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Article

Comparison of Connected Power Domination Number with Connected Equitable Power Domination Number

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Abstract: This paper investigates the strategic placement of monitoring units in networks by exploring the structural relationships between different graph domination parameters. We introduce a comparative analysis between the Connected Power Domination Number (CPDN) and the newly proposed Connected Equitable Power Domination Number (CEPDN) across several distinct graph families, including paths, cycles, stars, wheels, ladders, bipartite graphs, combs, and twig graphs. By establishing exact values for both parameters across these configurations, we evaluate the precise mathematical trade-off between network monitoring efficiency and workload equitability constraints. Our results demonstrate that while equitable constraints match standard propagation efficiency in highly symmetric topologies, they significantly alter monitor requirements in asymmetric frameworks like ladder and twig structures. These findings provide foundational insights for designing balanced, fault-tolerant network monitoring systems.
Keywords: Equitable Power Domination, Connected Power Domination, Power domination set, Connected Equitable Power Domination.

1. INTRODUCTION

Sampathkumar and Walikar first proposed the idea of connected dominating sets (CDS), which states that a dominating set $D \subseteq V(G)$ in a graph Gis a CDS if the induced subgraph $\langle D \rangle$ is connected. The connected domination number, represented by $\gamma_c(G)$, is the smallest size of a connected dominating set in a graph G. Connected dominating sets are only found in connected graphs. The inequality $\gamma(G) \leq \gamma_c(G)$ is well known, and it is frequently strict. Practical requirements in electrical power systems, specifically the necessity to reduce the number of measuring units needed to efficiently monitor a whole network, gave rise to the idea of power domination in graphs. Power domination models the strategic placement of Phase Measurement Units (PMUs) in power grids, which are capable of directly monitoring a node and indirectly observing other components through propagation rules derived from Kirchhoff's laws. Due to the high cost of PMUs, determining their optimal number and placement has been a crucial topic of research in electrical engineering. Boris Brimkov has demonstrated that, for certain graph classes, such as block graphs or cactus graphs, the power domination number can be computed in linear time using decomposition techniques depending on cut-vertices and structural properties of these graphs, even though it is known to be NP-hard to compute in general. Furthermore, power domination number, connected power domination number (CPDN), or power propagation time computations have been presented using integer programming formulations. In recent developments, the concept of Connected Equitable Power Domination has emerged as a natural extension of CPDN. While CPDN ensures connectivity and complete coverage of the graph, the Connected Equitable Power Domination Number (CEPDN) introduces an equity constraint by requiring that each dominated vertex be adjacent to a dominating vertex with a degree difference of at most one. This condition encourages balanced domination, avoiding overburdened nodes and promoting fairness in the network. The importance of CPDN and CEPDN extends beyond theoretical interest, with key applications in: (i) Electrical Networks**:** Efficient and balanced placement of PMUs, maintaining network connectivity and minimizing vulnerability by avoiding concentrated monitoring points. (ii) Resource Distribution**:** Strategic positioning of sensors, communication relays, or aid centers to ensure equitable load distribution across the network. (iii) Social Network Analysis**:** Identifying influential and connected subgroups that can effectively oversee or influence others without significant imbalance. (iv) Disaster Management: Locating critical hubs for equitable emergency resource deployment to maximize coverage and maintain connectivity during crises. This study aims to determine and compare CPDN and CEPDN for certain classes of graphs. By analyzing these parameters, we provide insights into their structural behavior and practical relevance, contributing to both theoretical graph theory and real-world applications in network monitoring and optimization. In the present paper, we determine the CPDN and the CEPDN for certain classes of graphs involving paths, stars, cycles, wheels, ladders, bipartite graphs, combs, and twig graphs, and provide a comprehensive comparative analysis between them.

