8 and *a₃₃* = 0.2. We suppose also that there is a satisfied agent *E₄* having *a₄₃* = 0.8. It is interesting to agent *E₁* to contact agents *E₂*, *E₃*, *E₄* by sending them requests to decrease their aspiration level evaluations *a₂₃*, *a₃₃* and *a₄₃* while respecting their σ values. In fact, if *E₂*, *E₃*, *E₄* agree about that, there is more chance that *a₃^{avg}* decreases towards the value of 0.4 which decrease the dissatisfaction level of *E₁*. Thus, the aim of communicating with other agents is to bring closer the collective opinion towards *E₁* preferences. Agent *E₃* seems to be interested with such proposal since driving *a₃^{avg}* towards *a₁₃* = 0.4 is equivalent to driving *a₃^{avg}* towards *a₃₃* = 0.2. However, in order to convince *E₃* to accept his/her request, *E₁* should promise *E₃* that he/she will decrease his/her aspiration level evaluation *a₁₃* if *E₃* accepts the proposal and performs the decrease. However, in real world situation, a DM may be a liar. So, if *E₁* lies to *E₃* then *E₃* marks *E₁* as a liar and does not accept his/her future proposals. Besides, if *E₁* takes his/her promise, *E₃* marks *E₁* as a trustworthy agent and accepts his/her future requests. On

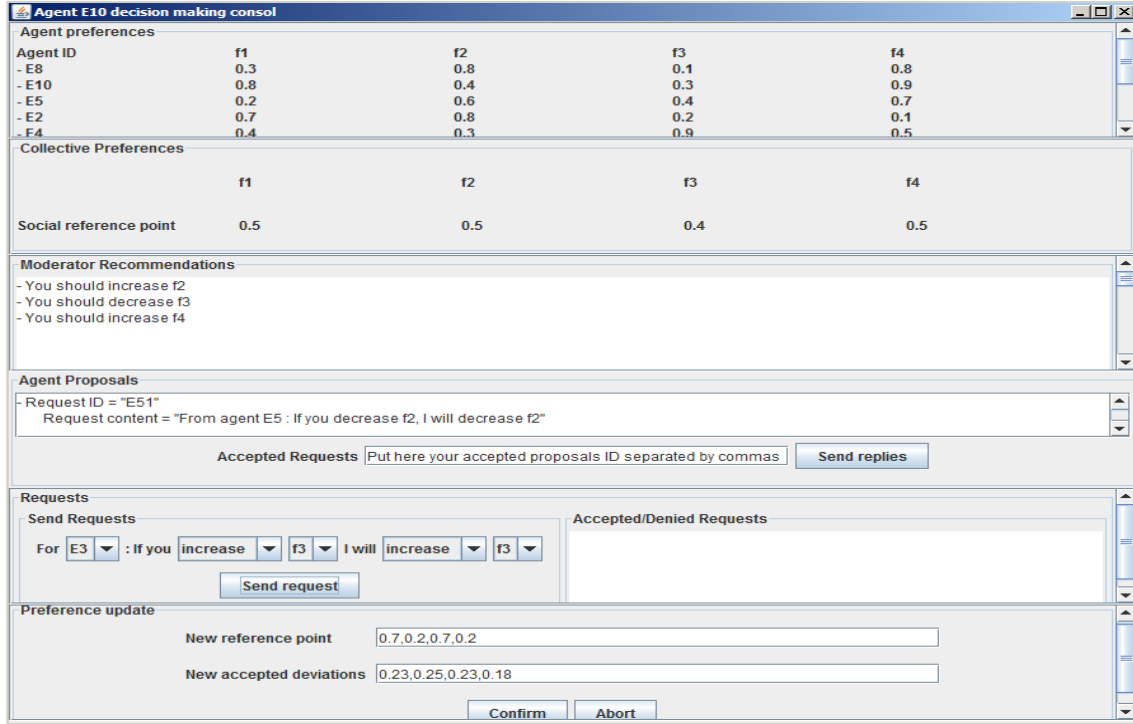


Figure 2. NSS-GPA decision making consol.

the contrary to agent E_3 , agent E_2 is not interested to such request since decreasing a_3^{avg} makes the SRP roll away from his/her preferences. Consequently, E_2 simply rejects the request. The satisfied agent E_4 may be interested in accepting E_1 proposal in order to improve the consensus level and hence augmenting the probability of ending the negotiation sooner with a satisfied state.

C. Implementation details

In this subsection, we give some important implementation details of our system. NSS-GPA is implemented by using the Java Agent DEvelopment framework (JADE) [7] and the ECLIPSE programming tool [2]. Each Assistant agent has a set of cyclic behaviors allowing it to perceive its environment, to communicate with other agents and to update its preferences. The Moderator agent also has a set of cyclic behaviors allowing him to perceive the DM agents' preferences, to produce then broadcast the advice rules for them, to detect then punish manipulators and to control the negotiation process based on the control parameters α , β and $MaxIter$.

