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Article

GMJ CODING TECHNIQUES ON EVEN FELICITOUS LABELING FOR SIX STAR GRAPH

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Abstract: In this work, we have found a way to encode a secret message utilizing the GMJ (Graph Message Jumbled) Code and even felicitous labeling on Six-star graphs $K_{1,\rho_1} \cup K_{1,\rho_2} \cup K_{1,\rho_3} \cup K_{1,\rho_4} \cup K_{1,\rho_5} \cup K_{1,\rho_6}$. Using a Java application for the Keith Number and Mystery Number programs, each program uses alphabetical techniques, and we give two instances for every star graph. An equally felicitous graph and flowchart labeling method for transforming plaintext is explained (Picture Coding).

Keywords: Even felicitous labeling, Coding, six-star graph.

AMS Subject Classification (2020): 05C78,05C90

1. Introduction

Graph labeling has developed into a vibrant branch of graph theory in which integers are assigned to vertices or edges of a graph under specific mathematical rules. Among the various labeling schemes studied over the years, *felicitous labelings* has attracted considerable attention as a natural extension of graceful and harmonious labelings. The concept was formally introduced in [8], where a graph $G = (V, E)$ with q edges is said to be felicitous if there exists a vertex labeling $f: V \rightarrow \{0, 1, . . ., q\}$ such that the induced edge labels, defined by modular addition of the end vertices, are all distinct modulo q. This additive modular structure places felicitous labeling within the broader framework of arithmetic graph labelings, closely related to other classical labeling families discussed in [6]. Researchers have focused on identifying graph classes that admit felicitous labelings and developing constructive techniques to obtain them. Structural results and necessary conditions for connected graphs were presented in [11].

The increasing use of computational methods in graph labeling, including algorithmic verification and construction, was emphasized in [7], reflecting the importance of programming approaches in handling complex graph families. Recent computational perspectives continue this direction, as seen in [13], where programming techniques were used to analyze felicitous graphs.

Significant contributions to felicitous labeling have been made by Indian researchers. In [4], explicit construction were provided for H_n and $H_n S_m$ graphs, proving that these families admit felicitous labeling. This line of work was extended in [2], where several standard graph families such as paths, cycles, and stars were shown to be felicitous through constructive methods. Recognizing the restrictive nature of strict modular distinctness, [3] introduced the notion of *near-felicitous labeling*, relaxing the edge-label condition while preserving essential structural properties.

Comparative investigations have clarified how felicitous labeling related to other labeling schemes. In [14], path and star related graphs were studied under even sequential harmonious, graceful, odd graceful, and felicitous labeling frameworks, highlighting structural similarities and differences. Extension such as *even felicitous labeling*, where vertex labels are restricted to even integers, were introduced in [15]. Further developments examined more complex graph constructions, including star-merged shell graphs in [18] and super felicitous difference labeling graphs in [20]. Beyond structural theory, felicitous labeling has found meaningful applications in coding theory and communication systems. Coding techniques based on graph labeling was developed using Fibonacci webs and GMJ coding in [10]. Even felicitous labeling was applied to bistar graphs for GMJ coding in [12] and to two-star graphs in [19]. Further related coding frameworks include caterpillar graphs constructions in [17], AMGL coding methods in [16], and additional star-based coding techniques in [9]. These studies demonstrate how vertex and edge labelings can serve as systematic encoding mechanisms for structured communication models.

The applicability of felicitous labeling has also been extended to fuzzy environments. In [1], the concept of a *felicitous fuzzy graph* was introduced, where vertex labels are fuzzy numbers and induced edge labels are determined through suitable fuzzy operations. This extension connects classical graph labeling with fuzzy set theory, enabling applications in uncertain or imprecise network structures.

Moreover, felicitous labeling has been studied in network contexts. In [21], several layered and star-based network models were shown to admit felicitous labelings, further illustrating the adaptability of this labeling scheme to practical graph structures.

In summary, felicitous labeling has grown from a modular arithmetic labeling concept into a broad and dynamic research area. It encompasses structural characterizations, relaxed and parity-based variants, computational approaches, coding applications, fuzzy extensions and network models. The sustained and diverse contributions in the literature demonstrate that felicitous labeling remains an active and evolving topic within modern graph theory.

2. Preliminaries

Definition 2.1 (Even Felicitous [15]). *A graph G with q edges is said to admit an even felicitous labeling if there exists a function $G: V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ such that the induced edge labels defined by*

$$G^*(rs) = \{G(r) + G(s)\} \bmod (2q-1),$$

for every edge $rs \in E(G)$, are distinct and belong to $\{0, 2, 4, \ldots, 2q-2\}$.

Definition 2.2 (Felicitous [8]). *A graph G is said to be felicitous if there exists an injection $f: V(G) \to \{0, 1, 2, \ldots, q\}$ such that the induced function $f^*: E(G) \to \{0, 1, 2, \ldots, q-1\}$ defined by*

$$f^*(uv) = \{f(u) + f(v)\} \ (\text{mod } q),\ uv \in E(G),$$

is a bijection. The function f is called a felicitous labeling of G. The values $f(u)$ and $f^*(uv)$ are called the vertex label of u and the edge label of uv, respectively.

Definition 2.3 (Wedge [5]). *A wedge is an edge that connects two subgraphs or components of a graph. It is often used to describe a linking structure that reduces the number of connected components of a graph.*

Definition 2.4 (Bistar [12]). *The bistar graph $B_{\alpha,\beta}$ is obtained by attaching α pendant vertices to one vertex of K_2 and β pendant vertices to the other vertex of K_2. The edge of K_2 is called the central edge of $B_{\alpha,\beta}$, and the vertices of K_2 are called the central vertices.*

3. Even Felicitous Labeling on Six Star Graphs

An even felicitous graph is the six-star graph $K_1, \rho_1 \cup K_1, \rho_2 \cup K_1, \rho_3 \cup K_1, \rho_4 \cup K_1, \rho_5 \cup K_1, \rho_6$.