2. Preliminaries

If the subgraph induced by D, represented by $G[D]$, is connected, then the dominating set D in graph G is said to be connected. A connected dominating set exists if and only if G is connected, since a dominating set must contain at least one vertex from each component of G. All connected dominating sets of G have a minimum cardinality, which is called the connected domination number, denoted by $\gamma_c(G)$. If the induced subgraph $G[S]$ is connected and the set $S \subseteq V$ is a power dominating set for a connected graph $G = (V, E)$, then it is referred to as a connected power dominating set (CPDS). The Connected Power Domination Number (CPDN), denoted by $\gamma_{P,c}(G)$, is defined as the minimum cardinality of a CPDS in the graph G. If every vertex $v \in V - S$ is adjacent to at least one vertex $u \in S$ where $|d(u) - d(v)| \leq 1$, and the subgraph $G[S]$ is connected, then an equitable power dominating set $S \subseteq V$ in G is called a connected equitable power dominating set (CEPDS).

Before evaluating specific graph structures, we establish a fundamental theoretical hierarchy that defines the behavior of Connected Equitable Power Domination for any connected graph G. This structural relationship validates the mathematical trade-off introduced by combining connectivity requirements with degree equity constraints.

Theorem 1

For any connected graph $G = (V, E)$, the parameters satisfy the following inequality: $\gamma_{P,c}(G) \leq \gamma_{eP,c}(G) \leq \gamma_c(G)$.

Proof

Let $S_1 \subseteq V$ be a minimum Connected Equitable Power Dominating Set (CEPDS) of G, such that $|S_1| = \gamma_{eP,c}(G)$. By definition, S_1 must satisfy three conditions: it must be a power dominating set, its induced subgraph $G[S_1]$ must be connected, and every vertex $v \in V - S_1$ must satisfy the equity condition $| d(u) - d(v) | \leq 1$ for at least one adjacent $u \in S_1$. If we remove the third condition (the degree equity constraint), S_1 remains a valid power dominating set that induces a connected subgraph. Thus, any valid CEPDS is fundamentally a valid Connected Power Dominating Set (CPDS). Since a minimum CPDS has a cardinality of $\gamma_{P,c}(G)$, it follows strictly that: $\gamma_{P,c}(G) \leq \gamma_{eP,c}(G)$. Next, let $S_2 \subseteq V$ be a minimum standard Connected Dominating Set (CDS) of G, where $|S_2| = \gamma_c(G)$. Since standard domination initially observes every vertex in $V - S_2$ in the very first step, the propagation rules of power domination are not even required to achieve complete coverage. Furthermore, because S_2 dominates the entire graph traditionally, the degree difference step does not limit its coverage scope under standard propagation. Because power domination rules allow for wider structural observation with equal or fewer vertices, a minimal standard connected dominating set represents an upper bound constraint on equitable power propagation. Therefore, the cardinality of a minimum CEPDS cannot exceed that of a standard CDS, yielding: $\gamma_{eP,c}(G) \leq \gamma_c(G)$. Combining both inequalities, we establish the universal structural hierarchy: $\gamma_{P,c}(G) \leq \gamma_{eP,c}(G) \leq \gamma_c(G)$.

3. RESULT

Connected Domination Number

1. [5] Let S_n be a star graph, then $\gamma_c(S_n) = 1, n \geq 3$.
2. Let CB_n be a comb graph, then $\gamma_c(CB_n) = n, n \geq 2$.
3. Let T_n be the Twig graph, then $\gamma_c(T_n) = n, n \geq 1$.
4. Let $W_{1,n}$ be a wheel graph, then $\gamma_c(W_{1,n}) = 1, n \geq 3$.

Connected Power Domination Number

1. [2] Let C_n be a cycle, then $\gamma_{P,c}(C_n) = 1, n \geq 3$.
2. [2] Let S_n be a star graph, then $\gamma_{P,c}(S_n) = 1, n \geq 3$.
3. [2] Let P_n be a path graph, then $\gamma_{P,c}(P_n) = 1, n \geq 2$.
4. Let CB_n be a comb graph, then $\gamma_{P,c}(CB_n) = 1, n \geq 2$.

