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Article

Extending Hesitant Fuzzy Graphs to Bipolar Hesitant Frameworks for Capturing Positive and Negative Uncertainty

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Abstract: Hesitant fuzzy graphs provide a flexible framework for representing uncertainty through multiple possible membership values assigned to vertices and edges. However, classical hesitant fuzzy graphs are restricted to positive hesitant information and do not incorporate negative or conflicting influence. In this paper, we introduce a Bipolar Hesitant Fuzzy Graph (BHFG) framework that extends hesitant fuzzy graphs by simultaneously incorporating positive and negative hesitant membership degrees. Formal definitions and consistency conditions are established, and it is proved that every hesitant fuzzy graph can be embedded in the bipolar hesitant framework, while the converse does not hold. Fundamental structural operations, including complement, union, and Cartesian product, are investigated and shown to preserve the bipolar hesitant structure. A bipolar hesitant uncertainty measure is proposed to quantify combined positive and negative hesitation. An illustrative application to a conflicting social relationship network demonstrates the enhanced expressive capability of the proposed framework compared to classical hesitant fuzzy graphs. The results confirm that the bipolar hesitant model provides a more comprehensive representation of networks involving cooperative and conflicting interactions.
Keywords: - Bipolar hesitant fuzzy graph, Hesitant fuzzy graph; Uncertainty modeling, Graph operations, Social network analysis, Bipolar fuzzy sets.

1. Introduction

Uncertainty modeling has become an essential component in the mathematical analysis of complex systems. Since the introduction of fuzzy sets by Zadeh [1], numerous extensions have been developed to represent different forms of imprecision and vagueness arising in real-world problems. Graph-theoretic structures incorporating fuzzy information have been particularly useful in representing uncertain relationships among interconnected entities. Fuzzy graphs, introduced by Rosenfeld [2], laid the foundation for integrating fuzzy set theory with graph theory, and have since evolved into various advanced models.

Hesitant fuzzy sets, proposed by Torra [3], allow the representation of situations in which several possible membership degrees are available for a single element. Further developments in hesitant fuzzy aggregation and decomposition theories were studied in [4,5], enhancing their applicability in decision-making and information fusion contexts. These ideas naturally extended to hesitant fuzzy graphs, where vertices and edges are assigned sets of possible membership values rather than single degrees [6,9]. Structural properties and applications of hesitant fuzzy graphs have been investigated in various directions, including decision-making and network analysis [21].

In recent years, more generalized fuzzy graph models have been introduced to address additional layers of uncertainty. These include bipolar-valued hesitant fuzzy sets [7], neutrosophic graph structures [8], complex fuzzy graphs [10], vague graph models [11,19], and complex hesitant fuzzy graph extensions [12,15]. Surveys and structural investigations on hybrid fuzzy graph models further demonstrate the growing interest in richer uncertainty representations [20]. Moreover, bipolar-valued hesitant fuzzy graphs and their applications have been explored in social and network-based environments [13,16,17], and extended hesitant frameworks have been applied in transportation and optimization problems under uncertainty [18].

Despite these advancements, classical hesitant fuzzy graphs remain restricted to positive hesitant information. They do not incorporate negative influence or opposition within the same structural framework. However, many practical systems, such as social interactions involving trust and distrust, collaborative environments with cooperation and conflict, and decision systems with benefits and risks, require simultaneous representation of both acceptance and rejection under uncertainty. While bipolar and hesitant concepts have individually been studied [7,13], a systematic structural extension of hesitant fuzzy graphs into a unified bipolar hesitant graph framework has not been fully developed.

To address this limitation, we propose an extension of hesitant fuzzy graphs to a bipolar hesitant framework. The proposed Bipolar Hesitant Fuzzy Graph (BHFG) associates each vertex and edge with sets of positive membership values in the interval [0,1] and negative membership values in the interval [-1,0]. This dual hesitant representation enables the modeling of both supportive and conflicting interactions within a unified mathematical structure.

