International Journal of Computer Information Systems and Industrial Management Applications.
ISSN 2150-7988 Volume6(2014) pp.89-101

A Meta-Ontology for Modeling Fuzzy Ontologies and its Use in Classification Tasks based on Fuzzy Rules

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Abstract: **Ontologies have been employed in applications that require semantic information representation and processing. However, traditional ontologies are not particularly suitable to express fuzzy or vague information, which often occurs in human vocabulary as well as in several application domains. To deal with this limitation, concepts from the Fuzzy Set Theory can be incorporated into ontologies making it possible to represent and reason over fuzzy or vague knowledge. In this context, this paper proposes *Fuzz-Onto*, a meta-ontology for representing fuzzy ontologies which, so far, models fuzzy concepts, fuzzy relationships and fuzzy properties. In particular, the representation of fuzzy properties and linguistic terms makes it possible to combine fuzzy modeling in ontologies with existing fuzzy rule-based classification methods. The paper also presents a case study in the knowledge domain of scientific documents as an instantiation of the modeling-inference articulation.**

Keywords: Knowledge Representation, Fuzzy Ontology, Fuzzy Set Theory, Fuzzy Rule-Based Reasoning, Classification methods

I. Introduction

Ontologies have been widely used in applications regarding knowledge representation and reasoning. A remarkable example is the Semantic Web [1], with a number of applications (e.g. [2, 3, 4]) using ontologies to deal with the semantics of content and services over the Web. In the context of computer and information sciences, an ontology defines a set of representational primitives with which to model a domain of knowledge or discourse. Its representational primitives are typically concepts (also known as classes or sets), attributes (or properties), and relationships (or relations among class members) [5]. In this sense, ontologies have been applied to improve communication and semantic information processing among humans and computational systems.

However, traditional ontologies may not be suitable when modeling domains in which concepts are not *precisely* defined. For instance, it is difficult to represent linguistic terms like *young*, *dark*, *hot*, *large* and *thick*, as they involve the so-called fuzzy or vague concepts for which a clear and precise definition is not possible [6, 7]. Therefore, it is necessary to extend traditional ontologies by incorporating to them ways to model fuzzy concepts [8] in order to represent and reason over vague or fuzzy information; such extensions can be characterized as fuzzy ontologies.

Fuzzy ontologies have received much attention from some research areas, such as ontology matching, data integration, multimedia information processing, natural language interfaces, among others [7]. In general, these applications analyze fuzzy concepts, fuzzy relationships and linguistic characteristics represented in ontologies in order to handle vagueness, a very common characteristic of human vocabulary. Fuzzy ontologies are also important for text mining and information retrieval applications, since the integration of imprecise concepts with ontologies makes it possible to represent, retrieve and rank documents according to a degree of relevance to the user query. Some applications of fuzzy ontologies in these areas can be found in [9, 10, 11, 12].

In this sense, a number of proposals have been developed to represent and reason over fuzzy ontologies, ranging from fuzzy hierarchical structures (fuzzy taxonomies) to more complex representations that include linguistic terms and rules. Fuzzy taxonomies represent the fuzzy *is-a* relationship, which defines that a concept is more generic or more specific than other to a certain degree. Some studies [13, 14, 15, 16] propose more elaborated representations composed of classes, individuals, relationships and axioms, resulting in extensions of the *Web Ontology Language* (OWL) [17] to represent fuzzy concepts and fuzzy relationships in ontologies. In addition, some proposals [6, 9, 18, 19, 20] can handle linguistic terms in ontologies making it possible to represent linguistic values of an attribute; for instance, the attribute *age* characterized by the terms *young*, *middle age* or *old*, each one defined by a specific fuzzy set. Some studies, by increasing the expressiveness of fuzzy ontologies, investigate the integration of fuzzy rules and ontologies, focusing on rules containing either degrees of truth [21, 22] or linguistic terms [23, 24, 25].

In particular, a relevant point in the fuzzy extensions of ontologies is handling information that is closely related to the vagueness of the human language and reasoning, by means of linguistic terms and rules. By integrating ontologies and rules containing linguistic terms, it is possible to apply reasoning methods often used in fuzzy rule-based reasoning systems, such as the interpolation, Mamdani and classification methods [26]. In this sense, a reasoner modeled according to these fuzzy reasoning methods can perform relevant inferences based on imprecise information represented in fuzzy ontologies, improving the expressiveness of the domain representation and eventually inducing new knowledge. Specifically, classification methods based on fuzzy rules can be useful in applications that require the categorization of individuals into classes of an ontology. Textual documents, for instance, can be categorized into concepts of a domain based on fuzzy rules associated to groups of terms occurring in their contents, contributing to organize large-scale document repositories. In this context, this work is focused on reasoning mechanisms associated to fuzzy ontologies in order to classify individuals based on fuzzy rules and linguistic terms.

A practical issue related to using ontologies, pointed out by Lukasiewicz and Straccia [7], has to do with making available more implementations of fuzzy ontology approaches, especially of those involving scalable formalisms. There are proposals which extend existing ontology languages with fuzzy elements. However, some of them are only theoretical studies [13, 6, 27] with no inference engines available and others [14, 23] are intrusive approaches that modify the structure of ontology languages. Consequently, an additional effort is required to adapt applications to the modified languages; this however has the side effect of loosing backward compatibility due to the introduction of elements incompatible with existing reasoners and technologies for ontology-based applications. Given this difficulty, it is interesting to investigate a non-intrusive model that could represent fuzzy information using constructors provided by traditional ontology languages, so that the existing applications, reasoners and technologies can be straightforwardly reused. There are recent proposals [28, 20, 24, 25] following this direction, by applying fuzzy ontologies in practical situations. However, none of them classify individuals based on fuzzy rules as it is done in Fuzzy Rule-Based Classification Systems (FRBCSs), which perform inferences that can also be useful for ontology-based applications.

In order to address these issues, this paper proposes and describes *Fuzz-Onto*, a meta-ontology that represents fuzzy concepts, fuzzy relationships, fuzzy properties and linguistic terms, making it possible to handle fuzzy classification rules. This is a non-intrusive approach that models fuzzy elements in an abstract representation, using constructors provided by traditional ontology languages, in order to be inherited and/or instantiated by domain ontologies. Moreover, *Fuzz-Onto* considers not only representational issues but also fuzzy reasoning methods for the classification of individuals based on rules containing linguistic terms. Aiming at describing the main ideas that support *Fuzz-Onto*, this paper is organized as follows. Section II describes related work regarding fuzzy ontologies, including a brief discussion about expressiveness, reasoning and some practical issues. Section III presents *Fuzz-Onto*, the proposed meta-ontology for fuzzy ontology representation, followed by a case study involving scientific document classification (Section IV). Finally, Section V concludes this paper and points out future directions of the on-going research.

