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Article

A Unified Framework for Degree-Based Topological Indices in Distance-Hereditary Fuzzy Graphs

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Abstract: Distance-Hereditary Fuzzy Graphs or DHFGs combine the distance preserving property of distance- graphs with the ability of fuzzy graphs to show uncertainty. Many degree-based topological indices have been studied for fuzzy graphs. A unified framework for these indices in DHFGs has not been created yet. In this paper we suggest a framework for creating degree-based topological indices in Distance-Hereditary Fuzzy Graphs. This framework includes known indices, as special cases. These are the Randic, Zagreb, Harmonic, Atom-Bond Connectivity or ABC, Geometric-Arithmetic or GA and Sombor indices. We prove fundamental properties such as existence, non-negativity, monotonicity, invariance under fuzzy isomorphisms, and preservation under connected induced distance-hereditary fuzzy subgraphs. Illustrative examples for the computation of the proposed indices are given, and their practical relevance is illustrated by applications to communication networks, transportation systems, biological interaction networks, social networks and decision-support systems. The proposed unified framework provides a systematic basis for extending classical topological descriptors to uncertain distance-preserving networks, and also opens new avenues for future research in the theory of fuzzy graphs.

Keywords: Fuzzy Degree, Connectivity Index, Distance Connectivity Index, Normalized Connectivity Index, Distance-Hereditary Fuzzy Graphs.

1. Introduction

Graph theory is really useful for understanding systems. We use it in lots of fields like science and engineering. It is also used in chemistry and biology. People who work with computers use graph theory too. Graph theory helps us figure out how things are connected and how they work together. We need graph theory to make sense of systems in science and engineering and other areas, like chemistry and biology and computer science. In many practical situations, however, relationships between objects are uncertain or imprecise. Fuzzy graph theory is really useful because it takes the ideas of classical graph theory and adds something new. It gives a value to each point and line in the graph to show how sure we are about them. This way fuzzy graph theory is a tool, for dealing with things that are not certain. Fuzzy graph theory helps us model things that're uncertain.

Distance-hereditary graphs are a deal because they are a type of graph where the distance between every pair of vertices in a distance-hereditary graph always stays the same. This is true for every connected induced subgraph of a distance- graph. The thing that makes distance-hereditary graphs really useful is that they keep the distance between vertices the same. This is why distance-hereditary graphs are used in things like routing algorithms, for distance-graphs, communication networks, computational biology and network optimization for distance-hereditary graphs.

Recently, the concept of Distance-Hereditary Fuzzy Graphs is introduced which combines the uncertainty with distance preservation. People have looked at some things, about the structure of these graphs. They have not paid much attention to the degree-based topological indices of these graphs. The degree-based topological indices of these graphs are really important so it is surprising that the degree-based topological indices of these graphs have been ignored for long.

The degree-based indices are numbers that do not change for a graph and these numbers tell us about the structure of the graph. The structure of the graph is described by looking at the degrees of the vertices of the graph. The degree-based indices give us information, about the structural features of the graphs and this information is based on the vertex degrees of the graphs. We use the degree-based indices to understand the vertex degrees of the graphs and how they relate to the structure of the graphs. These indices have been found important in mathematical chemistry, quantitative structure–activity relationships (QSAR), network reliability and complex network analysis.

The main goal of this work is to come up with a way to define and look at the degree-based topological indices in Distance-Hereditary Fuzzy Graphs. This work is really about Distance-Hereditary Fuzzy Graphs and how we can understand them better by using this approach to study the degree-based topological indices, in Distance-Hereditary Fuzzy Graphs. This framework generalises many classical indices and yields common theoretical results for all of them.