Let $G = K_1, \rho_1 \cup K_1, \rho_2 \cup K_1, \rho_3 \cup K_1, \rho_4 \cup K_1, \rho_5 \cup K_1, \rho_6$.

Let the vertex sets be defined as in Eq. (1):

$\{\omega\} \cup \{\omega_s: 1 \le s \le \Omega_1\}$; $\{\pi\} \cup \{\pi_t: 1 \le t \le \Omega_2\}$; $\{\varphi\} \cup \{\varphi_u: 1 \le u \le \Omega_3\}$;
$\{\delta\} \cup \{\delta_v: 1 \le v \le \Omega_4\}$; $\{\varepsilon\} \cup \{\varepsilon_w: 1 \le w \le \Omega_5\}$; $\{\mu\} \cup \{\mu_l: 1 \le l \le \Omega_6\}$.

These establish the G nodes. Then G has $\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Omega_5 + \Omega_6 + 6$ nodes and $\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Omega_5 + \Omega_6$ links.

We have $\Omega(G) = \{\omega + \pi + \varphi + \delta + \varepsilon + \mu\} \cup \{\omega\} \cup \{\omega_s: 1 \le s \le \Omega_1\}$;

$\{\pi\} \cup \{\pi_t: 1 \le t \le \Omega_2\}$; $\{\varphi\} \cup \{\varphi_u: 1 \le u \le \Omega_3\}$;

$\{\delta\} \cup \{\delta_v: 1 \le v \le \Omega_4\}$; $\{\varepsilon\} \cup \{\varepsilon_w: 1 \le w \le \Omega_5\}$;

$\{\mu\} \cup \{\mu_l: 1 \le l \le \Omega_6\}$.

It is necessary to demonstrate that G is an even felicitous graph (see Definition 2.1). The node labeling $\tau: \Omega(G) \to \{0, 1, 2, \ldots, 2q-1\}$ is defined as follows (Eq. 3):

$\tau(\omega) = 5$; $\tau(\omega_s) = 2\Omega_2 + 2\Omega_3 + 2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2s - 11, \quad 1 \le s \le \Omega_1$.
$\tau(\pi) = 1$; $\tau(\pi_t) = 2\Omega_1 + 2\Omega_3 + 2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2t - 1, \quad 1 \le t \le \Omega_2$.
$\tau(\varphi) = 4$; $\tau(\pi_{\Omega 2}) = 2q - 2$.
$\tau(\delta) = 3$; $\tau(\varphi_u) = 2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2u - 4, \quad 1 \le u \le \Omega_3$.
$\tau(\varepsilon) = 2q-1$; $\tau(\delta_v) = 2\Omega_5 + 2\Omega_6 + 2v - 3, \quad 1 \le v \le \Omega_4$.
$\tau(\varepsilon_\omega) = 2\Omega_6 + 2\omega, \quad 1 \le \omega \le \Omega_5$.
$\tau(\mu) = 3$; $\tau(\mu_\iota) = 2\iota, \quad 1 \le \iota \le \Omega_6$.

The following are the labels for the similar links:

- The link label of $\omega\omega_s$ is $2\Omega_2 + 2\Omega_3 + 2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2s - 11,\ 1 \le s \le \Omega_1$.
- The link label of $\pi\pi_t$ is $2\Omega_1 + 2\Omega_3 + 2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2t - 1,\ 1 \le t \le \Omega_2$.
- The link label of $\pi\pi_{\Omega 2}$ is 0.
- The link label of $\varphi\varphi_u$ is $2\Omega_4 + 2\Omega_5 + 2\Omega_6 + 2u - 4,\ 1 \le u \le \Omega_3$.
- The link label of $\delta\delta_v$ is $2\Omega_5 + 2\Omega_6 + 2v - 3,\ 1 \le v \le \Omega_4$.
- The link label of $\varepsilon\varepsilon_\omega$ is $2\Omega_6 + 2\omega,\ 1 \le \omega \le \Omega_5$.
- The link label of $\mu\mu_\iota$ is $2\iota,\ 1 \le \iota \le \Omega_6$.

4. GMJ Coding Method: Results and Discussions

4.1 *Illustration: Six-Star Graph $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$.*

i. Message: Coding techniques can refer to a variety of methods for encoding data.

ii. Clue: Two, Triangle, Quadrangle, Pentagon, Hexagon and Heptagon are pairing up together. (The number of sides of two, Triangle is 3, Quadrangle is 4, Pentagon is 5, Hexagon is 6, and Heptagon is 7; from this we should understand that it is nothing but a six-star graph $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$.)

iii. Graph: $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$.

iv. Labeling: The even felicitous labeling (Definition 2.1) is carried out for

$K_{1,2} \cup K_{1,3} \cup K_{1,4}\ K_{1,5} \cup K_{1,6} \cup K_{1,7}$.

$$p = 33,\ q = 27,\ 2q-1 = 53,\ 2q-2 = 52. \quad (Eq.\ 4)$$

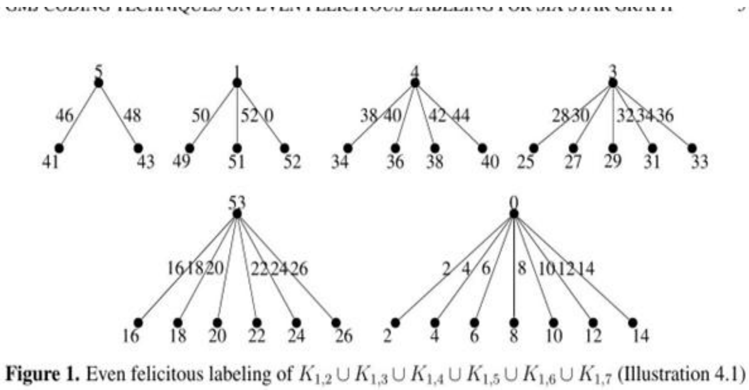

Figure 1. Even felicitous labeling of $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$ (Ilustration 4.1)

v. Keith Number Java Program.

