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Article

A Hybrid Analytical, Numerical, and Machine Learning Framework for Health Management, Resource Allocation, Operational Efficiency, Risk Forecasting, and Strategic Decision-Making in Nepal

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Abstract: We consider the problem of optimal health resource allocation in a setting where disease dynamics are governed by compartmental epidemiological models, demand is predicted by black-box machine learning forecasts, and resource deployment is constrained by geography, budget, and infrastructure fragility. This problem arises concretely in Nepal, where mountainous topography, climate-sensitive disease outbreaks, and decentralized governance create a decision environment too complex for siloed methodologies [1,2,3]. We pose the following question: Given that disease dynamics follow SEIR processes, demand is predicted by non-linear ML models, and resources are constrained by road networks and budget, what is the optimal allocation policy, and can we bound its suboptimality?

We formalize this as a coupled SEIR-MILP stochastic program and prove that the general problem is NP-hard (Theorem 1). We then develop the Sequential Decomposition with Learning-Based Oracles (SD-LO) algorithm, which iterates between (i) maximum-likelihood estimation of epidemiological parameters, (ii) training of a Temporal Fusion Transformer with an epidemiologically structured loss, and (iii) Benders decomposition of a distributionally robust MILP whose right-hand side is informed by the ML oracle. We prove that if the ML predictor is Lipschitz continuous with bounded error ϵ, the optimality gap of the coupled system is $O(\epsilon\sqrt{T})$ where T is the planning horizon (Theorem 2), and that SD-LO converges to a local optimum of the coupled problem (Theorem 3).

We validate the framework on a computational study designed around Nepal's 77-district health system, using DHIS2 epidemiological records (2015–2024), NASA POWER climate rasters, OpenStreetMap road networks, and Logistics Management Information System (LMIS) pharmaceutical flow data. On held-out 2023–2024 test data, SD-LO achieves a 12.7% MAPE on dengue forecasting (vs. 24.1% for ARIMA and 15.3% for pure ML without epidemiological structure), reduces pharmaceutical stock-out rates from 12.3% to 4.1%, and yields a 31% cost reduction over heuristic allocation. An ablation study demonstrates that removing the epidemiological layer increases DALYs lost by +180, removing the OR layer by +520, and removing the ML layer by +340. The full MILP with 12,000 variables solves in 4.3 seconds using Gurobi 10.0; the SD-LO outer loop converges in 18 iterations (average 2.1 minutes per iteration).

Keywords: Epidemiological-ML optimization, Benders decomposition, distributionally robust optimization, health resource allocation, Nepal, SEIR, Temporal Fusion Transformer, stochastic programming

1. Introduction

1.1 The Problem

Nepal's health system operates under constraints that are simultaneously epidemiological, geographic, climatic, and economic [1,2]. The country spans elevations from 60 meters to 8,849 meters, with 77 districts connected by road networks that are seasonally disrupted by monsoon landslides and winter snow [3,4]. Disease dynamics—whether dengue in the Terai, tuberculosis in urban centers, or respiratory outbreaks in the Himalayas—follow compartmental processes that are modulated by climate and population mobility [5,6]. Resource allocation decisions (where to place beds, how to route vaccines, whom to staff) must be made before demand is realized, yet demand itself is a stochastic function of transmission dynamics that are only partially observable [7,8].

This creates a coupled prediction-optimization problem that existing methodologies fail to solve adequately:

- Pure epidemiological models (SEIR, metapopulation) capture mechanistic transmission but struggle with high-dimensional exogenous drivers (climate, mobility, behavioral heterogeneity) [9,10].
- Pure machine learning models (TFT, XGBoost) capture non-linear patterns but ignore physical transmission constraints, producing forecasts that are statistically accurate but epidemiologically implausible [11,12].
- Pure operations research models (MILP, stochastic programming) assume demand is exogenous and stationary, failing when demand is endogenous to the disease process itself [13,14].

We therefore pose the following falsifiable technical question:

How does one optimally allocate finite medical resources (beds, staff, pharmaceuticals, diagnostic capacity) across a geographically constrained network when demand is generated by a coupled SEIR-ML dynamical system, and what guarantees can be provided on the quality of the resulting policy?

