

# On Equitable Dominator Coloring of Splitting Graphs from Specific Graph Classes

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**Abstract:** . An equitable dominator coloring (EDC) of a graph  $G$  is a proper coloring of  $G$  where each vertex dominates every other vertex of at least one color class and the cardinalities of the color classes differ by a maximum of one. The equitable dominator chromatic number of  $G$ , which is represented by  $\chi_{ed}$ , is the minimum number of colors desired for a dominator coloring of  $G$ . In this paper, we calculate the dominator chromatic number of splitting graphs of specific graph classes.

**Keywords:** Dominator chromatic number, Dominator coloring, Splitting Graph, complete graph, Star graph, Ladder Rung Graph, Banana tree, Sun let graph, Comb Graph, Chromatic number.

## 1. INTRODUCTION

This study explores finite, undirected, and simple graphs, which consist of distinct vertices and edges without loops or multiple connections. These graphs have a well-defined structure with a limited number of elements. The focus is on their characteristics, significance, and practical applications.

A *dominating set* in graph theory is a set of vertices  $D$  in a graph  $G = (V, E)$  such that each vertex in  $V$  is either in  $D$  or next to a minimum of one vertex in  $D$ .

Graph coloring is the process of giving a graph's edges or vertices a color while adhering to specific rules. Vertex coloring, the most popular type of graph coloring, adds a color to each vertex so that no two neighboring vertices have the same color. For this kind of coloring, the chromatic number of the graph is the smallest amount of colors needed. Hedetniemi [2] proposed the notion of dominator coloring, frequently referred to as  $\chi_d$  coloring. Gera [1] discovered a correlation between dominator coloring of graphs and chromatic and domination numbers. The Equitable dominator coloring of graph is a new concept which is introduced and clearly discussed by Pheba Sarah George, S. Madhumita and Sudev Nadubath [5] and proposed the notion if equitable dominator coloring is referred as  $\chi_{ed}$ .

In a graph  $G$ , a dominator coloring occurs when each vertex ( $u$ ) either dominates or is dominated by at least one vertex of the same color.

For a graph  $G$ , an equitable dominator coloring [5] is a proper coloring of  $G$  in which every vertex  $v$  in the vertex set  $V(G)$  dominates at least one color class. The color classes vary by no more than one size.

"Proper" coloring is defined as one in which no two neighboring vertices have the same color.

The splitting graph [4] of a graph  $G$ , denoted as  $S'(G)$ , is obtained by adding duplicate vertex  $v'$  for each vertex  $v \in V(G)$ , and connecting  $v'$  to all the neighbors of  $v$  in  $G$ .

## 2. RESULTS AND DISCUSSION

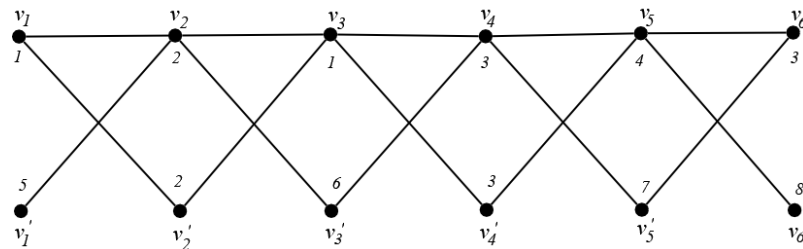
*Theorem 1.* For  $n \geq 4$ ,  $\chi_{ed}[S'(P_n)] = n + 2$

*Proof:* Consider a path graph  $P_n$  with the edge set  $E(P_n) = \{(v_i, v_{i+1}) \mid 1 \leq i \leq n-1\}$  and vertices set  $V(P_n) = v_1, v_2, v_3, \dots, v_n$ . By adding new vertices  $v'_1, v'_2, v'_3, \dots, v'_n$ , the path graph's splitting graph is generated. Pendant vertices  $v'_1$  and  $v'_n$  are present in the splitting graph of route graph  $S'(P_n)$ . These vertices can be given unique colors, and they either dominate their own color class or the color class assigned to the nearby vertices,  $v_2$  and  $v_{n-1}$ , respectively. In light of this, we derive the subsequent coloring patterns such that

$$C(V_j) = \begin{cases} C_j - \left\lfloor \frac{j}{3} \right\rfloor - 1, & j \equiv 0 \pmod{3} \\ C_j - \left\lfloor \frac{j}{3} \right\rfloor, & j \equiv 1, 2 \pmod{3} \end{cases}$$

Assume that the color class assigned by vertices  $v_i$  and  $v'_i$  is the same where  $i = 2, 4, 6, \dots$ . The other vertices of  $v'_i$  is assigned by unique color class and it dominates themselves. The colors assigned to the other vertices are used no more than twice. Therefore, both the dominator and equitable requirements are met.