II. Related Work on Fuzzy Ontologies

There is a number of proposals that extend ontologies to the fuzzy case. Some approaches consider fuzzy taxonomies; others also extend representational primitives such as fuzzy concepts, fuzzy relationships, fuzzy properties, and some include modifiers and fuzzy rules. Before describing research work related to fuzzy ontologies, some correspondences between these elements and definitions from the Fuzzy Set Theory [8] are considered.

Fuzzy taxonomies represent fuzzy hierarchies of concepts related by specialization (or generalization) to a certain degree. Fuzzy concepts (or fuzzy classes) and fuzzy relationships correspond to definitions of fuzzy sets and fuzzy relations, respectively. A fuzzy property is a property or attribute that can be characterized by linguistic terms represented by fuzzy sets, according to the definition of linguistic variable. Linguistic terms can be defined by fuzzy sets with parameterized membership functions, such as the triangular and trapezoidal functions [26]. A fuzzy rule generally refers to an *if-then* rule that contains either fuzzy classes, fuzzy relationships or fuzzy properties in the antecedent and consequent parts. Specifically, fuzzy classification rules contain a conjunction of fuzzy properties and their linguistic values in the antecedent and a class in the consequent part.

In general, the research on fuzzy taxonomies is related to applications and the development of methods for automatic learning from textual data or structured databases. The *Personalized Abstract Search Service* [29] analyzes terms organized in a fuzzy hierarchy based on the co-occurrence of terms in scientific documents from the Computer Science area. Lee *et al.* [30] propose a fuzzy taxonomy to support the summarization of news in the Chinese language, classifying summary sentences with membership degrees to different events. Leite and Ricarte [31] as well as Pereira *et al.* [32] apply fuzzy taxonomies to retrieve documents according to their relevance to specific concepts. Although fuzzy taxonomies support the representation of two very important properties of fuzzy concepts, i.e., generalization and specialization, they are unable to provide the means for representing other relation properties, such as the transitive, symmetric and reflexive properties required by some domains.

Some proposals consider fuzzy classes and fuzzy relationships in ontologies, resulting in extensions of the *Web Ontology Language* (OWL) [17] based on Fuzzy Description Logics (Fuzzy DLs). For instance, FOWL [13] represents fuzzy classes, fuzzy relationships and extends some axioms of the OWL according to the fuzzy DL $\mathcal{ALC}$ [33]. Based on f-$\mathcal{SHIN}$, Stoilos *et al.* [14] extend the OWL with fuzzy elements resulting in the Fuzzy OWL, providing support to extended reasoning tasks via the FiRE reasoner [15]. Pan *et al.* [16] provide scalable reasoning services for the f-DL-Lite [34], which is able to derive inferences from fuzzy concepts and fuzzy relationships. Although these proposals are expressive with regard to fuzzy extensions, they usually modify the syntax of the original language (OWL) and, as a consequence, loose compatibility with existing tools for ontology-based applications. Such a feature can make it difficult to apply these approaches in practical situations.

The representation of linguistic characteristics in ontologies have been investigated by some proposals. The fuzzy $\mathcal{SHOIN}(D)$ [6] extends the Description Logic $\mathcal{SHOIN}(D)$, regarding fuzzy concrete predicates, modifiers and the subsumption of concepts hold to a certain degree. Calegari and Ciucci [9] propose the language Fuzzy-OWL based on fuzzy $\mathcal{SHOIN}(D)$, by modifying some features such as allowing modifiers to be applied to fuzzy relationships and restricting cardinality axioms only to the classical Boolean case. Bobillo and Straccia [19] model fuzzy data types and linguistic hedges in the fuzzy DL $\mathcal{ALCF}(D)$, which provides support to generalized fuzzy operations from several fuzzy logics (e.g. Zadeh, Product, Lukasiewicz). Bobillo *et al.* [35] propose a crisp representation of the fuzzy DL $\mathcal{SROIQ}(D)$ which is compatible to crisp ontology reasoners, including support to fuzzy concrete predicates based on trapezoidal and triangular membership functions. However, as these membership functions are mapped to crisp intervals in a crisp ontology language, further reasoning should be implemented to obtain the partial membership degree of a particular property value, which could be further used for classification. Bobillo and Straccia [36] define an ontology to represent the fuzzy DL $\mathcal{SROIQ}(D)$ in an OWL syntax and also provide parsers to different fuzzy DL reasoners. More recently, Bobillo and Straccia [20] have proposed a fuzzy extension of OWL 2 with fuzzy properties and modifiers based on the fuzzy DL $\mathcal{SROIQ}(D)$ [28]. Proposals that deal with fuzzy properties and modifiers are interesting as they can express linguistic characteristics common in the human vocabulary, although some of them [6, 9] do not support fuzzy reasoning.

Aiming to increase expressiveness, several studies are investigating how to combine fuzzy rules and ontologies. In general, fuzzy rules can be interpreted considering a degree of truth instead of being strictly true or false. Some proposals focus on rules containing explicit degrees of truth that indicate which conditions or rules are more relevant to perform inferences. For example, f-SWRL [21] and Vague-SWRL [37] extend the SWRL Rule Language [38] with degrees of truth assigned to atoms in antecedents and consequents of rules. Similarly, Damásio *et al.* [22] extend the RuleML language to represent degrees of truth assigned to conditions and rules; these proposals, however, do not support rules with linguistic terms.

To handle linguistic characteristics related to vagueness, some studies are dealing with rules containing fuzzy properties and linguistic terms. The *fuzzyDL* reasoner [23] is based on the Fuzzy Description Logic $\mathcal{SHIF}(D)$ with support to fuzzy data types, parameterized membership functions (trapezoidal, triangular, linear) and fuzzy modifiers. *FuzzyDL* also handles fuzzy rules and implements reasoning mechanisms based on the Mamdani model and defuzzification methods. Bobillo *et al.* [39] consider ontology and fuzzy rules according to the Mamdani model, applied to a balanced scorecard system to support decision making in the business management context. Reformat and Ly [25] define a framework to *computing with words* systems based on ontologies, providing an abstract model that represent fuzzy properties and parameterized membership functions in domain ontologies. This framework also handles rules with linguistic terms; the reasoning mechanism, however, considers Boolean conditions in the antecedent of rules.

Another proposal that represents fuzzy properties and linguistic terms in an abstract model is the SWRL-F approach [24], which does not modify the SWRL language syntax, maintaining the compatibility to the existing tools and reasoners. To perform inferences, Wlodarczyk *et al.* [24] developed a plug-in to the Protg tool [40] that considers a reasoning procedure based on fuzzy controllers. OWL-FC [41] also represents linguistic variables and fuzzy rules through a high-level specification model for fuzzy control systems that enables links to domain ontology concepts. Bragaglia *et al.* [42] propose a hybrid reasoner that integrates existing reasoners for rules (Drools), traditional ontologies (Pellet) and fuzzy ontologies (*fuzzyDL*). In their work, fuzzy rules contain linguistic terms that are checked by the *fuzzyDL* reasoner, and reasoning mechanisms are based on the *modus ponens* inference rule from fuzzy logics.

