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Article

# Matrix Implementation on Oscillating Polar Fuzzy Graph

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**Abstract:** This article introduces the matrix implementation on Oscillating Polar Fuzzy Graphs. It defines the order, size, degree and edge matrix representation of oscillating polar fuzzy graph, supported by examples. Furthermore, various theorems are validated using the properties of matrix representation.
**Keywords:** Fuzzy Graph, Oscillating polar Fuzzy Graph, Edge matrix implementation on OPFG, Adjacency matrix.

## 1. Introduction

Graph Theory was first introduced by Leonard Euler in 1736. In 1965, Lofti A. Zadeh introduced the concept of fuzzy logic through the proposal of fuzzy set theory. Ten years later, in 1975, Azriel Rosenfeld developed fuzzy graph theory, which was earlier defined by Kauffmann in 1973. In 1977, Thomson defined fuzzy matrices, which are particularly useful for dealing with uncertain situations. Later, Mythili V, Vasanthi J, and R. Roop Rekha introduced the matrix representation of fuzzy graphs. In 1994, Zhang introduced bipolar fuzzy sets, which were generalised by Juanjuan Chen et al. into m-polar fuzzy sets. In 2014, Chen extended this to define m-polar fuzzy graphs.

T. Pathinathan and J. Jesintha Rosline explored the matrix representation of double-layered fuzzy graphs and their properties, while Jon Arockiaraj J and V. Chandrasekaran focused on double layered complete fuzzy graphs. Anthoni Amali A and J. Jesintha Rosline introduced the concept of oscillating polar fuzzy graphs (OPFG) and their applications.

The paper discusses the matrix implementation of OPFG, including its order, size, degree of an OPFGs with examples and provides a detailed explanation of their edge matrix implementation supported by illustrative examples.

Chapter 2 provides basic definitions of fuzzy graphs needed to prove theorems. Chapter 3 defines the matrix implementation on Oscillating Polar Fuzzy Graphs (OPFG), including their or-der, size, and degree. Chapter 4 presents proof of some theorems and Chapter 5, Conclusion is given as this implementation offers promising advancements in various fields.

## 2. Preliminaries

Definition 2.1 [14]

A fuzzy set (class) A in X is characterized by a membership (characteristic) function $f_A(x)$ which associates with each point in X a real number in the interval [0, 1], with the value of $f_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A. When A is a set in the ordinary sense of the term, its membership function can take only two values 0 and 1, with $f_A(x) = 1$ or 0 according as x does or does not belong to A. Thus, in this case $f_A(x)$reduces to the familiar characteristic function of a set A.

Definition 2.2: [7]

A fuzzy Graph $G = (v, \sigma, \mu)$is a triple consisting of a non-empty finite set $v$ together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\forall\, x, y \in V, \mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

Definition:2.3 [9]

Let $G(\sigma, \mu)$be a fuzzy graph, the order of G is defined as

$$O(G) = \sum_{u \in V} \sigma(u)$$

Definition:2.4 [9]

Let $G(\sigma, \mu)$be a fuzzy graph, the size of G is defined as

$$S(G) = \sum_{u,v \in V;\, u \neq v} \mu(u, v)$$

Definition:2.5 [9]

Let $G(\sigma, \mu)$be a fuzzy graph, the degree of a vertex $u$ in G is defined as

$$d(u) = \sum_{v \in V;\, u \neq v} \mu(u, v)$$

Definition:2.6 [9]

A fuzzy graph $G(\sigma, \mu)$ with the fuzzy relation $\mu$ to be reflexive and symmetric is completely determined by the fuzzy matrix $M_{G,}$ where

$$(M_G)_{ij} = \begin{cases} \mu(v_i, v_j) \; if \; i \neq j \\ \sigma(v_i) \quad if \; i = j \end{cases}$$

Definition:2.7 [1]