*Example 1.* Let  $S'(P_6)$  and its equitable dominator coloring as displayed in the Figure 1.



**FIGURE 1.** Consider  $S'(P_6)$  and its EDC.

*Theorem 2.* For  $n \geq 3$ ,  $\chi_{ed}[S'(S_n)] = n + 1$

*Proof:* Suppose that  $S_n$  is a star graph defined on  $n+1$  vertices, with vertex set consists  $V(S_n) = v_j, 1 \leq j \leq n$ . The degree of  $v_1$  is  $n$  and  $v_j, 1 \leq j \leq n$  vertices have degree 1. For every initial vertex  $v_i$  in a graph  $G$ , represented by  $S'(S_n)$ , a new vertex  $v'_i$  is added to create the splitting graph.

Let us first observe the structure:

The set of vertices is  $V[S'(S_n)] = \{v_1, v_2, \dots, v_n\} \cup \{v'_1, v'_2, \dots, v'_n\}$ , where  $V = 2(n+1)$

Define a coloring function

$\phi = V[S'(S_n)] \rightarrow C$ , where  $C = \{c_1, c_2, \dots, c_n\}$  is set of  $n+1$  distinct colours. Assign  $\phi(v_1) = c_1, \phi(v_i) = c_i, \forall 1 \leq i \leq n$  and  $\phi(v'_1) = c_1, \phi(v'_i) = c_i \forall 1 \leq i \leq n$ . This results in a mapping where for

every  $i$ , the pair  $(v_i, v'_i)$  share the identical color since there is no edge exists between them satisfying the proper coloring requirement.

We have to show  $\forall uv \in E[S'(S_n)], \phi(u) = \phi(v)$

We consider all edge types

$$\phi(v_0) = c_1 \neq c_i = \phi(v_i)$$

$$\phi(v'_0) = c_1 \neq c_i = \phi(v_i)$$

$$\phi(v'_i) = c_i \neq c_1 = \phi(v_0)$$

Since no neighboring vertices have the same color  $\phi$  is a proper coloring. Each color class  $C_j = \phi^{-1}(C_j)$  must be a dominating set. For  $j = 0, C_1 = \{v_0, v'_0\}$ . Since both  $v_0$  and  $v'_0$  are adjacent to all  $v_i, c_1$  dominates  $V[S'(S_n)] \setminus c_1$ . For  $j \geq 1, C_j = \{v_j, v'_j\}$ . Since  $v_j \sim v_j$  and  $v'_j \sim v_j, C_j$  dominates at least  $v_1$ , hence satisfies the dominates requirements.

Each and every color class  $c_j$  satisfies:

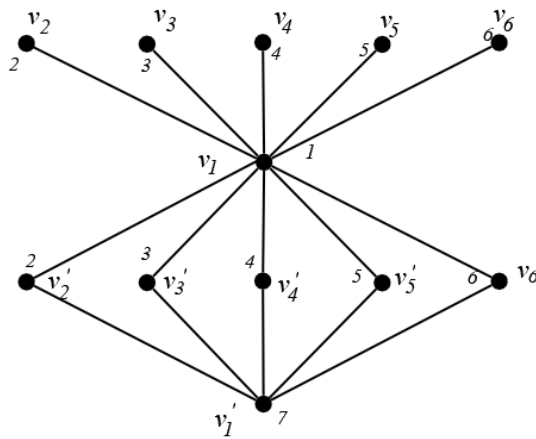
$$|c_j| = 2, \forall j \in \{0, 1, 2, \dots, n\}$$

Thus

$$\max_{ij} ||c_i| - |c_j|| = 0 \leq 1$$

Equitable condition is satisfied. Assume that a dominator coloring with fewer than  $n + 1$  colors exists for contradiction. Then, two different initial leaf vertices  $v_i, v_j$  or their matching splitting vertices  $v'_i, v'_j$  must share at least one color. However, the central vertex  $v_i$  which is also near to  $v_j$  and  $v'_j$  is bounded by both  $v_i$  and  $v'_j$ . Therefore, the correct coloring constraint would be broken by any attempt to combine color classes. Therefore, the bare minimum of colors needed is  $n + 1$ .