The following are this paper's primary contributions. First, we formally define Bipolar Hesitant Fuzzy Graphs and establish their structural consistency conditions. Second, we prove that hesitant fuzzy graphs arise as a special case of the proposed framework, thereby demonstrating that BHFGs constitute a strict extension. Third, we investigate fundamental structural properties and graph operations within the bipolar hesitant setting. Lastly, an example application is given to show the usefulness of the suggested extension in modeling networks with both positive and negative uncertainty.

2. Preliminary

Definition 2.1 ([3]). Let X be a non-empty set. A hesitant fuzzy set H on X is defined as a mapping $H: X \to \rho([0,1])$, where $\rho([0,1])$ denotes the collection of all finite subsets of the unit interval [0,1]. For each element $x \in X$, the value $H(x) \subseteq [0,1]$ represents a finite set of membership degrees that captures uncertainty across several possible evaluations.

Definition 2.2 ([4,5]). A hesitant fuzzy graph (HFG) is an ordered pair $G = (A, B)$,

where:

- $A: V \to P([0,1])$ assigns to each vertex a finite set of positive membership degrees, and
- $B: V \times V \to P([0,1])$ assigns to each edge a finite set of positive membership degrees,

such that for all $u, v \in V$,

$$max\, B(u,v) \leq min\{max\, A(u), max\, A(v)\}.$$

This condition ensures that the strength of an edge does not exceed the membership strength of its incident vertices. Figure 1 shows that hesitant fuzzy graph with positive membership degree to each vertex an edge.

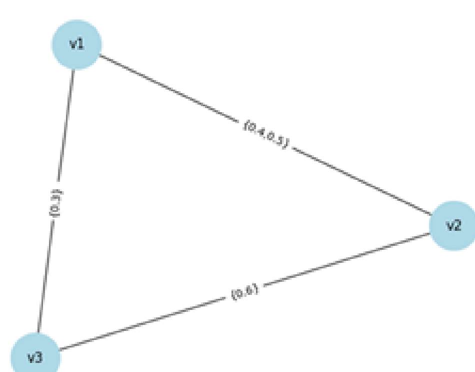

Figure 1. Hesitant Fuzzy Graph (HFG)

Definition 2.3 ([13]). Let X be a non-empty set. A bipolar hesitant fuzzy set B on X is defined as

$B(x) = (B^+(x), B^-(x))$,

where

$$B^+ : X \rightarrow P([0,1])$$

assigns a finite set of positive membership degrees, and

$$B^- : X \rightarrow P([-1,0])$$

assigns a finite set of negative membership degrees.

The set $B^+(x)$ reflects degrees of acceptance, while $B^-(x)$ reflects degrees of rejection or opposition.

Definition 2.4 ([7]). A Bipolar Hesitant Fuzzy Graph (BHFG) is an ordered triple

$$G = (V, A, B),$$

where:

- $A = (A^+, A^-)$ is a bipolar hesitant fuzzy set on V, and
- $B = (B^+, B^-)$ is a bipolar hesitant fuzzy set on $V \times V$,

reaching the following conditions for consistency for all $u, v \in V$

$$max\, B^+(u, v) \leq min\{max\, A^+(u), max\, A^+(v)\},$$

and

$$min\, B^-(u, v) \geq max\{min\, A^-(u), min\, A^-(v)\}.$$

The first condition restricts positive edge strength by vertex acceptance levels, while the second condition controls negative edge influence relative to vertex rejection levels. Figure 2 shows that bipolar hesitant fuzzy graph with positive and negative membership degree to each vertex and edge.

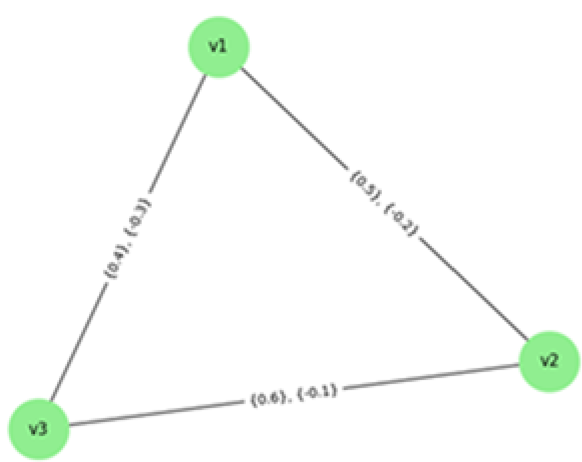

Figure 2. Bipolar Hesitant Fuzzy Graph (BHFG)

$If\ A^-(v) = \{0\}$ and $B^-(u, v) = \{0\}$ for all vertices and edges, then a BHFG reduces to a hesitant fuzzy graph.