An Oscillating Polar Fuzzy Graph $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ is defined to be pair of functions with vertex membership,$\sigma_{OP} = ({\sigma_{OP}}^N, {\sigma_{OP}}^P), \sigma_{OP}: v \to [-1,0], [0,1]$, and with edge membership is denoted with notation $\mu_{OP} = ({\mu_{OP}}^N, {\mu_{OP}}^P)$, $\mu_{OP}: v \times v \to [-1, 0], [0,1]$, for all values of $x, y \in V$, which lies between two equal opposite extremes of$[-1, 1]$, such that $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ is an Oscillating Polar Fuzzy Graph (OPFG) with underlying crisp graph, graph $G_{OP^*}: (V, \sigma_{OP^*}, \mu_{OP^*})$

$$(i.e) \quad {\mu_{OP}}^N(x, y) \geq \max\left({\mu_{OP}}^N(x), {\mu_{OP}}^N(y)\right)$$

$$\quad {\mu_{OP}}^P(x, y) \leq \min({\mu_{OP}}^P(x), {\mu_{OP}}^P(y)) \quad \forall x, y \in V$$

With an equal opposite extreme of the ranges [-1, 1]

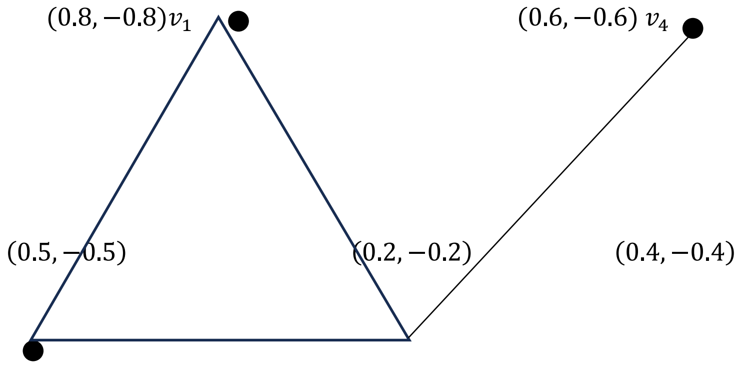
## 3. Definitions

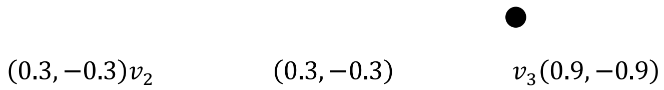
### *3.1 Matrix Implementation on Oscillating Polar Fuzzy Graph:*

The matrix Implementation of OPFG $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ is defined as,

$$A_{OPFG} = \begin{cases} ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_j) if \; (v_i, v_j) \epsilon \mu_{OP}, if \; i \neq j \\ ({\sigma_{OP}}^P, {\sigma_{OP}}^N)(v_i, v_j), \qquad if \; i = j \\ (0, 0), \qquad Otherwise \end{cases}$$

**Example**






The matrix representation of the above OPFG is,

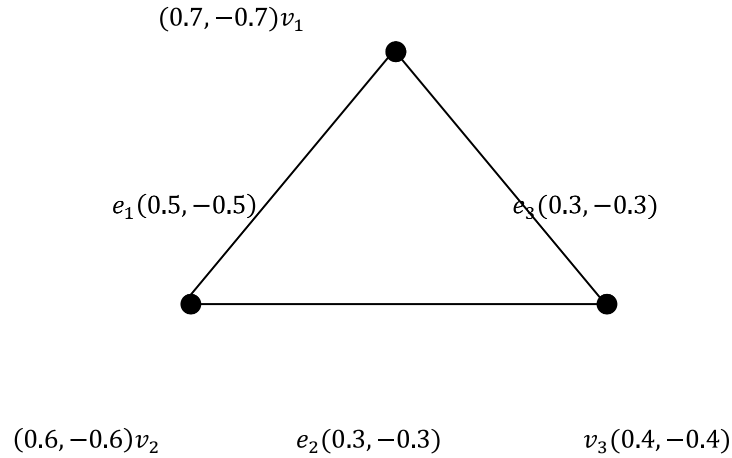
$$A_{OPFG} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad v_1 \quad & \quad v_2 \quad & \quad v_3 \quad & \quad v_4 \quad \end{array} \\ \begin{bmatrix} (0.8,-0.8) & (0.5,-0.5) & (0.2,-0.2) & (0,0) \\ (0.5,-0.5) & (0.6,-0.6) & (0.4,-0.4) & (0.5,-0.5) \\ (0.2,-0.2) & (0.4,-0.4) & (0.5,-0.5) & (0.3,-0.3) \\ (0,0) & (0.5,-0.5) & (0.3,-0.3) & (0.8,-0.8) \end{bmatrix} \end{array}$$