5. Let $W_{1,n}$ be a wheel graph, then $\gamma_{P,c}(W_{1,n}) = 1, n \geq 3$.
6. Let $K_{m,n}$ be a bipartite graph, then $\gamma_{P,c}(K_{m,n}) = 1, m \geq 1, n \geq 2$.

Connected Equitable Power Domination Number

1. Let $C_n$be a cycle, then$\gamma_{eP,c}(C_n) = 1, n \geq 3$.
2. Let $S_n$be a star graph, then $\gamma_{eP,c}(S_n) = 1, n \geq 3$.
3. Let P_n be a path graph, then $\gamma_{eP,c}(P_n) = 1, n \geq 2$.
4. [11] Let CB_n be a comb graph, then $\gamma_{eP,c}(CB_n) = n - 2, n \geq 2$.
5. Let $W_{1,n}$ be a wheel graph, then $\gamma_{eP,c}(W_{1,n}) = 1, n \geq 3$.
6. Let $K_{m,n}$ be a bipartite graph, then $\gamma_{eP,c}(K_{m,n}) = 1, m \geq 1, n \geq 2$.

Theorems on Connected Power Domination

Theorem 2

Suppose L_n be the ladder graph, then $\gamma_{P,c}(L_n) = 1, n \geq 2$.

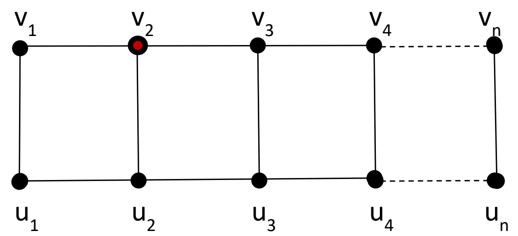

Figure 1. Connected Power Domination of Ladder Graph L_n.

Proof

The ladder graphL_n as shown in Fig. 1. The vertex set of the ladder graph comprisestwo parallel paths: $v_1, v_2, v_3, \ldots\ldots v_n$, $u_1, u_2, u_3, \ldots\ldots u_n$, where each u_i is connected to v_i for $i = 1,2, \ldots, n$, as well as consecutive vertices in both u-path and v-path are also adjacent. Consider the set $S = \{u_2\}$. According to the rules of power domination, the vertex u_2 initially dominates itself and its adjacent vertices: $u_1, u_3$and v_2. Then, through the propagation rule, the following sequence of observations occurs:u_1 observesv_1, v_2 observes v_3, u_3 observes v_3 and finally, v_3 observes v_4.This technique is repeated until every vertex in the graph is observed. This method keeps going until all of the vertices have been observed. $S = \{u_2\}$formspower dominating set for the ladder graph L_n. Hence, $\gamma_{P,c}(L_n) = 1,\ n \geq 2$.

Theorem 3

SupposeCB_n be the comb graph, then$\gamma_{P,c}(CB_n) = \begin{cases} 1, n \leq 3 \\ n - 2, n \geq 4 \end{cases}$

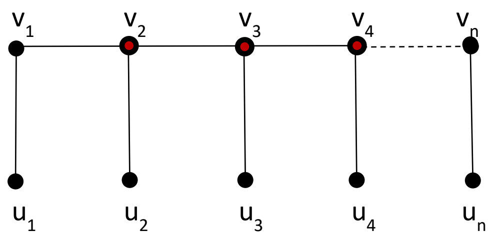

Figure 2. Connected Power Domination of Comb graph CB_n.