Assuming a minimization MOP, the system provides the DMs with the ideal objective vector in addition to the nadir one by using, for example, our MR-NSGA-II_N method [1]. In this way, each DM's aspiration level value lies in the interval $[f_i^{ideal}, f_i^{nadir}]$. After that, the system uses the normalized aspiration values (which lie in the interval [0,1]) in order to ensure that all mean/average gap values lie between 0 and 1. In this way, the negotiation can be well-controlled and the recommendations can be fairly produced based on the designed rules (cf. section II.B.1). Additionally, the system works with the normalized accepted deviation values which can be expressed as follows:

$$\sigma_{ji}^{norm} = \sigma_{ji} / (f_i^{nadir} - f_i^{ideal}) \quad (10)$$

We note that NSS-GPA imposes that $\sigma_{ji}^{norm} \in [0, \sigma_{max}^{norm}]$ in order to control manipulations where σ_{max}^{norm} is specified before the beginning of the negotiation by a human Moderator agent. At this stage, we can define a DM's manipulation analytically. Assuming a_{ji}^{t-1} an aspiration level fixed by DM agent E_j for the i^{th} objective at the previous negotiation round ($t-1$) and a_{ji}^t is the updated value of a_{ji}^{t-1} at the actual negotiation round (t). The update operation is said to be a manipulation if and only if:

$$|a_{ji}^t - a_{ji}^{t-1}| > \sigma_{ji}^{t-1} \quad (11)$$

where σ_{ji}^{t-1} is the accepted deviation fixed by E_j for the i^{th} objective at the generation ($t-1$).

III. First simulation results

A. Assessing W-NSS-GPA with equally important DMs

This subsection is devoted to describe a run of NSS-GPA on a case study with 10 arbitrary chosen DMs and 4 objectives. The 10 DMs are considered here to be equally important in order to illustrate the basic mechanism of W-NSS-GPA. Consequently, we use uniform weights for the DMs. α , β and $MaxIter$ are settled to 0.5, 0.8 and 50 respectively. σ_{max}^{norm} is set to 0.3. These parameters are fixed by a human Moderator agent. Table I shows the initial DM agents' reference points in addition to the accepted deviations (mentioned between parentheses). The DM interacts with the NSS-GPA through a guided user interface which is composed with six panels (cf. fig. 2):

- **Agent Preferences:** where the DM can see the reference points of all DM agents.
- **Collective Preferences:** where the DM can see the collective opinion (i.e., the SRP).
- **Moderator Recommendations:** where the DM receives the global advice rules from the Moderator agent.
- **Agent Proposals:** where the human DM receives proposals from other DM agents. The user can accept or deny such proposals.
- **Requests:** where the DM can send requests to other DM agents and can verify for each one of his/her sent requests whether it was accepted or denied.
- **Preference Update:** where the DM can update his/her reference point and his/her accepted deviation vector.

Based on the used parameter setting, the 10 human DMs have confronted their preferences, through NSS-GPA, while being supervised and guided by the Moderator software agent. Fig. 3 shows the parallel coordinate plots of the DMs' reference points (in addition to the SRP): (a) at the beginning of the negotiation process (cf. table I) and (b) at the end of this process. Fig. 3(a) shows how the initial preferences are so conflicting. In fact, there are large gaps between DMs' reference points themselves. Besides, these initial aspiration level vectors are so conflicting with the initial SRP (0.52,0.50,0.40,0.51). Fig. 3(b) illustrates the final reference points at the end of the negotiation. We see from this figure how the final reference points are less conflicting and so convergent towards the final SRP (0.479,0.560,0.307,0.642). We conclude that NSS-GPA has achieved a good consensus between the different DMs about a SRP. We can say that NSS-GPA has succeeded to bring closer DMs' preferences through the negotiation rounds. Table II shows some statics provided by our system that we call *profiling statistics* since they allow drawing the profile of each DM. These statistics are: (1) *NSR*: the number of requests sent by the DM, (2) *NAR*: the number of accepted requests, (3) *NDR*: the number of denied requests, (4) *NM*: the number of manipulations and (5) *NARec/NRRec*: the number of accepted recommendations divided by the number of received recommendations (the acceptance percentage is set between parentheses). From fig. 3(b), we see that DM4, DM7 and DM8 are the most satisfied

TABLE I. INITIAL DM'S ASPIRATION LEVELS (\pm ACCEPTED DEVIATIONS).

	f_1	f_2	f_3	f_4
DM1	0.8 (\pm 0.10)	0.1 (\pm 0.05)	0.4 (\pm 0.18)	0.2 (\pm 0.11)
DM2	0.4 (\pm 0.18)	0.9 (\pm 0.09)	0.2 (\pm 0.15)	0.5 (\pm 0.25)
DM3	0.6 (\pm 0.28)	0.5 (\pm 0.07)	0.7 (\pm 0.22)	0.4 (\pm 0.26)
DM4	0.1 (\pm 0.09)	0.4 (\pm 0.18)	0.1 (\pm 0.05)	0.6 (\pm 0.30)
DM5	0.4 (\pm 0.08)	0.3 (\pm 0.11)	0.9 (\pm 0.09)	0.5 (\pm 0.20)
DM6	0.8 (\pm 0.07)	0.7 (\pm 0.30)	0.1 (\pm 0.09)	0.2 (\pm 0.08)
DM7	0.2 (\pm 0.20)	0.6 (\pm 0.14)	0.4 (\pm 0.25)	0.7 (\pm 0.11)
DM8	0.9 (\pm 0.08)	0.4 (\pm 0.22)	0.3 (\pm 0.11)	0.9 (\pm 0.09)
DM9	0.3 (\pm 0.21)	0.9 (\pm 0.10)	0.2 (\pm 0.21)	0.8 (\pm 0.12)
DM10	0.7 (\pm 0.23)	0.2 (\pm 0.15)	0.7 (\pm 0.16)	0.3 (\pm 0.18)

TABLE II. DM PROFILING STATISTICS.