## *3.2 Edge matrix implementation of complete OPFG:*

For a complete OPFG $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$with the fuzzy relation to be reflexive, symmetric and transitive, the edge matrix implementation of $M_{OP_{(K\mu)}}$ is defined as,

$$M_{OP_{(K\mu)}} = \begin{cases} \left.\begin{matrix} \min\left({\mu_{OP}}^{P}(\,e_i), {\mu_{OP}}^{P}(\,e_j)\right), \\ \max\left({\mu_{OP}}^{N}(\,e_i), {\mu_{OP}}^{N}(\,e_j)\right) \end{matrix}\right\}, if\ v_i is\ the\ common\ verted\ betweene_i ande_j \\ ({\mu_{OP}}^{P}, {\mu_{OP}}^{N})(e_i, e_j)\,, \qquad if\ i = j \\ (0,0)\,, \qquad Otherwise \end{cases}$$

**Example**




The edge matrix implementation is given by

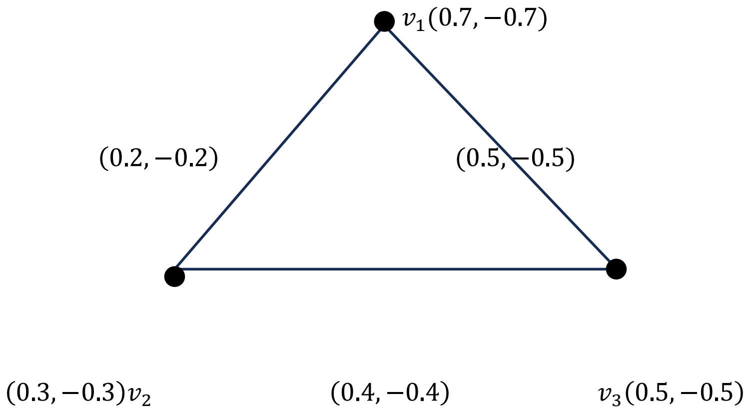
$$M_{OP_{(K\mu)}} = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} \quad e_1 \quad & \quad e_2 \quad & \quad e_3 \quad \end{array} \\ \begin{bmatrix} (0.5,-0.5) & (0.3,-0.3) & (0.3,-0.3) \\ (0.3,-0.3) & (0.3,-0.3) & (0.3,-0.3) \\ (0.3,-0.3) & (0.3,-0.3) & (0.3,-0.3) \end{bmatrix} \end{array}$$

**Definition 3.3**

Let $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ be an Oscillating Polar fuzzy graph, the order of $G_{OP}$ is defined as

$$O(G_{OP}) = \sum_{v \in V} ({\sigma_{OP}}^{P}, {\sigma_{OP}}^{N})(v), \forall v \in \sigma_{OP}$$

**Example**




$$O(G_{OP}) = \sum_{v_i \in V} ({\sigma_{OP}}^P, {\sigma_{OP}}^N)(v_i)$$

$$= (0.7 + 0.5 + 0.3, -0.7 - 0.5 - 0.3)$$

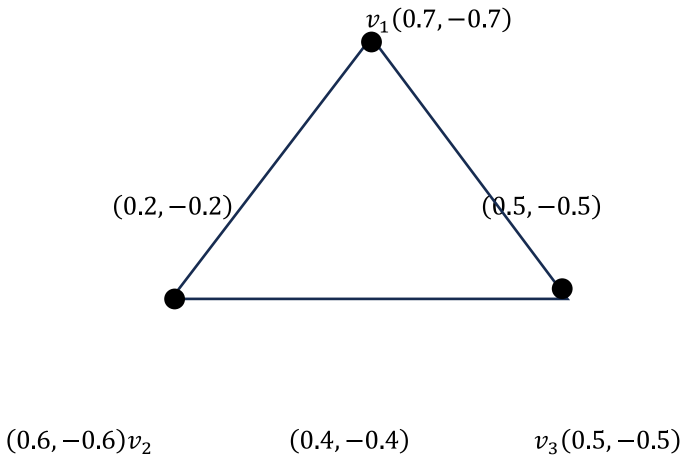
$$= (0.15, -0.15)$$

**Definition 3.4**

Let $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ be an Oscillating Polar fuzzy graph, the size of $G_{OP}$ defined as