Proof

Consider that $CB_n$is the comb graph depicted in Fig.2. The vertex set consists of a path $v_1, v_2, v_3, \ldots\ldots v_n, u_1, u_2, u_3, \ldots\ldots u_n$ with each vertex connected to a corresponding vertex forming a “tooth” at each vertex of the path. That is, for each $i \in \{1,2,3, \ldots, n\}$, the edge (u_i, v_i) exists. To determine the Connected Power Domination Number$\gamma_{P,c}(CB_n)$, consider the following construction: When $n \leq 3$, selecting a single vertex (e.g.v_2

) is sufficient to observe all vertices through domination and propagation. Hence$\gamma_{P,c}(C_n) = 1$, for $n \leq 3$. For $n \geq 4$, select the vertices $\{v_{2,}v_{3,} \ldots\ldots v_{n-1}\}$ to form the set S. This subset induces a connected subgraph, satisfying the connectivity condition.Each$v_i \in S$dominates itself, u_i andits adjacent vertices in the path (i.e., $v_{i-1,}v_{i+1}$). Through initial observation and propagation, all $u_i$and v_i vertices are eventually observed. Consequently, $n-2$ is cardinality of the connected power dominating set.Hence $\gamma_{P,c}(CB_n) = \begin{cases} 1, n \leq 3 \\ n-2, n \geq 4 \end{cases}$

Theorem 4

Let T_n be the Twig graph, then $\gamma_{P,c}(T_n) = n$,$n \geq 1$.

Proof

A twig graph $T_n$consists of $3n+2$ vertices and $3n+1$ edges. The vertex set is given by:$V(T_n) = \{u_{1,}u_{2,}u_{3,} \ldots\ldots u_{3n+2}\}$. The structure of the twig graph is such that it resembles a sequence of small "branches" or "twigs"

emerging from a central path or spine. As shown in Figure 3, for example, the twig graph T_2 consists of 8 vertices and 7 edges.To compute CPDN$\gamma_{P,c}(T_n)$, consider case $n = 2$:Select the vertex $u_2 \in S$. According to the power dominationrule,

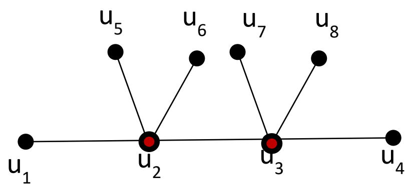

Figure 3. Connected Power Domination of Twig graph T_n.

$u_2$dominates u_1, u_5, u_6, and u_3.To complete the domination, select $u_2 \in S$. Through propagation, the remaining vertices are observed.Since the sub graph induced by $S = \{u_{2,}u_{3,}\}$ is connected and all vertices are observed, this is a valid connected power dominating set. Hence, $\gamma_{P,c}(T_n) = 2$. In general, for any n, one vertex per twig branch must be selected to ensure observation of all vertices and connectivity. Thus, for each twig segment, selecting one vertex ensures all others can be observed via domination and propagation. As a result, the number of vertices required in connected power dominating set is exactly n.Therefore, CPDN for twig graph:
$\gamma_{P,c}(T_n) = n$.

Theorems on Connected Equitable Power Domination

Theorem 5

Suppose L_n be the ladder graph, then$\gamma_{eP,c}(L_n) = n-2$ for $n \geq 2$.

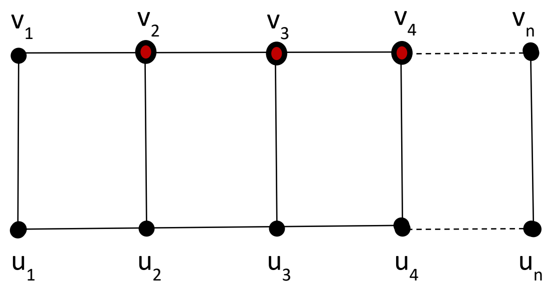

Figure 4. Connected Equitable Power Domination of Ladder Graph L_n.