*Example 2.* Consider  $S'(S_6)$  and its equitable dominator coloring as displayed in the Figure 2.



**FIGURE 2.** Consider  $S'(S_6)$  and its EDC.

*Theorem 3.* For  $n \geq 3$ ,  $\chi_{ed}[S'(T_n)] = 4n - 6$

*Proof:* Let's define the twig graph  $G$  as follows:

$V(G) = B \cup L$ , where  $B = \{b_1, b_2, \dots, b_{n-1}\}$  is the set of backbone vertices forming a path  $P_{n-1}$   
 $L = \{l_1, l_2, \dots, l_{n-1}\}$  is the set of leaf vertices, each attached to exactly one  $b_i$   
 $E(G) = \{b_i b_{i+1} \mid 1 \leq i \leq n-2\} \cup \{b_i l_i \mid 1 \leq i \leq n-1\}$ . Thus,  $G$  has the following total number of vertices:  
 $|V(G)| = |B| + |L| = 2(n-1)$ . Let  $\phi: V(G) \rightarrow \{c_1, c_2, \dots, c_k\}$  be a vertex coloring that is pleasing. For any edge  
 $uv \in E(G)$  there exists  $E(G), \phi(u) \neq \phi(v)$ . Every color class  $c_i = \phi^{-1}(C_i)$  is a dominating set, i.e., every  
vertex  $v \in V(G) \setminus C_i$  has at least one neighbor in  $C_i$ .

Assign each backbone vertex  $b_i$  a unique color  $C_i$ , for  $i = 1, 2, \dots, n-1$

$$\phi(b_i) = C_i$$

This assures that neighboring backbone vertices are colored differently. For each leaf  $l_i$ , assign the same color as its neighbor  $b_i$

$$\phi(l_i) = \phi(b_i) = C_i$$

This is valid because  $l_i \sim b_i$ , but  $l_i$  has no other neighbors. The proper coloring condition is not broken because no two leaf vertices are adjacent. There are color differences between adjacent vertices  $b_i b_{i+1}$ .

Assign a new color  $C_i \neq C_i$  to every leaf  $l_i$  Now,  $\phi(b_i) = C_i$ ,  $\phi(l_i) = C_i$ ,  $C_i \neq C_i$ . Each and every color class  $C_i = \{b_i\}$  dominates  $b_{i-1}, b_{i+1}, l_i$  (for  $1 < i < n-1$ ). Each leaf color class  $C_i = \{l_i\}$  only dominates  $b_i$ , so every color class dominates at least one vertex. There are  $n-1$  backbone vertices and  $n-1$  leaves  $\rightarrow 2(n-1)$  vertices. We use  $2(n-1)$  distinct colors, each class has size 1  $\rightarrow$  all sizes are equal  $\Rightarrow$  equitability holds.

$$\chi_{ed}[S'(T_n)] = 4n - 6.$$

*Example 3.* Consider  $S'(T_6)$  and its equitable dominator coloring as shown in the Figure 3.

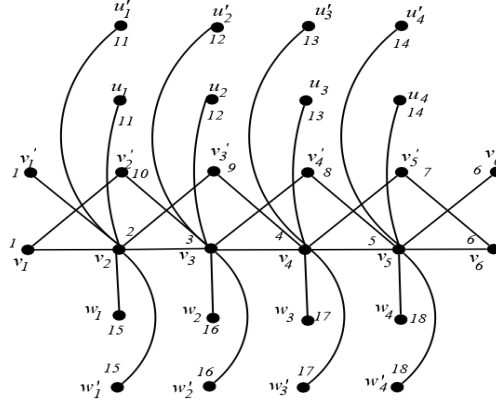


FIGURE 3. Consider  $S'(T_6)$  and its EDC.

*Theorem 4.* For  $m \geq 2, n \geq 3$ ,  $\chi_{ed}[S'(B_{(m,n)})] = (m \times n) + \frac{m!}{(m-1)!} + 2$ .

*Proof:* Suppose that  $B_{(m,n)}$  be a banana graph defined on  $mn + 1$  vertices, with vertex set consists  $V(B_{(m,n)}) = v_k, 1 \leq k \leq mn$ . The degree of centre vertices is  $n$  and  $v_k, 1 \leq k \leq mn$ . vertices have degree 1. For every original vertex  $v_k$  of a graph  $G$ , a new vertex  $v'_k$  is added to create the splitting graph, which is represented by  $S'(B_{(m,n)})$ .