From a practical point of view, some expressive proposals [13, 6, 27, 22] do not provide fuzzy inference engines to applications, making it difficult to use fuzzy ontologies in real-world situations. Some approaches [21, 14, 23] modify the syntax of ontology languages to introduce fuzzy elements, resulting in languages that, generally, are incompatible with resources currently available for ontology-based applications. Modifying languages demands an additional effort to adapt existing applications and reasoners to the extended languages, usually an unfeasible task that may impair their effective use. There are proposals [28, 36, 20, 24, 25, 41] that investigate non-intrusive methods to represent fuzzy ontologies, which do not modify the underlying ontology languages. An interesting non-intrusive strategy is defining an abstract model based on conventional ontology languages, so that fuzzy elements can be inherited and/or instantiated by particular domain ontologies. Going into this direction, some studies [36, 25, 24, 41] have developed expressive models for representing fuzzy ontologies, however they do not consider the classification of individuals based on fuzzy rules as it is done in FRBCSs.

In addition to practical issues, there is a research stream in the literature to increase the expressiveness of fuzzy ontologies. Regarding the mentioned fuzzy extensions of ontologies, linguistic terms and rules deserve special attention since they are able to express information related to the vagueness of

real-world situations as well as human language and reasoning. Moreover, by combining ontologies and rules with linguistic terms, it is possible to apply approximate reasoning methods often used in Fuzzy Rule-Based Systems, such as the Interpolation, Mamdani [26, 43] and also the classification methods from FRBCSs [44]. However, several proposals neither support fuzzy rules and linguistic characteristics nor provide fuzzy reasoning mechanisms based on methods available for fuzzy inference systems. In particular, none of the related works have exploited the classification based on reasoning methods of FRBCSs, which could be useful to applications that require the categorization of individuals into classes of an ontology. To this purpose, the Classical and General Fuzzy Reasoning Methods [44] could be employed to classify individuals of an ontology.
Following this line of thoughts, this paper proposes and describes *Fuzz-Onto*, a meta-ontology to represent fuzzy ontologies, handling linguistic characteristics and fuzzy classification rules. In addition to the representational issues, this proposal deals with fuzzy reasoning methods from FRBCSs in order to classify individuals of ontologies. Next section describes *Fuzz-Onto* and its main features for representing and processing fuzzy ontologies.

III. The *Fuzz-Onto* Meta-Ontology

Fuzz-Onto represents some concepts from the Fuzzy Set Theory using representational primitives commonly found in traditional ontology languages, namely concepts, binary relationships, attributes, concrete domains and individuals. Concrete domains are related to the representation of specific domains in Description Logics, including data types such as numerical types, strings, among others [45], associated to the values of attributes. Concrete individuals are instances of concrete domains, representing specific concrete values, e.g. a specific number or string. Individuals of concepts, also known as abstract individuals, are members of concepts. The five primitives have shown to be sufficient for the purposes of *Fuzz-Onto*.
As a meta-ontology, *Fuzz-Onto* can be approached as an abstract model to be instantiated into domain ontologies in order to represent fuzzy concepts, fuzzy relationships and fuzzy properties. In this paper, previous work [46, 47] is extended in order to refine the representation of fuzzy concepts, fuzzy relationships and fuzzy properties in *Fuzz-Onto*. The elements of *Fuzz-Onto* are identified with the prefix *fuz:*. Subsections III-A, III-B and III-C describe how fuzzy concepts, fuzzy relationships and fuzzy properties are represented in *Fuzz-Onto* respectively and Subsection III-D describes how fuzzy classification rules are supported.

A. Representing Fuzzy Concepts

In *Fuzz-Onto* fuzzy concepts correspond to fuzzy sets of individuals defined over a discrete domain. *Fuzz-Onto* considers a vocabulary to represent fuzzy concepts, modeled by using representational primitives of ontologies (indicated in parenthesis):

- ***fuz:Individual*** (concept): represents a set of abstract individuals of the domain ontology which involve fuzziness in their definition (i.e. are instances of a fuzzy concept and/or are involved in fuzzy relationships);
- ***fuz:FuzzyConcept*** (concept): represents atomic fuzzy concepts defined in a discrete domain. If an atomic concept in a domain ontology is a fuzzy concept, it should be subsumed by *fuz:FuzzyConcept* to denote that its individuals belong to the concept with a certain membership degree in $[0, 1]$;
- ***fuz:FuzzyConceptAssociation*** (concept): associates an individual to its fuzzy concept and respective membership degree. In a domain ontology, each instance of *fuz:FuzzyConceptAssociation* should be related to a fuzzy concept by the *fuz:hasFuzzyConcept* relationship, to an individual by the *fuz:hasFuzzyMembership* relationship and to a membership degree by the *fuz:hasMembershipDegree* attribute;
- ***fuz:hasFuzzyConcept*** (relationship): associates an instance of *fuz:FuzzyConceptAssociation* to a fuzzy concept;
- ***fuz:hasFuzzyMembership*** (relationship): associates an individual of the domain ontology to an instance of *fuz:FuzzyConceptAssociation*;
- ***fuz:hasMembershipDegree*** (attribute): defines the membership degree represented as a real number in the interval $[0, 1]$.
- ***fuz:Real*** (concrete domain): represents values in the domain of real numbers.

In what follows, graphical representations of ontologies will have the notation:

- Concepts or classes: white ellipse-like nodes;
- Binary relationships: directed arcs between ellipse-like nodes;
- Attributes: directed arcs between an ellipse-like node and a box;
- Concrete domains (e.g. real): white boxes;
- Concrete individuals: grey boxes;
- Individuals of concepts (abstract individuals): grey ellipse-like nodes.

The upper part of Figure 1 shows a graphical representation of the elements involved in the representation of fuzzy concepts in *Fuzz-Onto*. The lower part shows an example of its instantiation, producing a domain ontology that describes cars and their characteristics (elements identified by the prefix *car:*). Instances of the concept *fuz:FuzzyConceptAssociation* are responsible for associating individuals of the domain ontology to their correspondent fuzzy concepts. In the domain ontology the instance of *fuz:FuzzyConceptAssociation* named *car:fordFocusMembershipToSportCar* represents that the domain-specific individual *car:fordFocus* has a membership degree of 0.7 in the fuzzy concept *car:SportCar*.