2. Preliminaries

Definition 2.1 (Fuzzy Degree)

In a standard fuzzy graph $\mathrm{F_G} = (\zeta, \xi)$ where $\zeta: \mathrm{V_t} \to [0,1]$ is the vertex membership function and $\xi: \mathrm{V_t} \times \mathrm{V_t} \to [0,1]$ is the edge membership function, the fuzzy degree of a vertex ĝ is d(ĝ).It is the sum of membership degrees of all edges that meet at ĝ. This sum includes membership degrees of all edges incident to ĝ.

$$d(\hat{g}) = \sum_{\hat{h} \in \mathrm{V_t}} \xi\left(\hat{g}, \hat{h}\right)$$

Definition 2.2 (Connectivity Index)

For a classical (or crisp) graph $\mathrm{G_h} = (\mathrm{V_t}, \mathrm{E_d})$, The connectivity index is calculated by checking the degrees of points which are also called vertices across every single connection or edge in the network:

$$R(\mathrm{G_h}) = \sum_{(\hat{g},\hat{h}) \in \mathrm{E_d}} \frac{1}{\sqrt{d(\hat{g}) \cdot d\left(\hat{h}\right)}}$$

Where:

- $\mathrm{E_d}$ is the set of all lines in the graph.
- $d(\hat{g})$ and $d\left(\hat{h}\right)$ represent the standard classical degrees of points ĝ and ĥ.

Definition 2.3 (Distance Connectivity Index)

The distance connectivity index $J(\mathrm{G_h})$ is calculated by summing a value over all adjacent pairs of vertices (edges), normalized by the graph's size and cycle attributes:

$$J(\mathrm{G_h}) = \frac{m}{\xi + 1} \sum_{(\hat{g},\hat{h}) \in \mathrm{E_d}} \frac{1}{\sqrt{D(\hat{g}) \cdot D\left(\hat{h}\right)}}$$

Definition 2.4 (Normalized Connectivity Index)

In fuzzy networks, the raw Connectivity Index (CI) evaluates the global path strength of a network by summing the maximum flow capability between all pairs. The number of pairs of the number of points increases quickly as the number of points grows.

To normalize this, the index is divided by the maximum possible number of unique vertex pairs:

$$NCI(G) = \frac{2}{n(n-1)} \sum_{ĝ,ĥ \in V} CON_G(ĝ, ĥ)$$

Where:

- n is the total number of points in the fuzzy graph.
- $CON_G(ĝ, ĥ)$ is the strength of connectedness (the maximum path capacity) between vertex u and vertex v.
- The Bounded Range: Because edge and vertex weights in fuzzy graphs fall in $[0,1]$, the resulting $NCI(G)$ is strictly bounded within $[0,1]$. A value closer to 1 implies a highly reliable, strongly interconnected fuzzy network.

$$NCI(G) = \frac{CI(G)}{\sum_{ĝĥ} \xi(ĝ, ĥ)}.$$

3. Main Results

Definition 3.1(Connectivity Index of D-HDFG)

For a DHFG

$$\mathrm{F_G} = (\zeta, \xi)$$

define the connectivity index as

$$CI(\mathrm{F_G}) = \sum_{ĝĥ \in \mathrm{E_d}} \frac{\xi(ĝ, ĥ)}{\sqrt{d(ĝ)d(ĥ)}}.$$

This extends the classical Randić connectivity index to DHFGs.

Definition 3.2(Distance Connectivity Index of D-HFG)

Define

$$DCI(G) = \sum_{ĝĥ \in E} \frac{\xi(ĝ, ĥ)}{D(ĝ, ĥ) + 1} \cdot \frac{1}{\sqrt{d(ĝ)d(ĥ)}}.$$

Where $D(ĝ, ĥ)$ is the fuzzy shortest-path distance.

Definition 3.4 (Normalized Connectivity Index of D-HFG)

The Normalized Connectivity Index (N CI) of D-HFG is defined as

$$NCI(G) = \frac{CI(G)}{\sum_{ĝĥ} \xi(ĝ, ĥ)}.$$

Where the Bounded Range between edge and vertex weights in fuzzy graphs fall in $[0,1]$, the resulting $NCI(G)$ is strictly bounded within $[0,1]$. A value closer to 1 implies a highly reliable, strongly interconnected fuzzy network.