Let us first observe the structure:

The set of vertices is  $V[S'(B_{(m,n)})] = \{v_1, v_2, \dots, v_k\} \cup \{v'_1, v'_2, \dots, v'_k\}$  where  $V = 2(mn + 1)$ . Define a coloring function  $\phi = V[S'(B_{(m,n)})] \rightarrow C$ , where  $C = \{c_1, c_2, \dots, c_n\}$  is set of  $mn + 1$  distinct colours. Assign  $\phi(v_1) = c_1, \phi(v_k) = c_k, \forall 1 < k < n$  and  $\phi(v'_1) = c_1, \phi(v'_k) = c_k, \forall 1 < k < n$ . This results in a mapping where, for every  $k$ , the pair  $(v_k, v'_k)$  share the identical color since there is no edge exists between them in satisfying the proper coloring requirement.

We have to show  $\forall uv \in E[S'(B_{(m,n)})], \phi(u) = \phi(v)$  then  $\phi$  is a proper coloring, no nearby vertex has the same color.

For  $k = 0, C_1 = \{v_0, v'_0\}$  Since both  $v_0$  and  $v'_0$  are adjacent to all  $v_k$ ,  $C_1$  dominates  $V[S'(B_{(m,n)})] \setminus C_1$ .

For  $k \geq 1, C_k = \{v_k, v'_k\}$ . Since  $v_k \sim v_k$  and  $v'_k \sim v_k, C_k$  dominates at least  $v_1$ , hence satisfies the dominates requirements. Equitable condition is satisfied. Assume that a dominator coloring with fewer than  $mn + 1$  colors exists for contradiction. Then, two different initial leaf vertices  $v_k, v_j$  or their matching splitting vertices  $v'_k, v'_j$  must share at least one color. However, the central vertex  $v_1, v_n, v_m$  which is also near to  $v'_k, v'_j$  is bounded by both  $v_k, v_j$ . Therefore, the correct coloring constraint would be broken by any attempt to combine color classes.

Therefore, the bare minimum of colors needed is  $mn + 1$ . By combining the contributions from  $m.n$  clique vertices, factorial arrangements within cliques, and the central dominating vertices, the equitable dominator chromatic number is

$$\chi_{ed}[S'(B_{(m,n)})] = (m \times n) + \frac{m!}{(m-1)!} + 2.$$

Example 4. Consider  $[S'(B_{(2,6)})]$  and its equitable dominator coloring as depicted in the Figure 4.

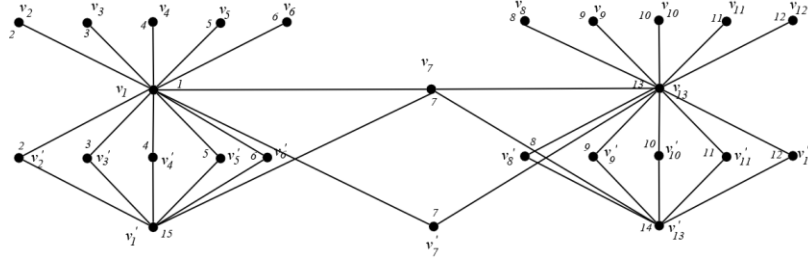


FIGURE 4. Consider  $[S'(B_{(2,6)})]$  and its EDC.

Theorem 5. For  $n \geq 2$ ,  $\chi_{ed}[S'(K_n)] = \begin{cases} \frac{3(n+1)}{2} - 1 & \text{if } n = \text{odd} \\ \frac{3n}{2} & \text{if } n = \text{even} \end{cases}$

Proof: With  $n$  vertices  $\{v_1, v_2, v_3, \dots, v_{n-1}\}$ , let  $K_n$  be the entire graph in which each pair of vertices is nearby. Its set of edges is:

$$E(K_n) = \{(v_i, v_j) \mid 1 \leq i \leq j \leq n\} \quad \text{and} \quad |E(K_n)| = \binom{n}{2}.$$

The splitting graph  $S'(K_n)$  introduces a new vertex  $v'_i$  from each  $v_i \in V(K_n)$  connected only to  $v_i$ . The edge set and vertex set of  $S'(K_n)$  are  $E[S'(K_n)] = \{(v_i, v_j) \mid 1 \leq i \leq j \leq n\} \cup \{(v'_i, v'_j) \mid 1 \leq i \leq j \leq n\}$  and  $V[S'(K_n)] = \{(v_1, v_2, \dots, v_n) \cup (v'_1, v'_2, \dots, v'_n)\}$ .

