

A STUDY OF FERROFLUID SPACE - TIME ADMITTING CONFORMAL MOTIONS

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Abstract: The space time comprising of relativistic charged perfect fluid with infinite electric conductivity and variable magnetic permeability (Ferrofluid) is analyzed in context of geometrical symmetry known as conformal motion. It is proved that the magnitude of the magnetic fields in ferrofluid space-time is conserved along the expansion-free flow lines if and only if the magnetic permeability is constant along the flow. Moreover, for isotropic pressure, the conservation of matter density and the magnetic permeability follow simultaneously along the expansion free flow lines. Also the necessary and sufficient conditions for the magnetic lines to be divergence free are obtained. Further for the ferrofluid system admitting a group of conformal motion, the necessary and sufficient conditions for special conformal motion are derived.

Keywords: Relativistic ferrofluid, conformal motion, Maxwell equations.

1. INTRODUCTION

The specific geometric symmetries like motions and collineations are generally introduced to examine the intricate structure of conservation laws in general relativity. The impact of such symmetries in exploiting the internal structure of fluid space times is revealed through the works of Davis and Katzin [1], Katzin, Levine and Davis [2], Katzin, Levine and Davis [3], Collinson [4], Grenberg [5], Asgekar and Date [6]. Asgekar et al. [7].

According to Oliver and Devis (1977), a one parameter group of continuous infinitesimal transformations describe by $\bar{x}^a = x^a + \xi^a \delta t$, is said to exhibit a group of conformal motions if

$$\mathcal{L}_\xi g_{ab} = 2\psi g_{ab} \quad (1.1)$$

Where, ψ is scalar function of space – time co-ordinates and $\mathcal{L}_\xi$ denotes lie derivative along vector ξ . These conditions (1.1) are said to represent

- (a) Homothetic motions if $\psi = \text{constant}$,
- (b) Motion if $\psi = 0$,
- (c) Special conformal killing vector if $\psi_{;ab} = 0, \psi_{;a} \neq 0$.

The interrelationship between collinsion and motion are studied by Katzin et.al.[2], Maartens and Maharaj [8] and Duggal [9].

This works is devoted to study of geometrodynamical structure of ferrofluid space-time admitting conformal motions.

The section II deals with the tensorial description of ferrofluid along with the related field equations. The consequences of Bianchi identities are also examined. The interactions of conformal motions with ferrofluid space-time are considered in section III. Moreover, the Non-geodesic deviation equations are special feature of this section.



2. GEOMETRODYNAMICAL PRELIMINARIES

The geometry and dynamics of the space – time is coupled with well – known Einstein field equations through the tensor equation

$$R_{ab} - \frac{1}{2}Rg_{ab} = -\kappa T_{ab} \quad (2.1)$$

[Note: - For empty space – time $T_{ab} = 0$ this implies $R_{ab} = 0$.]

Here R_{ab} , R , g_{ab} and T_{ab} are the Ricci tensor, Ricci scalar, metric tensor of the general relativity and the stress energy tensor of the ferrofluid distribution, respectively.

Ferrofluid system:

Accordingly, the Relativistic magneto hydrodynamical theory comprises the stress–energy tensor embodying the perfect fluid tensor and the electromagnetic field tensor with infinitely conducting charged relativistic fluid and variable magnetic permeability. This system is known as ferrofluid system and is introduced by Cissako in 1978 [10]. It is given by the stress– energy tensor

$$T_{ab} = (\rho + p + \mu h^2)u_a u_b - \left(p + \frac{1}{2}\mu h^2\right)g_{ab} - \mu h^2 H_a H_b \quad (2.2)$$

Here ρ is matter energy density, p is the isotropic pressure, u_a is unit flow vector (4 –velocity of the fluid), μ is the variable magnetic permeability, H_a is the unit space like magnetic field vector, h^2 is the magnitude of the magnetic field vector H_a and

$$u^a u_b = 1, u^a H_a = 0, H^a H_a = -1, \left(H_a = \frac{h_a}{h}\right), h_a h^a = -h^2 \Rightarrow h_a = h H_a \quad (2.3)$$