1.2 Contributions

Our contributions are fourfold:

- **Problem Formalization.** We define the coupled SEIR-MILP stochastic program and prove its NP-hardness via reduction from the Capacitated Facility Location Problem (Theorem 1).
- **Algorithm.** We propose the Sequential Decomposition with Learning-Based Oracles (SD-LO) algorithm, which integrates maximum-likelihood epidemiological fitting, a custom TFT with epidemiologically structured loss, and Benders decomposition for the MILP. We prove convergence to a local optimum (Theorem 3).
- **Approximation Guarantees.** We derive a bound on the optimality gap of the stochastic program as a function of ML prediction error ϵ and horizon T (Theorem 2).
- **Empirical Validation.** We design and execute a computational study on Nepal's 77-district system using real-world data architectures (DHIS2, LMIS, NASA POWER, OSM), demonstrating superiority over three benchmarks and presenting full ablation and sensitivity analyses [15,16].

1.3 Scope and Limitations

This article focuses on the resource allocation and risk forecasting domains, where the coupling between epidemiology, ML, and optimization is mathematically deepest [17,18]. We develop the theory and algorithms fully for these domains; companion work addresses health management dashboards, NLP for clinical notes, and computer vision for diagnostics, which are beyond the scope of this paper.

2. Related Work

2.1 Epidemiological-Operational Research Integration

The integration of compartmental models with operations research has been explored primarily in vaccine allocation [9] and hospital surge planning [10]. However, these works typically assume deterministic demand or simple stochastic extensions. Patel et al. [14] coupled SEIR dynamics with linear programming for COVID-19 bed allocation, but did not incorporate ML-based demand forecasting or prove complexity results. Our work is the first to embed a black-box ML oracle within the constraints of a stochastic MILP and prove approximation bounds.

2.2 Machine Learning for Disease Forecasting

Deep learning for epidemiological forecasting has advanced substantially, with Temporal Fusion Transformers (TFT) demonstrating state-of-the-art performance on multiple time-series benchmarks [11]. However, TFT and related models are trained purely on historical patterns, lacking mechanistic constraints. Recent work by Wang et al. [12] incorporated physical constraints into neural ODEs for epidemiology, but did not connect to resource optimization. Our SD-LO algorithm uses the SEIR structure as an inductive bias in the ML loss function, creating a mechanistically grounded predictor [19,20].

2.3 Distributionally Robust Optimization in Healthcare

Distributionally Robust Optimization (DRO) has been applied to hospital scheduling [13] and inventory management [21], typically using Wasserstein or χ^2 ambiguity sets. These works assume demand is drawn from an exogenous distribution. Our framework extends DRO to the setting where the demand distribution is itself generated by a learned dynamical system (SEIR-ML), requiring new proof techniques for finite-sample guarantees [22,23].

2.4 Health Systems in Mountainous LMICs

Research on Nepal's health system has documented access inequities [4], pharmaceutical supply chain fragility [5], and climate-sensitive disease expansion [6]. However, no prior work has proposed an integrated algorithmic framework with provable guarantees for this context [1,2].

2.5 End-to-End Learning and Optimization

Recent advances in differentiable optimization and end-to-end learning [24,25,26] have enabled gradient-based tuning of optimization problem parameters. Ferber et al. [27] and Wilder et al. [28] demonstrated methods for melding machine learning with combinatorial optimization. Our SD-LO algorithm differs in that it handles dynamical systems constraints (SEIR) and uses decomposition rather than full differentiability, which is necessary for the scale of public health resource allocation.

3. Problem Formulation

3.1 Notation and System Definition

Let $\mathcal{D} = \{1, \dots, D\}$ denote the set of districts, indexed by d. We set $D = 77$ corresponding to Nepal's administrative districts [3]. Let $\mathcal{T} = \{1, \dots, T\}$ denote the planning horizon in weeks. For our computational study, $T = 52$ (one year). The system state at time t comprises:

- **Epidemiological state:** $\mathbf{S}_t = \left(S_{d,t}, E_{d,t}, I_{d,t}, R_{d,t}\right)_{d\in\mathcal{D}} \in \mathbb{R}_+^{4D}$, representing susceptible, exposed, infected, and recovered populations.
- **Control variables:** $\mathbf{u}_t = \left(u_{d,t}^{\text{bed}}, u_{d,t}^{\text{staff}}, u_{d,t}^{\text{pharm}}\right)_{d\in\mathcal{D}} \in \mathcal{U}_t$, where $\mathcal{U}_t$ encodes budget, road network, cold chain, and capacity constraints.
- **Exogenous covariates:** $\mathbf{x}_{d,t} = \left(\mathbf{c}_{d,t}, \mathbf{m}_{d,t}, \boldsymbol{\theta}_d\right)$, where $\mathbf{c}_{d,t}$ is climate (temperature, precipitation), $\mathbf{m}_{d,t}$ is mobility, and $\boldsymbol{\theta}_d$ is geographic fixed effects (elevation, road density).

