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Qualitative Directions in Egocentric and Allocentric Spatial Reference Frames

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***Abstract*: In this paper, we have presented a formalism for representation of and reasoning about directions in a qualitative way in two spatial reference frames, namely, an egocentric reference frame and an allocentric reference frame. Qualitative direction has been separated from dimensionality of spatial objects and as such, the formalism may be used to represent qualitative direction in a dimension-independent way. The formalism uses fewer numbers of base relations than existing formalisms and granularity can be refined easily. Algorithms for finding composition and converse of base relations have been presented. An example is presented to show how formal grammar can be used in conjunction with JEPD sets of qualitative relations for recognition of motion events involving any number of entities.**

***Keywords*:** QSR, spatial reference frames, qualitative relation algebra, formal grammar, composition, converse**.**

I. Introduction

In qualitative spatial reasoning (QSR), spatial objects having directions have been abstracted in different ways. In QSR literature, we find that directed line segments, oriented points etc. have been used as abstractions for such physical objects. A recent survey on Qualitative Spatial Reasoning can be found in [1]. In these formalisms, point and lines are the basic abstractions for spatial objects. In certain applications, it becomes necessary to reason about directions of objects in a qualitative way, without focusing on the dimensionality of these objects. In this paper, we have discussed qualitative directions of physical objects without any regard to their shape and size.

Two important formalisms that can be used for representation of directed objects in a qualitative way are dipole relation algebra [2] and oriented point algebra [3], [4]. In the first one, spatial entities are abstracted as directed line segments and relations are defined in terms of position of the end points of the dipoles (whether on left or on right). Since spatial location of end points is part of the definition, it will not be suitable for higher dimensional objects.

Oriented point algebra is an excellent formalism for representation of qualitative direction of objects abstracted as points. In this formalism, orientation of a point defines a direction called front and other directions like **frontleft**, **frontright** etc. are defined accordingly. Here, we find that qualitative directions are expressed in terms of spatial orientation labels like **FrontLeft**, **FrontRight** etc. For a two or higher dimensional object, the model of direction labels can be quite different. For example, a possible spatial orientation model for two dimensional objects abstracted as rectangles has been studied in [5] and we find that many more direction labels become relevant.

In this paper, we have proposed qualitative direction algebras in two different spatial frames of reference. Qualitative direction is separated from the issues of spatial location and dimensionality. We have proposed a direction model that does not use spatial location labels for representing qualitative direction. The direction relation labels, that we have proposed, are closer to our cognitive perception of object locomotion. Binary JEPD relations have been presented for expressing qualitative direction and algorithms for finding their composition and converse have been outlined. The issue of spatio-temporal continuity of these base relations has also been addressed.

Rest of the paper is organized as follows : in section II, a brief introduction about spatial reference frames is presented, section III and IV discuss qualitative relations with respect to egocentric reference frame, section V discusses qualitative direction relations in allocentric reference frame, in section VI, the theoretical basis for an application of qualitative direction algebra is presented, section VII explains an example and finally the paper is concluded in section VIII with an indication of future work in this regard.

II. Spatial Reference Frame

From a cognitive point of view, Tversky [6] advocates that people's spatial mental models use only two basic perspectives - locating elements relative to one another from a point of view or locating an element to a higher order environmental feature or reference frame. The first of these corresponds to an egocentric frame of reference and the second corresponds to an allocentric frame of reference or survey perspective. Spatial reference frame is an important issue when we want to represent qualitative direction [7]. In literature, mention has been made of allocentric and egocentric [8], [9], extrinsic, intrinsic and deictic frames of reference [10]. In an allocentric frame of reference, direction of an object is specified with respect to a fixed external frame of reference or coordinate system. For example, this may be north-south directions or may be X-Y coordinate system in a two dimensional plane. In the egocentric case, there is no such external frame. We represent direction with respect to an intrinsic axis of orientation imposed by physical configuration of the object. In the intrinsic case, the coordinate system is determined by some inherent characteristic of the reference object like its topology, size or shape. Deictic frame is imposed by an external observer. The external reference frame is synonymous with the allocentric case. In this paper, we have treated qualitative direction in an egocentric (intrinsic) spatial reference frame. In such a reference frame, we assume that a spatial object can be directed along an axis and this direction defines a coordinate system.

III. Qualitative Direction Algebra: Egocentric Reference Frame

A. Qualitative Direction Relations

Definition: A Direction Line is a directed line segment in a two dimensional plane having a direction **dir** and magnitude **m**.

Definition: Direction Region: - Let l_1 and l_2 be two direction lines having directions dir_1 and dir_2 respectively and having a point of intersection **o**. Let Θ be the angle between dir_1 and dir_2 in an anticlockwise direction. Then, a direction region defines a set of direction lines that originate at **o** and the direction of any such line is bounded by the angle Θ from dir_1 in an anticlockwise direction.

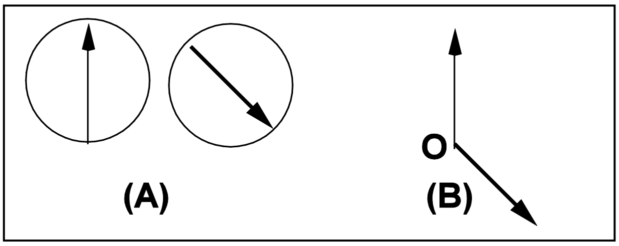

Figure 1. Object Directions using Direction Lines

In Figure 1, we have shown two objects whose egocentric heading is indicated by arrowheads (part **A**). In part (**B**), two direction lines are drawn parallel to the direction of the two objects.

Qualitative spatial reasoning (QSR) introduces as many abstractions as are needed for a particular application. In this respect, it is different from fuzzy computation. Categories in fuzzy approach are approximations of real values, while categories in QSR depend on application requirement [1]. We start with four qualitative direction relations, namely, **Same**, **Opposite**, **LR** and **RL**. Intuitive meaning ***of A Same B*** is that the objects are directed in the same direction. In **LeftToRight** relation (abbreviated as **LR**), one object is directed in a left-to-right orientation with respect to the other and in **RightToLeft** (abbreviated as **RL**), the situation is just the opposite. These major qualitative direction relations are illustrated in Figure 2. We want to emphasize the fact that **LR** and **RL** are in no way related with spatial orientation of the objects. If an object moves along its egocentric direction, its course of motion divides the two dimensional plane into two parts, one to the left and the other to the right. Now, if the second object moves in **LR** direction, its course of motion is from the left to the right with respect to the first and intersects the first at 90 degree.