B. Representing Fuzzy Relationships

In *Fuzz-Onto* fuzzy relationships correspond to atomic fuzzy binary relations defined over a discrete domain. In addition to the elements *fuz:Individual*, *fuz:hasMembershipDegree* and *fuz:Real* described previously, *Fuzz-Onto* represents fuzzy relationships using the following vocabulary:

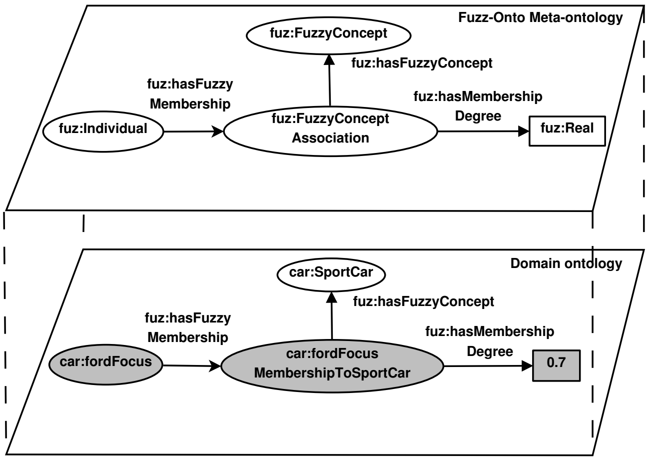

Figure. 1: Representing a fuzzy concept using *Fuzz-Onto*.

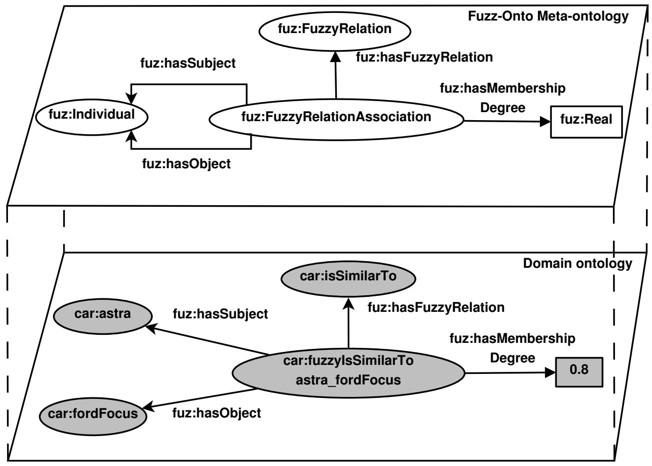

Figure. 2: Representing a fuzzy relationship using *Fuzz-Onto*.

- ***fuz:FuzzyRelation*** (concept): represents atomic fuzzy binary relations. If an atomic relationship in a domain ontology is a fuzzy relationship, it should be an instance of *fuz:FuzzyRelation* to denote that pairs of individuals can be related to each other to a certain membership degree in $[0, 1]$ describing the strength of their relationship;

- ***fuz:FuzzyRelationAssociation*** (concept): associates two individuals to a fuzzy relation and the respective membership degree. In a domain ontology each instance of *fuz:FuzzyRelationAssociation* should be related to a fuzzy relation by the *fuz:hasFuzzyRelation* relationship, to two individuals by both the *fuz:hasSubject* and the *fuz:hasObject* relationships and to a membership degree by the *fuz:hasMembershipDegree* attribute;

- ***fuz:hasFuzzyRelation*** (relationship): associates an instance of *fuz:FuzzyRelationAssociation* to a fuzzy relation;

- ***fuz:hasSubject*** (relationship): associates an instance of *fuz:FuzzyRelationAssociation* to the individual that represents the subject of a fuzzy binary relationship;

- ***fuz:hasObject*** (relationship): associates an instance of *fuz:FuzzyRelationAssociation* to the individual that represents the object of a fuzzy binary relationship.

The upper part of Figure 2 is a graphical representation of fuzzy binary relationships in *Fuzz-Onto*. The lower part instantiates *Fuzz-Onto* in a domain ontology that describes cars. Instances of the concept *fuz:FuzzyRelationAssociation* assign a membership degree to a fuzzy relationship between two individuals (subject and object of the relationship). In the domain ontology the instance of *fuz:FuzzyRelationAssociation* named *car:fuzzyIsSimilarToastra_fordFocus* represents that the domain-specific individual *car:astra* is related to another individual *car:fordFocus* by the fuzzy relationship *car:isSimilarTo* with the membership degree of 0.8 expressing the strength of their relationship.

C. Representing Fuzzy Properties

In order to model fuzzy properties *Fuzz-Onto* was extended with elements inspired by the model proposed by Reformat and Ly [25] to represent linguistic variables, linguistic terms and parameterized membership functions. Besides the elements *fuz:hasMembershipDegree* and *fuz:Real* previously described, the vocabulary for representing fuzzy properties also includes:

- ***fuz:FuzzyVariable*** (concept): represents linguistic variables which can be characterized by linguistic values represented by fuzzy sets. Each instance of *fuz:FuzzyVariable* should be associated to a fuzzy property by the *fuz:hasFuzzyProperty* relationship and to one or more linguistic terms by the *fuz:hasFuzzyTerm* relationship;

- ***fuz:FuzzyProperty*** (concept): represents fuzzy properties which correspond to attributes of the ontology defined in a continuous domain (e.g. real) that can be characterized by linguistic values. If an attribute in a domain ontology is a fuzzy property, it should be an instance of *fuz:FuzzyProperty* and also be related to a fuzzy variable (*fuz:FuzzyVariable*) to denote that its values can be linguistic terms represented by fuzzy sets;

- ***fuz:hasFuzzyProperty*** (relationship): associates an instance of *fuz:FuzzyVariable* to a fuzzy property;

- ***fuz:FuzzyTerm*** (concept): represents linguistic values that are associated to linguistic variables. In a domain ontology, each instance of *fuz:FuzzyTerm* should be related to a linguistic variable by the *fuz:hasFuzzyTerm* relationship and to a fuzzy set, defined by a parameterized membership function, by the *fuz:hasMembershipFunction* relationship;

- ***fuz:hasFuzzyTerm*** (relationship): associates an instance of *fuz:FuzzyVariable* to a linguistic term;

- ***fuz:MembershipFunction*** (concept): represents the membership function of the fuzzy set representing a linguistic term. So far, *Fuzz-Onto* represents two different types of membership functions (subclasses *fuz:Triangular* and *fuz:Trapezoidal*); it is possible to include other types by adding subclasses to the *fuz:MembershipFunction* concept. In *Fuzz-Onto*, membership functions are parameterized, so they should be related to parameters depending on the function (e.g. three parameters for triangular functions and four parameters for trapezoidal functions) by relationships such as *fuz:leftParameter*, *fuz:centerParameter* and *fuz:rightParameter*;

- ***fuz:hasMembershipFunction*** (relationship): associates a linguistic term to its corresponding membership function;