3. Methodology and Results

Extension of Hesitant Fuzzy Graphs to Bipolar Hesitant Frameworks

Theorem 3.1

Every Hesitant Fuzzy Graph is a Bipolar Hesitant Fuzzy Graph.

Proof

Let $G = (A, B)$ be a hesitant fuzzy graph on the vertex set V, where

$$A: V \rightarrow P([0,1])$$

and

$$B: V \times V \rightarrow P([0,1])$$

satisfy the consistency condition

$$max\, B(u, v) \leq min\{max\, A(u), max\, A(v)\}$$

for all $u, v \in Vu$,

Define two mappings:

$$A^+(v) = A(v), \qquad A^-(v) = \{0\}$$

$$B^+(u, v) = B(u, v), B^-(u, v) = \{0\},$$

for all $u, v \in V$,

Since $\{0\} \subseteq [-1,0]$, the negative membership condition becomes

$$min\, B^-(u, v) = 0 \geq max\{min\, A^-(u), min\, A^-(v)\} = 0,$$

which holds trivially.

Moreover, the positive consistency condition reduces to the original hesitant fuzzy graph condition. Therefore, G satisfies all requirements of a Bipolar Hesitant Fuzzy Graph.

Hence every hesitant fuzzy graph can be viewed as a special case of a bipolar hesitant fuzzy graph.

Theorem 3.2

The class of Bipolar Hesitant Fuzzy Graphs strictly contains the class of Hesitant Fuzzy Graphs.

Proof

From Theorem 3.1, every hesitant fuzzy graph can be represented as a bipolar hesitant fuzzy graph. Thus,

$$HFG \subseteq BHFG.$$

To prove strict inclusion, it suffices to construct a BHFG that is not an HFG.

Let $V = \{v_1, v_2\}$ Define:

$$A^+(v_1) = \{0.7\}, A^-(v_1) = \{-0.4\},$$

$$A^+(v_2) = \{0.6\}, A^-(v_2) = \{-0.3\}.$$

Define the edge:

$$B^+(v_1, v_2) = \{0.5\}, B^-(v_1, v_2) = \{-0.35\}.$$

The consistency conditions are satisfied since

$$0.5 \leq min\{0.7,0.6\} = 0.6,$$

and

$$-0.35 \geq max\{-0.4, -0.3\} = -0.3.$$

Thus, the structure forms a valid BHFG.

However, since the vertices and edge possess non-zero negative membership values, it cannot be represented as a hesitant fuzzy graph, which allows only positive hesitant memberships.

Therefore, there exists a BHFG that is not an HFG, implying

$$HFG \subsetneq BHFG.$$

Theorem 3.3 (Characterization of Reduction)

A Bipolar Hesitant Fuzzy Graph reduces to a Hesitant Fuzzy Graph if and only if

$$A^{-}(v) = \{0\} \quad and \quad B^{-}(u,v) = \{0\}$$

for all vertices and edges.

Proof

If the negative hesitant memberships are equal to $\{0\}$, then the bipolar structure contains only positive hesitant information. Hence, it coincides with the definition of a hesitant fuzzy graph.

Conversely, if a BHFG reduces to a hesitant fuzzy graph, then no non-zero negative membership values can exist. Therefore, all negative hesitant sets must be equal to $\{0\}$.

Corollary 3.1

Bipolar Hesitant Fuzzy Graphs provide a strictly richer representational framework than Hesitant Fuzzy Graphs for modelling uncertainty.