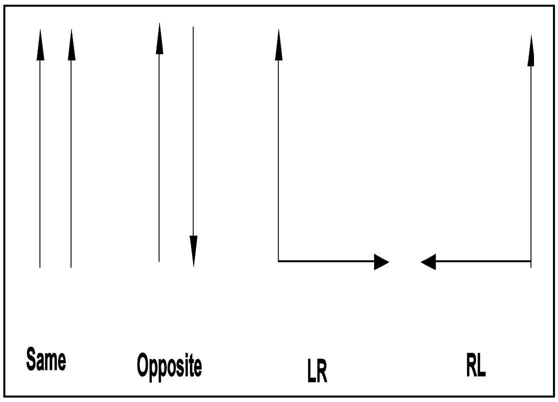

Figure 2. The Major Direction Relations

In Qualitative Spatial Reasoning, it is common to introduce abstractions and to discretize the domain under consideration. In order to refine each of the above relations, a span of 45 degrees counterclockwise is denoted by a **+** and the same in clockwise direction is denoted by a **-**. So, if the direction of the primary object makes an angle less than or equal to 45 degrees anticlockwise with the direction of the reference object, the resulting relation is **Same+** and in the counterclockwise case, it is **Same-**. Similar convention can be followed to arrive at relations like **Opposite+**, **Opposite-** , **LR+**, **LR-** , **RL+** and **RL-**. Thus, we have obtained twelve base relations. At this level of granularity, changes in direction are noticed after a threshold of 45 degrees.

In Table 1, these base relations are enumerated along with direction regions in terms of angles of the bounding direction lines and in Figure 3, the relations are illustrated. Direction regions are expressed in terms of positive angles measured in counterclockwise direction for all relations.

B. Granularity

The wheel shown in Figure 3 divides 360 degrees into eight regions, each having a span of 45 degrees. The number of divisions can be used to express the granularity of the algebra. We denote it as $\mathbf{QDA_8}$ to express the fact that it is Qualitative Direction Algebra (**QDA**) with granularity equal to **8**.

For certain applications, we may have to observe smaller changes in direction and as such, further refinement of qualitative direction relations may be necessary. We will explain below the refinement of base relations one level further and show how we can arrive at QDA16. Similar process can be repeated further for additional refinements. For granularity refinement, we equally divide the **+** and **-** direction regions. For example, if we divide the **+** region for which the angle range is] 0, 45], we obtain two direction regions of span 22.5 degrees each. The region] 0, 22.5] is denoted by the symbol $+^1$ and the region] 22.5, 45] is denoted by $+^2$. As a result, we get twenty base relations that are listed in Table 2. This time, change in direction is noticed after a threshold of 22.5 degrees. These refined relations are shown in Figure 4.

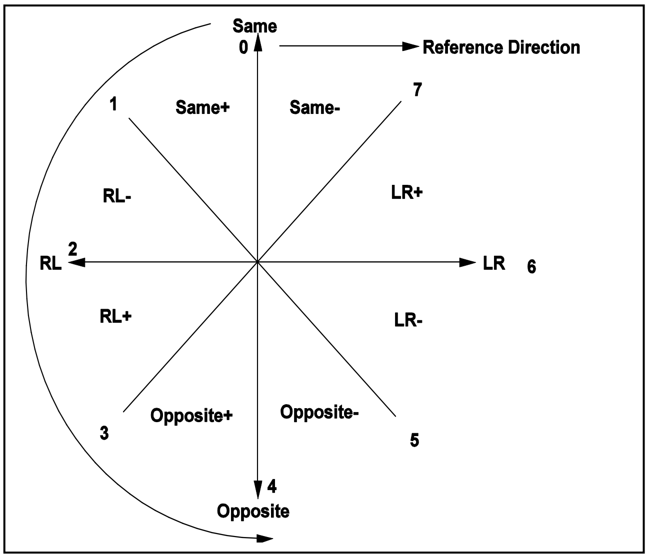

Figure 3. Direction Relations

Sl No	Base Relation	Angle Range	Converse
1	Same	[0,0]	Same
2	Same+	]0,45]	Same-
3	Same-	]315,360[	Same+
4	Opposite	[180,180]	Opposite
5	Opposite+	]180,225]	Opposite-
6	Opposite-	]135,180[	Opposite+
7	lr	[270,270]	rl
8	lr+	]270,315]	rl-
9	lr-	]225,270[	rl+
10	rl	[90,90]	lr
11	rl+	]90,135]	lr-
12	rl-	]45,90[	lr+

Table 1. Base Relations for Qualitative Direction

In order to apply constraint based reasoning to a set of spatial relations, we develop a partition scheme for the objects in the domain under consideration [11] and arrive at a set of Jointly Exhaustive Pairwise Disjoint (JEPD) base relations. General relations are obtained by taking the power set of base relations, with top, bottom, union, intersection and complement of relations defined in the set theoretic way [11]. Moreover, an identity relation and a converse operation on base relations must be provided.

For the set of base relations introduced earlier, **Same** is the identity relation. Each relation is closed under converse operation. The converses for the base relations were listed in Table 1 and in Table 2.

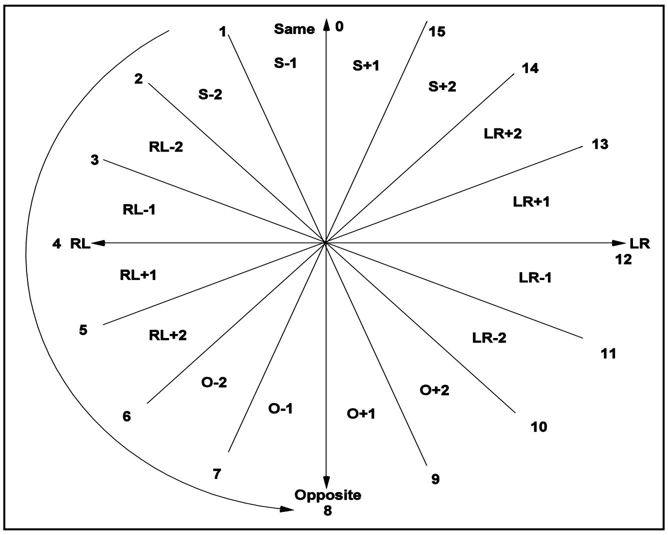

Figure 4. Direction Relations: One Level Refined

Sl No	Base Relation	Angle Range	Converse
1	Same	[0,0]	Same
2	Same$+^1$	]0,22.5]	Same$-^1$
3	Same$+^2$	]22.5,45]	Same$-^2$
4	Same$-^1$	]337.5,360[	Same$+^1$
5	Same$-^2$	]315,337.5]	Same$+^2$
6	Opposite	[180,180]	Opposite
7	Opposite$+^1$	]180,202.5]	Opposite$-^1$
8	Opposite$+^2$	]202.5,225]	Opposite$-^2$
9	Opposite$-^1$	]157.5,180[	Opposite$+^1$
10	Opposite$-^2$	]135,157.5]	Opposite$+^2$
11	lr	[270,270]	rl
12	lr$+^1$	]270,292.5]	rl$-^1$
13	lr$+^2$	]292.5,315]	rl$-^2$
14	lr$-^1$	]247.5,270[	rl$+^1$
15	lr$-^2$	]225,247.5]	rl$+^2$
16	rl	[90,90]	lr
17	rl$+^1$	]90,112.5]	lr$-^1$
18	rl$+^2$	]112.5,135]	lr$-^2$
19	rl$-^1$	]67.5,90[	lr$+^1$
20	rl$-^2$	]45,67.5]	lr$+^2$

Table 2. Refined Base Relations for Qualitative Direction

For finding the direction relation that holds between directions of two spatial objects, one of these objects is considered as a reference. A direction line parallel to the direction of the reference object is drawn and this direction line can be designated as line **0**. The direction relation wheel can now be drawn with respect to this line according to the granularity level under consideration. Then, the direction line corresponding to the direction of the primary object is drawn. The direction region in which this line falls tells us the direction relation of the primary with respect to the reference.