Further for the fluid without energy flux and without a preferred direction of anisotropy, the energy-momentum tensor expression is due to Mason and Maartens [11]

$$T_{ab} = (\rho + P)u_a u_b - P g_{ab} + \pi_{ab} \quad (2.4)$$

Where π_{ab} is the anisotropic pressure tensor such that

$$\pi^a_b = 0 = \pi_{ab} u^b \quad (2.5)$$

The stress energy tensor characterizing the relativistic anisotropic perfect fluid is given by Herrera et.al [12] in the form

$$T_{ab} = (\rho + P_T)u_a u_b - P_T g_{ab} + (P_R - P_T)X_a X_b \quad (2.6)$$

$$\text{By inserting the values } P = \frac{1}{3}(P_R + 2P_T) \quad (2.7)$$

$$\text{And } \pi_{ab} = (P_R - P_T)\left(\frac{1}{3}g_{ab} - \frac{1}{3}u_a u_b + X_a X_b\right) \quad (2.8)$$

In the equation (2.4)

Here X_a is unit space like vector orthogonal to u_a , P_R is the radial pressure in the direction of X_a , P_T is the tangential pressure in the direction perpendicular to u_a and X_a both. On similar lines by readjustment of the terms in the expression (2.2) we get

$$T_{ab} = (\rho + P_T + 2m)u_a u_b - (P_T + m)g_{ab} + (P_R - P_T - 2m)H_a H_b \quad (2.9)$$

It follows from the expression (2.6) and (2.9) that the stress energy tensor given by the expression (2.9) represents relativistic anisotropic ferrofluid system with

$$\mu h^2 = 2m \quad (2.10)$$

and P_R is radial pressure in the direction of H_a , P_T is the tangential pressure in the direction perpendicular to both u_a and H_a .

On the similar lines by readjustment of terms in the expression (2.2) we get

$$T^{ab} = (\rho + P_T + 2m)u^a u^b - (P_T + m)g^{ab} + (P_R - P_T - 2m)H^a H^b$$

The gradient of the velocity vector u_a can be decomposed (Greenberg) [5] in terms of parameter θ , the acceleration vector $\dot{u}_a = u_{a;b}u_b$, the shear tensor σ_{ab} and rotation tensor ω_{ab} as

$$u_{a;b} = \dot{u}_a u_b + \omega_{ab} + \sigma_{ab} + \frac{1}{3}\theta h_{ab} \quad (2.11)$$

where $h_{ab} = g_{ab} - u_a u_b$ is the 3 – space projection operator and (;) represents covariant derivative.

Maxwell equation:

The magnetic field present in the ferrofluid system is subject to satisfy Maxwell equations (Ray and Banerjee) [13].

$$[\mu h(u^a H^b - u^b H^a)]_{;b} = 0 \quad (2.12)$$

The equation (2.3), we have

$$u^a_{;b} u_a = 0 \quad (2.13)$$

$$H^a_{;b} u_a = -u_{a;b} H^a \quad (2.14)$$

$$H^a_{;b} H_a = 0 \quad (2.15)$$

The equation (2.11) leads to the following consequences

$$h\mu_{;b} H^b + \mu h_{;b} H^b + \mu h H^b_{;b} + \mu h \dot{u}_a H^a = 0 \quad (2.16)$$

$$\mu h \sigma_{ab} H^a H^b + \frac{2}{3} \mu h \theta + h\mu + \mu h = 0 \quad (2.17)$$

$$\Rightarrow u_{a;b} H^a H^b = -\mu^{-1} \mu_{;b} u^b - h^{-1} h_{;b} u^b - \theta$$

$$u_{a;b} H^a H^b = -\mu^{-1} \dot{\mu} - h^{-1} \dot{h} - \theta \quad (2.18)$$

Equation (2.16) and (2.17) exhibit the connection between kinematical parameters and the magnetic field present in ferrofluid space time.

Local Conservation laws:

The well-known Bianchi identities with Einstein's field equations (2.1) gives rise to local energy balance equation formulated as the vanishing of the divergence of the stress energy tensor,

$$T^{ab}_{;b} = 0 \quad (2.19)$$

The equation (2.19) with (2.9) provides the equation of continuity and stream line equation in the form

$$(\rho + P_R)\theta + [\rho_{;b} + \Delta(\mu^{-1} \mu_{;b} + h^{-1} h_{;b}) - \frac{1}{2} h^2 \mu_{;b}] u^b = 0 \quad (2.20)$$

$$(\rho + P_T + 2m)\dot{u}^a - (P_T + m)_{;b} h^{ab} + (\Delta - 2m)_{;b} H^a H^b + (\Delta - 2m)(H^a H^b)_{;b} - \frac{1}{2} \dot{\mu} h^2 u^a = 0 \quad (2.21)$$

Where $\Delta = P_R - P_T$

From the equation of continuity (2.20) the following trivial result are claimed.