Proof

Let $L_n$represent ladder graph as shown in Fig. 4. The vertices of $L_n$consist of two parallel paths: Horizontal vertices $u_{1,}u_{2,}u_{3,} \ldots\ldots u_n$, Corresponding top vertices $v_{1,}v_{2,}v_{3,} \ldots\ldots v_n$. To construct a Connected

Equitable Power Dominating Set (CEPD set):Start by choosing the vertex$u_2 \in S$. According to the rules of power dominationu_2 directly dominates $u_1, u_3$and v_2. Then,u_1 observes v_1.Now consider vertex u_3, which has two unobserved adjacent vertices. To maintain both connectedness and degree equity($| d(u) - d(v) | \leq 1$), include $u_3 \in S$.u_3 dominates v_2 and u_4 .Vertex v_3 observes v_2 , and u_4 again has two unobserved neighbors. So, include $u_4 \in S$ to continue the process.Continue this pattern, adding $u_i \in S$for $i = 2$ to $n - 1$, thereby ensuring all vertices are eventually observed under the rules of connected equitable power domination.Thus, CEPD Number of ladder graphL_n, for $n \geq 2$, is:$\gamma_{eP,c}(L_n) = n - 2$.

Theorem 6

SupposeCB_n be a comb graph, then $\gamma_{eP,c}(CB_n) = \begin{cases} 1, n \leq 3 \\ n-2, n \geq 4 \end{cases}$

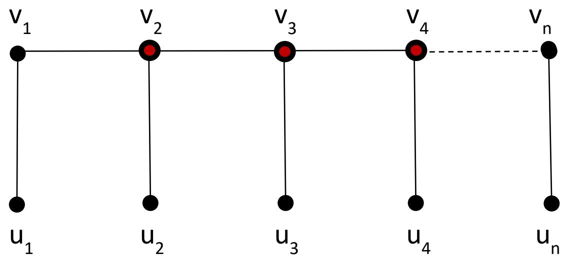

Figure 5. Connected Equitable Power Domination of Comb graph CB_n.

Proof

Let CB_n denote the comb graph, as illustrated in Fig. 5. The vertex set consists of a horizontal backbone path $v_1, v_2, v_3, \dots\dots v_n$ and a corresponding vertical "tooth" vertex $u_1, u_2, u_3, \dots\dots u_n$ connected to each v_i via the edge (u_i, v_i). To determine the Connected Equitable Power Domination Number $\gamma_{eP,c}(CB_n)$, we consider two cases based on the network size: (i) For $n \leq 3$: Selecting a single central backbone vertex, such as $S = \{v_2\}$, is sufficient. Vertex v_2 directly dominates its neighbors under equitable constraints, and the remaining vertices are completely observed via standard power propagation rules. Thus, $\gamma_{eP,c}(CB_n) = 1$ for $n \leq 3$. (ii) For $n \geq 4$: We define the dominating set by selecting the inner backbone path vertices, setting $S = \{v_2, v_3, \dots\dots v_{n-1}\}$. The induced subgraph $G[S]$ is trivially connected because it forms a contiguous sub-path along the backbone of the comb. Each chosen vertex $v_i \in S$ directly dominates itself, its corresponding vertical tooth u_i, and its adjacent path neighbors v_{i-1} and v_{i+1} under strict degree equity constraints. Through this initial domination and subsequent power propagation steps, all remaining peripheral vertices are completely observed. Because the minimum cardinality of this set requires exactly $n - 2$ vertices, we establish that $\gamma_{eP,c}(CB_n) = n - 2$ for $n \geq 4$.

Theorem 7

Suppose T_n be the twig graph, $\gamma_{eP,c}(T_n) = n$ for $n \geq 1$.

Proof

In Fig. 3, the twig graph T_2 is illustrated. The graph T_2 consists of the vertices:$u_1, u_2, u_3, \dots u_8$,which follow a branched structure where each node connects to others in a "twig-like" extension. To determine the Connected Equitable Power Dominating Set (CEPD set) for T_2, consider the following process, select the vertex$u_2 \in S$,according to power domination rulesu_2 dominatesu_1, u_3, u_6 ,u_7. Next u_3, having unobserved neighbors (such asu_4, u_8), must be added to the set. Thus, add$u_3 \in S$. Now, u_3 dominates u_4, u_8. Hence, the minimal connected power dominating set is$S = \{u_2, u_3\}$ and the induced sub graph$G[S]$is connected.Therefore,for $n = 2$,$\gamma_{eP,c}(T_2) = 2$. In general, for a twig graphT_n , the pattern follows:$\gamma_{eP,c}(T_n) = n$.