4. Structural Properties and Fundamental Operations of BHFGs

In this section, we investigate fundamental operations on Bipolar Hesitant Fuzzy Graphs and examine their structural behaviour. Throughout this section, let $G = (V, A, B)$ be a Bipolar Hesitant Fuzzy Graph.

4.1 Complement of a Bipolar Hesitant Fuzzy Graph

Let $G = (V, A, B)$ be a BHFG. The complement of G, denoted by G^c, is defined as

$$G^c = (V, A^c, B^c),$$

where for each vertex $v \in V$:

$$A^{c+}(v) = \{1 - \alpha \mid \alpha \in A^{+}(v)\},$$

$$A^{c-}(v) = \{-1 - \beta \mid \beta \in A^{-}(v)\},$$

and similarly for edges:

$$B^{c+}(u,v) = \{1 - \alpha \mid \alpha \in B^{+}(u,v)\},$$

$$B^{c-}(u,v) = \{-1 - \beta \mid \beta \in B^{-}(u,v)\}.$$

Theorem 4.1

The complement of a Bipolar Hesitant Fuzzy Graph is again a Bipolar Hesitant Fuzzy Graph.

Proof

Let $G = (V, A, B)$ be a BHFG satisfying

$$\max B^{+}(u,v) \le \min\{\max A^{+}(u), \max A^{+}(v)\},$$

$$min\, B^{-}(u,v) \ge max\{min\, A^{-}(u), min\, A^{-}(v)\}.$$

For the complement graph G^c, consider the positive part:

$$max\, B^{c+}(u,v) = 1 - min\, B^{+}(u,v),$$

and

$$max\, A^{c+}(u) = 1 - min\, A^{+}(u).$$

Since subtraction by 1 reverses inequalities, the required consistency conditions remain satisfied. A similar argument holds for the negative part.

Therefore, G^c satisfies the consistency conditions of a BHFG.

4.2 Union of Two BHFGs

Let $G_1 = (V, A_1, B_1)$ and $G_2 = (V, A_2, B_2)$ be two BHFGs on the same vertex set. Their union $G = G1 \cup G2$ is defined by:

$$A^{+}(v) = A_1^{+}(v) \cup A_2^{+}(v),$$

$$A^{-}(v) = A_1^{-}(v) \cup A_2^{-}(v),$$

and similarly for edges.

The consistency conditions follow from the fact that maxima and minima are preserved under set union.

4.3 Cartesian Product of Bipolar Hesitant Fuzzy Graphs

Let

$$G_1 = (V_1, A_1, B_1),$$

be two BHFGs. Their Cartesian product

$G = G1\square G2$

has vertex set $V = V1 \times V2$, with memberships defined as

$$A^{+}(u,v) = min\{A_1^{+}(u), A_2^{+}(v)\},$$

$$A^{-}(u,v) = max\{A_1^{-}(u), A_2^{-}(v)\}.$$

Edges are defined following classical Cartesian product adjacency with memberships defined analogously. Figure 3 shows that Cartesian product of two bipolar hesitant fuzzy graph with membership degree.

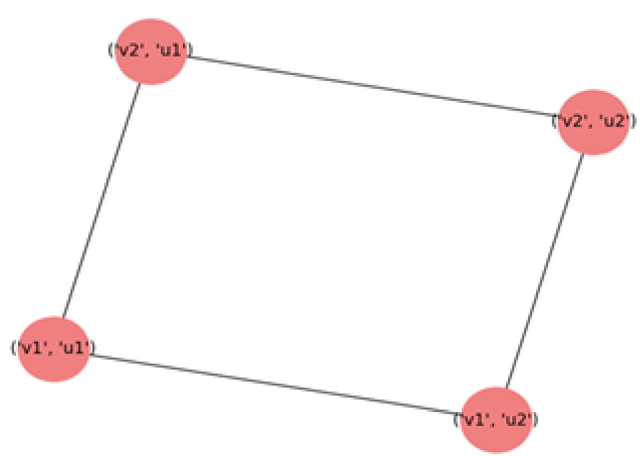

Figure 3. Cartesian Product of Two Graphs

Theorem 4.2

The Cartesian product of two Bipolar Hesitant Fuzzy Graphs is a Bipolar Hesitant Fuzzy Graph.