Claim (I): If $\rho = \text{constant}$ along expression free flow lines then

$$\mu_{;b} u^b = 0 \Rightarrow h_{;b} u^b = 0 \quad (2.22)$$

This means that the magnitude of the magnetic field is conserved along the expansion free flow lines if and only if the magnetic permeability is constant along that flow.

Claim (II): For isotropic case $\Delta = 0$,

$$\rho_{;b}u^b = 0 \Leftrightarrow \theta = 0 \text{ and } \mu_{;b}u^b = 0 \quad (2.23)$$

This direct that for isotropic pressure the conservation of matter density and magnetic permeability follows simultaneously along the expansion free flow lines.

On transvecing equation (2.21) with H_a gives

$$(\rho + P_R)H^a_{;b} + [(P_R - m)_{;b} + (\rho + P_T + 2m)(\mu^{-1}\mu_{;b} + h^{-1}h_{;b})]H^b = 0 \quad (2.24)$$

(vide (2.3), (2.13), (2.16))

From the above equations (2.24), it is observed that if $P_R = m$ then

$$H^a_{;b} = 0 \Leftrightarrow (\mu_{;b} + h_{;b})H^b = 0 \quad (2.25)$$

By using (2.16), the equation (2.24) is written as

$$(\rho + P_R)\dot{u}_a H^a - [(P_R - m)_{;b} + (\Delta - 2m)(\mu^{-1}\mu_{;b} + h^{-1}h_{;b})]H^b = 0 \quad (2.26)$$

On utilizing the equation (2.26) it is claimed that if $\Delta = 2m$ then

$$\dot{u}_a H^a = 0 \Leftrightarrow P_{R;b}H^b = m_{;b}H^b \quad (2.27)$$

Thus we get $\dot{u}_a H^a = 0 \Leftrightarrow (m - P_R)_{;a}H^a = 0$

This means that $m - P_R$ is invariant along magnetic lines if and only if magnetic lines are normal to acceleration.

Thus the necessary and sufficient condition for magnetic lines to be

- (I) divergence free (vide (2.24))
- (II) orthogonal to acceleration vector (vide (2.26)), are obtained.

3. CONFORMAL MOTION:

A space-time admits a ConfM generated by a Conformal Killing vector (CKV) $\bar{\xi}$ if

$$\mathcal{L}_{\bar{\xi}}g_{ab} = 2\psi g_{ab} \quad (3.1)$$

By using $ds^2 = g_{ab}dx^a dx^b$
(3.2)

And the following result $\mathcal{L}_{\bar{\xi}}dx^a = 0$ (3.3)

Obtained by Yano [14], we write $\mathcal{L}_{\bar{\xi}}u^a = -\psi u^a$ (3.4)

Consequently, $\mathcal{L}_{\bar{\xi}}u_a = \psi u_a$
(3.5)

By making use of this process one can also write for unit space like magnetic field vector $\bar{H}$

$$\mathcal{L}_{\bar{\xi}}H_a = \psi H_a \quad (3.6)$$

The value of the lie derivative of Ricci tensor along with vector field $\bar{\xi}$ is written in the form (Herrera et.al)[12]

$$\mathcal{L}_{\bar{\xi}}R_{ab} = \frac{1}{2}g^{cd}[(\mathcal{L}_{\bar{\xi}}g_{cd})_{;ab} - (\mathcal{L}_{\bar{\xi}}g_{bd})_{;ca} - (\mathcal{L}_{\bar{\xi}}g_{ad})_{;cb} + (\mathcal{L}_{\bar{\xi}}g_{ab})_{;cd}] \quad (3.7)$$

By using the condition (3.1) it leads to

$$E_{\xi}R_{ab} = \frac{1}{2}g^{cd}[(\psi g_{cd})_{;ab} - (\psi g_{bd})_{;ca} - (\psi g_{ad})_{;cb} + (\psi g_{ab})_{;cd}] \quad (3.8)$$