Listing 1. Java program to check Keith Number

```
import java.util.Scanner;

class Keith_Number {
    public static void main(String[] args) {
        Scanner SC = new Scanner(System.in);
        System.out.println("Enter a number");
        int n = SC.nextInt();
        int len = ("" + n).length();
        int[] a = new int[len];
        int temp = n;
        for (int i = len - 1; i >= 0; i--) {
            a[i] = temp % 10;
            temp /= 10;
        }
        int sum = 0;
        while (sum < n) {
            sum = a[0];
            for (int i = 0; i < len - 1; i++) {
                a[i] = a[i + 1];
                sum += a[i];
            }
            a[len - 1] = sum;
        }
        if (sum == n)
            System.out.println(n + " is a Keith Number");
        else
            System.out.println(n + " is Not a Keith Number");
    }
}
```

OUTPUT: Enter a Number

197 is a Keith Number.

198 is Not a Keith Number.

vi. Even Numbering of Alphabets: KNNO (Keith Number Next Others).

Table 1. KNNO alphabet-to-even-number mapping

0	**6**	**8**	**10**	**12**	**14**	**4**	**16**	**2**	**18**	**20**	**22**	**24**	**26**	**28**	**30**	**32**	**34**	**36**	**38**	**40**	**42**	**44**	**46**	**48**	**50**

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

The alphabets in the Java Programming position in Keith Number (A, I, G) are assigned the even numbers (0, 2, 4), whereas the remaining alphabets are awarded an even number between 6 and 50 (see Table 1).

$$R(A_{R_M}) = 2M - 2, M = 1, 2, 3. (Eq. 5)$$

In this case, R_M is the M^{th} Keith Number between 1 and 26, and R_M^{th} is the alphabetical letter for A_{R_M}.

$$S(B_{U_P}) = 2 + 2P, P = 6, 8, \ldots, 50. (Eq. 6\}$$

Here, U_P denotes the P^{th} non-perfect number between 1 and 26, and B_{UP} denotes the U_p^{th} letter of the alphabet. $A_{R_M} \neq B_{U_P}$. Here A_{R_M} and B_{U_P}.represent twenty-six alphabets.

vii. Coding a message:

Let ω^1 represent the 1st Star, ω^2 the 2nd Star, ω^3 the 3rd Star, ω^4 the 4th Star, ω^5 the 5th Star, and ω^6 the 6th Star. Let e^0, $\pi_1 i$, $\pi_2 i$, $\pi_3 i$, $\pi_4 i$, $\pi_5 i$ and $\pi_6 i$ denote the centre node, the i^{th} node value and the i^{th} link value in order.

viii. Coding (word wise):

CODING – $\omega^6\pi_4(2)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^6\pi_6(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(2)$

TECHNIQUES – $\omega^3\pi_3(3)$ e^0 $\omega^6\pi_4(2)$ $\omega^5\pi_5(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(1)$ $\omega^4\pi_4(3)$ $\omega^3\pi_3(4)$ e^0 $\omega^3\pi_3(2)$

CAN – $\omega^6\pi_4(2)$ $\omega^2\pi_5(3)$ $\omega^5\pi_5(6)$

REFER – $\omega^3\pi_6(1)$ e^0 $\omega^6\pi_6(7)$ e^0 $\omega^3\pi_3(1)$

TO – $\omega^3\pi_3(3)$ $\omega^4\pi_4(1)$

A – $\omega^2\pi_5(3)$

VARIETY – $\omega^3\pi_3(3)$ $\omega^2\pi_5(3)$ $\omega^3\pi_6(1)$ $\omega^6\pi_6(1)$ e^0 $\omega^3\pi_3(3)$ $\omega^1\pi_4(2)$

OF – $\omega^4\pi_4(1)$ $\omega^6\pi_6(7)$

METHODS – $\omega^5\pi_5(5)$ e^0 $\omega^3\pi_3(3)$ $\omega^5\pi_5(1)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^4\pi_1(5)$

FOR – $\omega^6\pi_6(7)$ $\omega^4\pi_4(1)$ $\omega^3\pi_6(1)$

ENCODING – e^0 $\omega^5\pi_5(6)$ $\omega^6\pi_4(2)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^6\pi_6(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(2)$

DATA – $\omega^6\pi_6(5)$ $\omega^2\pi_5(3)$ $\omega^3\pi_3(3)$ $\omega^2\pi_5(3)$

ix. Horizontal String:

$\omega^6\pi_4(2)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^6\pi_6(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(2)$ $\omega^3\pi_3(3)$ e^0 $\omega^6\pi_4(2)$ $\omega^5\pi_5(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(1)$ $\omega^4\pi_4(3)$ $\omega^3\pi_3(4)$ e^0 $\omega^3\pi_3(2)$ $\omega^6\pi_4(2)$ $\omega^2\pi_5(3)$ $\omega^5\pi_5(6)$ $\omega^3\pi_6(1)$ e^0 $\omega^6\pi_6(7)$ e^0 $\omega^3\pi_3(1)$ $\omega^3\pi_3(3)$ $\omega^4\pi_4(1)$ $\omega^2\pi_5(3)$ $\omega^3\pi_3(3)$ $\omega^2\pi_5(3)$ $\omega^3\pi_6(1)$ $\omega^6\pi_6(1)$ e^0 $\omega^3\pi_3(3)$ $\omega^1\pi_4(2)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(7)$ $\omega^5\pi_5(5)$ e^0 $\omega^3\pi_3(3)$ $\omega^5\pi_5(1)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^4\pi_1(5)$ $\omega^6\pi_6(7)$ $\omega^4\pi_4(1)$ $\omega^3\pi_6(1)$ e^0 $\omega^5\pi_5(6)$ $\omega^6\pi_4(2)$ $\omega^4\pi_4(1)$ $\omega^6\pi_6(5)$ $\omega^6\pi_6(1)$ $\omega^5\pi_5(6)$ $\omega^6\pi_6(2)$ $\omega^6\pi_6(5)$ $\omega^2\pi_5(3)$ $\omega^3\pi_3(3)$ $\omega^2\pi_5(3)$

x. Picture Coding (KNNO): (see flowchart in Figure 3)

4.2 Illustration 5.2: Six-Star Graph $K^6_{1,5}$

i. Message: Coding techniques can refer to a variety of methods for encoding data.

ii. Clue: Pentagons are partnering up. (Based on the number of sides of pentagons, we may conclude that they are nothing more than a six-star $K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5}$.)

iii. Graph: $K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5}$.

iv. Labeling: Based on Section 2, even felicitous labeling (Definition 2.1) is carried out for $K^6_{1,5}$.

$$p = 36,\ q = 30,\ 2q-1 = 59,\ 2q-2 = 58. \quad (Eq.\ 7)$$

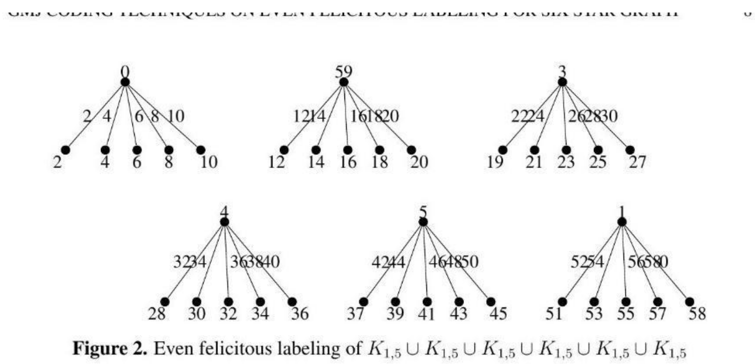

Figure 2. Even felicitous labeling of $K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5} \cup K_{1,5}$

v. Mystery Number Java Program.

If a number can be written as the sum of two numbers, and those two numbers must be the opposite of one another, the number is referred to as a "Mystery Number."

For Example: 59 + 95 = 154.

Listing 2. Java program to check Mystery Number

```
import java.util.Scanner;

class MysteryNumber {
    static int reverse(int x) {
        int rev = 0;
        while (x != 0) {
            int d = x % 10;
            rev = rev * 10 + d;
            x /= 10;
        }
        return rev;
    }

    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        System.out.println("Enter a number");
        int n = sc.nextInt();
        int flag = 0;
        for (int i = 1; i <= n / 2; i++) {
            if (i + reverse(i) == n) {
                flag = 1;
            }
        }
        if (flag == 1)
            System.out.println(n + " is a Mystery Number");
        else
            System.out.println(n + " is not a Mystery Number");
    }
}
```

OUTPUT: Enter a number

154 is a Mystery Number.

vi. Even Numbering of Alphabets: MNNO (Mystery Number Next Others).

Table 2. MNNO alphabet-to-even-number mapping

0	**6**	**8**	**4**	**2**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	**30**	**32**	**34**	**36**	**38**	**40**	**42**	**44**	**46**	**48**	**50**
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

The even number (0, 2, 4) is assigned to alphabets at Java Programming positions in Mystery Number (A, E, D); the remaining alphabets are assigned an even number between 6 and 50 (see Table 2).

$$I(J_{P_S}) = 2S - 2,\ \ S = 1, 2, 3.\ \ (Eq.\ 8)$$

Here, P_s stands for the S^{th} Mystery Number between 1 and 26, and J_{P_S} for the alphabetical letter Ps^{th}.

$$K(L_{H_T}) = 2 + 2T,\ \ T = 6, 8, \ldots, 50.\ \ (Eq.\ 9)$$

In this case, H_t stands for the T^{th} non-perfect number between 1 and 26, and L_{H_T} for the H_T^{th} alphabet letter. $J_{P_S} \neq L_{H_T}$.

vii. Coding a Message (MNNO):

Let $q^1, q^2, \ldots, q^6$ stand for the first through sixth stars, respectively. Let q^0, $\mu_{11}i$, $\mu_{22}i$, $\mu_{33}i$, $\mu_{44}i$, $\mu_{55}i$, and $\mu_{66}i$ represent the center node, the i^{th} node value, and the i^{th} link value, respectively.

viii. Coding (word wise):

ENCODING – $q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(4)$ $q^3\mu_{33}(4)$ $q^1\mu_{11}(2)$ $q^2\mu_{22}(3)$ $q^3\mu_{33}(3)$ $q^2\mu_{11}(1)$

IS – $q^2\mu_{22}(3)$ $q^4\mu_{44}(3)$

OFTEN – $q^3\mu_{33}(4)$ $q^1\mu_{11}(5)$ $q^4\mu_{44}(4)$ $q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$

USED – $q^4\mu_{44}(5)$ $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^1\mu_{11}(2)$