Theorem 3.5

Every Distance-Hereditary Fuzzy Graph possesses a finite connectivity index.

Proof

Since every edge membership satisfies

$$0 \leq \xi(\hat{g}, \hat{h}) \leq 1,$$

and the graph contains finitely many vertices and edges, each summand is finite. Hence the total sum is finite.

Theorem 3.6

The connectivity index is invariant under connected induced fuzzy subgraphs that preserve fuzzy distances.

Proof

Since DHFGs preserve shortest distances,

$$D_H(\hat{g}, \hat{h}) = D_G(\hat{g}, \hat{h}),$$

for every connected induced fuzzy subgraph.

Therefore, every term in the connectivity index remains unchanged, proving invariance.

Theorem 3.7

If every edge membership increases while the graph structure remains unchanged, then the connectivity index increases.

Proof

Suppose

$$\xi_1(\hat{g}, \hat{h}) \leq \xi_2(\hat{g}, \hat{h}).$$

Each summand increases, implying $CI(G_1) \leq CI(G_2)$. Hence the index is monotonic.

Theorem 3.8

For a complete DHFG, $CI(G)$ attains its maximum value.

Proof

A complete graph has the maximum number of edges and highest possible fuzzy degrees. Therefore, the connectivity index is maximal.

Theorem 3.9

For a tree DHFG, $CI(G)$ is minimal among connected DHFGs having the same number of vertices.

Theorem 3.10

The Distance Connectivity Index satisfies $DCI(G) \leq CI(G)$.

Proof

Since $\frac{1}{D(\hat{g},\hat{h})+1} \leq 1$, every summand in $DCI(G)$ is bounded above by the corresponding summand in $CI(G)$.

Definition: 3.11

Let G be equal to (V, ζ, ξ) be a fuzzy graph that is distance-hereditary.

For every vertex $\hat{g}$, ∈V, its fuzzy degree is

$$d(\hat{g}) = \sum_{\hat{h} \in V} \xi(\hat{g}, \hat{h}).$$

Describe a continuous function

$$\phi: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+.$$

The definition of the General Degree-Based Topological Index is

$$TI_\phi(G) = \sum_{\hat{g}\hat{h} \in E} \xi(\hat{g}, \hat{h})\, \phi(d(\hat{g}), d(\hat{h})).$$

Theorem 3.12

Every finite Distance-Hereditary Fuzzy Graph possesses a finite generalized topological index.

Proof

Let

$F_G = (V_t, \zeta, \xi)$ be a finite Distance-Hereditary Fuzzy Graph (DHFG), where $V_t$is the finite vertex set, $\zeta: V_t \to [0,1]$ is the vertex membership function, and $\xi: V_t \times V_t \to [0,1]$ is the edge membership function satisfying

$$\xi(\hat{g}, \hat{h}) \le \min\{\zeta(\hat{g}), \zeta(\hat{h})\}, \forall\, \hat{g}, \hat{h} \in V_t.$$

Since $G$is finite, let

$$\mid V_t \mid = n \text{and} \mid E \mid = m,$$

where $m < \infty$.

The fuzzy degree of a vertex $u$is defined by

$$d(\hat{g}) = \sum_{v \in V} \xi(\hat{g}, \hat{h}).$$

Because each edge membership satisfies

$$0 \le \xi(\hat{g}, \hat{h}) \le 1,$$

and each vertex is adjacent to at most $n - 1$vertices, it follows that

$$0 \le d(\hat{g}) \le n - 1.$$

Hence every fuzzy degree is finite.

Now define the generalized degree-based topological index by

$$TI_\phi(G) = \sum_{uv \in E} \xi(\hat{g}, \hat{h})\, \phi(d(\hat{g}), d(\hat{h})),$$

where

$$\phi: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$$

is a real-valued function that is finite for all finite arguments.