	<i>NSR</i>	<i>NAR</i>	<i>NDR</i>	<i>NM</i>	<i>NARec/NRRec</i>
DM1	61	37	14	1	15/36 (41.67%)
DM2	8	3	5	5	18/29 (62.07%)
DM3	42	23	19	2	12/18 (66.67%)
DM4	63	48	15	0	17/26 (65.38%)
DM5	11	6	5	5	19/31 (61.29%)
DM6	21	9	12	4	14/33 (42.42%)
DM7	81	69	12	0	21/28 (75.00%)
DM8	89	76	13	0	22/25 (88.00%)
DM9	14	6	8	3	26/38 (68.42%)
DM10	54	26	28	1	22/28 (78.57%)

DMs since their final reference points are the nearest to the final SRP. The satisfaction of such DMs may be explained by the obtained results in table II. In fact, these satisfied agents are the most communicating agents since they have large values for the *NSR* statistic. Additionally, DM4, DM7 and DM8 have succeeded to have a large number of accepted requests, they are said to be the most trustworthy agents. DM2 and DM5 are detected as manipulators according to the *NM* values (for the 4-objective case, $NM = 5$ means that the DM agent is a manipulator). DM2 and DM5 are the most dissatisfied DM agents according to fig. 3(b). This observation could be explained not only by their manipulation behavior but also by their poor communication with other DM agents (cf. *NSR* values from table II). Intuitively, for a particular DM, the larger his/her *NARec/NRRec* ratio value is, the greater his/her satisfaction level is. However, this is not sufficient since the DM's satisfaction depends also on its communication skills and attitudes. For example, the ratio *NARec/NRRec* of DM9 is greater than DM1 one. Nevertheless, from fig. 3(b), we see that DM1 reference point is nearer to the final SRP than DM9 one. This observation can be explained by the superiority of DM1 over DM9 in terms of communication skills and trustworthiness (cf. *NSR*, *NAR* and *NDR* values from table II). We can say that NSS-GPA favors communicating DMs over non-communicating ones.

Once the DMs' negotiation is performed, we can search for the Pareto-optimal ROI of the considered MOP by running any reference point-based EMOA (e.g., r-NSGA-II [4] and PBEA [14]) with the final SRP obtained by NSS-GPA. We choose to perform a run of r-NSGA-II with the final SRP (0.479, 0.560, 0.307, 0.642) on the four-objective DTLZ2 minimization test problem (which is described in [8]). The population size and the number of generations are set to 200 and 500 respectively. The parameter δ which controls the breadth of the obtained ROI [4] is set to 0.3. Fig. 4 shows the parallel coordinates plot of the obtained preferred solutions. From this figure, we remark that, although the objective values lie in [0,1], most obtained solutions are concentrated near the reference point designed with a dashed bold gray line which would be the region closest to the final SRP furnished by NSS-GPA. When computing $\sum_{i=1}^n f_i^2$ for all obtained solutions, the values are found to lie within [1.051, 1.311], thereby meaning that all solutions are near the true Pareto region (since Pareto optimal

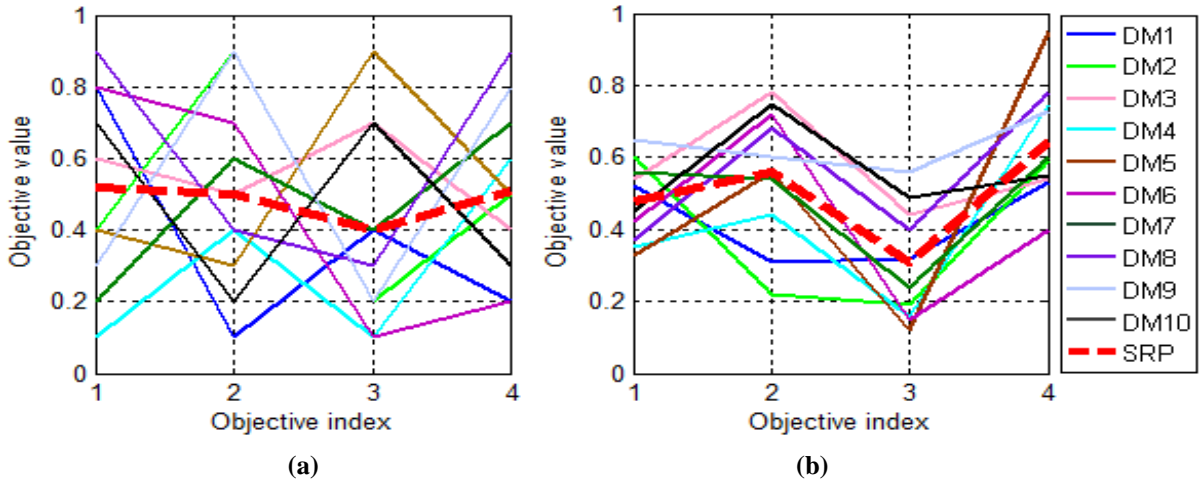


Figure 3. DMs' preferences: (a) Initial aspiration level vectors and (b) final aspiration level vectors.