All angles are measured counterclockwise and direction relations in terms of angle ranges have been listed before.

We would like to present an algorithm for finding the converse of any qualitative direction relation. Let us assume that ***A dr B***, where **A** and **B** are spatial objects and ***dr*** is the direction relation holding between their directions. Intuitively, for finding the converse of ***dr,*** we should know the number of rotations we should give to the direction line of **A** to get back to the direction line of **B**. This is because of the fact that the converse expresses the relation of **B** with respect to **A**. So, for finding the converse relation, the direction line of **A** will be taken as line **0**. Moreover, we should know the relation that results after these many rotations. An algorithm is presented below for finding the converse of a qualitative direction relation.

```
Algorithm Converse(R, m)
R is the relation whose converse has to
be returned and m is the granularity
BEGIN
1. n := Calc_Rot_Conv(R)
2. If   (R=='Same'  ||  R=='Opposite'  ||  R=='LR'
          || R=='RL') Then
        Begin
              Conv_Rel := Find_Rel(n , n)
        End
    Else
       Begin
              Max_rot  :=  n
              Min_rot   := n-1
              Conv_Rel  := Find_Rel( Min_rot , Max_rot)
        End
END
```

In the above algorithm, the function **Calc_Rot_Conv** returns the number of rotations needed to align the direction line of the primary object with that of the reference. The bottom and top lines for the relation are retrieved into local variables **p** and **q**. **p** denotes the index of the bottom line and **q** denotes the index of the top line. Since the function returns **m - p**, we understand that the maximum required number of rotations is returned by the function. This returned value gets stored in the local variable **n** inside the function **Converse**. If the relation is one of **Same**, **Opposite**, **LR** or **RL**, then we know that the direction line of the primary will not fall in a direction region. It will align with one of the lines (at one of the angles 90, 180, 270 or 360 degrees measured counterclockwise) in the direction wheel. For example, let us consider $\mathbf{QDA_8}$ and let ***A Opposite B*** hold. Then, the direction line of the primary aligns with the line at an angle of 180 degrees in the direction wheel. The value of <**p, q>** will be **<4, 4>**. The value returned by **Calc_Rot_Conv** will be **4**. Inside the function **Converse**, a call will be made as **Find_Rel (4, 4)**. The relation whose bottom and top lines are **(4, 4)** is **Opposite**. So, the converse of **Opposite** is computed as **Opposite**.

For a discussion of other type of relations, let us consider **RL+**. The bottom and top lines for this relation will be returned as **<p, q > = <2, 3>**. The value returned from **Calc_Rot_Conv** will be **8 - 2** i.e. **6**. Inside the function **Converse**, **Max_rot** will be **6** and **Min_rot** will be **5**. This time, there will be a call like **Find_Rel (5, 6)**. The relation for which $\mathbf{dir_{bottom}}$ is **5** and dir_{top} is **6** is **LR-**. So, the converse of **RL+** is **LR-**.

An outline of the Calc_Rot_Conv function is given below:

```
Algorithm Calc_Rot_Conv(R,m)
The function Get_Lines gives the bottom
and top direction lines associated with
the relation R
BEGIN
1. < p, q>  :=  Get_Lines(Rel)
2. Return   m-p
END
```

We assume that the function **Find_Rel** retrieves the appropriate relation from a hash table depending on the pair of integers passed to it. Every direction relation can be represented by a pair of integers **(i, j)** where **i** is the integer corresponding to dir_{bottom} and **j** is the integer corresponding to dir_{top}. For example, when the pair **(0, 0)** is passed, the retrieved relation is **Same**, when **(0, 1)** is passed, the retrieved relation is **Same+** and so on. An outline of the algorithm for the **Find_Rel** function is given below. The algorithm takes care of the fact that sometimes (while computing composition of relations) the second argument can be two more than the first and the local variables **c** and **d** are used to control this. The algorithm will return a quadruple of relations. Let us assume that this quadruple is of the form **<R, Q, S, T>**. In this quadruple, only non-null entries are meaningful. For example, if we call **Find_Rel (2, 2)**, then only one relation is returned and this is available in **R**. If the call is like **Find_Rel (4, 5)**, then also a single relation is returned in **R**. The parameters **Q, S and T** become meaningful when **max** is **min + 2**.

```
Algorithm Find_Rel( min , max)
BEGIN
1. c:=d:=-1 ;  R:= Q := S := T := Null
2. If   (min==max) Then
3. Begin
4.    R :=  From_Hash(min,min)
5.    Return <R, Q, S, T>
6. End
7. Else If (max==min+1) Then
8. Begin
9.     R := From_Hash(min,max)
10.   Return <R,Q,S,T>
11.Else If (max==min+2) Then
12. Begin
13.   <a,b> := <min,min+1>
14. If  (min+1==0 || min+1==2
15.   || min+1==4 || min+1==6) Then
16.    <c,d> := <min+1 , min+1>
17.    <e,f> := <min+1 , max>
18.    Q := Get_From_Hash(a,b)
19.   If (c!=-1) Then
20.   Begin
21.      S := From_Hash(c,d)
22.      T := From_Hash(e , f)
```

23. **End**
24. Return <R, Q, S, T>
END

For constraint based reasoning, set theoretic composition of base relations is an important issue. We would present a simple algorithm for composition of base relations. Let **A**, **B** and **C** be three objects such that **A Rel1 B** and **B Rel2 C** hold. We want to find **Rel1 º Rel2**, where **º** denotes set theoretic composition. For this, direction relation wheel is drawn with respect to the direction of **B**. Since the relation of **A** to **B** is already known, we can identify the direction line or direction region for **A**. The remaining task is to fix **C** in the wheel. The relation **Rel2** is given, but that expresses the relation of **B** with respect to **C**. So, we take the converse of **Rel2** and identify the direction line or region for **C** with respect to **B**. Now, to find the composition we need to compute the number of rotations required to align direction of **C** with that of **A** and find the resulting relation. For any relation **Rel,** let us denote the lower line of its direction region as Rel_{Bottom} and the corresponding upper line as Rel_{Top}. Then, any relation can be expressed as an ordered pair of the form (Rel_{Bottom}, Rel_{Top}). For example, in the Figure 3, the relation **Opposite+** can be specified as **(4, 5)** and **Same** as **(0, 0)**.