Proof

The minimum and maximum operators preserve the membership bounds and maintain the required consistency inequalities. Hence, the Cartesian product satisfies the defining conditions of a BHFG.

4.4 Bipolar Hesitant Uncertainty Measure (Original Contribution)

To quantify the degree of uncertainty in the bipolar hesitant environment, we introduce the following measure.

Let $G = (V, A, B)$ be a BHFG. The Bipolar Hesitant Uncertainty Measure of a vertex $v \in V$ is defined as

$$U(v) = (max\, A^+(v) - min\, A^+(v)) + |max\, A^-(v) - min\, A^-(v)|.$$

Similarly, for an edge (u, v):

$$U(u, v) = (max\, B^+(u, v) - min\, B^+(u, v)) + |\, max\, B^-(u, v) - min\, B^-(u, v)\, |.$$

This measure captures the combined spread of positive and negative hesitation.

Proposition 4.1

For any vertex $v \in V$,

$$0 \le U(v) \le 2.$$

Proof

Since positive memberships lie in [0,1],

$$0 \le max\, A^+(v) - min\, A^+(v) \le 1.$$

Since negative memberships lie in $[-1,0]$,

$$0 \le |\, max\, A^-(v) - min\, A^-(v)\, | \le 1.$$

Adding both parts gives

$$0 \le U(v) \le 2.$$

4.5 Monotonicity Property

Proposition 4.2

If $G1 \subseteq G2$ such that

$A_1^+(v) \subseteq A_2^+(v), A_1^-(v) \subseteq A_2^-(v),$

then

$$U_{G_1}(v) \le U_{G_2}(v)$$

This shows that uncertainty increases with larger hesitant sets.

4.6 Reduction Property

Proposition 4.3

If

$$A^-(v) = \{0\}, \qquad B^-(u, v) = \{0\}$$

for all vertices and edges, then all structural properties of BHFGs reduce to those of hesitant fuzzy graphs.

Proposition 4.4 (Comparison with HFG Uncertainty)

Let G be a hesitant fuzzy graph embedded in the bipolar hesitant framework. Then for every vertex v,

$$U_{BHFG}(v) \ge U_{HFG}(v).$$

This result reinforces that Bipolar Hesitant Fuzzy Graphs form a genuine extension of hesitant fuzzy graphs while preserving their classical properties.

5. Illustrative Application: Modeling Conflicting Social Relationships

We look at a tiny social interaction network to show the usefulness of the suggested framework. Instead of creating a large-scale model, the goal of this example is to demonstrate how bipolar hesitant information goes above traditional hesitant fuzzy graphs in terms of expressive power.

5.1 Network Description

Let

$$V = \{v_1, v_2, v_3\}$$

represent three individuals in a working group.

- v_1: Team leader
- v_2: Senior member
- v_3: Junior member

Relationships are not always just supportive in real life. Disagreement and cooperation can coexist. This supports the requirement for both positive and negative hesitancy representation at the same time.

5.2 Hesitant Fuzzy Graph Representation

First, consider the hesitant fuzzy graph model.

Vertex memberships:

$$A(v_1) = \{0.7,0.8\}$$

$$A(v_2) = \{0.6\}$$

$$A(v_3) = \{0.5,0.6\}$$

Edge memberships:

$$B(v_1, v_2) = \{0.5,0.6\}$$

$$B(v_2, v_3) = \{0.4\}$$

$$B(v_1, v_3) = \{0.3,0.4\}$$

This structure reflects hesitation in cooperation levels. However, it assumes that all interactions are positively oriented.

There is no way to indicate conflict, disagreement, or resistance.

5.3 Bipolar Hesitant Fuzzy Graph Representation

Now we extend the same network into the bipolar hesitant framework.