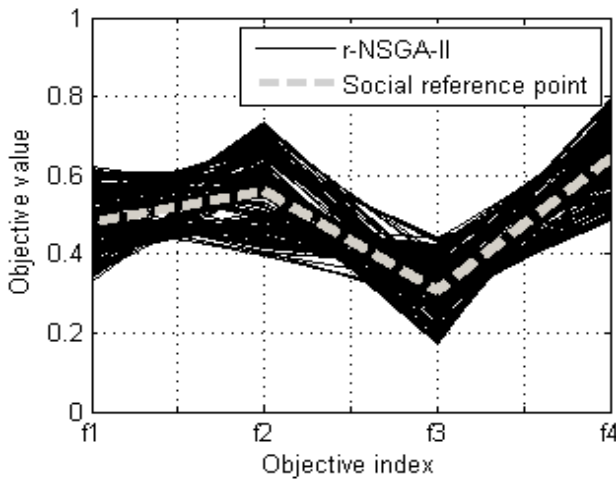


Figure 4. The r-NSGA-II ROI with the final SRP provided by NSS-GPA (0.479, 0.560, 0.307, 0.642) on the four-objective DTLZ2.

solutions of DTLZ2 satisfy $\sum_{i=1}^n f_i^2 = 1$ [8]). We can say that r-NSGA-II has provided a *social ROI* and, as noted in Section I, each non-dominated solution picked from this region is considered as a satisfying solution for each of the considered DMs.

B. Assessing W-NSS-GPA with non-equally important DMs

The goal of this subsection is to show a run of W-NSS-GPA with non-equally important DMs. We retake the same case study used in the previous subsection. We use non-uniform weights for the different negotiators. These hierarchy weights are expressed by $w = (0.10, 0.04, 0.15, 0.05, 0.25, 0.03, 0.10, 0.08, 0.05, 0.15)$. According to this setting, DM5 is the most important DM with a hierarchy weight $w_5 = 0.25$. We use also the same parameter settings as previously noted. Fig. 5 shows the parallel coordinate plots of the DMs' reference points (in addition to the SRP): (a) at the beginning of the negotiation process (cf. table I) and (b) at the end of this process. We see, from fig. 5(b), how the final SRP (0.5791, 0.4422, 0.7017, 0.5691) is so close to the final reference points of the most important DMs, i.e., DM3, DM5 and DM10. Besides, less important DMs' reference points are farther from the final

SRP than the other ones. These observations demonstrate the effect of the weighting coefficient in handling the DMs' hierarchy. It is important to note that a very important DM may have final preferences which are so far from the social ones if he/she has bad behaviors and attitudes such as manipulation and/or poor communication skills. Besides, we would like to notice that a less important DM can have a final reference point that is so near to the final SRP if he/she is a good negotiator. In summary, we have shown that hierarchy weights are useful to express the importance of the different DMs in our system. However, the compliance with the W-NSS-GPA negotiation rules remains a key factor to have satisfying results for any kind of DM.

IV. Application to the Portfolio selection problem

In this section, we demonstrate the usefulness of W-NSS-GPA on a bi-objective Portfolio selection problem with practicalities [16]. In a Portfolio optimization problem with an asset universe of n securities, let x_i ($i = 1, \dots, n$) designate the initial capital proportion to be allocated to security i . Typically, there are two conflicting objectives: (1) minimize the Portfolio risk and (2) maximize the expected Portfolio return. These two objectives have received the most attention and such formulation is known as the mean-variance model of Markowitz [17]. The most basic form of this problem can be expressed as follows:

$$\begin{aligned}
 \text{Min } f_1(x) &= \sum_{i=1}^n \sum_{j=1}^n x_i \sigma_{ij} x_j \\
 \text{Max } f_2(x) &= \sum_{i=1}^n r_i x_i \\
 \text{Subject to } & \sum_{i=1}^n x_i = 1 \\
 & x_i \geq 0 \quad \forall i = 1, \dots, n
 \end{aligned} \tag{12}$$

The first objective is Portfolio risk that is usually computed from a $n \times n$ covariance matrix $[\sigma_{ij}]$. The second objective is expected Portfolio return as computed from a weighted sum of

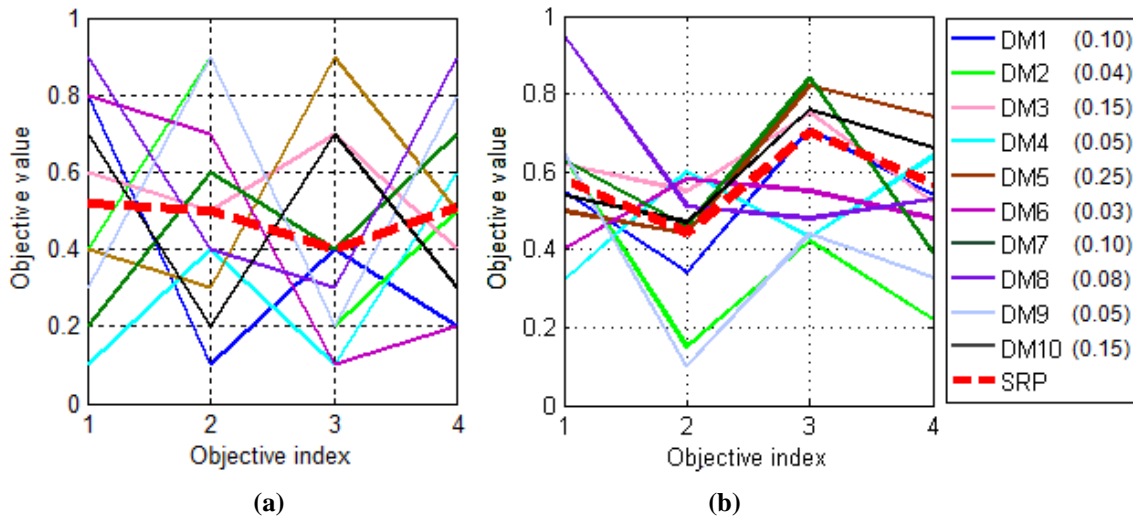


Figure 5. Non-equally important DMs’ preferences: (a) Initial aspiration level vectors and (b) final aspiration level vectors (the hierarchy weights are set between parathenses in the legend).