Algorithm Compose (Rel1, Rel2, m)
This algorithms computes composition of
rel1 and rel2. m is the granularity
BEGIN
1. S := Converse(Rel2 , m)
2. <p,q >:= Get_Lines(S)
3. <r, s>:= Get_Lines(Rel1)
4. <min,max>:=Calc_Rot_Comp(<p,q>,<r,s>)
5. Find_Rel (min, max)
END

The above algorithm returns a quadruple and the non-null entries this quadruple are placed in appropriate slots of the composition table.

IV. Conceptual Dependency

Conceptual dependency of a spatial relation defines a set of relations that may hold after this relation whenever a change is recorded. For example, if at any point of time **Same** is the qualitative direction relation that holds between directions of two objects, then it is not possible that this relation will change to **Opposite** whenever change in direction is noted. After the relation **Same**, the possible relations that may hold can be either **Same+** or **Same-**. This gives rise to a notion of spatio-temporal continuity which can be exploited in many applications. The relations that may hold after the current relation are termed as its conceptual neighbor(s). Conceptual neighbors are generally expressed by a graph where nodes represent relations and edges are drawn from a node to its conceptual neighbors. In Figure 5, conceptual dependency of 12 base relations for **QDA$_8$** is shown. When the direction relations are refined one level further in **QDA$_{16}$**, twenty base relations result. The conceptual dependency of these twenty base relations is shown in Figure 6.

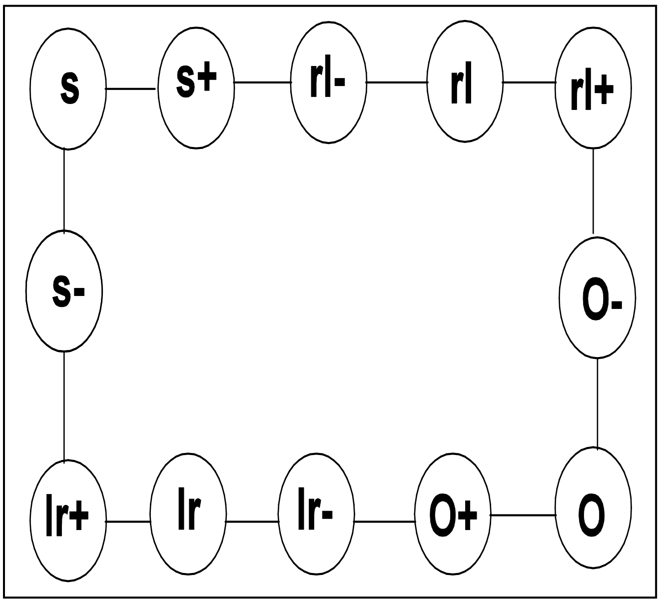

Figure 5. Conceptual Dependency of Base Relations for QDA_8: Egocentric

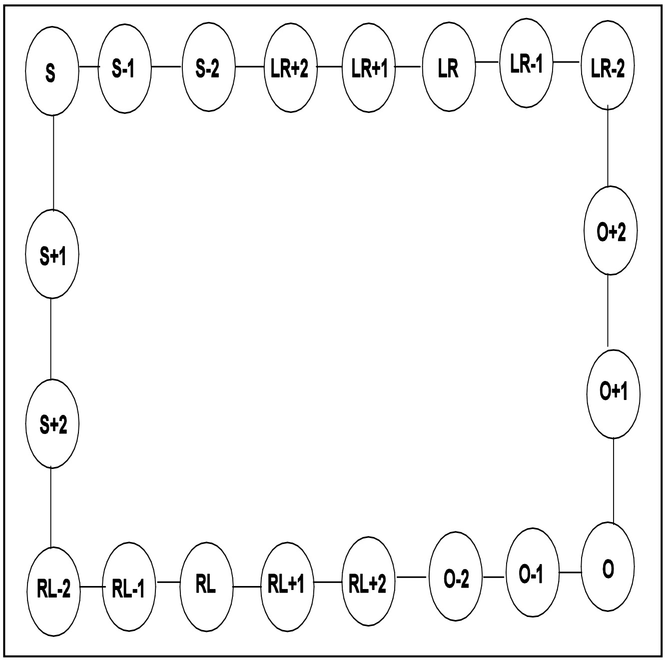

Figure 6. Conceptual Dependency of Base Relations for QDA_{16}: Egocentric

V. Qualitative Direction Algebra: Allocentric Reference Frame

We would like to use a similar idea for defining qualitative direction relations with respect to an external reference frame. This external frame may be north-south, east-west directions in geographic space or X-Y coordinate system or coordinate system with respect to any two arbitrary lines intersecting at right angles. We keep similar relation labels i.e. **Same**, **Opposite**, **LR** and **RL** as before. Though the intuitive idea remains same, definition of these relations will now differ. It is because of the fact that all the relations will be defined with respect to the external coordinate system.

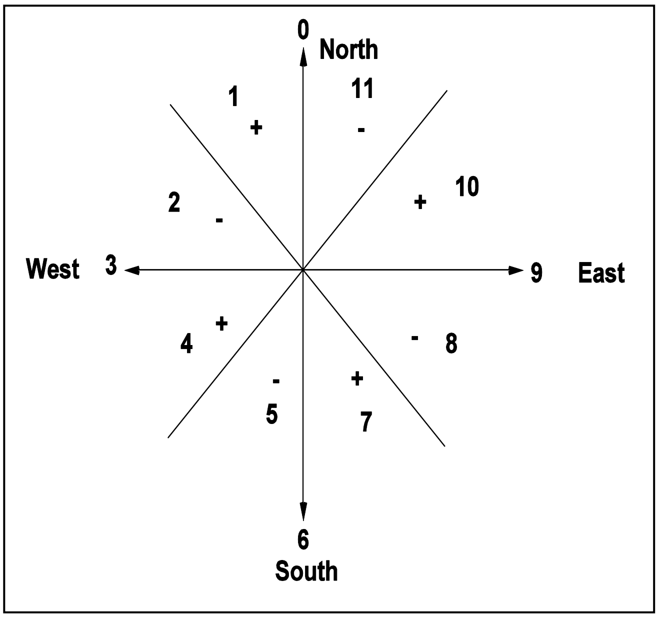

Figure 7. Coordinate System and Direction Regions

In Figure 7, north-south and east-west directions in geographic space have been shown as the reference frame. Right angles are divided into eight direction regions and the same interpretation for **+** and **-** is retained. Formally, we can define these relations as a set of ordered pairs. The first integer in the ordered pair indicates the direction of the primary and the second one indicates that of the reference. We present below the definition of four major qualitative direction labels. Each relation is defined as a union of Cartesian product of two sets of integers. At this point, these major direction relations are at a coarse level. The cases where the direction is inclined to an axis are also included in the definition.