Thus  $S'(K_n)$  has  $\binom{n}{2} + n$  edges and  $2n$  vertices. A dominating set  $D \subseteq S'(K_n)$  satisfies  $\forall u \in V(S'(K_n)) \setminus D, \exists v \in D$  such that  $(u, v) \in E[S'(K_n)]$ .

In  $S'(K_n)$ , each vertex  $v_i$  is either a dominator of itself or is adjacent and hence dominated by its counterpart  $v'_i \in S'(K_n)$ , i.e.,  $v'_i \in N(v_i)$ , where  $N(v_i)$  denotes the closed neighborhood of  $v_i$ .

To satisfy the dominator coloring condition where each color class induces a dominating set each vertex pair  $v(v_i, v_j)$  must be assigned to the same color class  $C_j \subseteq S'(K_n)$ . This ensures that for all  $u \in C_j$  therefore at least one vertex  $v \in C_j$  there exists  $u \in N(v_j)$  i.e.,  $C_j$  is a dominating set within itself. Consequently, the coloring  $\phi: V(S'(K_n)) \rightarrow \{1, 2, \dots, k\}$  must be constructed so that for each  $i, \phi(v_i) = \phi(v'_i)$ . This pairing ensures the dominator condition is fulfilled for every vertex in the graph under the given coloring.

Thus, the vertex set of  $[S'(K_n)]$  can be partitioned into  $n$  color classes:

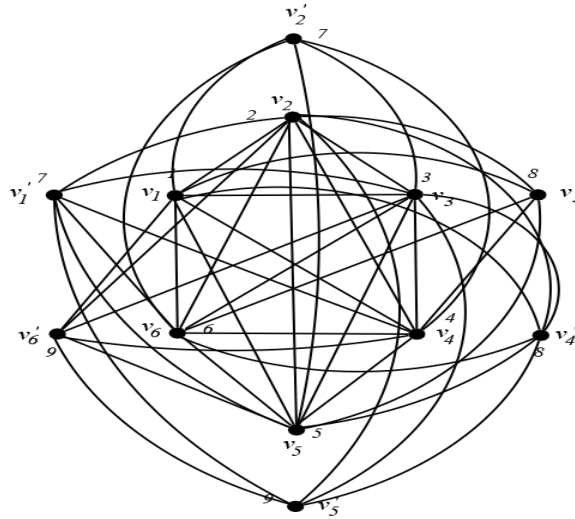
$$c_1, c_2, \dots, c_n \text{ Where } c_i = \{v_i, v'_i\} \text{ for } 1 \leq i \leq n.$$

Color class sizes must differ by no more than one in order for there to be equity. Since  $S'(K_n)$  has  $2n$  vertices and  $n$  color classes, each color class  $c_i$  has exactly 2 vertices:

$$|c_i| = 2, \forall i \in \{1, 2, \dots, n\}$$

This satisfies the equitable condition trivially. To confirm that  $\chi_{ed}[S'(K_n)]$  is the minimum number of coloring,  $K_n$  requires  $n$  colors for a proper coloring, as each vertex is adjacent to all others. The dominator condition is satisfied since each vertex  $v_i$  dominates both  $v_i$  and also  $v'_i$  and  $v'_i$  dominates  $v_i$ . With 2 vertices in each class, the equitable condition is also satisfied. From this we get  $\frac{3(n+1)}{2} - 1$  as equitable chromatic number when  $n$  is odd and  $\frac{3n}{2}$  if  $n$  is even.

*Example 5.* Consider  $[S'(K_6)]$  and its equitable dominator coloring as displayed in the Figure 5.



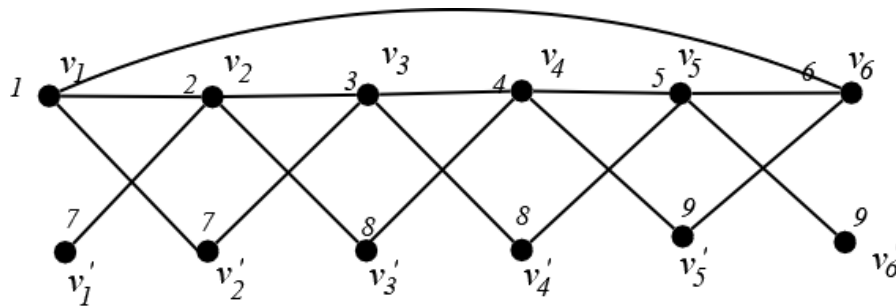
**FIGURE 5.** Consider  $[S'(K_6)]$  and its EDC.