Since $d(\hat{g})$and $d(\hat{h})$are finite, the value

$$\phi(d(\hat{g}), d(\hat{h}))$$

is finite for every edge $uv \in E$. Moreover,

$$0 \leq \xi(\hat{g}, \hat{h}) \leq 1,$$

so each summand

$$(\xi(\hat{g}, \hat{h})\phi(d(\hat{g}), d(\hat{h}))$$

is finite.

Finally, because the graph contains only finitely many edges ($m < \infty$), the generalized topological index is a finite sum of finite real numbers. Therefore,

$$\mid TI_{\phi}(G) \mid < \infty.$$

Hence every finite Distance-Hereditary Fuzzy Graph possesses a finite generalized topological index.

Remark: 3.13

It is crucial to assume that the function ϕ is finite on all pairings of finite degrees. For all degree-based indices derived from the unified framework, this theorem provides a fundamental existence result. For every finite Distance-Hereditary Fuzzy Graph, each of these indices is finite since it corresponds to a particular option of the function ϕ.

Theorem 3.14

The generalized degree-based topological index of a Distance-Hereditary Fuzzy Graph is non-negative.

Proof

Let

$$\mathrm{F_G} = (\mathrm{V_t}, \zeta, \xi)$$

be a finite Distance-Hereditary Fuzzy Graph (D-HFG).

The generalized degree-based topological index is defined as

$$TI_{\phi}(G) = \sum_{uv \in E} \xi(\hat{g}, \hat{h})\ \phi(d(\hat{g}), d(\hat{h})),$$

Where $d(\hat{g})$and $d(\hat{h})$denote the fuzzy degrees of the points $u$and v, respectively, $\phi: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$is a non-negative function. Since the edge membership values satisfy

$$0 \leq \xi(\hat{g}, \hat{h}) \leq 1,$$

it follows that

$$\xi(\hat{g}, \hat{h}) \geq 0.$$

Also, by assumption,

$$\phi(d(\hat{g}), d(\hat{h})) \geq 0.$$

Therefore, for every edge $\hat{g}\hat{h} \in E$,

$$\xi(\hat{g}, \hat{h})\ \phi(d(\hat{g}), d(\hat{h})) \geq 0.$$

Hence, every term in the summation defining $TI_{\phi}(G)$is non-negative.

Since the sum of finitely many non-negative real numbers is itself non-negative,

$$\sum_{uv\in E} \xi(\hat{g}, \hat{h})\, \phi(d(\hat{g}), d(\hat{h})) \geq 0.$$

Therefore,

$$TI_{\phi}(G) \geq 0.$$

Thus, the generalized degree-based topological index of every finite Distance-Hereditary Fuzzy Graph is non-negative.

$$\boxed{TI_{\phi}(G) \geq 0.}$$

Hence, the theorem is proved.

Remark:3.15

The condition that $\phi(x, y) \geq 0$is essential for this theorem. Consequently, each of these indices is non-negative for every finite Distance-Hereditary Fuzzy Graph under its respective domain of definition.

Theorem 3.16

If the function $\phi(x, y)$ is symmetric, then the generalized degree-based topological index $TI_{\phi}(G)$ is independent of the ordering of points.

Proof

Let

$$\mathrm{F_G} = (\mathrm{V_t}, \zeta, \xi)$$

be a finite Distance-Hereditary Fuzzy Graph, and let the generalized degree-based topological index be defined by $TI_{\phi}(\mathrm{F_G}) = \sum_{\hat{g}\hat{h}\in \mathrm{E_d}} \xi(\hat{g}, \hat{h})\, \phi(d(\hat{g}), d(\hat{h}))$,

Where $d(\hat{g})$and $d(\hat{h})$denote the fuzzy degrees of the points $\hat{g}$ and $\hat{h}$, $\xi(\hat{g}, \hat{h})$is the membership value of the edge $\hat{g}\hat{h}$, $\phi: \mathbb{R}^{+} \times \mathbb{R}^{+} \rightarrow \mathbb{R}^{+}$is a symmetric function.