Which on simplification gives

$$E_{\xi}R_{ab} = 2\psi_{;ab} + g_{ab}\square\psi \quad (3.9)$$

$$\text{Where } \square\psi = g^{cd}\psi_{;cd} \quad (3.10)$$

In similar way one can write

$$E_{\xi}R = 6\square\psi - 2R\psi \quad (3.11)$$

Thus from the equation (3.9) and (3.11) one obtain

$$E_{\xi}(R_{ab} - \frac{1}{2}Rg_{ab}) = 2[\psi_{;ab} - g_{ab}\square\psi] \quad (3.12)$$

Theorem: For the anisotropic ferrofluid system admitting a group of conformal motion along an arbitrary vector field ξ , then the following result are equivalent

- A) $\psi_{;ab} = 0$
- B) I) $E_{\xi}(\rho + m) + 2\psi(\rho + m) = 0$
 II) $E_{\xi}(P_R - m) + 2\psi(P_R - m) = 0$
 III) $E_{\xi}(P_T + m) + 2\psi(P_T + m) = 0$

Proof: The Einstein's field equation (2.1) with the use of expression (3.1), (3.5), (3.12) and with (2.9) generates the result

$$2[\psi_{;ab} - g_{ab}\square\psi] = -\kappa\{(u^a u^b)[E_{\xi}(\rho + P_T + 2m) + 2\psi(\rho + P_T + 2m)] - g^{ab}[E_{\xi}(P_T + m) + 2\psi(P_T + m)] + H^a H^b[E_{\xi}(\Delta - 2m) + 2\psi(\Delta - 2m)]\} \quad (3.13)$$

To prove (A) $\Rightarrow$ (B)

$$\text{By (A), } \psi_{;ab} = 0 \quad (3.14)$$

$$\Rightarrow \psi_{;ab}g^{ab} \equiv \square\psi = 0 \quad (3.15)$$

Hence L.H.S. of (3.13) is zero, which gives

$$-\kappa\{(u^a u^b)[E_{\xi}(\rho + P_T + 2m) + 2\psi(\rho + P_T + 2m)] - g^{ab}[E_{\xi}(P_T + m) + 2\psi(P_T + m)] + H^a H^b[E_{\xi}(\Delta - 2m) + 2\psi(\Delta - 2m)]\} = 0 \quad (3.16)$$

$$\text{Suppose } W^a \text{ is a unit space like vector such that } W^a W_a = -1, u^a W_a = 0 = H^a W_a \quad (3.17)$$

Transvecting (3.16) with $u^a u^b, H^a H^b, g^{ab}, W^a W^b$ yields

$$E_{\xi}(\rho + m) + 2\psi(\rho + m) = 0 \quad (3.18)$$

$$E_{\xi}(P_R - m) + 2\psi(P_R - m) = 0 \quad (3.19)$$

$$E_{\xi}(\rho - 2P_T - P_R) + 2\psi(\rho - 2P_T - P_R) = 0 \quad (3.20)$$

$$E_{\xi}(P_T + m) + 2\psi(P_T + m) = 0 \quad (3.21)$$

The rearrangement of the terms in the equation (3.20) and making the use of equation (3.18), (3.19) and (3.21) gives results in (B).

To prove (B) $\Rightarrow$ (A)

If the condition (B) is used in the right hand side of the equation (3.20) then one gets

$$2[\psi_{;ab} - g_{ab}\square\psi] = 0 \quad (3.22)$$

$$\Rightarrow \psi_{;ab} = 0$$

Hence the proof of the theorem is completed.

Remark 1: This theorem provides the necessary and sufficient dynamic condition for

$$\psi_{;ab} = 0.$$

Remark 2: The CKV condition (3.1) with (A) describes special conformal motion (Katzin et.al) [4].

Remark 3: For the special choice of conformal vector $\bar{\xi}$ along time like unit flow and space like magnetic unit vector, the value of $\bar{\xi}$ becomes zero. In other words, the conformal killing vector along u and along H turns to be killing vectors. In this case the theorem directs that ρ, P_R, P_T, m be invariant.