the individual security expected returns. The first constraint ensures the investment of all funds while the second one ensures the non-negativity of each investment. Such bi-objective problem gives rise to a front of optimal trade-off solutions which should be found to investigate the risk-return relationships. One way to solve this MOP is to convert it to a single objective problem using the ϵ -constraint method as follows:

$$\begin{aligned}
 \text{Min } f_1(x) &= \sum_{i=1}^n \sum_{j=1}^n x_i \sigma_{ij} x_j \\
 \text{Subject to } f_2(x) &= \sum_{i=1}^n r_i x_i \geq R \\
 \sum_{i=1}^n x_i &= 1 \\
 x_i &\geq 0 \quad \forall i = 1, \dots, n
 \end{aligned} \tag{13}$$

In order to generate a representative approximation of the Pareto front, the above quadratic problem is solved repetitively for many different values of R which corresponds to the minimal acceptable return value.

According to the study of Deb et al. [16], it can be expected that almost any solution of (13) contains many of its securities at the zero level, i.e., for many i , $x_i^* = 0$. It can be also expected, for at least a few securities that x_i^* is a very small quantity. However, to have a practical Portfolio, very small investments in any security may not be desired and are to be avoided. Thus, there is the practicality that, for any Portfolio to be of interest, there is to be a lower limit on any non-zero investment, i.e., either $x_i^* = 0$ (meaning no investment in the i^{th} security) or $x_i^* \geq \lambda$ (meaning that there is a minimum non-zero investment amount for the i^{th} security). There may also be an upper bound ω on the proportion of any security in any Portfolio. Unfortunately, the solution of (13) for any given R does not guarantee the possession of any of these characteristics.

In addition to the above, there is a second practicality and it is about the number of non-zero securities contained in the Portfolios along the Pareto front. Over this, a user may wish to exert control. To generate practical Portfolios, a user may be interested in specifying a given number of non-zero investments or a range in the number of non-zero investments a Portfolio have to contain. This is a cardinality constraint and it has also been the subject of some research attention [18, 19]. Taking both practicalities into account, we have the following bi-objective optimization problem:

$$\begin{aligned}
 \text{Min } f_1(x) &= \sum_{i=1}^n \sum_{j=1}^n x_i \sigma_{ij} x_j \\
 \text{Max } f_2(x) &= \sum_{i=1}^n r_i x_i \\
 \text{Subject to } \sum_{i=1}^n x_i &= 1
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 x_i &= 0 \text{ ou } \lambda \leq x_i \leq \omega \\
 d_{\min} &\leq d(x) \leq d_{\max} \\
 0 &< \lambda \leq \omega < 1
 \end{aligned}$$

where $d(x)$ is given as follows:

$$d(x) = \sum_{i=1}^n \begin{cases} 1 & \text{if } x_i > 0 \\ 0 & \text{if } x_i = 0 \end{cases} \tag{15}$$

Standard quadratic problem solvers face difficulties in the presence of discontinuities and other complexities. For instance, the second constraint, requires an “or” operation. While $x_i = 0$ or $x_i = \lambda$ are allowed, values between the two are not. This introduces discontinuities in the search space. The third constraint involves a parameter d which is defined by a discontinuous function of the decision variables given in (15). The second and third constraints make the application of standard quadratic problem solvers difficult which is not the case for the MOEAs [16].

After illustrating the problem details, we can now describe the case study concerning the application of our system

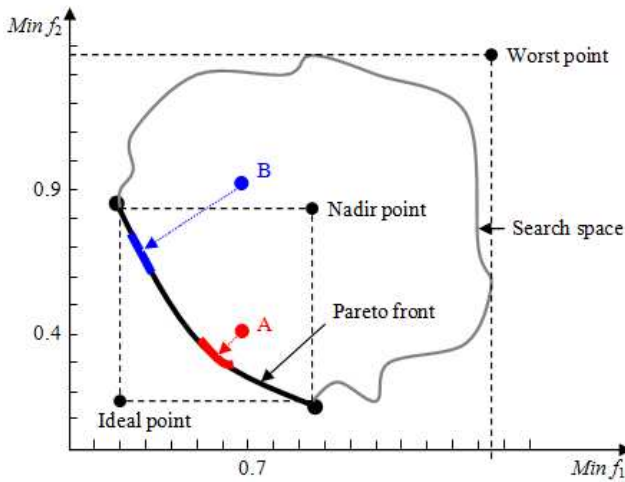


Figure 6. Importance of the nadir objective vector and the ideal one for DM's preference expression.