Same = **{** (0,1,11) X (0,1,11) , (8,9,10) X (8,9,10), (5,6,7) X (5,6,7), (2,3,4) X (2,3,4) **}**

Opposite = **{** (0,1,11) X (5,6,7) , (8,9,10) X (2,3,4), (5,6,7) X (0,1,11), (2,3,4) X (8,9,10) **}**

LR = **{** 8,9,10) X (0,1,11) , (5,6,7) X (8,9,10), (2,3,4) X (5,6,7), (0,1,11) X (2,3,4) **}**

RL = **{** (2,3,4) X (0,1,11) , (0,1,11) X (8,9,10), (8,9,10) X (5,6,7), (5,6,7) X (2,3,4) **}**

When we consider refinement of these relations in terms of **+** and **-**, we find a difference with the egocentric case. Now, direction is expressed with respect to the external coordinate system. So, we need a representation for the situation when direction is strictly along one of the axes of the external coordinate system. This case is indicated by using the symbol •. For example, the ordered pair (0, 0) finds a name **Same••** and (0, 6) finds a name **Opposite••**. Let us illustrate this by explaining the ordered pair (0, 0). There are four ways in which two objects can move in the **Same** direction parallel to an axis of projection. These are given by the ordered pairs (0, 0), (3, 3), (6, 6) and (9, 9). Actually, all these ordered pairs are included in the relation **Same••**. Similarly, the ordered pairs that indicate that the objects are directed along axes of projection in opposite direction are (0, 6), (3, 9), (6, 0) and (9, 3). These ordered pairs are included in the relation **Opposite••**. In this scheme for naming relations, we give two symbols after the major relation name from the set {•, +, -}.

Sl No.	Direction Relation	Set Theoretic Definition
1	Same••	{(0,0),(3,3),(6,6),(9,9)}
2	Same+•	{(1,0),(4,3),(7,6),(10,9)}
3	Same-•	{(11,0),(2,3),(5,6),(8,9)}
4	Same•+	{(0,1),(3,4),(6,7),(9,10)}
5	Same•-	{(0,11),(3,2),(6,5),(9,8)}
6	Same++	{(1,1),(10,10),(7,7),(4,4)}
7	Same--	{(11,11),(2,2),(5,5),(8,8)}
8	Same+-	{(1,11),(4,2),(7,5),(10,8)}
9	Same-+	{(11,1),(2,4),(5,7),(8,10)}
10	Opposite••	{(0,6),(3,9),(6,0),(9,3)}
11	Opposite+•	{(1,6),(4,9),(7,0),(10,3)}
12	Opposite-•	{(11,6),(2,9),(5,0),(8,3)}
13	Opposite•+	{(0,7),(3,10),(6,1),(9,4)}
14	Opposite•-	{(0,5),(3,8),(6,11),(9,2)}
15	Opposite++	{(1,7),(4,10),(7,1),(10,4)}
16	Opposite--	{(11,5),(2,8),(5,11),(8,2)}
17	Opposite+-	{(1,5),(4,8),(7,11),(10,2)}
18	Opposite-+	{(11,7),(2,10),(5,1),(8,4)}
19	lr••	{(0,9),(3,0),(6,3),(9,6)}
20	lr+•	{(1,9),(4,0),(7,3),(10,6)}
21	lr-•	{(11,9),(2,0),(5,3),(8,6)}
22	lr•+	{(0,10),(3,1),(6,4),(9,7)}
23	lr•-	{(0,8),(3,11),(6,2),(9,5)}
24	lr++	{(1,10),(4,1),(7,4),(10,7)}
25	lr--	{(11,8),(2,11),(5,2),(8,5)}
26	lr+-	{(1,8),(4,11),(7,2),(10,5)}
27	lr-+	{(11,10),(2,1),(5,4),(8,7)}
28	rl••	{(0,3),(3,6),(6,9),(9,0)}
29	rl+•	{(1,3),(4,6),(7,9),(10,0)}
30	rl-•	{(11,3),(2,6),(5,9),(8,0)}
31	rl•+	{(0,4),(3,7),(6,10),(9,1)}
32	rl•-	{(0,2),(3,5),(6,8),(9,11)}
33	rl++	{(1,4),(4,7),(7,10),(10,1)}
34	rl--	{(11,2),(2,5),(5,8),(8,11)}
35	rl+-	{(1,2),(4,5),(7,8),(10,11)}
36	rl-+	{(11,4),(2,7),(5,10),(8,1)}

Table 3. Direction Relations: Allocentric Frame of Reference

Then, the name of each relation will have three place holders. The first of these three place holders indicates the major direction relation; the second one indicates the direction information of the primary and the third indicates that of the reference. As an example, let us consider the **Opposite** relation. When we write **Opposite••**, we mean that objects are directed in opposite direction and direction of each object is strictly along an axis. The relation **Opposite+•** means that they are directed in opposite direction and direction of the primary is in the **+** direction region while the reference is directed strictly along an axis. Therefore, each of the four major relations introduced earlier will result in six different relations and we will have 36 base relations in total. These relations are listed in Table 3 along with their set theoretic definition in terms of direction regions and direction lines.

Granularity can be refined by equally dividing the **+** and **-** regions as shown in Figure 8. Following the same convention as we have done for the egocentric reference frame, the relation **Same**•$+^1$ will mean that the objects are directed in the same direction, the primary is directed strictly along an axis and reference direction lies in the $\mathbf{+^1}$ region with respect to the axis. Formally, it can be defined as {(0, 1), (5, 6), (10, 11), (15, 16)}.

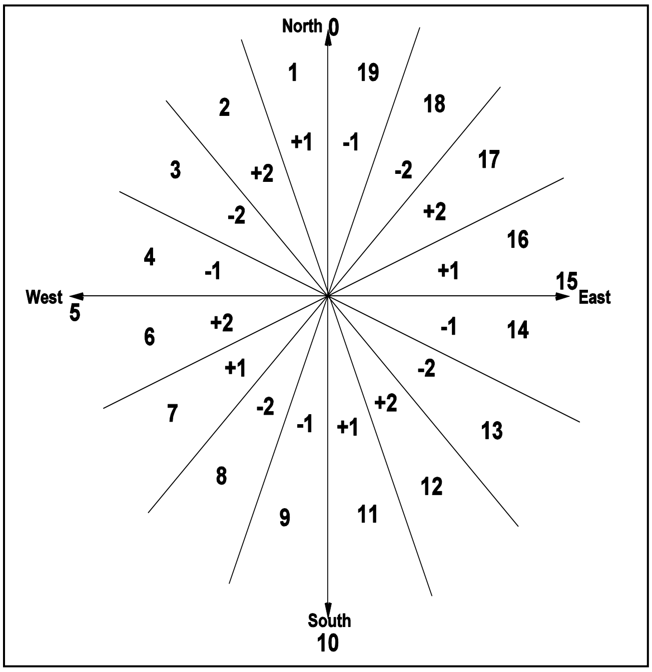

Figure 8. Refined Coordinate System and Direction Regions

The set of base relations is closed under composition and converse. The relation **Same••** is the identity relation. An important point to note is that since directions are expressed with respect to an external coordinate system, certain combinations will be ruled out while constructing the composition table. For example, let us assume that **A**, **B** and **C** are three objects such that ***A Rel1 B*** and ***B Rel2 C***. Now, if **Rel1** is **Same•+,** then the reference object is in the **+** region while the primary is directed strictly along an axis. In **Rel2**, the reference **B** is having a direction relation with **C**. Since **B** is already declared to be in **+** region, it is not possible that in **Rel2**, it can be treated in any different manner. So, it is not possible to assign some relations to **Rel2**.