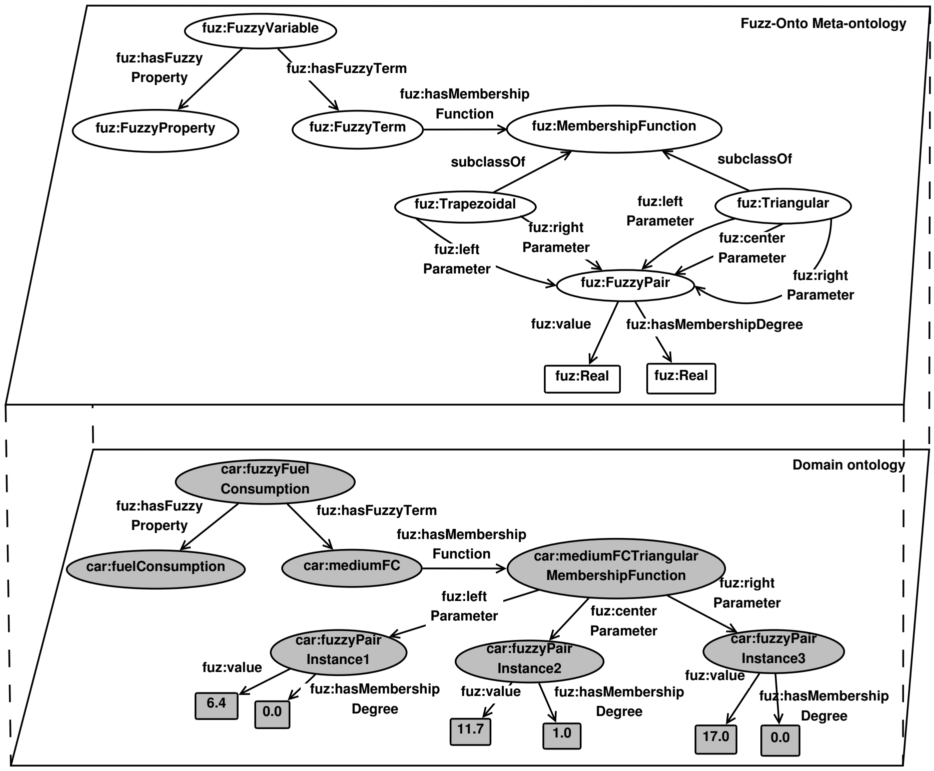

Figure. 3: Representing a fuzzy property and one of its linguistic terms using *Fuzz-Onto*.

- ***fuz:Trapezoidal*** (concept): represents trapezoidal membership functions, defined by four parameters corresponding to the corners of the trapezium: two parameters for representing the left side of the trapezium (two *fuz:leftParameter* relationships) and two parameters for the right side of the trapezium (two *fuz:rightParameter* relationships);

- ***fuz:Triangular*** (concept): represents triangular membership functions, defined by three parameters corresponding to the three corners of a triangle: (1) left corner (*fuz:leftParameter*), (2) center corner (*fuz:centerParameter*) and (3) right corner (*fuz:rightParameter*);

- ***fuz:FuzzyPair*** (concept): represents a pair (value, membership degree) which maps a value in the domain of a membership function to its membership degree. Each instance of *fuz:FuzzyPair* represents a parameter related to a membership function, and should always be associated to a numerical value in the domain of the function by the *fuz:value* attribute and to a membership degree by the *fuz:hasMembershipDegree* attribute;

- ***fuz:value*** (attribute): represents a numerical value in the domain of the membership function associated to an instance of *fuz:FuzzyPair*, represented as a real number (*fuz:Real*);

- ***fuz:leftParameter*** (relationship): associates a membership function to a parameter related to a left corner of the geometrical shape that graphically represents the function (i.e., triangular or trapezoidal). Thus, it should associate an instance of *fuz:MembershipFunction* to an instance of *fuz:FuzzyPair*, which represents the value of the parameter;

- ***fuz:centerParameter*** (relationship): associates a triangular membership function to a parameter related to the center corner of the triangle. Thus, it should associate an instance of *fuz:Triangular* to an instance of *fuz:FuzzyPair*, which represents the value of the parameter;

- ***fuz:rightParameter*** (relationship): associates a membership function to a parameter related to a right corner of the geometrical shape that graphically represents the function (i.e., triangular or trapezoidal). Thus, it should associate an instance of *fuz:MembershipFunction* to an instance of *fuz:FuzzyPair*, which represents the value of the parameter.

Figure 3 shows a graphical representation of fuzzy properties in *Fuzz-Onto*. The upper part of the figure shows the elements *fuz:FuzzyVariable*, *fuz:FuzzyProperty*, *fuz:FuzzyTerm* and *fuz:MembershipFunction* used to represent linguistic variables and linguistic terms.

The lower part of Figure 3 illustrates a domain ontology about cars. It describes an instantiation of *Fuzz-Onto* that models the fuzzy property named *car:fuelConsumption* representing the consumption in kilometers per liter of fuel (km/l). The fuzzy property *car:fuelConsumption* should be related to an instance of *fuz:FuzzyVariable* (*car:fuzzyFuelConsumption*) in order to be characterized by linguistic values. A possible linguistic value is *medium* represented by the instance of *fuz:FuzzyTerm* named *car:mediumFC*, associated to the individual *car:fuzzyFuelConsumption*. In the domain ontology, *car:mediumFC* is represented by a fuzzy set with a triangular membership function denoted by the instance of *fuz:Triangular* named *car:mediumFCTriangularMembershipFunction*. Its three parameters are represented as instances of *fuz:FuzzyPair* (*car:fuzzyPairIn stance1*, *car:fuzzyPairInstance2* and *car:-fuzzyPairInstance3*), each one associating a value in the domain of the property *car:fuelConsumption* to its membership degree according to the function *car:mediumFCTriangular-MembershipFunction*.

The method employed by *Fuzz-Onto* to represent fuzzy concepts, fuzzy relationships and fuzzy properties is based on abstract representations, thus it is independent of ontology language. Therefore, *Fuzz-Onto* can be instantiated using traditional ontology languages that model representational primitives such as concepts, individuals, attributes, concrete domains and binary relationships. This is a relevant contribution in comparison to some related work presented in Section II that extend the syntax of current ontology languages to represent vagueness.

D. Supporting Fuzzy Classification Rules

The fuzzy properties and linguistic terms modeled using *Fuzz-Onto* can be used in fuzzy classification rules, similarly to what it is done in the Fuzzy Rule-Based Classification Systems. Generally, rules are combined with elements represented in ontologies, increasing the expressiveness of the domain representation. In particular, dealing with fuzzy classification rules makes it possible to use the Classical and General Fuzzy Reasoning Methods [44], so that individuals of an ontology can be classified according to the values of their fuzzy properties. Considering the example about cars (Figure 3), a domain ontology can be combined with fuzzy rules that classify individuals (instances of cars) based on fuzzy properties, such as fuel consumption, power and weight (Listing 1).