W-NSS-GPA on this practical bi-objective constrained Portfolio selection problem. We consider an instance used in [16] with 88 securities, $\lambda = 0.005$, $\omega = 0.04$ and $d \in [30, 45]$. In order to facilitate the task of preference expression, we should provide the DMs with the ideal point and the nadir one [1]. Indeed, each aspiration level lies between the ideal value and the nadir one. Fig. 6, demonstrates the effects of expressing: (1) a reference point in the region delimited by the ideal point and the nadir one $A(0.7, 0.4)$ and (2) a reference point outside of this region $B(0.7, 0.9)$. We see, from this figure, that although the two reference points have the same aspiration level value for the first objective, they provide quite different ROIs. This observation emphasizes the importance of the nadir point and the ideal one in the preference expression process. The ideal point can be easily obtained by minimizing each objective function *individually* which is not the case for the nadir point. For this reason, we apply our algorithm MR-NSGA-II_N [1] in order to estimate the nadir point. We note that we use, throughout this experiment, the repair mechanisms proposed in [16] in order to generate feasible solutions when initializing the population and generating the children. The used parameter setting is described as follows: population size = 200, number of generations = 800, crossover probability = 0.9, crossover (SBX operator) distribution index = 10, mutation probability = 0.1, polynomial mutation index = 50 and $\varepsilon = 0.0005$. For the SQP-LS used in our algorithm MR-NSGA-II_N, the termination criterion is: (1) the norm of descent direction $\|d\| = 10^{-8}$ or (2) the number of allowed iterations $\mu = 40$ is elapsed. The obtained nadir point approximation is *NADIR* (0.0024, 0.0102). For the bi-objective case, the ideal point can be deduced from the nadir one; however the opposite is not true. Thus, we use the outlier solutions found by MR-NSGA-II_N in order to find the ideal point approximation which is found to be *IDEAL* (0.000123, 0.0238). We supply the 10 DMs involved in this experiment with these values in order to express their reference points in addition to their accepted deviation vectors. The used parameter setting for NSS-GPA is as follows: α , β and *MaxIter* are settled to 0.4, 0.75 and 50

respectively. σ_{\max}^{norm} is set to 0.3. These parameters are fixed by a human Moderator agent. Consequently, the risk aspiration levels should lie in the interval $[0.000123, 0.002400]$ with an accepted deviation of $(0.002400 - 0.000123) * 0.30 = 0.000683$. The return aspiration levels should lie between $[0.0102, 0.0238]$ with an accepted deviation of $(0.0238 - 0.0102) * 0.30 = 0.00408$. Table IV shows the initial DMs' preferences in addition to their importance levels. We remark, from this table, that there are several kinds of DMs (risk-averse investors, risk-neutral investors, risk-seeking investors) with different wishes of return which makes the initial DMs' reference points so conflicting. Based on the used parameter settings, the 10 DMs have confronted their preferences through W-NSS-GPA. Fig. 7 shows a confrontation, in the risk-return space, between the: (1) the initial DMs' reference points and (2) the final ones. We see, from this figure, how the initial preferences are so conflicting not only between themselves but also with the initial SRP (0.001559, 0.018273). Fig. 7(b) illustrates the final reference points at the end of the negotiation. We see, from this figure, how the final reference points are less conflicting and so convergent towards the final SRP (0.000955, 0.0166) with respect not only to the DM's preferences but also to the hierarchy weighting coefficients. We conclude that W-NSS-GPA has achieved a good consensus between the different DMs about a SRP. We can say that NSS-GPA has succeeded to bring closer the DMs' risk-return aspiration level vectors. We remark also from fig. 7 that: (1) most DMs' who have decreased their risk aspiration values have also decreased their return risk and (2) most DMs' who have increased their risk aspiration values have also increased their return aspiration values. These two observations emphasize the fact that higher return is usually obtained with higher risk.

We can now apply our reference point-based EMO algorithm r-NSGA-II [3] with the final SRP in order to find the social ROI for the considered instance of the practical Portfolio selection problem. We use a population size of 300 and a number of generations of 1500. The non-r-dominance threshold δ which controls the breadth of the ROI is set to 0.3. We note that we use the same repair mechanisms proposed in [16] in order to generate feasible solutions. Fig. 8 shows the obtained social ROI designed with green triangles. We see, from this figure, how this region is composed with the nearest Pareto optimal Portfolios to the social reference point in the risk-return space. Besides, we remark that surprisingly the social ROI corresponds to the knee region composed with the worthiest Portfolios in terms of risk-return trade-off [20, 21].

From a computational viewpoint, we note that the customized NSGA-II approach (without using local search) of Deb et al. [16] needs about $1.5 \cdot 10^6$ FEs to approximate the whole Pareto front according to the experimental results presented in the corresponding paper. However, our approach, which uses MR-NSGA-II_N for estimating the nadir point and then r-NSGA-II to approximate the social ROI, requires about $0.7 \cdot 10^6$ FEs. This observation emphasizes the computational efficiency of our approach regarding the use of a general-purpose MOEA (without any preference-based mechanism) and then selecting a Portfolio in a posteriori manner.