$$S(G_{OP}) = \sum_{v_i, v_j \in V} ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_j), \forall\, v_i, v_j \in \mu_{OP}$$

**Example**




$$S(G_{OP}) = \sum_{v_i, v_j \in V} ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_j)$$

$$= (0.2 + 0.5 + 0.4, -0.2 - 0.5 - 0.4)$$

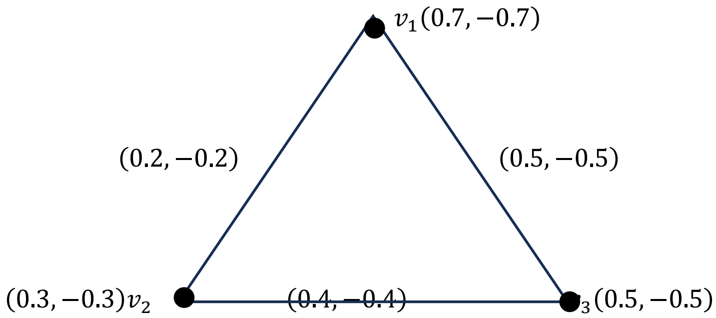
$$= (0.11, -0.11)$$

**Definition:3.5**

Let $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$ be an Oscillating Polar fuzzy graph, the degree of a vertex $v_i$ in $G_{OP}$ is defined as

$$d_{G_{OP}}(v_i) = \sum_{\substack{v_i \neq v_j \\ v_j \in V}} ({\mu_{OP}}^P, {\mu_{OP}}^P)(v_i, v_j)$$

**Example**




$$d_{G_{OP}}(v_i) = \sum_{\substack{v_i \neq v_j \\ v_j \in V}} ({\mu_{OP}}^P, {\mu_{OP}}^P)(v_i, v_j)$$

$$d(v_1) = (0.2 + 0.5, -0.2 - 0.5)$$
$$= (0.7, -0.7)$$

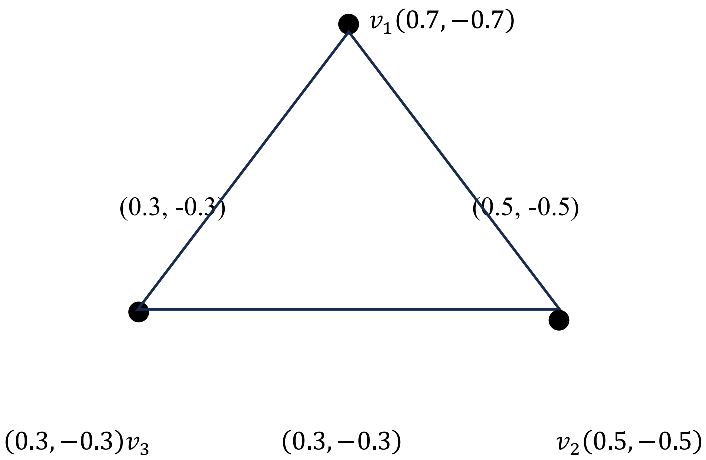
**Definition 3.6**

A complete Oscillating polar fuzzy graph $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$with the fuzzy relation $\mu_{OP}$ to be reflexive and symmetric is completely determined by the Oscillating polar fuzzy matrix

$$M_{OP(K\sigma)} = \begin{cases} \left({\mu_{OP}}^P(v_i, v_j), {\mu_{OP}}^N(v_i, v_j)\right) & if \ \ i \neq j \\ ({\sigma_{OP}}^P(v_i), {\sigma_{OP}}^N(v_j)) & if \ \ i = j \end{cases}$$