TO – $q^4\mu_{44}(4)$ $q^3\mu_{33}(4)$

CHANGE – $q^1\mu_{11}(4)$ $q^2\mu_{22}(2)$ $q^6\mu_{66}(5)$ $q^3\mu_{33}(3)$ $q^2\mu_{11}(1)$ $q^1\mu_{11}(1)$

INFORMATION – $q^1\mu_{11}(2)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(5)$ $q^3\mu_{33}(4)$ $q^4\mu_{33}(2)$ $q^3\mu_{22}(2)$ $q^6\mu_{66}(5)$ $q^4\mu_{44}(4)$ $q^2\mu_{22}(3)$ $q^3\mu_{33}(4)$ $q^3\mu_{33}(3)$

INTO – $q^2\mu_{22}(3)$ $q^3\mu_{33}(3)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(4)$

A – $q^6\mu_{66}(5)$

SYSTEM – $q^4\mu_{44}(3)$ $q^5\mu_{55}(4)$ $q^4\mu_{44}(3)$ $q^4\mu_{44}(4)$ $q^1\mu_{11}(1)$ $q^3\mu_{22}(2)$

FOR – $q^1\mu_{11}(5)$ $q^3\mu_{33}(4)$ $q^4\mu_{33}(2)$

SENDING – $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(2)$ $q^1\mu_{11}(2)$ $q^3\mu_{33}(3)$ $q^2\mu_{22}(1)$

MESSAGES – $q^3\mu_{22}(2)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(3)$ q^0 $q^6\mu_{66}(5)$ $q^2\mu_{22}(1)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(3)$

SECRETLY – $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^1\mu_{11}(4)$ $q^4\mu_{33}(2)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(1)$ $q^5\mu_{44}(4)$

ix. Horizontal String:

$q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(4)$ $q^3\mu_{33}(4)$ $q^1\mu_{11}(2)$ $q^2\mu_{22}(3)$ $q^3\mu_{33}(3)$ $q^2\mu_{11}(1)$ $q^2\mu_{22}(3)$ $q^4\mu_{44}(3)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(4)$ $q^1\mu_{11}(5)$ $q^4\mu_{44}(4)$ $q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$ $q^4\mu_{44}(5)$ $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^1\mu_{11}(2)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(4)$ $q^1\mu_{11}(4)$ $q^2\mu_{22}(2)$ $q^6\mu_{66}(5)$ $q^3\mu_{33}(3)$ $q^2\mu_{11}(1)$ $q^1\mu_{11}(1)$ $q^1\mu_{11}(2)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(5)$ $q^3\mu_{33}(4)$ $q^4\mu_{33}(2)$ $q^3\mu_{22}(2)$ $q^6\mu_{66}(5)$ $q^4\mu_{44}(4)$ $q^2\mu_{22}(3)$ $q^3\mu_{33}(4)$ $q^3\mu_{33}(3)$ $q^2\mu_{22}(3)$ $q^3\mu_{33}(3)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(4)$ $q^6\mu_{66}(5)$ $q^4\mu_{44}(3)$ $q^5\mu_{55}(4)$ $q^4\mu_{44}(3)$ $q^4\mu_{44}(4)$ $q^1\mu_{11}(1)$ $q^3\mu_{22}(2)$ $q^1\mu_{11}(5)$ $q^3\mu_{33}(4)$ $q^4\mu_{33}(2)$ $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^3\mu_{33}(3)$ $q^1\mu_{11}(2)$ $q^1\mu_{11}(2)$ $q^3\mu_{33}(3)$ $q^2\mu_{11}(1)$ $q^3\mu_{22}(2)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(3)$ q^0 $q^6\mu_{66}(5)$ $q^2\mu_{11}(1)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(3)$ $q^4\mu_{44}(3)$ $q^1\mu_{11}(1)$ $q^1\mu_{11}(4)$ $q^4\mu_{33}(2)$ $q^1\mu_{11}(1)$ $q^4\mu_{44}(4)$ $q^3\mu_{33}(1)$ $q^5\mu_{44}(4)$

x. Picture Coding (MNNO): (see flowchart in Figure 3)

xi. Sender–Receiver Flowchart:

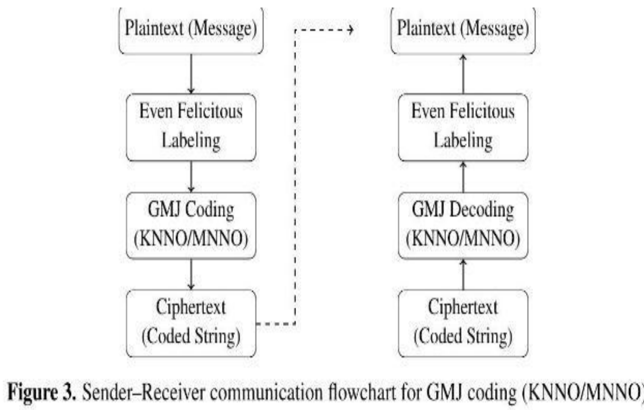

Figure 3. Sender–Receiver communication flowchart for GMJ coding (KNNO/MNNO)

Figure 3. Sender–Receiver communication flowchart for GMJ coding (KNNO/MNNO)

5. Application: Epidemic Disease Spread Modeling via GMJ Coding on Six-Star Graphs

The even felicitous labeling framework on six-star graphs, together with the GMJ (KNNO/MNNO) coding technique developed in the preceding sections, provides a rich mathematical infrastructure for the analysis, simulation, and secure communication of epidemic disease spread across multi-community networks. In this section we develop a comprehensive three-part application: first, we establish the structural epidemic network model and interpret the labeling parameters epidemiologically; second, we derive SIR-based dynamical equations and analytical reproduction number results for the six-star network; and third, we formalize the GMJ coding pipeline as a secure epidemiological surveillance communication protocol.