Note: It is observed from the condition $\psi_{;ab} = 0$ that $\psi_{;a} = 0$ is globally time like and its integrability condition leads to

$$T^a{}_b \psi_{;a} = \frac{1}{2} R \psi_{;b} \quad (3.23)$$

This implies that $\psi_{;b}$ is time like eigen vector for stress energy tensor with corresponding eigen value $-\frac{R}{2}$. (Coley and Tupper, 1991) [15] for the anisotropic system described by the stress energy tensor

$$T^{ab} = (\rho + P_T + 2m)u^a u^b - (P_T + m)g^{ab} + (P_R - P_T - 2m)H^a H^b$$

the equation (3.23) provides the results

$$\begin{aligned} & (\rho + P_T + 2m)u^a u_b \psi_{;a} - (P_T + m)g^a{}_b \psi_{;a} + (\Delta - 2m)H^a H_b \psi_{;a} \\ &= \frac{\kappa}{2} [(\rho + P_T + 2m - 4P_T - 4m - P_R + P_T + 2m)\psi_{;b}] \\ &= \frac{\kappa}{2} [(\rho - P_R - 2P_T)\psi_{;b}] \end{aligned} \quad (3.24)$$

Also $\psi_{;b}$ cannot be parallel to u_a and H_a this implies that

$$u^a \psi_{;a} = 0 \quad (3.25)$$

$$H^a \psi_{;a} = 0 \quad (3.26)$$

$\Rightarrow$ vector $\psi_{;a}$ is perpendicular to both u^a and H^a .

Contracting (3.25) and (3.26) with $\psi_{;b}$ we get

$$\psi_{;a} \psi^{;a} = 0$$

$\Rightarrow$ Conharmonic conformal motion (Abdusattar, 1998) [16]

Remark: Conformal motion degenerate into Conharmonic motion.

INTEGRABILITY CONDITION FOR ANISOTROPIC FERROFLUID:

Let ξ be the conformal killing vector satisfying the condition

$$\xi_{i;j} + \xi_{j;i} = 2\psi g_{ij} \quad (3.27)$$

We have Ricci identities

$$\xi_{k;ij} - \xi_{k;ji} = \xi_n R^n{}_{kij} \quad (3.28)$$

This equation (3.27) and (3.28) implies with simplification

$$\xi_{k;ij} + \xi_{i;jk} + \xi_{j;ki} = \psi_{;i}g_{kj} + \psi_{;j}g_{ik} + \psi_{;k}g_{ji} \quad (3.29)$$

Also by adjusting the terms in equation (3.29), we get

$$\xi_{k;ij} - (\xi_{j;ki} - \xi_{j;ik}) = \psi_{;i}g_{kj} + \psi_{;j}g_{ik} - \psi_{;k}g_{ji} \quad (3.30)$$

Due to Ricci identities (3.28), the equation (3.30) provides

$$\xi_{k;ij} = -\xi_n R^n{}_{ijk} + \{\psi_{;i}g_{kj} + \psi_{;j}g_{ik} - \psi_{;k}g_{ji}\} \quad (3.31)$$

Contracting (3.31) with ξ^i , yields

$$\xi^i \xi_{k;ij} = \psi_{;i} \xi^i g_{kj} + \psi_{;j} \xi_k - \psi_{;k} \xi_j \quad (3.32)$$

Remark: These provide the integrability conditions for Conformal killing vector $\bar{\xi}$.

Non-geodesic deviation equation or Jacobi equation:

The comparison Newtonian gravitational potential with the general relativistic gravitational potential directs that the relative acceleration of two neighbouring bodies would be related to some components of Riemann tensor.

In order to investigate this relation more precisely we shall examine the behavior of a congruence of time like curves with time like unit tangent vector V .

If suppose $\lambda(t)$ is a curve with tangent vector $Z = (\frac{\partial}{\partial t})_\lambda$. Then one may construct a family $\lambda(t, s)$ of curves by moving each point of the curve $\lambda(t)$, a distance s along the integral curves of V . Hence we write $D/\partial s Z^a = V^a{}_{,b} Z^b$ (3.33)