Since $G$is an undirected fuzzy gaph, the edge membership function satisfies

$$\xi(\hat{g}, \hat{h}) = \xi(\hat{h}, \hat{g}),$$

and the edge $\hat{g}\hat{h}$ is identical to the edge $\hat{h}\hat{g}$.

Now consider any edge $\hat{g}\hat{h} \in \mathrm{E_d}$. Its contribution to the generalized topological index is

$$\xi\left(\hat{g}, \hat{h}\right) \phi\left(d(\hat{g}), d\left(\hat{h}\right)\right).$$

If the ordering of the points is reversed, the contribution becomes

$$\xi\left(\hat{h}\hat{g}\right) \phi\left(d\left(\hat{h}\right), d(\hat{g})\right)$$

Using the symmetry of the edge membership function and the symmetry of ϕ, we obtain

$$\xi\left(\hat{h}\hat{g}\right) \phi\left(d\left(\hat{h}\right), d(\hat{g})\right) = \xi\left(\hat{g}, \hat{h}\right) \phi\left(d(\hat{g}), d\left(\hat{h}\right)\right).$$

Thus, reversing the order of the points does not change the value contributed by any edge.

Since this argument holds for every edge in the graph, every term in the summation remains unchanged under any ordering of the vertices. Therefore,

$$\sum_{\hat{g}\hat{h}\in E_d} \xi(\hat{g},\hat{h})\,\phi\left(d(\hat{g}),d(\hat{h})\right) = \sum_{\hat{h}\hat{g}\in E_d} \xi(\hat{h}\hat{g})\,\phi\left(d(\hat{h}),d(\hat{g})\right).$$

Hence, $TI_\phi(F_G)$ is independent of the ordering of the points.

Therefore, $TI_\phi(F_G)$ is invariant under any ordering of the vertices.

Hence, the theorem is proved.

Remark:3.17

The symmetry condition guarantees that each edge's two endpoints are treated equally by the generalised framework. As a result, the symmetric defining functions of many popular degree-based topological indices make them independent of vertex ordering. Consequently, on undirected Distance-Hereditary Fuzzy Graphs, all of these indices are well-defined and independent of the vertex labelling or ordering.

Theorem 3.18

Let $F_{G_1} = (V_t, \zeta, \xi_1)$and $F_{G_2} = (V_t, \zeta, \xi_2)$be two Distance-Hereditary Fuzzy Graphs having the same vertex set and the same graph structure. If

$$\xi_1(\hat{g},\hat{h}) \leq \xi_2(\hat{g},\hat{h}), \forall\, \hat{g}\hat{h} \in E_d,$$

and

$$\phi(x,y) \geq 0$$

for all non-negative real numbers $x$and y, then

$$TI_\phi(F_{G_1}) \leq TI_\phi(F_{G_2}).$$

Hence, the generalized degree-based topological index is non-decreasing with respect to edge membership values.

Proof

Let

$F_{G_1} = (V_t, \zeta, \xi_1)$ and $F_{G_2} = (V_t, \zeta, \xi_2)$

be two Distance-Hereditary Fuzzy Graphs having the same set of vertices and the same edge set. Assume that

$$\xi_1(\hat{g},\hat{h}) \leq \xi_2(\hat{g},\hat{h}), \forall\, \hat{g}\hat{h} \in E_d$$

The generalized degree-based topological index of a DHFG is defined by

$$TI_\phi(F_G) = \sum_{\hat{g}\hat{h}\in E_d} \xi(\hat{g},\hat{h})\,\phi(d(\hat{g}),d(\hat{h})),$$

where

$$d(\hat{g}) = \sum_{\hat{h}\in V_t} \xi(\hat{g},\hat{h})$$

is the fuzzy degree of the vertex u.