We would like to elaborate on details of finding converse and composition of the set of base relations introduced in Table 3. Converse of the major unrefined direction relations (i.e. **Same**, **Opposite**, **LR** and **RL**) is computed from their semantics and are listed in Table 4 (named as **CONV**).

Sl No	Base Relation	Converse of Base Relation
1	Same	Same
2	Opposite	Opposite
3	lr	rl
4	rl	lr

Table 4. Converses of Major Direction Relations(CONV)

A trivial algorithm, given below, can be used to find the converse of any relation.

Algorithm Converse_Allocentric (Rab)

In the relation 'Rab', R is one of

the major relations and a and b can

be one symbol in the set $\{+,-, \bullet, +^1, +^2,-^1,-^2\}$

Here, CONV is a table name

BEGIN

1. S := CONV[R]

2. Return Sab

END

For composition, at first we use semantics of the major relations to find their composition. Table 5 (named as CTM) lists the composition of major direction relations in the allocentric reference frame.

	Same	Opposite	lr	rl
Same	Same	Opposite	lr	rl
Opposite	Opposite	Same	rl	lr
lr	lr	rl	Opposite	Same
rl	rl	lr	Same	Opposite

Table 5. Composition of Major Direction Relations (CTM)

The following algorithm can be used for finding the set theoretic composition of any relations.

Algorithm Compose_Allocentric (Rab, Scd)

In the relations 'Rab' and 'Scd', R and S are the major relations

and a, b, c, d can be any symbol in the set $\{+,-,\bullet, +^1,+^2,-^1,-^2\}$

BEGIN

1. **If** (b != c) **Then** Return NULL

2. **Else**

3. Return CTM[R, S] ad

END

VI. Theoretical Basis for Presenting an Example on Motion Event Recognition

A. Qualitative Spatial Reasoning

QSR is a knowledge representation technique that represents knowledge at a level close to human cognition. Quantitative information is precise and accurate, but such information may not always be useful from a cognitive viewpoint. For example, instead of giving latitude and longitude values for Kochi, it is easier to understand if we say "Kochi is to the north-east from my hometown". In **QSR**, different aspects of space are treated in a qualitative way. Aspects of space that have been treated in a qualitative way are topology, orientation, direction, distance, shape etc. [12]. Each aspect of space is represented by a Jointly Exhaustive Pairwise Disjoint (**JEPD**) set of binary qualitative relations. When a spatial aspect is represented by such a JEPD set, each value in the domain belongs to exactly one relation and all the values in the domain are obtained by taking a union of the relations. These qualitative abstractions, expressed in the form of binary qualitative relations, make knowledge representation very close to the way human beings perceive things [15]. QSR is different from fuzzy systems that are widely used to solve different types of problems [13], [14]. Unlike fuzzy systems, in QSR, we introduce as many abstractions as are needed for a particular application.

In QSR, qualitative relations in a JEPD set embody a notion of spatio-temporal continuity. This notion was first formalized as conceptual neighborhood in [16]. Two relations are conceptual neighbors if one can be transformed into the other by continuous change. For example, let us consider qualitative distance. A JEPD set for representing qualitative distance may be {**veryclose, close, near, far, veryfar**}. Conceptual neighbors of **close** are **veryclose** and **near**. This is because when the objects are **close**, then possible changes occur when they move closer or move apart. In order to be **veryfar** from **near**, the distance between the objects will change to **near**, then to **far** and finally to **veryfar**.

B. Modeling a Motion event

A motion event has certain features. These features may be spatial orientation of the objects, direction, velocity, distance etc. Each such feature of a motion event is represented by a set of binary qualitative relations. For example, let us consider a motion event of two cars moving on a street. The features we may select for such an event may be direction in which cars are moving, distance between them, spatial orientation of one car with respect to the other, the size of the two cars etc. We may describe such an event as "a big car crossing a small car from opposite at a very close distance". Then, we need to have a qualitative abstraction for each of the features of this motion event. For doing this, we have used the concept of a temporal state. A motion event between two objects (one is reference and the other is primary) during an observational interval is modeled as a temporal state. Such a state has been termed as a primitive event. In such a state, each of the qualitative motion features has a value and these values are invariant during the state. For our example, we have two qualitative features, namely, direction and orientation. Spatial objects in our example are abstracted as directed points. For representation of spatial orientation of such point objects, we have used a projection based model illustrated in Figure 9. The set of qualitative direction relation QDA_8 is denoted by **D** in our subsequent discussions and the set of spatial orientation relations, as illustrated in Figure 9, is denoted as **S**. This JEPD set **S** = {**Front, Back, Left, Right, FrontRight, FrontLeft, BackRight, BackLeft**}.

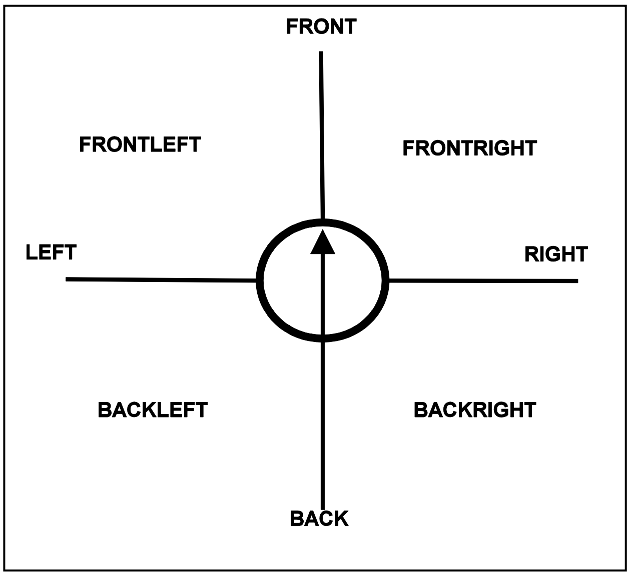

Figure 9. Spatial Orientation Model

Definition: Primitive Event: Let **D** and **S** be two sets of JEPD qualitative binary spatial relations, where **D** is the set of qualitative direction relations QDA_8 and S is the set of spatial orientation relations. Then, a primitive event between a primary and a reference is denoted by an ordered pair of the form $< d, s >$ such that $< d, s > \in$ **D** X **S**. A primitive event holds for a finite interval of time.

The use of QSR in modeling a temporal state has an implication. Allowable state transitions can be formally defined. In ordered pair representation of primitive event, each element of the pair is a value of some binary qualitative spatial relation. Therefore, the spatio-temporal continuity of these relations naturally extends to primitive events as well. We can think of primitive events that are conceptual neighbors of a particular event.