Listing 1: Fuzzy rules to classify cars.

```
If fuelConsumption is high and power is high
  and weight is low then class is SportCar

If fuelConsumption is medium and power is high
  and weight is high then class is UtilityVehicle

If fuelConsumption is low and power is low
  and weight is low then class is CompactCar
```

The rules in Listing 1 contain a conjunction of fuzzy propositions (composed of a fuzzy property and a linguistic value) in the antecedent part and a concept from the ontology in the consequent part. The linguistic values *low*, *medium* and *high* should be defined in the domain ontology by using the elements *fuz:FuzzyVariable*, *fuz:FuzzyTerm* and the subclasses of *fuz:MembershipFunction*, as it was illustrated in Figure 3 for the property fuel consumption and its linguistic term *medium* (*car:mediumFC*). Therefore, the rules refer to linguistic terms described in the ontology using *Fuzz-Onto*. The classes in the consequent part are also defined in the ontology and can be either crisp or fuzzy.

To support reasoning over fuzzy classification rules, a reasoner was implemented based on the Classical and the General Fuzzy Reasoning Methods, commonly used in Fuzzy Rule-Based Classification Systems. Although a rule can have a fuzzy concept in the consequent part, both reasoning methods will infer crisp classes. As the reasoner implements both reasoning methods, it is possible to select which one is more suitable depending on the application.

Some practical aspects were considered for the development of the reasoner, such as the ontology language for instantiating the *Fuzz-Onto* meta-ontology and the tools and frameworks available for applications. Regarding the ontology language, OWL [17] was chosen because it is officially recommended by the *World Wide Web Consortium* for Semantic Web applications, and has plenty of resources available for developing ontology-based applications. For the implementation of the reasoner, Jena [48] was used since it is an open-source framework based on the OWL and also provides support for traditional rule-based reasoning. Thus, Jena was used to implement the fuzzy reasoning methods considering fuzzy properties and linguistic terms modeled in OWL. Notice that the choice for OWL and Jena was just a matter of implementation; other languages and technologies could be employed as well. Listing 2 shows the fuzzy rules involving fuel consumption, power and weight, represented using the Jena rule syntax.

Listing 2: Fuzzy classification rules in the Jena rule syntax.

```
[rule1: (?x car:fuelConsumption car:highFC),
(?x car:power car:highP),
(?x car:weight car:lowW) ->
(?x rdf:type car:SportCar)]

[rule2: (?x car:fuelConsumption car:mediumFC),
(?x car:power car:highP),
(?x car:weight car:highW) ->
(?x rdf:type car:UtilityVehicle)]

[rule3: (?x car:fuelConsumption car:lowFC),
(?x car:power car:lowP),
(?x car:weight car:lowW) ->
(?x rdf:type car:CompactCar)]
```

To show a real application of *Fuzz-Onto* and the implemented fuzzy reasoning methods, Section IV describes a case study involving the classification of textual documents considering categories, fuzzy properties and linguistic terms modeled in a fuzzy ontology. The categories of documents are inferred by applying fuzzy reasoning methods that analyze fuzzy properties and fuzzy rules extracted from the documents. Information retrieval applications can use such classification to improve query results.

IV. Case Study: Classification of Scientific Documents

Vagueness is intrinsically present in textual information, stressed also by the many different ways and levels of depth readers and writers approach a text. As previously discussed, ontologies can be combined with fuzzy set concepts to support techniques for handling vague information commonly present in many real-world applications. This section describes a case study reporting the articulation of *Fuzz-Onto* and fuzzy classification methods in a real-world scenario involving textual information.

The case study aimed at the classification of scientific documents contained in proceedings of the ACM Digital Library[1]. A collection of 100 documents was chosen from four Computer Science related subareas (25 per subarea) namely: (1) Mobile Multimedia (MM), (2) Virtual Reality (VR), (3) Data Management (DM) and (4) Software Engineering (SE). The subareas were modeled in an ontology based on the ACM Computing Classification System[2] (ACM Taxonomy), a classification reference for scientific documents in the Computer Science research area. The version of the ACM Taxonomy considered for the case study is provided by the IEEE Computer Society, which extends the original ACM version of

[1] http://dl.acm.org/

[2] http://www.computer.org/portal/web/publications/acmtaxonomy

Table 1: Document-cluster matrix generated by FCM.

C1	C2	C3	C4	C5	C6	C7	C8	C9	Class
0.1113	0.1106	0.1113	0.1113	0.1113	0.1107	0.1113	0.1113	0.1107	MM
0.1112	0.1112	0.1112	0.1112	0.1112	0.1109	0.1112	0.1112	0.1109	SE
0.1112	0.1109	0.1112	0.1112	0.1112	0.1110	0.1112	0.1112	0.1109	VR
0.0004	0.9965	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	SE
0.1117	0.1094	0.1117	0.1117	0.1117	0.1102	0.1117	0.1117	0.1100	DM
0.1112	0.1110	0.1112	0.1112	0.1112	0.1109	0.1112	0.1112	0.1110	VR

1998. Part of the ACM Taxonomy was modeled in the ontology considered for the case study, as shown in Figure 4.

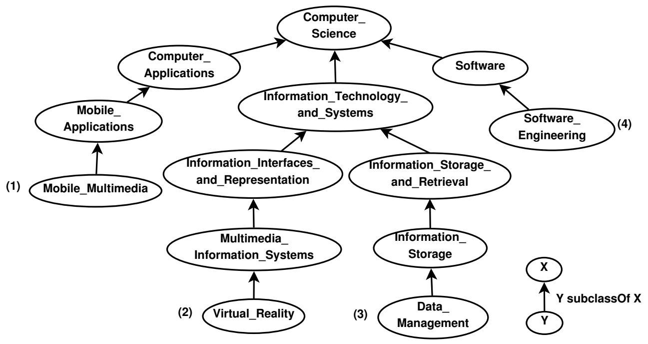

Figure. 4: Computer Science subareas modeled as classes in the ontology.

The case study, by articulating *Fuzz-Onto* with fuzzy classification methods, focused on:

1. modeling the fuzziness embedded in the description of documents using *Fuzz-Onto* and
2. providing a mechanism for inferring the class of a document using a fuzzy classification method based on fuzzy rules.

The employed methodology for this particular case study was organized into three main steps: (1) pre-processing the collection of documents aiming at extracting relevant information to characterize each of them and then, grouping them based on their similarities; (2) modeling the nine groups obtained in step (1) using *Fuzz-Onto* and finally (3) using fuzzy classification methods for classifying documents. The next three subsections detail each of them.

A. Document Pre-Processing

The preprocessing step firstly focuses on two concepts: (1) stopwords and (2) representative terms. Stopwords are words that are not relevant in the analysis of texts and usually consist of prepositions, pronouns, articles, interjections, among others. Representative terms are words that supposedly characterize the text.

The dataset containing the 100 documents was preprocessed using the Pretext tool [49]. Each document was stripped of stopwords and had its most representative terms identified (using 2-grams, i.e., terms represented by 2 consecutive words).