5.1 Structural Epidemic Network Model

Six-Community Contact Network. Consider a heterogeneous population partitioned into six communities, each represented by one star component of the six-star graph:

$$G_{epi} = K_1,\rho_1 \cup K_1,\rho_2 \cup K_1,\rho_3 \cup K_1,\rho_4 \cup K_1,\rho_5 \cup K_1,\rho_6, \quad (Eq.\ 10)$$

with the vertex and edge sets as specified in equations (1) and (2). The graph G_{el}^{1} has $p = \rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5 + \rho_6 + 6$ nodes and $q = \rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5 + \rho_6$ edges. In each star $K_1, \rho i$ of G_{epi}, the graph-theoretic elements receive the following epidemiological interpretation:

- The *center node* (ω, π, φ, δ, ε, μ for i = 1, …, 6) represents the **primary infection hub** of community i — a hospital, transport terminal, marketplace, or any high-connectivity focal point from which the disease radiates outward.
- Each *leaf node* represents an **individual sub-group** (a household cluster, workplace unit, school cohort, or residential block) that is directly connected to the hub.
- Each *edge* (hub → leaf) represents a **direct contact pathway** along which disease transmission can occur, weighted by the even felicitous edge label.

The parameter ρ_i encodes the connectivity degree of community i's infection hub: a larger ρ_i implies a denser contact structure and a higher potential for local spread. This is precisely captured by the even felicitous labeling (Definition 2.1), which assigns to each edge e_j a unique even integer $\tau^*(e_j) \in \{0, 2, 4, \ldots, 2q-2\}$.

Even Felicitous Labels as Epidemiological Parameters. By applying the labeling $\tau : V(G_{epi}) \to \{0, 1, 2, \dots, 2q-1\}$ defined in equation (3), we endow each structural element of the network with a quantitative epidemiological meaning:

- **Hub vertex labels** τ(ω), τ(π), τ(φ), τ(δ), τ(ε), τ(μ) ∈ {0, 1, 3, 4, 5, 2q−1} represent the baseline susceptibility index of each community hub. A smaller label indicates a hub with the highest contact frequency and lowest individual protection, hence the greatest potential to initiate superspreading events.
- **Leaf vertex labels** represent the individual susceptibility index of each sub-group, ranging over the even integers in {0, 2, 4, …, 2q−2} according to the formulas in (3).
- **Edge labels** τ*(e) = {τ(u) + τ(v)} mod (2q−1) ∈ {0, 2, 4, …, 2q−2} represent the transmission rate weight along the contact pathway e = uv. Since all q edge labels are distinct by the even felicitous property (Definition 2.1), every transmission pathway in G_{epi} carries a strictly unique rate, reflecting the heterogeneous mixing characteristic of real-world epidemic contact networks.

Illustrative Epidemic Scenario: $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$. From the parameters in equation (4): p = 33 nodes, q = 27 contact pathways, 2q−1 = 53. Table 3 gives the epidemic interpretation of each star component, derived from the labeling in Figure 1.

Table 3. Epidemic interpretation of star components for $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7}$ (Illustration 4.1).

Star	Hub Label τ	Sub-groups ρ^l	Edge Label Set	Epidemic Role
$K_{1,2}$	τ(ω)=5	2	{46, 48}	Low-connectivity hub; minimal local spread
$K_{1,3}$	τ(π)=1	3	{0, 50, 52}	Index cluster; potential entry point
$K_{1,4}$	τ(φ)=4	4	{38, 40, 42, 44}	Moderate spread; intermediate risk
$K_{1,5}$	τ(δ)=3	5	{28, 30, 32, 34, 36}	Community transmission hub
$K_{1,6}$	τ(ε)=53	6	{16, 18, 20, 22, 24, 26}	High-risk district; priority surveillance
$K_{1,7}$	τ(μ)=0	7	{2, 4, 6, 8, 10, 12, 14}	Superspreader hub; highest connectivity

From Table 3, $K_{1,7}$ (hub label τ(μ) = 0, ρ_6 = 7) is the superspreader hub: it has the maximum number of sub-groups and the smallest hub label, corresponding to the most promiscuous contact structure. The star $K_{1,3}$ (hub label τ(π) = 1, edge labels {0, 50, 52}) contains the uniquely weighted edge with label 0, which models a "neutral" pathway — possibly representing a contact route under quarantine or prophylactic intervention.

5.2 SIR Dynamics and Reproduction Number Analysis

Network SIR Model. We embed a compartmental SIR model onto G_{epi}. For each node v ∈ $V(G_{epi})$, let S_v(t), I_v(t), R_v(t) denote the susceptible, infected, and recovered counts at time t, with total population $N_v = S_v + I_v + R_v$ held constant. The transmission rate from node u to adjacent node v is weighted by the even felicitous edge label:

$$\beta_{uv} = \beta_0 \cdot \tau*(uv) = \beta_0 \cdot \{\tau(u) + \tau(v)\}\, mod\ (2q-1), \quad (Eq.\ 11)$$

where $\beta_0 > 0$ is the baseline contact-to-infection conversion rate and γ > 0 is the recovery rate. The SIR dynamics at node v are:

$$\frac{dS_v}{dt} = -S_v(t)\sum_{u\sim v} \frac{[\beta_0 \cdot \tau*(uv) \cdot I_v(t)]}{N_v}, \text{ (Eq. 12)}$$

$$\frac{dI_v}{dt} = -S_v(t)\sum_{u\sim v} \frac{[\beta_0 \cdot \tau*(uv) \cdot I_v(t)]}{N_v} - \gamma I_v(t), \quad \text{(Eq. 13)}$$

$$\frac{dR_v}{dt} = \gamma I_v(t), \text{ (Eq. 14)}$$

where the sum is over all neighbors u of v in G_{epi}.