**Example**:

Consider the complete OPFG with 3 vertices ($K_3$)




The matrix implementation with respect to vertices for the complete OPFG $G_{OP}$ is,

$v_1$ $v_2$ $v_3$

$$M_{OP_{(K\sigma)}} = \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \begin{bmatrix} (0.7,-0.7) & (0.5,-0.5) & (0.3,-0.3) \\ (0.5,-0.5) & (0.5,-0.5) & (0.3,-0.3) \\ (0.3,-0.3) & (0.3,-0.3) & (0.3,-0.3) \end{bmatrix}$$

IV. Theoretical Concept

**Theorem 4.1:**

The trace of the compete OPFG is equal to the sum of the order and size of the complete OPFG.

**Proof:**

Let $G_{OP}: (V, \sigma_{OP}, \mu_{OP})$be a complete OPFG.

$Trace\ (G_{OP})$ = sum of the diagonal entries in $M_{OP_{(K\sigma)}}$

$$= \sum_{i=n}^{n} ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_i)$$

$$= \sum_{v_i \in ({\sigma_{OP}}^P, {\sigma_{OP}}^N)} ({\sigma_{OP}}^P, {\sigma_{OP}}^N)(v_i)$$

$$= \sum_{\substack{v_i \in \sigma_{OP} \\ (v_i, v_i) \in \mu_{OP}}} \left( ({\sigma_{OP}}^P, {\sigma_{OP}}^N)(v_i) + ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_i) \right)$$

By the definition, we get

$$= \sum_{v_i \in \sigma_{OP}} ({\sigma_{OP}}^P, {\sigma_{OP}}^N)(v_i) + \sum_{(v_i, v_i) \in \mu_{OP}} ({\mu_{OP}}^P, {\mu_{OP}}^N)(v_i, v_i)$$

$$= Order(G_{OP}) + Size(G_{OP})$$

Hence the proof

**Theorem 4.2:**

Every matrix representation of complete OPFG $M_{OP_{(K\sigma)}}$ is transitive.

**Proof:**

We prove this theorem by an example,

By the definition of transitivity,

Let $x, y, z \in E,\ then\ \forall (x, y)(y, z)(z, x) \in E \times E,$

$\mu_R(x, z) \geq (\max(\min(\mu_R(x, y), \mu_R(y, z))),$

$${\mu_{OP}}^P(x, z) \geq \max\left(\min\left({\mu_{OP}}^P(x, y), {\mu_{OP}}^P(y, z)\right)\right)$$

$${\mu_{OP}}^N(x, z) \leq \min(\max\left({\mu_{OP}}^N(x, y), {\mu_{OP}}^N(y, z)\right))$$

Arc $(v_1, v_1)$

$\left({\mu_{OP}}^P(v_1, v_1) \wedge {\mu_{OP}}^P(v_1, v_1), {\mu_{OP}}^N(v_1, v_1) \wedge {\mu_{OP}}^N(v_1, v_1)\right)$

$$= (0.7 \wedge 0.7, -0.7 \wedge -0.7)$$

$$= (0.7, -0.7)$$

$\left({\mu_{OP}}^P(v_1, v_2) \wedge {\mu_{OP}}^P(v_2, v_1), {\mu_{OP}}^N(v_1, v_2) \wedge {\mu_{OP}}^N(v_2, v_1)\right)$

$$= (0.5 \wedge 0.5, -0.5 \wedge -0.5)$$

$$= (0.5, -0.5)$$

$\left(\mu_{OP}{}^{P}(v_1, v_3) \wedge \mu_{OP}{}^{P}(v_3, v_1), \mu_{OP}{}^{N}(v_1, v_1) \wedge \mu_{OP}{}^{N}(v_1, v_1)\right)$

$$= (0.3 \wedge 0.3\, , -0.3 \wedge -0.3)$$

$$= (0.3, -0.3)$$

$\therefore$ $(max(0.7, 0.5, 0.3), \min(-0.7, -0.5, -0.3)) = (0.7, -0.7)$ Proceeding in the same way, it satisfies the transitivity.