The 100 document collection was then represented as a two-dimensional matrix ($d - t$ matrix), each line corresponding to a document and each column to a specific representative term. The cells were filled with the value of the $tf\text{-}idf$ (Term Frequency-Inverse Document Frequency) metric, which represents the ratio between the frequency of a particular term in the collection and the inverse of the frequency of this term in the document.

The obtained $d - t$ matrix, however, was very sparse and had an excessive number of columns. These characteristics can make the analysis process computationally expensive and, sometimes, even impossible, and frequently cause a negative impact on the outcome of some algorithms used for knowledge extraction.

To circumvent the problem, the dimensionality of this matrix was reduced by clustering the documents and generating a document-cluster matrix, according to the approach proposed by Nogueira *et al.* [50, 51]. The goal in applying a clustering algorithm is to reveal groups of similar documents that are likely to refer to the same topic. Since the Fuzzy C-Means (FCM) Clustering algorithm [52] is used, the documents can be associated to more than one topic with different degrees, reflecting the intrinsic vagueness that characterizes textual information and favoring the textual organization. The FCM was used having as parameter values: fuzzification rate = 1.25 and convergence rate = 0.01. The best number of clusters was chosen using an extension of a simplified version of the Average Silhouette Width Criterion [53], named Fuzzy Silhouette (FS) [54].

Table 1 presents a snippet of the document-cluster matrix. Each row represents a document. Columns C1 to C9 present the membership degrees of each document in each cluster, which are meant to represent the compatibility of the document with the topic associated to each cluster. Note that the last column contains the document class that corresponds to the class defined in the ontology (Figure 4), where SE stands for Software Engineering, VR for Virtual Reality, DM for Data Management and MM for Mobile Multimedia. The topic associated with each cluster is identified by a set of descriptors obtained from the method proposed by Nogueira *et al.* [55, 56]. Table 2 shows examples of descriptors per cluster.

The compatibility of documents in relation to the identified clusters can be represented by means of linguistic terms, which resembles in a more appropriate way the indication of importance given by human beings. For example, a document can be *very much*, or a *bit* compatible with a particular topic. Moreover, expressing the importance of documents with relation to a topic using linguistic terms allows the generation of fuzzy rules that can be used for document classification.

The following subsection describes how the ontology is modeled by instantiating *Fuzz-Onto* elements to represent clusters and their linguistic terms.

Table 2: Fuzzy clusters descriptors

Cluster	Descriptors
C1	wireless, networks, virtual, reality
C2	engineering, methods, automated, software
C3	mobile, computing, access, control
C4	peer, virtual, environments
C5	software, engineering, virtual, reality
C6	reality, applications, mixed
C7	performance, evaluation, wireless, networks
C8	mobile, computing, virtual, reality
C9	queries, published, match

B. Fuzzy Ontology Modeling

After pre-processing the document collection, the following step is modeling fuzzy properties and linguistic terms in the ontology. In this case study fuzzy properties correspond to the association between documents and the identified clusters. Fuzzy properties are named C1 to C9, corresponding to the clusters identified by FCM. The properties can be characterized by five linguistic terms: *very low*, *low*, *medium*, *high* and *very high*, defined by uniformly distributed triangular membership functions (Figure 5).

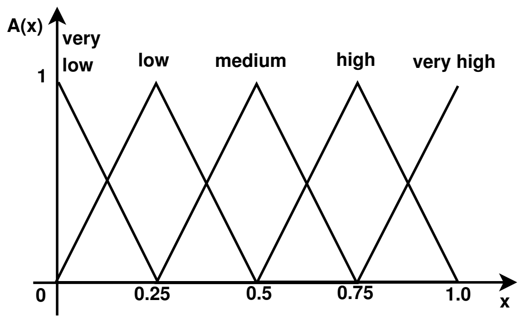

Figure. 5: Fuzzy sets representing the linguistic terms used for characterizing the fuzzy properties C1 to C9.

In the ontology, the linguistic terms *very low*, *low*, *medium*, *high* and *very high* can be represented only once as they have the same parameters for all fuzzy properties. Listing 3 illustrates how the linguistic term *medium* is represented by instantiating *Fuzz-Onto* in OWL, based on the representation described in Section III-C. Note that the fuzzy set parameters in the listing correspond to those ones illustrated in Figure 5. The other four linguistic terms are similarly represented.
Listing 4 shows the OWL representation of the fuzzy property C1, corresponding to the cluster C1, as an attribute of the ontology that associates a document to a real number (membership degree to C1) represented by the datatype *float*. To be characterized by linguistic terms, C1 should be a fuzzy property, so it is subsumed by the *Fuzz-Onto* element *fuz:FuzzyProperty*. The association of C1 to its possible linguistic values is done by an instance of *fuz:FuzzyVariable* (*C1_fuzzy_variable*), according to *Fuzz-Onto*. The other fuzzy properties (C2 to C9) are represented in a similar way.

Listing 3: OWL representation of the linguistic term *medium* using *Fuzz-Onto*.

```
<fuz:FuzzyTerm rdf:ID="medium">
  <fuz:hasMembershipFunction>
    <fuz:Triangular>
      <fuz:leftParameter>
        <fuz:fuzzyPair>
          <fuz:value rdf:datatype="&xsd;float">
              0.25
          </fuz:value>
          <fuz:hasMembershipDegree rdf:datatype="&xsd;float">
              0.0
          </fuz:hasMembershipDegree>
        </fuz:fuzzyPair>
      </fuz:leftParameter>
      <fuz:centerParameter>
        <fuz:fuzzyPair>
          <fuz:value rdf:datatype="&xsd;float">
              0.5
          </fuz:value>
          <fuz:hasMembershipDegree rdf:datatype="&xsd;float">
              1.0
          </fuz:hasMembershipDegree>
        </fuz:fuzzyPair>
      </fuz:centerParameter>
      <fuz:rightParameter>
        <fuz:fuzzyPair>
          <fuz:value rdf:datatype="&xsd;float">
              0.75
          </fuz:value>
          <fuz:hasMembershipDegree rdf:datatype="&xsd;float">
              0.0
          </fuz:hasMembershipDegree>
        </fuz:fuzzyPair>
      </fuz:rightParameter>
    </fuz:Triangular>
  </fuz:hasMembershipFunction>
</fuz:FuzzyTerm>
```

Listing 4: OWL representation of the fuzzy property C1 associated to its linguistic values.

```
<owl:DatatypeProperty rdf:ID="C1">
   <rdfs:range rdf:resource="&xsd;float" />
   <rdfs:subPropertyOf rdf:resource="&fuz;FuzzyProperty"/>
</owl:DatatypeProperty>

<fuz:FuzzyVariable rdf:ID="C1_fuzzy_variable">
   <fuz:hasFuzzyProperty rdf:resource="#C1" />
   <fuz:hasFuzzyTerm rdf:resource="#very_low" />
   <fuz:hasFuzzyTerm rdf:resource="#low" />
   <fuz:hasFuzzyTerm rdf:resource="#medium" />
   <fuz:hasFuzzyTerm rdf:resource="#high" />
   <fuz:hasFuzzyTerm rdf:resource="#very_high" />
</fuz:FuzzyVariable>
```

Once all fuzzy properties (C1 to C9) and their linguistic terms were represented in the ontology using *Fuzz-Onto*, they could be used in fuzzy classification rules as described in Subsection IV-C.