*Theorem 6.* For  $n \geq 2$ ,  $\chi_{ed}[S'(C_n)] = n + \left\lfloor \frac{n+1}{2} \right\rfloor$ .

*Proof:* Let  $[S'(C_n)]$  denote a regular graph of degree that affects the equitable partition and dominator coloring. Since  $C_n$  is a cyclic graph, any vertex in has isomorphic structural properties. The symmetric structure of the graph eases the study of the equitable partitions, but on the other hand makes it much harder to ensure the uniqueness of domination requirements. The following inequality is satisfied by the equitable dominator chromatic number.

Given the  $3n$  vertex structure of  $[S'(C_n)]$ , we analyze these constraints. At least  $n$  colors are needed to ensure domination. As every vertices in a  $C_n$ , needs a dedicated color due to their shared adjacency constraints. And  $v_i$  and  $v'_{i+1}$  having the same coloring because they are independent sets. After ensuring domination, the remaining vertices (if any) must be partitioned equitably among the color classes, requiring an additional  $\left\lfloor \frac{n+1}{2} \right\rfloor$  colors. Divide  $V[S'(C_n)] = V(C_n) \cup V'$  into blocks corresponding to the adjacency neighborhoods of each  $v_i$  and  $v'_i$ . Assign the first  $n$  colors to the dominating vertices in  $V(C_n)$ , ensuring any vertex in  $[S'(C_n)]$  is dominated. Use an additional  $\left\lfloor \frac{n+1}{2} \right\rfloor$  colors to equitably partition the duplicates and ensure compliance with the equitable size rule. Each color class contains a dominator vertex that dominates all other vertices in the same class. By symmetry and regularity, this ensures all domination and equitable constraints are satisfied. Thus give the  $\chi_{ed}[S'(C_n)] = n + \left\lfloor \frac{n+1}{2} \right\rfloor$ .

*Example 6.* Consider  $[S'(C_6)]$  and its equitable dominator coloring as shown in the Figure 6.



**FIGURE 6.** Consider  $[S'(C_6)]$  and its EDC.

*Theorem 7.* For  $n \geq 3$ ,  $\chi_{ed}[S'(S_n)] = \begin{cases} \frac{5n+1}{2} & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases}$

*Proof:* Let  $S_n = (V, E)$  be a sunlet graph with  $|V| = 2n$ . The splitting graph of  $G$ , represented as  $[S'(S_n)]$  is achieved by adding a duplicate vertex.  $v'$  for any vertex  $u \in v$ , such that  $v'$  is adjacent to all the neighbors of  $v$  in  $G$ , but not to  $v$  itself. Since each of the original  $2n$  vertices has a duplicate, the total number of vertices in  $[S'(S_n)]$  is:

$$|V(S_n)| = 2|v| = 4n$$

It retains certain bipartite characteristics because it includes both the original and replicated structures, but the extra adjacency adds complexity. In order to provide equitable coloring, the resulting graph structure offers a coloring pattern that grows linearly with  $n$ .

When  $n$  is even the number of vertices (divided into color classes) suggests that an equitable dominator coloring requires

$$\chi_{ed}[S'(S_n)] = \frac{5n}{2}$$

When  $n$  is odd, the parity imbalance slightly increases the required number of colors:

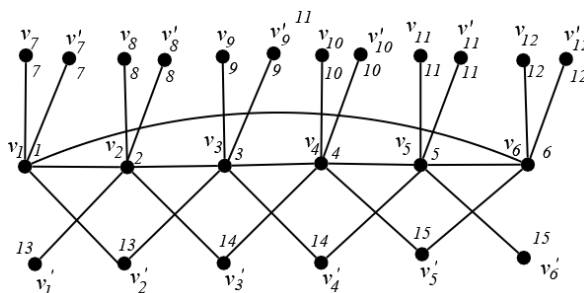
$$\chi_{ed}[S'(S_n)] = \frac{5n+1}{2}$$

This is due to the necessity of ensuring that all vertices dominate atleast one color class while maintaining equitable partitions.