**Theorem 4.3**

The edge matrix implementation of a complete OPFG is symmetric.

**Proof:**

Let $M_{OP_{(K\mu)}}$ is an edge matrix representation of complete OPFG.

$$M_{OP_{(K\mu)}\,i,j} = (\mu_{OP}{}^{P}, \mu_{OP}{}^{N})(e_i, e_j)$$

$$= (\mu_{OP}{}^{P}, \mu_{OP}{}^{N})(e_j, e_i)$$

$\Rightarrow \mu_{OP}$ is symmetric

$$= M_{OP_{(K\mu)}\,j,i}$$

$$\Rightarrow \left(M_{OP_{(K\mu)}}\right)^{T} = M_{OP_{(K\mu)}}$$

Hence the proof

**Theorem 4.4**

The matrix implementation of every undirected OPFG $G_{OP} : (V, \sigma_{OP}, \mu_{OP})$ is symmetric.

**Proof:**

Let $G_{OP} : (V, \sigma_{OP}, \mu_{OP})$ be the undirected simple OPFG.

$$\Longrightarrow (\mu_{OP}{}^{P}, \mu_{OP}{}^{N})(v_1, v_2) = (\mu_{OP}{}^{P}, \mu_{OP}{}^{N})(v_2, v_1)$$

Let $A_{OPFG} = (a_{ij}, -a_{ij})$be the OPF adjacency matrix of $G_{OP}$.

The membership values of $A_{OPFG}$ of any row and column are same.

$$\Longrightarrow A_{OPFG} = (A_{OPFG})^{t}$$

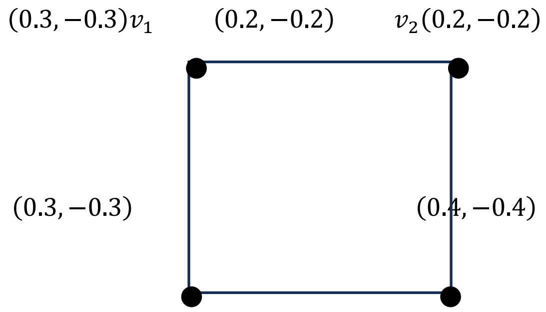
$A_{OPFG}$ is symmetric.

**Remark:**

The sum of all the entries in the matrix of an OPFG except diagonal element is equal to twice the sum of the membership value of the edges.

**Proof:**

We will prove this by an example:



$(0.7,-0.7)v_4 \qquad (0.5,-0.5) \qquad v_3(0.6,-0.6)$

The OPF adjacency matrix implementation of the above graph is

$$A_{OPFG} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} (0.3,-0.3) & (0.2,-0.2) & (0.0,0.0) & (0.3,-0.3) \\ (0.2,-0.2) & (0.2,-0.2) & (0.4,-0.4) & (0.0,0.0) \\ (0.0,0.0) & (0.4,-0.4) & (0.6,-0.6) & (0.5,-0.5) \\ (0.3,-0.3) & (0.0,0.0) & (0.5,-0.5) & (0.7,-0.7) \end{bmatrix} \end{array}$$

The sum of all the entries except diagonals

$$= (0.2+0.3+0.2+0.4+0.4+0.5+0.3+0.5, -0.2-0.3-0.2-0.4-0.4-0.5-0.3-0.5)$$

$$= (2.8,-2.8) \qquad (1)$$

Sum of the membership values of the edges

$$= (0.2+0.3+0.4+0.5, -0.2-0.3-0.4-0.5)$$

$$= (1.4,-1.4) \qquad (2)$$

From Equation (1) and (2), we say that, the sum of all the entries except diagonals of an OPFG is equal to twice the membership values of the edges.

## 4. Conclusion

In this paper, we present the matrix implementation on Oscillating Polar Fuzzy Graphs (OPFG) and analyse the results using the edge matrix representation, order, size, and vertex degree of OPFGs. several theorems are proved based on these definitions. This implementation has the potential to be extended for application in computer programming, image classification, and artificial intelligence.

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