Definition: Neighboring Primitive Event: Let T_1 be an interval during which primitive event **e** holds and let T_2 be an interval during which primitive event **v** holds (with T_1 meets T_2). Here, **meets** is an Allen's Interval Algebra relation. Let **e** be of the form $<d_1, s_1>$ and let **v** be of the form $<d_2, s_2>$. Then, **v** is a neighboring event of **e iff** d_2 is a conceptual neighbor of d_1 and s_2 is a conceptual neighbor of s_1.

Allen [17] showed that thirteen JEPD binary qualitative relations can be defined between two intervals of time. These relations are known as Allen's interval algebra relations and in Figure 10, these relations are illustrated. A motion event can be as short as a primitive event or it may last for a longer duration. If it is composed of multiple primitive events, then this sequence of primitive events must exhibit spatio-temporal continuity.

Definition: Composite Event- A composite event **C1**, expressed as $e_1 \bullet e_2 \bullet \ldots \bullet e_n$, where each e_i is a primitive event, is a sequence of primitive events such that for any two primitive events e_i and e_{i+1} in the sequence , $i < n$, e_{i+1} is a neighboring event of e_i. Here, • is the concatenation operator.

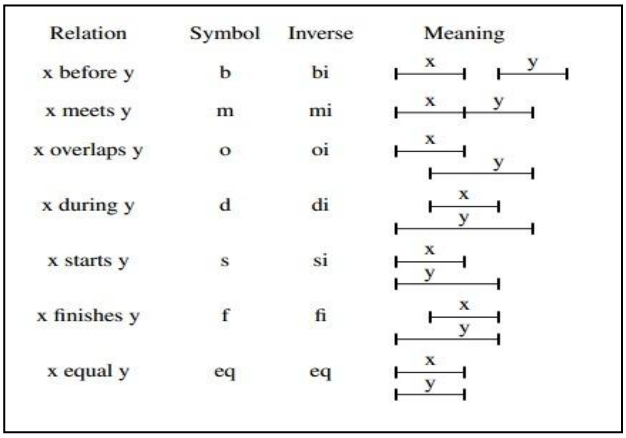

Figure 10. Allen's Interval Algebra Relations

C. Recognition of a Composite Event

A composite event is constructed by appending primitive events using the concatenation operator. Such an event can be recognized by a regular grammar.

Definition: Regular Grammar- A regular grammar G = (V, T, P, S) where S is the start symbol of the grammar, V is the set of non-terminals, T is the set of terminals and P is the set of productions of the form $\mathbf{A \rightarrow a\ \ B}$ or $\mathbf{A \rightarrow B\ \ a}$, where $\mathbf{a \in T}$ and $\mathbf{B \in V}$.

In a regular grammar, the right hand side of any production contains a single terminal followed by a non-terminal or a single non-terminal followed by a terminal and the left hand side contains a single non-terminal.

Lemma: A composite motion event **C1** is recognized by a regular grammar G = (V , T , P , S) where S is the start symbol , P is the set of productions of the form $\mathbf{X \rightarrow e\ Y}$ or $\mathbf{X \rightarrow \epsilon}$ where X and Y are non-terminals and **e** is a primitive motion event , T is the set of terminals such that each terminal is a primitive event and V is the set of non-terminals.

Proof: Let **C1** be of the form $\mathbf{e_1\ e_2\ ...\ e_i\ e_{i+1}\ ...\ e_n}$. We can construct a finite state automaton for recognizing the sequence such that the FSA makes a transition from one state S_i to S_{i+1} when the primitive event e_i is encountered. Let S_1 be the initial state and S_{n+1} be the final state. Then, from the equivalence of FSA and regular grammar, we construct the grammar such that the productions are of the form $S_i \rightarrow e_i\ Si_{+1}$ and finally $S_{n+1} \rightarrow \epsilon$.

D. Implication of JEPDness

We would like to emphasize the fact that the set of relations we choose for representing a qualitative aspect of a motion event must be Jointly Exhaustive and Pairwise Disjoint (JEPD). The importance of JEPDness is outlined in the following definitions and lemmas.

Definition: Exhaustive Set of Primitive Events- Let α be the set of primitive events and each entry in α is of the form <d, s > where each $\mathbf{d} \in \mathbf{D}$ and $\mathbf{s} \in \mathbf{S}$. Then, α is exhaustive if each **D** is Jointly Exhaustive (JE) and **S** is Jointly Exhaustive.

The following lemma underlines the importance of exhaustiveness.

Lemma: A regular grammar G = (V, T, P, S) can recognize any composite motion event C1 only when the set of primitive events is exhaustive.

Proof: Let α be the set of primitive events and C1 be a composite event of the form $e_1\ e_2\ ...\ e_i\ v\ e_{i+1}\ ...\ e_n$. Let **v** be a primitive event such that **v** does not belong to α. Since **v** is not in the set of primitive events defined, there cannot be any production of the form $\mathbf{X \rightarrow v\ Y}$ in P. Hence, when a sequence of primitive events, with v contained in it, is encountered, it cannot be parsed by the grammar. So, when α, the defined set of primitive events is exhaustive i.e. it includes all possible primitive events, then only any **C1** can be parsed by the grammar.

Lemma: A single primitive event holds at any point of time if D is Pairwise Disjoint (PD) and **S** is pairwise Disjoint (PD).

Proof: Let αbe the set of defined primitive events. Let us assume that some **D** is not pairwise disjoint. Then, we can find two relations d_1, $d_2 \in$ D such that both can hold at the same point of time. This implies that two primitive events $<d_1, s_1>$ and $< d_2, s_1>$ can hold during the same time interval. Therefore, in order to have a single primitive event holding at any point of time, the set **D** must be pairwise disjoint. Similar argument can be given by assuming that the set **S** is not Pairwise Disjoint (PD).

Lemma: Composite motion events can be recognized if a single primitive event holds during any interval of time.

Proof: Let us assume that e**, E, v** is a composite event such that $\mathbf{e, v \in\ \alpha}$, where αis the set of defined primitive events. **E** is a set of primitive events such that all the events in **E** hold during the same time interval and let E = {e1, e_2, . . . en}. Let G be the regular grammar defined to recognize this event. G = (V, T, P, S) where the symbols bear their usual meaning. In order to parse events in **E**, we include productions of the form $\mathbf{X \rightarrow e_1\ A_1\ |\ e2\ A_2\ |\ .\ .\ .\ e_n\ A_n}$. During parsing, let the parser non-deterministically select some e_i in **E** and parse the sequence looking for some defined composite event. If this e_i is not contained in the list of primitive events that define the composite event to be recognized, then parsing fails. Since our parsing technique is deterministic, **E** must include only a single primitive event.