Hence one may interpret Z to represent as separation between two initial points along two neighbouring curves. The projection of Z into the orthogonal space of V is denoted by $\perp Z^a = h^a{}_b Z^b$ with the projection operator $h^a{}_b = \delta^a{}_b - V^a V_b$. In this case of fluid one can regard $\perp Z$ (projection of Z) as the distance between the neighbouring particles with the fluid as measured in their rest frame. Hence we write

$$\perp \frac{D}{\partial s} (\perp Z^a) = V^a{}_{,b} \perp Z^b \quad (3.34)$$

This gives the rate of change of separation of two infinitemally neighbouring curves as measured in the rest space. This result further generates the following equation

$$h^a{}_b \left(\frac{D}{\partial s} \right) \left(h^b{}_c \left(\frac{D}{\partial s} \right) \perp Z^c \right) = R^a{}_{bcd} h^c{}_e Z^e V^b V^d - h^a{}_b \dot{V}^b{}_{;c} h^c{}_e Z^e - \dot{V}^a \dot{V}_b h^b{}_e Z^e \quad (3.35)$$

This is the Non-geodesic deviation equation or Jacobi equation (Hawking and Ellis)[17]

Let us write

$$A = R^a{}_{bcd} h^c{}_e Z^e V^b V^d \quad (3.36)$$

$$B = h^a{}_b \dot{V}^b{}_{;c} h^c{}_e Z^e = h^a{}_b \dot{V}^b{}_{;c} h^c{}_e \quad (3.37)$$

$$C = \dot{V}^a \dot{V}_b h^b{}_e Z^e \quad (3.38)$$

Therefore, expression (3.35) can be written as

$$h^a{}_b \left(\frac{D}{\partial s} \right) \left(h^b{}_c \left(\frac{D}{\partial s} \right) \perp Z^c \right) = A - B - C \quad (3.39)$$

And the Weyl conformal curvature tensor expression

$$C^a{}_{bcd} = R^a{}_{bcd} - (1/2)[g^a{}_c R_{bd} - g^a{}_d R_{bc} + g_{bd} R^a{}_c] + (1/6)R[g^a{}_d g_{bc} - g^a{}_c g_{bd}] \quad (3.40)$$

We have

$$A = E^a{}_c Z^c - (1/2)[R_{bd}V^bV^dZ^a - R_{bc}V^aV^bZ^c + R^a{}_cZ^c] - (1/6)RZ^a \quad (3.41)$$

Where $E_{ac} = C_{abcd}V^bV^d$ is the electric type tensor.

For anisotropic ferrofluid,

$$R_{ab} = -\kappa[(\rho + P_T + 2m)V_aV_b - (1/2)(\rho - P_R + 2m)g_{ab} + (P_R - P_T - 2m)H_aH_b] \quad (3.42)$$

Thus we get from (3.41) and (3.42)

$$A = E^a{}_c Z^c + (1/6)\kappa(\rho + 2P_R + P_T)Z^a + (1/2)\kappa(P_R - P_T - 2m)H^aH_cZ^c \quad (3.43)$$

Hence the final expression for (3.35) becomes

$$h^a{}_b \left(\frac{D}{\partial s} \right) \left(h^b{}_c \left(\frac{D}{\partial s} \right) \perp Z^c \right) = E^a{}_c Z^c + (1/6)\kappa(\rho + 2P_R + P_T)Z^a + (1/2)\kappa(P_R - P_T - 2m)H^aH_cZ^c - h^a{}_b \dot{V}^b{}_{;c} h^c{}_e Z^e - \dot{V}^a \dot{V}_b h^b{}_e Z^e \quad (3.44)$$

(vide (3.37), (3.38), (3.39))

This is Jacobi equation for anisotropic ferrofluid.

Remark 1: In case of conformal killing vector $V, \dot{V}^a = 0$ then the equation (3.44) reduces to

$$h^a{}_b \left(\frac{D}{\partial s} \right) \left(h^b{}_c \left(\frac{D}{\partial s} \right) \perp Z^c \right) = E^a{}_c Z^c + (1/6)\kappa(\rho + 2P_R + P_T)Z^a + (1/2)\kappa(P_R - P_T - 2m)H^aH_cZ^c \quad (3.45)$$

This represents a geodesic deviation equation in ferrofluid system.

Remark 2: The effect of electric type tensor $E^a{}_c$ on the relative acceleration is depicted through (3.44)

CONCLUSION

This paper investigates the space-time of anisotropic ferrofluid admitting a group of conformal motions in the section III. In section I derived some preliminary data. For the anisotropic ferrofluid derived the integrability condition. Also the necessary and sufficient conditions for special conformal motion are to be derived. At the last Non-geodesic deviation equation or Jacobi equation is derived.

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