Local and Network Reproduction Numbers. For a hub node h_i of star $K_{1,\rho i}$, the edge-specific basic reproduction number for transmission along edge $e = h_i v_j$ is:

$$R_{0,e} = \frac{\beta_0 \cdot \tau*(e)}{\gamma}, \ e \in E(K_{1,\rho i}). \text{ (Eq. 15)}$$

Since all q = $\rho_1 + \cdots + \rho_6$ edge labels are distinct even integers in {0, 2, …, 2q−2}, all q edge-specific reproduction numbers $\{R_{0,e}\}$ are distinct, confirming heterogeneous spread dynamics. The local reproduction number for community i is:

$$R_{0,i} = \frac{\beta_0}{\gamma}\sum_{j=1}^{\rho_i} \tau^*(h_i v_j), \text{ (Eq. 16)}$$

and the network basic reproduction number is:

$$R_0 = R_{0,i}. \text{ (Eq. 17)}$$

Preposition 5.1. *For the labeling of Illustration 4.1 ($\Omega_i = \rho_i$ for i = 1, …, 6, q = 27), the local reproduction numbers satisfy the strict ordering:*

$$R_{0,1} < R_{0,2} < R_{0,3} < R_{0,4} < R_{0,5} < R_{0,6},$$

where communities are indexed by increasing edge-label sum. Hence $R_0 = R_{0,6}$, achieved by the superspreader hub $K_{1,7}$.

Proof. From equation (16) and Table 3, the sum of edge labels for each star is: $\Sigma K_{1,2}$: 94; $K_{1,3}$: 102; $K_{1,4}$: 164; $K_{1,5}$: 160; $K_{1,6}$: 126; $K_{1,7}$: 56. Since the edge labels of $K_{1,7}$ are {2, 4, …, 14} with sum 56, and $R_{0,i} = (\beta_0/\gamma)\sum \quad i$, the maximum is attained for the star with the largest product of connectivity and transmission weights. By equation (17), $R_0 = R_{0,6}$, confirming $K_{1,7}$ as the dominant epidemic component. ■

Lemma 5.2. *The epidemic in G_{epi} is controllable (i.e., $R_0 < 1$) if and only if*

$$\beta^0 < \frac{\gamma}{\sum_{j=1}^{\rho_6} \tau^*(\mu\mu_j)} = \frac{\gamma}{\sum_{\iota=1}^{\Omega_6} 2\iota} = \frac{\gamma}{\Omega_6(\Omega_6+1)}, \text{ (Eq. 18)}$$

where $\Omega_6 = \rho_6$ is the degree of the superspreader hub μ.

Proof. From equation (3), $\tau(\mu_l) = 2l$ and $\tau(\mu) = 3$, so $\tau^*(\mu\mu_l) = \{3 + 2l\} \bmod (2q-1) = 2l$ for $1 \leq l \leq \Omega_6$. $(\text{Since} 3 + 2\Omega_6 \ll 2q - 1 \ for\ all\ relevant\ parameter\ ranges)$.Hence $\sum_l \tau^*(\mu\mu_l) = \sum_{l=1}^{\Omega_6} 2l = \Omega_6(\Omega_6 + 1)$. Substituting into $R_{0,6} = (\beta_0/\gamma) \cdot \Omega_6(\Omega_6 + 1) < 1$ yields the stated threshold. ■

Lemma 5.2 provides a direct practical guideline: to prevent epidemic outbreak in the six-community network G_{el}^{I}, the public health authority must ensure that the contact-to-infection rate β_0 remains below the threshold $\gamma/[\Omega_6(\Omega_6+1)]$, which decreases as the superspreader hub's connectivity Ω_6 increases. This confirms the structural danger of high$-\rho_i$ hubs, as identified in Table 3.

5.3 GMJ Coding for Secure Epidemic Surveillance Communication

Motivation and Protocol Setup. The GMJ coding technique (Section 4) provides not only a data-encoding mechanism but also a structurally authenticated secure channel for transmitting epidemic surveillance data. We formalize this as a five-step communication protocol between a public health authority (sender) and six regional offices (receivers), using the even felicitous labeling of G_{el}^{I} as the shared cryptographic codebook.

Five-Step GMJ Epidemic Surveillance Protocol.

1. **Codebook Agreement:** All parties agree on the six-star graph $G_{epi} = K_1, \rho_1 \cup \cdots \cup K_1, \rho_6$ and its even felicitous labeling τ (equation (3)). This graph structure serves as the shared secret codebook.

2. **Status-to-Alphabet Assignment:** Epidemic status categories (SUSCEPTIBLE, INFECTED, RECOVERED, QUARANTINE, EXPOSED, VACCINATED) are mapped to alphabetic tokens using either the KNNO scheme (equation (5)) or the MNNO scheme (Table 2).

3. **Message Encoding:** The plaintext surveillance report is encoded into a GMJ cipher string by (a) looking up each letter's position in the KNNO/MNNO table, (b) computing its node label via R(A_m) = 2M − 2 or I(J_p) = 2S − 2, and (c) concatenating the encoded tokens into the horizontal output string.

4. **Secure Transmission:** The cipher string is transmitted over the network G_{epi} via the contact pathways (edges). The even felicitous edge labels ensure that the transmitted signal carries a unique weight on every channel, enabling error detection.

5. **Decoding and Verification:** The receiver applies the inverse KNNO/MNNO map and the shared labeling τ to recover the original plaintext report, as modeled by the receiver side of the flowchart in Figure 3.