VII. Presenting an Example

A. Spatial Orientation Model

Qualitative direction relations can be used to represent and reason about motion events of directional entities. For such applications, formalisms are required for representation and recognition of motion events in an input stream. We will discuss an example where spatial objects will be abstracted as directed points and qualitative direction of such points will be represented by $\mathbf{QDA_8}$ in an egocentric spatial reference frame. Though, we can use qualitative direction as the only feature of a motion event, it would be more meaningful if we bring in spatial location of the objects. For this, we use a well-known projection based spatial orientation model for point like objects. This model is shown in Figure 9. Direction of motion of the object (shown using the arrow) sets up spatial orientation regions marked by qualitative labels like **Front, FrontRight, FrontLeft, Back, BackRight, BackLeft, Left and Right**. For convenience, we name these relations as **F, FR, FL, B, BR, BL, L and R** respectively. Then, we have a set of eight JEPD binary qualitative relations for modeling spatial orientation of a primary object with respect to a reference object. Since spatial objects are abstracted as points, the primary can be located in exactly one of these regions. Moreover, any location of a primary object can only be in one of these regions. We would use two qualitative features for modeling a motion event. These are spatial orientation and qualitative direction of motion. So, for representation of the events, we use two JEPD sets of binary qualitative relations. Out of these one is QDA_8 in an egocentric reference frame and the other is the set of spatial orientation relations {F, FR, FL, B, BR, BL, L, R} for the projection based model introduced above.

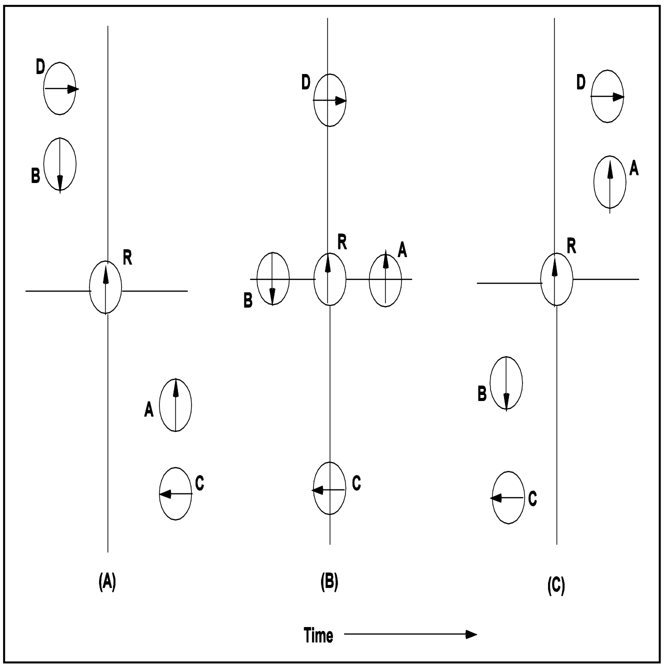

Figure 11. Motion Events of Directed Points

B. Representation of Motion Events in the Example

In Figure 11, we have shown five objects **A, B, C, D** and a reference **R**. Three snapshots in time are shown and in each snapshot, spatial orientation and direction of the objects with respect to the reference **R** are depicted. From the point of human cognition, we can say that the object **A** overtakes **R** during motion, **B** crosses from opposite, **C** moves from right to left in the back and **D** crosses from left to right in the front. In Figure 11a, the primitive motion event that holds between **A** and **R** is $\mathbf{P_A}$ = <BR, Same>, that between **B** and **R** is $\mathbf{P_B}$ = < FL, Opposite>, that between **C** and **R** is $\mathbf{P_C}$ = < BR, rl> and the primitive event between **D** and **R** is $\mathbf{P_D}$ = <FL, lr>. Over the extended interval of time, the composite motion events can be represented as $\mathbf{C_A}$ = <BR, Same> <R, Same> < FR, Same> (between **A** and **R**), $\mathbf{C_B}$ = <FL, Opposite> <L, Opposite> < BL, Opposite> (between **B** and **R**), $\mathbf{C_C}$ = <BR, rl> < B, rl> < BL, rl> (between **C** and **R**) and $\mathbf{C_D}$ = <FL, lr> <F, lr> <FR, lr> (between **D** and **R**). It is important to note that a composite motion event is still binary as it is expressed between a primary object and a reference.

C. Recognition of Motion Events in the Example

It has been proved that when spatio-temporal continuity of JEPD sets of binary qualitative relations, in the form of conceptual dependency, is combined with formal grammars, the resulting grammars are regular. We would like to show this fact for the example presented above. There are four composite events, namely, $\mathbf{C_A}$, $\mathbf{C_B}$, $\mathbf{C_C}$ and $\mathbf{C_D}$ in the example shown in Figure 11. Out of these, the event $\mathbf{C_A}$ can be recognized using the grammar:

$S_1 \rightarrow$ <BR, Same> P_1
$P_1 \rightarrow$ <R, Same> P_2
$P_2 \rightarrow$ <FR, Same> P_3
$P_3 \rightarrow \epsilon$

The composite event $\mathbf{C_B}$ can be recognized by the following regular grammar:

$S_2 \rightarrow$ <FL, Opposite> Q_1
$Q_1 \rightarrow$ <L, Opposite> Q_2
$Q_2 \rightarrow$ <BL, Opposite> Q_3
$Q_3 \rightarrow \epsilon$

The event $\mathbf{C_C}$ is recognized by:

$S_3 \rightarrow$ <BR, rl> R_1
$R_1 \rightarrow$ <B, rl> R_2
$R_2 \rightarrow$ <BL, rl> R_3
$R_3 \rightarrow \epsilon$

Finally, the event $\mathbf{C_D}$ between the primary entity **D** and the reference **R** can be recognized by the following grammar:

$S_4 \rightarrow$ <FL, lr> X_1
$X_1 \rightarrow$ <F, lr> X_2
$X_2 \rightarrow$ <FR, lr> X_3
$X_3 \rightarrow \epsilon$

Though such regular grammars can be used for recognition, for effectiveness and completeness, a full-fledged programming language should be designed for recognition of motion events of directional entities. Such an effort has been undertaken by the authors for the design of a qualitative language using the concepts of typed set theory. In this language, qualitative terms about motion events of directional objects can be encoded and this representation is very close to human cognition. Language features are designed in such a way that motion event among any number of spatial objects can be represented and recognized by creating hierarchies of abstractions.

VIII. Conclusion

In this paper, a qualitative direction algebra is proposed for representation of and reasoning about qualitative directions of spatial objects in a dimension independent way. Existing formalisms combine dimensionality into definition of relations and as a result of this, these become unsuitable when the dimension scales up. We have defined qualitative spatial relations for $\mathbf{QDA_8}$ and $\mathbf{QDA_{16}}$. Algorithms for finding the converse and composition of the base relations have been presented. It has been shown that in this formalism, it is very easy to move to a finer granularity depending on application requirement. This finer granularity is realized using fewer base relations than existing formalisms. A limitation of this approach might be the fact that it ascertains the directions in a deterministic way. At any point of time, it is assumed that we know the angles between the directions certainly. There is no element of probability involved here. Future work includes completion of the design of a qualitative language for representation and recognition of motion events of directional objects.

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