Thus, the  $\chi_{ed}(G)$  of the splitting graph of a sunlet graph is

$$\chi_{ed}[S'(S_n)] = \begin{cases} \frac{5n+1}{2} & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases}$$

*Example 7.* Consider  $[S'(S_6)]$  and its equitable dominator coloring as shown in the Figure 7.



**FIGURE 7.** Consider  $[S'(S_6)]$  and its EDC.

*Theorem 8.* For  $n \geq 3$ ,  $\chi_{ed}[S'(P_n^+)] = 3n$

*Proof:* A splitting graph of comb graph is denoted by  $[S'(P_n^+)]$ . Assign colors  $1, 2, 3, \dots, 3n$  cyclically for proper coloring. Each vertex gets one color out of three possible colors. Assign the color  $1, 2, 3 \dots 2n$  for  $v_i, v'_i$  vertices where  $i = 1, 2, \dots, n$  and the vertices having  $2n+1$  color class for both  $u_i, u'_i, 1 \leq i \leq n$ . The result is a 3-color cycle repeated for  $3n$  vertices, maintaining equitable distribution. Additionally, there is a dominating vertex (from the spine or teeth) in every single vertex class. Coloring is accurate since no two adjacent vertices are the same color. There is an equal distribution of the three colors. Thus, it is determined that the splitting graph's  $\chi_{ed}(G)$  for a comb graph is  $3n$ .

*Example 8.* Consider  $[S'(P_5^+)]$  and its equitable dominator coloring as shown in the Figure 8.

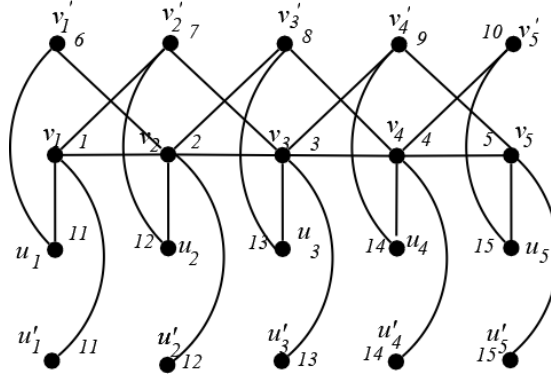


FIGURE 8. Consider  $[S'(P_5^+)]$  and its EDC.

Theorem 9. For  $n, m \geq 2$ ,  $\chi_{ed}[S'(B_{(n,m)})] = 2n + 4$ .

Proof: Let  $B_{(n,m)}$  be the bistar graph. Assign unique colors to the vertices which is  $v_1, v'_1, v_{n+1}, v'_{n+1}$  and then assign the color classes by the following procedure to obtain the equitable dominator coloring

$$C_n = \begin{cases} v_i, 2 \leq i \leq n \\ v'_i, 2 \leq i \leq n \end{cases}$$

In an equitable dominator coloring, where atleast one color class dominates any vertex, this is the minimal number of color classes. Any two color classes have a maximum size difference of one. To satisfy the equitable condition, distribute the  $2n+4$  colors therefore, the color classes are balanced in size. For example, consider  $n$  colors for the original leaves.  $n$  colors for the duplicate leaves. Two extra colors for the center and its replication. By structure, it meets all of the requirements for an equitable dominator color. Thus, the proof is complete.

Example 9. Consider  $[S'(B_{(2,5)})]$  and its equitable dominator coloring as illustrated in the Figure 9.

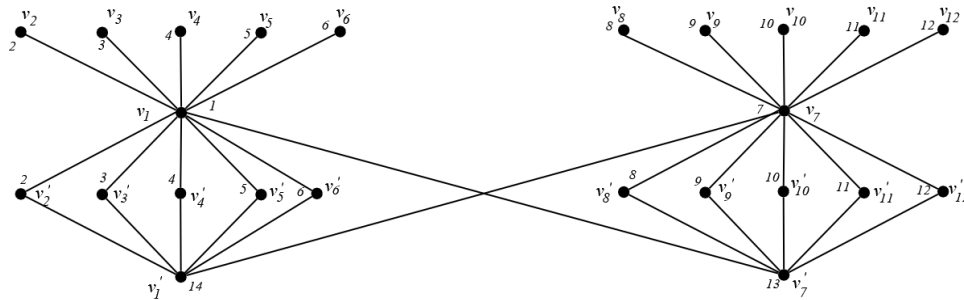


FIGURE 9. Consider  $[S'(B_{(2,5)})]$  and its EDC.