Table 1. Comparison CDN, CPDN and CEPDN.

GRAPH	CDN	CPDN	CEPDN	Observations
Cycle "graph	$n-2, if n \geq 3$	$1, if n \geq 3$	$1, if n \geq 3$	**CPDN = CEPDN < CDN**

Path graph	$\begin{cases} 1, if n \leq 2 \\ n-2, if n > 2 \end{cases}$	$1, if n \geq 2$	$1, if n \geq 2$	**CPDN = CEPDN < CDN**
Star graph	$1, if n \geq 3$	$1, if n \geq 3$	$1, if n \geq 3$	**All equal due to central domination**
Comb graph	$n, if n \geq 2$	$1, if n \leq 3$ $n-2, if n \geq 4$	$1, if n \leq 3$ $n-2, if n \geq 4$	**CPDN = CEPDN < CDN**
Wheel graph	$1, if n \geq 3$	$1, if n \geq 3$	$1, if n \geq 3$	**Center vertex dominates entire graph**
Bipartite graph	$\begin{cases} 1, if either m or n = 1 \\ 2, \quad if m, n \geq 2 \end{cases}$	$1, if m \geq 1, n \geq 2$	$1, if m \geq 1, n \geq 2$	**CPDN =CEPDN" for $m \geq 1, n \geq 2$**
Ladder graph	$n, if n \geq 2$	$1, if n \geq 2$	$n-2, if n \geq 2$	**CDN>CEPDN > CPDN**
Twig graph	$n, if n \geq 1$	$n, if n \geq 1$	$n, if n \geq 1$	**CPDN = CEPDN =CDN**

5. OBSERVATION

Validation of General Bounds (Theorem 1)

The calculated values across the selected graph families perfectly validate the universal structural hierarchy established in Theorem 1 $(\gamma_{P,c}(G) \leq \gamma_{eP,c}(G) \leq \gamma_c(G))$. The results demonstrate how different structural symmetries cause these bounds to either collapse into equalities or diverge.

1. **Equality of CPDN and CEPDN:** In most standard graphs (cycle, path, star, wheel, bipartite), CPDN and CEPDN are equal. This shows that the bounds collapse $(\gamma_{P,c}(G) = \gamma_{eP,c}(G))$, proving that power domination naturally distributes the monitoring load equitably due to either structural symmetry or the dominance power of central vertices (like in stars and wheels).
2. **Deviation in Ladder and Twig Graphs:** In ladder graphs, although the CPDN is 1, the CEPDN increases to $n-2$, indicating a strong impact of the equitable constraint where the parameter strictly climbs away from the lower bound. In twig graphs, CEPDN is equal to CDN $(\gamma_{eP,c}(G) = \gamma_c(G))$, showing that the parameter hits the absolute upper bound, leaving no advantage in using power domination when equitability is enforced.

6. CONCLUSION

As established by our general structural hierarchy $(\gamma_{P,c}(G) \leq \gamma_{eP,c}(G) \leq \gamma_c(G))$, the Connected Equitable Power Domination Number (CEPDN) is naturally bounded below by the standard connected power domination number and bounded above by the traditional connected domination number. The comprehensive comparison across various graph families conducted in this study reveals a critical structural trade-off: while these parameters completely coincide for highly symmetric or centrally-dominated graph families (such as cycles, paths, stars, and wheels), they sharply diverge in asymmetric or irregular structures like ladder and twig networks. This divergence underscores a foundational principle in network monitoring architecture—enforcing architectural equity and balancing monitor workloads introduces a mathematically quantifiable cost to optimal monitoring efficiency. These theoretical bounds and precise calculations provide valuable architectural insights for electrical grids, sensor layouts, and communication frameworks where maintaining network connectivity and preventing localized node fatigue are simultaneously critical. Future research will explore these parameters within broader structural configurations, such as graph products and network operations under dynamic load constraints..

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