Positive memberships:

$$A^+(v1) = \{0.7,0.8\}, A^+(v2) = \{0.6\}, A^+(v3) = \{0.5,0.6\}$$

Negative memberships:

$$A^-(v1) = \{-0.2\}$$

$$A^-(v2) = \{-0.3, -0.2\}$$

$$A^-(v3) = \{-0.1\}$$

For edges:

$$B^+(v_1, v_2) = \{0.5\},\ \ B^-(v_1, v_2) = \{-0.2\}$$

$$B^+(v_2, v_3) = \{0.4\},\ \ B^-(v_2, v_3) = \{-0.3\}$$
$$B^+(v_1, v_3) = \{0.3\},\ \ B^-(v_1, v_3) = \{-0.2\}$$

Here, negative memberships represent tension, disagreement, or resistance.

This reflects a more realistic social structure.

5.4 Computation of Bipolar Hesitant Uncertainty

Recall the uncertainty measure:

$$U(v) = (max\, A^+(v) - min\, A^+(v)) + |\ max\, A^-(v) - min\, A^-(v)\ |$$

For v_1:

Positive spread:

$$0.8 - 0.7 = 0.1$$

Negative spread:

$$|\ -0.2 - (-0.2)\ | = 0$$

Thus,

$$U(v_1) = 0.1$$

For v_2:

Positive spread:

$$0.6 - 0.6 = 0$$

Negative spread:

$$|\ -0.2 - (-0.3)\ | = 0.1$$

Thus,

$$U(v_2) = 0.1$$

For v_3:

Positive spread:

$$0.6 - 0.5 = 0.1$$

Negative spread:

$$|\ -0.1 - (-0.1)\ | = 0$$

Thus,

$$U(v_3) = 0.1$$

5.5 Comparison with Hesitant Fuzzy Graph

In the hesitant fuzzy graph model, uncertainty only comes from positive spread.

For example:

$$U_{HFG}(v_2) = 0$$

But in the bipolar model:

$$U_{BHFG}(v_2) = 0.1$$

This difference arises from negative hesitation, which the classical hesitant fuzzy graph cannot represent.

Thus, the bipolar hesitant framework reveals hidden structural variability that would otherwise remain undetected.

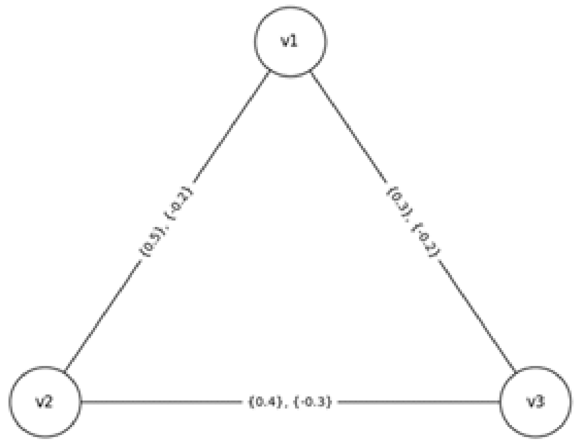

Figure 4. Bipolar Hesitant Fuzzy Graph (v1,v2,v3)

5.6 Interpretation

The example shows that the suggested extension is more than just a new parameter. It significantly alters the graph model's capacity for description. The bipolar hesitant structure provides a more accurate picture of reality in networks where conflict and cooperation coexist as shown in Figure 4.

The added negative hesitant component alters the uncertainty distribution across vertices, even in such a small example. This implies that within the suggested concept, larger real-world systems might show even more significant structural differences.

6. Conclusion

In this study, we introduced a bipolar hesitant extension of hesitant fuzzy graphs designed to encapsulate both positive and negative uncertainties within a cohesive framework. Although hesitant fuzzy graphs effectively model variations in positive membership values, they fail to consider conflicting or detrimental influences. The newly proposed Bipolar Hesitant Fuzzy Graph (BHFG) framework addresses this gap by integrating hesitant degrees of both acceptance and rejection simultaneously. It was formally demonstrated that every hesitant fuzzy graph can be integrated into the bipolar hesitant framework, with this integration being strict. Key structural operations were analyzed and shown to maintain the integrity of the bipolar hesitant structure. An uncertainty metric was presented to measure the combined positive and negative hesitation. The example related to social relationships illustrated that the bipolar hesitant framework offers a more profound structural understanding.

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