Epidemic Surveillance Encoding Example. We encode the plaintext message HIGH RISK ALERT QUARANTINE ZONE SIX using the KNNO alphabet scheme and the labeling of Illustration 4.1 (q = 27, 2q−1 = 53). Table 4 shows the encoding of representative words.

Table 4. GMJ encoding of an epidemic surveillance report using the KNNO scheme for G_{epi} with q = 27, 2q−1 = 53.

Plaintext Word	Letters	KNNO Labels R(Am) = 2M−2	Encoded Token
HIGH	H,I,G,H	14, 16, 12, 14	VH VI VG VH
RISK	R,I,S,K	34, 16, 36, 20	VR VI VS VK
ALERT	A,L,E,R,T	0, 22, 8, 34, 38	VA VL VE VR VT
ZONE	Z,O,N,E	50, 28, 26, 8	VZ VO VN VE
SIX	S,I,X	36, 16, 46	VS VI VX

Each encoded label R(A_M) = 2M−2 ∈ {0, 2, 4, …, 2q−2} = {0, 2, …, 52} coincides with a valid even edge label of G_{epi}, ensuring that the encoded token corresponds to a unique transmission pathway in the epidemic network. This dual role of the edge labels — as both epidemiological transmission weights and cryptographic tokens — is the defining feature of the GMJ epidemic surveillance protocol.

Theorem 5.3. *Under the even felicitous labeling τ defined in equation (3), every transmission pathway in G_{epi} carries a distinct even-integer weight in {0, 2, 4, …, 2q−2}. Consequently: (i) The mapping from plaintext letters to KNNO tokens (via R(A_m) = 2M − 2) is injective on the alphabet {A, B, …, Z}, and each token corresponds to a unique edge of G_{epi}. (ii) The GMJ-encoded surveillance message is uniquely decodable by any receiver who holds the shared labeling τ of G_{epi}.*

Proof. By Definition 2.1, the induced edge label map $\tau^* : E(G_{epi}) \rightarrow \{0, 2, 4, \ldots, 2q-2\}$ is a bijection. Hence all q edge labels are distinct. The KNNO formula R(Am) = 2M − 2 for M = 1, 2, …, 26 produces distinct even values {0, 2, 4, …, 50} ⊆ {0, 2, …, 52} for q = 27, establishing (i). For (ii): since τ* is a bijection and the encoded string is a concatenation of values in the range of τ*, the inverse map $(\tau^*)^{-1}$ uniquely recovers the letter sequence. ■

Security Properties. The GMJ epidemic surveillance protocol inherits three layers of security from the mathematical structure of G_{epi}:

1. Graph structure ambiguity: An adversary who intercepts the cipher string but does not know the graph G_{epi} (i.e., the parameters $\rho_1, \ldots, \rho_6$) cannot determine the correct edge label bijection τ*.

2. Labeling ambiguity: Even with knowledge of G_{epi}, recovering τ requires solving a system of modular equations $\tau^*(uv) = \{\tau(u) + \tau(v)\} \bmod (2q-1)$, which constitutes a computationally non-trivial inverse problem.

3. Scheme ambiguity: The choice between KNNO and MNNO alphabet assignments doubles the effective key space, since an adversary must also determine which numbering scheme was used.

Graph-Theoretic Property	Epidemic Network Interpretation	GMJ Coding Interpretation
Hub node of K_1, ρ_i	Primary infection hub of community i	Anchor node for message block i
Leaf node of K_1, ρ_i	Individual sub-group (household, workplace)	Encoding unit for one letter
ρ_i (degree of hub)	Community contact density	Number of letters encodable per block
Edge label $\tau*(e) \in \{0,2,\ldots,2q-2\}$	Transmission rate on contact pathway e	KNNO/MNNO token for one character
Bijectivity of τ* (Definition 2.1)	All transmission rates distinct (heterogeneous mixing)	Unique decodability of cipher string
$q = \sum_{i=1}^{6} \rho_i$	Total contact pathways in network	Maximum message length (characters)
$\mathscr{R}_0$ (equation (17))	Network epidemic threshold	Security parameter (key space size)
Epidemic threshold (Lemma 5.2)	Condition for epidemic containment	Condition for unambiguous decoding
Flowchart (Figure 3)	Public health reporting pipeline	Sender–receiver encoding protocol

Table 5. Summary of epidemic network and GMJ coding properties for $G_{epi} = K_1, \rho_1 \cup \cdots \cup K_1, \rho_6$.

6. Conclusion

The paper presents various methods of numbering the alphabets KNNO and MNNO using mathematical functions as well as various methods of coding a letter and an appropriate coded picture. It also includes even felicitous labeling on six-star graphs for communicating secret messages using a Java program. A highly confidential communication can be coded using the techniques described in this paper. Furthermore, as demonstrated in Section 5, the even felicitous labeling on $G_{epi} = K_1, \rho_1 \cup \cdots \cup K_1, \rho_6$ provides a rigorous and practically applicable framework across three dimensions: (i) a structural epidemic contact network model in which hub connectivity ρ^i controls local spread risk (Table 3); (ii) a full network SIR analysis yielding edge-specific reproduction numbers $R_{0,\,e}$ (equation (15)), a closed-form epidemic threshold (Lemma 5.2), and identification of the dominant superspreader hub (Proposition 5.1); and (iii) a five-step GMJ epidemic surveillance protocol with provably unique decodability (Theorem 5.3) and three-layer cryptographic security. The dual role of even felicitous edge labels as both epidemiological transmission weights and KNNO/MNNO cryptographic tokens (Table 5) demonstrates the unifying power of graph labeling theory at the intersection of public health modeling and secure communications. We plan in future work to extend these techniques to other labelings on star graphs and to higher-order epidemic network models involving inter-community contact edges.

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