Theorem 10. For  $m \geq 2, n \geq 3$ ,  $\chi_{ed}[S'(K_{(m,n)})] = 2m + 1$

Proof: Let the complete bipartite graphs be represented as  $(K_{(m,n)})$  where  $m$  and  $n$  represent the number of vertices in the two disjoint sets of the graph. The complete bipartite splitting graph is referred as  $[S'(K_{(m,n)})]$ . Each vertex in the  $m$ -partite set of  $(K_{(m,n)})$  has a unique duplicate vertex. Likewise, each vertex in the  $n$ -partite set of

$(K_{(m,n)})$  has its own duplicate vertex. To establish an equitable dominator coloring, assign the color utilizing the following strategy.

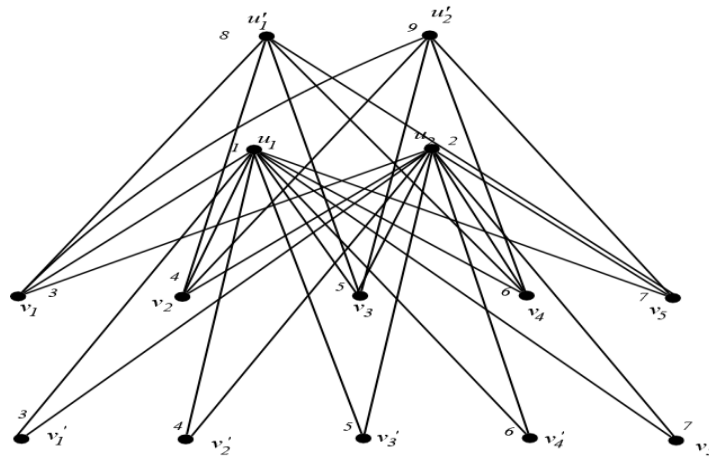
$$C_n = \begin{cases} v_i, & \text{where } i = n+2 \\ v'_i, & \text{where } i = n+2 \end{cases}$$

$$C_i = \begin{cases} u_i, & \text{where } i = 1, 2, \dots, n \\ u'_i, & \text{where } i = n+3, n+4, \dots, j \end{cases}$$

This is possible because all these vertices are adjacent to each vertex in the  $m$ -partite set and its duplicates. Hence, the total number of colors required is  $= m + m + 1 = 2m + 1$ .

$$\chi_{ed}[S'(K_{(m,n)})] = 2m + 1.$$

*Example 10.* Consider  $[S'(K_{(2,5)})]$  and its equitable dominator coloring as depicted in the Figure 10.



**FIGURE 10.** Consider  $[S'(K_{(2,5)})]$  and its EDC.

### 3. CONCLUSION

In this study, we demonstrate about the equitable dominator coloring of splitting graphs for several well-known graph families. We aim to determine the equitable dominator chromatic number for these graph classes and explore the algorithmic complexity of finding dominator colorings for general splitting graphs. The results offer valuable insights into the relationship between equitable dominator colorings and graph structure. Future work can explore optimized algorithms for specific graph types and further investigate the theoretical limits of EDC.

### References

1. P. S. George, S. Madumitha, and S. Naduvath, "Equitable dominator coloring of graphs," J. Interconnect. Netw., arXiv preprint, arXiv:2408.14374 (2024).
2. R. M. Gera, S. Horton, and C. Rasmussen, "Dominator colorings and safe clique partitions," Congr. Numer. 181, 19–32 (2006).
3. T. W. Haynes, S. T. Hedetniemi, and P. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, New York, 1998).
4. G. Jayaraman, D. Muthuramakrishnan, and K. Manikandan, "Equitable total chromatic number of splitting graph," Proyecciones J. Math. 38(4), 699–705 (2019).
5. P. S. George, S. Madhumitha, and S. Naduvath, "Equitable dominator coloring of graph," arXiv preprint, arXiv:2408.14374 [math.CO] (2024). <https://arxiv.org/abs/2408.14374>

6. S. Sudha and G. M. Raja, "Equitable coloring of prisms and the generalized Petersen graphs," *J. Math. Comput. Sci.* 2(2), 105–112 (2014).
7. P. Suganya and R. Mary Jeya Jothi, "Dominator chromatic number of some graph classes," *Int. J. Comput. Appl. Math.* 12(1), (2017).