The total population of district d is N_d, constant over the planning horizon.

3.2 The Epidemiological Layer

Disease dynamics follow a metapopulation SEIR model with control-modulated transmission. For each district $d \in \mathcal{D}$ and time $t \in \mathcal{T}$:

$$\frac{dS_{d,t}}{dt} = -\beta_{d,t}\left(\mathbf{u}_t, \mathbf{x}_{d,t}\right)\frac{S_{d,t}I_{d,t}}{N_d} + \mu\left(N_d - S_{d,t}\right) - \sum_{d\prime\neq d} \rho_{dd\prime} S_{d,t} I_{d\prime,t} \tag{1}$$

$$\frac{dE_{d,t}}{dt} = \beta_{d,t}(\mathbf{u}_t, \mathbf{x}_{d,t}) \frac{S_{d,t} I_{d,t}}{N_d} + \sum_{d' \neq d} \rho_{dd'} S_{d,t} I_{d',t} - \sigma E_{d,t} \tag{2}$$

$$\frac{dI_{d,t}}{dt} = \sigma E_{d,t} - \gamma I_{d,t} - \delta(u_{d,t}^{\text{bed}}) I_{d,t} \tag{3}$$

$$\frac{dR_{d,t}}{dt} = \gamma I_{d,t} + \delta(u_{d,t}^{\text{bed}}) I_{d,t} \tag{4}$$

where:

- $\beta_{d,t}$ is the effective contact rate, potentially reduced by public health interventions funded through $\mathbf{u}_t$
- $\rho_{dd'}$ is the mobility-driven coupling between districts, estimated from mobile network data and road traffic counts
- σ is the inverse mean latent period (we assume $\sigma = 1/5.2$ days^{-1} for dengue)
- γ is the inverse mean infectious period (we assume $\gamma = 1/4.7$ days^{-1} for dengue)
- $\delta(u_{d,t}^{\text{bed}})$ is the case fatality reduction due to treatment capacity, modeled as:

$$\delta(u) = \delta_0(1 - e^{-\eta u}) \tag{5}$$

where δ_0 is the baseline untreated fatality rate and η is the treatment efficacy parameter.

The function $f_{\text{SEIR}}(\mathbf{S}_t, \mathbf{u}_t, \mathbf{x}_t; \boldsymbol{\theta})$ compactly denotes this system, with $\boldsymbol{\theta} = (\sigma, \gamma, \delta_0, \eta, \rho, \mu)$.

For numerical integration, we employ the implicit-explicit (IMEX) scheme with time step $\Delta t = 1$ day:

$$\begin{aligned} S_{d,t+1} = S_{d,t} - \Delta t \cdot \beta_{d,t} \frac{S_{d,t} I_{d,t}}{N_d} + \Delta t \cdot \mu(N_d - S_{d,t}) \\ - \Delta t \sum_{d' \neq d} \rho_{dd'} S_{d,t} I_{d',t} \end{aligned} \tag{6}$$

$$E_{d,t+1} = \frac{E_{d,t} + \Delta t \cdot \beta_{d,t} \frac{S_{d,t} I_{d,t}}{N_d} + \Delta t \sum_{d' \neq d} \rho_{dd'} S_{d,t} I_{d',t}}{1 + \Delta t \cdot \sigma} \tag{7}$$

$$I_{d,t+1} = \frac{I_{d,t} + \Delta t \cdot \sigma E_{d,t}}{1 + \Delta t \cdot \left(\gamma + \delta(u_{d,t}^{\text{bed}})\right)} \tag{8}$$

$$R_{d,t+1} = R_{d,t} + \Delta t \cdot \gamma I_{d,t} + \Delta t \cdot \delta(u_{d,t}^{\text{bed}}) I_{d,t} \tag{9}$$

The conservation equation $S_{d,t} + E_{d,t} + I_{d,t} + R_{d,t} = N_d$ is preserved up to machine precision under this scheme.

3.3 The Machine Learning Layer

The ML layer predicts the effective contact rate $\beta_{d,t}$ and the resulting incidence $I_{d,t+h}$ using a Temporal Fusion Transformer g_ϕ with parameters ϕ:

$$\hat{I}_{d,t+h} = g_\phi(I_{d,\leq t}, \mathbf{c}_{d,\leq t}, \mathbf{m}_{d,\leq t}, \mathbf{h}_{d,t}) \tag{10}$$

where $\mathbf{h}_{d,t}$ is the static covariate embedding (elevation, population density, health facility density) and $I_{d,\leq t}$ is the historical incidence trajectory. The TFT architecture comprises variable selection networks, gated residual networks, and multi-head self-attention [11].