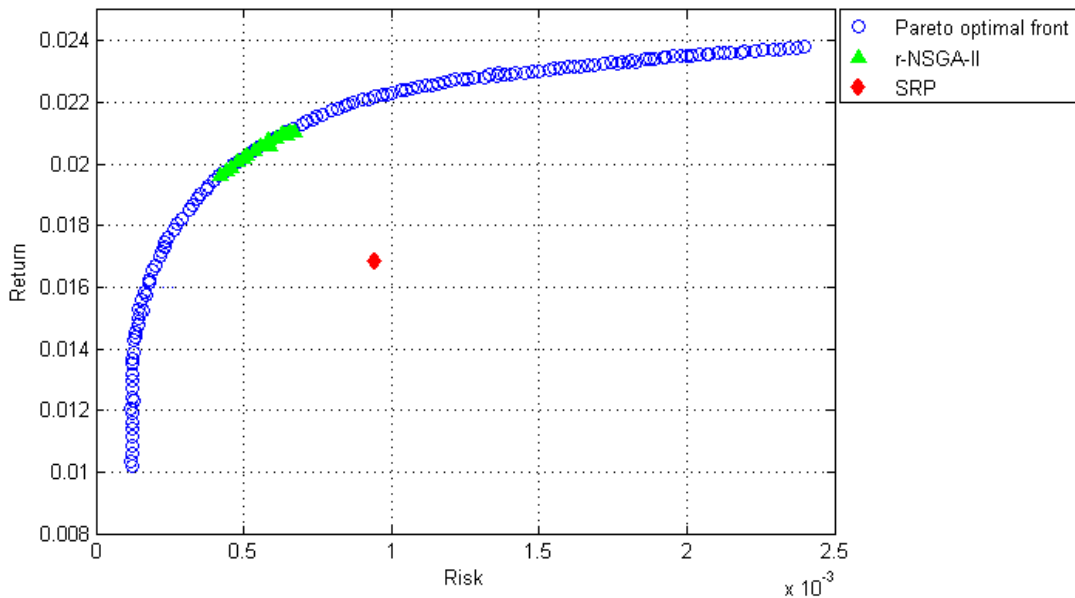


Figure 8. Obtained social ROI by running r-NSGA-II with the final SRP (0.000955, 0.0166) on the practical portfolio selection problem.

TABLE IV. INITIAL DM'S ASPIRATION LEVELS (\pm ACCEPTED DEVIATIONS) FOR THE PRACTICAL PORTFOLIO SELECTION PROBLEM.

	Objectives		w_j
	Risk	Return	
DM1	0.002300 (\pm 0.000132)	0.0197 (\pm 0.0018)	0.12
DM2	0.000648 (\pm 0.000465)	0.0114 (\pm 0.0033)	0.04
DM3	0.001526 (\pm 0.000206)	0.0200 (\pm 0.0030)	0.14
DM4	0.001276 (\pm 0.000369)	0.0188 (\pm 0.0023)	0.05
DM5	0.002235 (\pm 0.000103)	0.0219 (\pm 0.0028)	0.25
DM6	0.001730 (\pm 0.000476)	0.0182 (\pm 0.0024)	0.03
DM7	0.000925 (\pm 0.000258)	0.0178 (\pm 0.0012)	0.11
DM8	0.001667 (\pm 0.000587)	0.0155 (\pm 0.0018)	0.08
DM9	0.001335 (\pm 0.000355)	0.0140 (\pm 0.0011)	0.05
DM10	0.000524 (\pm 0.000405)	0.0138 (\pm 0.0027)	0.13

V. Conclusions and future works

Obtaining a social ROI that corresponds to a set of non-equally important DMs can be seen as the result of a sequential hybridization between an agent-based system and an EMOA. The first component helps the DMs to negotiate their preferences while the second one approximates the desired region from which the final decision to realize will be chosen. W-NSS-GPA is demonstrated to: (1) discourage dictatorship, manipulation and untrustworthiness behaviors and (2) encourage communication between the negotiators. Moreover, our system has been shown to handle DMs' hierarchy through the use of weights. The assessment of W-NSS-GPA has been performed through three steps. In subsection III.A, we demonstrated how our system takes into account the behavioral attitudes of the different DMs. This fact ensures the fairness of the negotiation. After that, in section III.B, we showed how W-NSS-GPA respects the DMs' hierarchy. It is important to note that even if the DM is so important, he/she should have good communication skills and behaviors in order

to achieve a high satisfaction level. Finally, in section IV, we illustrated the usefulness of our system through its application to a practical instance of the portfolio selection problem. This work can be extended in several ways. Firstly, W-NSS-GPA can be enriched by the notion of coalition where DMs' with similar preferences and goals form coalitions to improve their own utilities. Secondly, an omitted problematic in the EMO community is interactive multi-objective optimization with a group of DMs. Hence, it would be interesting to handle such a problematic by collaboration scheme between W-NSS-GPA and r-NSGA-II. In this way, we can exploit information issued from the search space to build advice/argumentation rules. Finally, since DM's preferences are usually expressed by fuzzy linguistic terms, it would be a challenging topic to handle DMs' negotiation with fuzzy aspiration levels.

References

- [1] S. Bechikh, L. Ben Said, and K. Ghédira, "Estimating nadir point in multi-objective optimization using mobile reference points," In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC'10)*, pp. 2129–2137, IEEE Press, 2010.
- [2] F. L. Bellifemine, G. Caire, and D. Greenwood, *Developing multi-agent systems with JADE*, Wiley, USA, 2007.
- [3] S. Bechikh, L. Ben Said, and K. Ghédira, "Negotiating DMs' reference points for group preference-based evolutionary multi-objective optimization," In *Proceedings of the IEEE international conference on Hybrid Intelligent Systems (HIS'11)*, pp. 377–382, IEEE press, 2011.
- [4] L. Ben Said, S. Bechikh, and K. Ghédira, "The r-dominance: a new dominance relation for interactive evolutionary multi-criteria decision making," *IEEE Transactions on Evolutionary Computation*, vol. 14, no. 5, pp. 801–818, 2010.
- [5] C. A. Coello Coello, G. B. Lamont, and D. A. Van Veldhuizen, *Evolutionary algorithms for solving*