Since

$$\xi_1(\hat{g}, \hat{h}) \leq \xi_2(\hat{g}, \hat{h}),$$

the fuzzy degrees satisfy

$$d_1(\hat{g}) \leq d_2(\hat{g}), d_1(\hat{h}) \leq d_2(\hat{h}),$$

for every vertex.

Assume that the function

$$\phi: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$$

is non-negative, that is,

$$\phi(x, y) \geq 0$$

for all $x, y \geq 0$.

Therefore, for every edge uv,

$$\xi_1(\hat{g}, \hat{h})(\hat{g}, \hat{h})\ \phi(d_1(\hat{g}), d_1(\hat{h})) \leq \xi_2(\hat{g}, \hat{h})(\hat{g}, \hat{h})\ \phi(d_2(\hat{g}), d_2(\hat{h})),$$

since both the edge membership values and the corresponding values of $\phi$are non-negative.

Summing over all edges of the graph yields

$$\sum_{\hat{g}\hat{h}\in E_d} \xi_1\,(\hat{g}, \hat{h})\ \phi(d_1(\hat{g}), d_1(\hat{h})) \leq \sum_{\hat{g}\hat{h}\in E_d} \xi_2\,(\hat{g}, \hat{h})\ \phi(d_2(\hat{g}), d_2(\hat{h})).$$

Hence,

$$TI_\phi(F_{G_1}) \leq TI_\phi(F_{G_2}).$$

Therefore,

$$\boxed{TI_\phi(F_{G_1}) \leq TI_\phi(F_{G_2}),}$$

This means that the generalized degree based index does not decrease when the edge membership values go up and the graph structure stays the same. Hence, the theorem is proved.

Remark 3.19

As long as the function ϕ is non-negative and non-decreasing in each of its arguments, the aforementioned proof is valid. Because the vertex degrees d(ĝ) rely on the edge membership values, the additional monotonicity assumption is crucial. Therefore, without adding constraints to the graph or changing the assumptions, the theory does not automatically hold for those indices. The theorem becomes mathematically rigorous when the monotonicity assumption on ϕ is included.

4. Uses

The following domains can use the suggested unified framework: • Communication Networks: Maintain shortest-path distances while assessing network connectivity and dependability in the face of unpredictable link strengths. • Transportation Systems: Evaluate the resilience and effectiveness of logistics, rail, and road networks under unpredictable travel circumstances. • Mathematical Chemistry: Use generalised topological descriptors to model chemical structures with unknown bond strengths. • Biological Networks: Examine gene regulatory networks with ambiguous relationships and protein-protein interactions. • Social Networks: Assess impact and community structure in networks with different levels of relationship strength. • Decision-Support Systems: In multi-criteria decision-making, these systems depict ambiguous relationships between criteria and options. • Artificial Intelligence: Enhance reasoning in uncertain knowledge graphs and semantic networks with broadened topological measurements.

5. Conclusions

We propose a unified mathematical framework for degree based topological indices in Distance-Hereditary Fuzzy Graphs in this paper. In this paper, we propose a generalised index defined by a parameterised function of fuzzy vertex degrees, which unifies several classical descriptors including the Randić, Zagreb, Harmonic, Geometric–Arithmetic, Atom–Bond Connectivity and Sombor indices. It is shown that these indices have fundamental theoretical properties such as finiteness, non-negativity, symmetry, monotonicity and invariance under isomorphisms of fuzzy graphs, and thus provide a common theoretical basis for these indices. The proposed framework allows to study easily many topological descriptors and also allows to develop new indices by choosing suitable degree functions. Moreover, its practical significance is reflected in its applicability to uncertain communication, transportation, biological, and social networks. The unified approach provides a solid base for future work on topological descriptors, optimisation methods, and advanced applications of Distance-Hereditary Fuzzy Graphs in network science.

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