C. Rule Extraction and Document Classification

To extract fuzzy rules from the documents, Nogueira *et al.* [50] developed a mechanism based on the document-cluster matrix (Table 1) so that it is possible to classify documents using a smaller search space. According to this mechanism, the rules were generated by applying the well-known Wang&Mendell (WM) [57] method over the document-cluster matrix considering the five fuzzy sets presented in Figure 5.
Each fuzzy rule generated for this particular case study has in its antecedent a conjunction of nine fuzzy propositions and in its consequent a class. Thus, documents can be classified based on their association to clusters, characterized by linguistic terms. Since each cluster is described by a set of descriptors (Table 2) the rules support the fuzzy interpretation of documents, in addition to the classification task.

After extracting fuzzy rules, fuzzy reasoning methods can be applied to classify individuals in the ontology. In this case study, individuals correspond to scientific documents from the ACM collection, which can be classified considering the classes presented in Figure 4. Some tests were carried out based on the reasoner and the ACM collection. Attempting to achieve an estimate error closer to the actual error, a 5-fold cross validation method was used. To evaluate the performance of the classification, the correct classification rate baseline ME was used, where $ME = 25\%$ is the Majority Error of the collection. The correct classification rate obtained using the Classical Fuzzy Reasoning method was 44%. For the tests, the generated rules were modeled using the Jena rule syntax, taking into account fuzzy properties and linguistic terms represented in the ontology. For illustration purposes, Listing 5 shows the seven rules generated in a specific fold.

Listing 5: Fuzzy rules generated by Wang&Mendell method.

```
[rule1:
(?x C1 very_high), (?x C2 very_low), (?x C3 very_high),
(?x C4 very_high), (?x C5 very_high), (?x C6 very_low),
(?x C7 very_high), (?x C8 very_high), (?x C9 very_low)
-> (?x rdf:type Virtual_Reality)]

[rule2:
(?x C1 very_low), (?x C2 very_high), (?x C3 very_low),
(?x C4 very_low), (?x C5 very_low), (?x C6 very_low),
(?x C7 very_low), (?x C8 very_low), (?x C9 very_low)
-> (?x rdf:type Software_Engineering)]

[rule3:
(?x C1 very_high), (?x C2 very_low), (?x C3 very_high),
(?x C4 very_high), (?x C5 very_high), (?x C6 very_low),
(?x C7 very_high), (?x C8 very_high), (?x C9 very_low)
-> (?x rdf:type Data_Management)]

[rule4:
(?x C1 very_low), (?x C2 very_low), (?x C3 very_low),
(?x C4 very_low), (?x C5 very_low), (?x C6 very_low),
(?x C7 very_low), (?x C8 very_low), (?x C9 very_high)
-> (?x rdf:type Data_Management)]

[rule5:
(?x C1 very_high), (?x C2 very_low), (?x C3 very_high),
(?x C4 very_high), (?x C5 very_high), (?x C6 very_low),
(?x C7 very_high), (?x C8 very_high), (?x C9 very_low)
-> (?x rdf:type Mobile_Multimedia)]

[rule6:
(?x C1 very_low), (?x C2 very_low), (?x C3 very_low),
(?x C4 very_low), (?x C5 very_low), (?x C6 very_high),
(?x C7 very_low), (?x C8 very_low), (?x C9 very_low)
-> (?x rdf:type Data_Management)]

[rule7:
(?x C1 very_high), (?x C2 very_low), (?x C3 very_high),
(?x C4 very_high), (?x C5 very_high), (?x C6 very_low),
(?x C7 very_high), (?x C8 very_high), (?x C9 very_low)
-> (?x rdf:type Software_Engineering)]
```

It is important to highlight that this proposal does not intend to evaluate results regarding the classification methods, since they are well-known algorithms used in Fuzzy Rule-Based Classification Systems. Moreover, the *Fuzz-Onto* abstract representation does not interfere in the classification performance of these methods. The main goal of the research developed and described in this paper is to show that *Fuzz-Onto* is a meta-ontology of easy understanding and use that can be successfully applied for modeling vagueness in ontologies. As an extra advantage, it can be articulated to fuzzy rule-based inference methods, a feature that makes it a very convenient fuzzy modeling tool for ontology-based applications. Finally, this case study illustrated that *Fuzz-Onto* contributed to represent the vagueness present in textual documents by means of fuzzy ontologies and fuzzy classification rules. As the meta-ontology focuses on the instantiation of fuzzy elements instead of extending ontology languages, it saves time and effort during fuzzy ontology modeling.

V. Conclusion and Future Work

In this paper the *Fuzz-Onto* meta-ontology for representing fuzzy elements in ontologies was proposed and described. Based on the proposed model the paper discusses the viability of representing vagueness by means of fuzzy concepts, fuzzy relationships and fuzzy properties characterized by linguistic terms in ontologies. The paper investigates how fuzzy properties modeled using *Fuzz-Onto* can be used in fuzzy classification rules and how fuzzy reasoning methods can be considered for classifying individuals of ontologies according to values of their fuzzy properties.

As *Fuzz-Onto* is an abstract representation, it can be instantiated using any traditional ontology language that models the basic representational primitives. Therefore, fuzzy concepts, fuzzy relationships and fuzzy properties can be represented in domain ontologies in a straightforward way, by instantiating and inheriting fuzzy elements from *Fuzz-Onto*.

Regarding implementation issues, *Fuzz-Onto* was instantiated using Semantic Web technologies (OWL and Jena), so that it can be also used in Semantic Web applications. Furthermore, two fuzzy reasoning methods (Classical and General) were implemented providing an articulation of fuzzy modeling in ontologies and fuzzy rule-based classification. Using this platform a case study involving the description and classification of scientific documents using fuzzy concepts was presented.

The results obtained from the case study show that the management of vagueness is a promising approach not only for classification but also for organization of the textual documents. However, more tests should be carried out towards this kind of application. Regarding the classification of documents, it is relevant to investigate the impact of the parameters related to the document pre-processing and the rule extraction steps in the document organization.

As a continuation of the work described in this paper, a few other applications of *Fuzz-Onto* are being planned to explore its fuzzy modeling capabilities as well as to identify how far it can be further extended. For example, fuzzy complex concepts and modifiers can be considered as extensions of *Fuzz-Onto*, along with other types of rules and their respective fuzzy reasoning methods (for instance the Mamdani method), as well as defuzzification methods.

Acknowledgments

This work is supported by CAPES Brazilian research agency (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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