- multi-Objective problems*, 2nd Ed., Springer, New York, 2007.
- [6] V. Conitzer, *Computational aspects of preference aggregation*, PhD thesis, Carnegie Mellon University, USA, 2006.
- [7] J. d'Anjou, S. Fairbrother, D. Kehn, J. Kellerman, and P. McCarthy, *The Java developer's guide to Eclipse*, Addison-Wesley, USA, 2004.
- [8] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multiobjective optimization test problems," In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC'02)*, pp. 825–830, IEEE Press, 2002.
- [9] E. Herrera-Viedma, S. Alonso, F. Chiclana, and F. Herrera, "A consensus model for group decision making with incomplete fuzzy preference relations," *IEEE Transactions on Systems, Man and Cybernetics, Part B*, vol. 37, no. 1, pp. 176–189, 2007.
- [10] K. Miettinen, *Nonlinear multiobjective optimization*, Kluwer Academic Publishers, USA, 1999.
- [11] M. Morge and P. Beaune, "A Negotiation support system based on a multi-agent system: specificity and preference relations on arguments," In *Proceedings of the ACM Symposium on Applied Computing (ACM SAC'04)*, pp. 474–478, ACM Press, 2004.
- [12] J. Pfeiffer, U. Golle, and F. Rothlauf, "Reference point based multi-objective evolutionary algorithms for group decisions," In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'08)*, pp. 697–704, ACM Press, 2008.
- [13] S. Saint and J. R. Lawson, *Rules for reaching consensus: a modern approach to decision making*, Jossey-Bass, San Francisco, CA, 1994.
- [14] L. Thiele, K. Miettinen, P. J. Korhonen, and J. Molina, "A preference based evolutionary algorithm for multiobjective optimization," *Evolutionary Computation*, vol. 17, no. 3, pp. 411–436, 2009.
- [15] L. Xia and V. Conitzer, "A sufficient condition for voting rules to be frequently manipulable," In *Proceedings of the ACM Conference on Electronic Commerce*, pp. 99–108, ACM Press, 2008.
- [16] K. Deb, R. E. Steuer, R. Tewari, and Ra. Tewari, "Bi-objective portfolio optimization using a customized hybrid NSGA-II procedure," In *Proceedings of the international conference on Evolutionary Multi-criterion Optimization (EMO'11)*, pp. 358–373, LNCS, Springer, 2011.
- [17] H. M. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77–91, 1952.
- [18] M. Stein, J. Branke, and H. Schmeck, "Efficient implementation of an active set algorithm for large-scale portfolio selection," *Computers & Operations Research*, vol. 35, no. 12, pp. 3945–3961, 2008.
- [19] F. Streichert, and M. Tanaka-Yamawaki, "The effect of local search on the constrained portfolio selection problem," In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC'06)*, pp. 2368–2374, IEEE press, 2006.
- [20] S. Bechikh, L. Ben Said, and K. Ghédira, "Searching for knee regions in multi-objective optimization using mobile reference points," In *Proceedings of the ACM symposium on Applied Computing (Best Paper Award of ACM SAC'10)*, pp. 1118–1125, ACM Press, 2010.
- [21] S. Bechikh, L. Ben Said, and K. Ghédira, "Searching for knee regions of the Pareto front using mobile reference points," *Soft Computing– A Fusion of Foundations, Methodologies and Applications*, vol. 15, no. 9, pp. 1807–1823, Springer, 2011.
- [22] S. Bechikh, N. Belgasmi, L. Ben Said, and K. Ghédira, "PHC-NSGA-II: A novel multi-objective memetic algorithm for continuous optimization," In *Proceedings of the IEEE International Conference on Tools with Artificial Intelligence (ICTAI'08)*, pp. 180–189, 2008.

Author Biographies



Slim Bechikh has received the B.S. degree in computer science applied to management and the M.S. degree in modelling in 2006 and 2008 respectively from the High Institute of Management of Tunis, University of Tunis, Tunisia. He is currently working towards the Ph.D. degree in computer science applied to management within the Optimization Strategies and Intelligent Computing laboratory (SOIE) (Tunisia). His research interests include multi-criteria decision making, evolutionary computation, metaheuristics, memetic computing, group decision making, multi-agent systems and Portfolio optimization.



Lamjed Ben Said has received the B.S. degree in computer science applied to management from the High Institute of Management of Tunis, University of Tunis, Tunisia, in 1998. He has obtained the M.S. degree and the Ph.D. degree in computer science from the University of Paris VI, France, in 1999 and 2003 respectively. He was a research fellow for three years at France Telecom (R&D). Dr. Ben Said is actually an Associate Professor at the High Institute of Management of Tunis (Tunisia). He is now also the scientific coordinator of Optimization Strategies and Intelligent Computing laboratory (SOIE) (Tunisia) and the general secretary of the Tunisian Association of Artificial Intelligence. His research interests cover multi-agent systems, multi-criteria decision making, evolutionary computation, supply chain management and behavioral economics.



Khaled Ghédira has received the Engineer degree in hydraulic from ENSEEIHT, the Engineer degree in computer science and applied mathematics from ENSIMAG, the M.S. degree then the Ph.D. degree in Artificial Intelligence from ENSAE (France) in 1983, 1986, 1990 and 1993 respectively. He also obtained the HDR degree from the National School of Computer Studies of Tunis (ENSI), in 1999. He was fellow research at the Institute of Computer Science and Artificial Intelligence of Neuchâtel (Switzerland) and a consultant at British Telecom. He is currently a Professor of Artificial Intelligence at the High Institute of Management of Tunis, University of Tunis, Tunisia. He is currently also the president of the Tunisian Association of Artificial Intelligence and the Head of the Intelligent Information Engineering Laboratory (Tunisia). Professor Ghédira has authored and co-authored more than 180 international journal and conference research papers. He has written one text book on combinatorial optimization. He is also the Editor-in-chief of the Mediterranean Journal of Artificial Intelligence (MJAI) and a member of several program committees relative to various conferences and journals. His research areas include multi-agent systems, constraint satisfaction problems, combinatorial optimization, supply chain management, multi-criteria decision making, and metaheuristics.