Let $\mathbf{z}_{d,t} \in \mathbb{R}^p$ denote the concatenated input features at time t. The variable selection network computes:

$$\mathbf{v}_{d,t} = \text{softmax}(W_v \mathbf{z}_{d,t} + b_v) \tag{11}$$

$$\tilde{\mathbf{z}}_{d,t} = \sum_{j=1}^{p} v_{d,t,j} \cdot W_{z,j}\mathbf{z}_{d,t,j} \tag{12}$$

The gated residual network with skip connection is:

$$\mathbf{r}_{d,t} = \text{LayerNorm}\left(\tilde{\mathbf{z}}_{d,t} + \text{GLU}\left(W_r\tilde{\mathbf{z}}_{d,t} + b_r\right)\right) \tag{13}$$

where $\text{GLU}(x) = \sigma\left(W_g x\right) \odot x$ is the gated linear unit.

The multi-head self-attention over the temporal dimension is:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^\top}{\sqrt{d_k}}\right)V \tag{14}$$

with $Q = W_Q\mathbf{r}_{d,\le t}$, $K = W_K\mathbf{r}_{d,\le t}$, $V = W_V\mathbf{r}_{d,\le t}$.

Crucially, we train g_ϕ with a mechanistically structured loss:

$$\mathcal{L}_{\text{hybrid}}(\phi) = \underbrace{\sum_{d,t}\left(I_{d,t} - \hat{I}_{d,t}\right)^2}_{\text{forecast accuracy}} + \lambda \underbrace{\sum_{d,t}\left(\frac{d\hat{S}_{d,t}}{dt} + \beta_{d,t}^{\text{ML}}\frac{\hat{S}_{d,t}\hat{I}_{d,t}}{N_d}\right)^2}_{\text{SEIR consistency}} \tag{15}$$

where $\hat{S}_{d,t}$ is derived from $\hat{I}_{d,t}$ via the SEIR conservation equations, and $\beta_{d,t}^{\text{ML}}$ is extracted from the TFT's attention weights over climate features. This loss ensures the ML predictions respect susceptible depletion and epidemic wave structure.

3.4 The Optimization Layer

The optimization layer selects controls $\mathbf{u}_t$ to minimize expected costs subject to the predicted demand and operational constraints. The objective function is:

$$\begin{aligned}\min_{\mathbf{u}\in\mathcal{U}} \mathbb{E}_{\xi\sim P}\Bigg[&\sum_{t=1}^{T}\sum_{d\in\mathcal{D}} c_d^{\text{fixed}}\, u_{d,t}^{\text{bed}} + c_d^{\text{var}}\max\left(0, \hat{I}_{d,t} - \kappa u_{d,t}^{\text{bed}}\right) \\ &+ c_d^{\text{trans}}\sum_{d'\neq d} z_{dd',t}\Bigg]\end{aligned} \tag{16}$$

where:

- $z_{dd',t}$ is the pharmaceutical shipment volume from depot d' to district d at time t
- κ is the bed-to-case conversion rate (we assume $\kappa = 0.15$)
- P is the demand distribution induced by the SEIR-ML system

The feasible region $\mathcal{U}$ encodes the following constraints:

Budget constraint:

$$\sum_{d} u_{d,t}^{\text{bed}} \le B_t^{\text{bed}} \quad \forall t \tag{17}$$

Pharmaceutical coverage constraint:

$$\sum_{d'} z_{dd',t} \ge \hat{I}_{d,t} \cdot \omega \quad \forall d, t \tag{18}$$

where $\omega = 2.3$ is the average pharmaceutical units per case.

Staffing constraints:

$$\sum_d u_{d,t}^{\text{staff}} \leq B_t^{\text{staff}} \quad \forall t \tag{19}$$

$$u_{d,t}^{\text{staff}} \leq \alpha u_{d,t}^{\text{bed}} \quad \forall d,t \tag{20}$$

where $\alpha = 0.4$ is the nurse-to-bed ratio.

Road network constraints:

$$z_{dd',t} \leq M \cdot y_{dd',t} \quad \forall d,d',t \tag{21}$$

$$\sum_{d'} y_{dd',t} \leq R_d \quad \forall d,t \tag{22}$$

where $y_{dd',t} \in \{0,1\}$ indicates whether a shipment is routed from d' to d, $M = 10^6$, and R_d is the number of vehicles available in district d.

Non-negativity and integrality:

$$\mathbf{u}_t \geq 0, \quad \mathbf{z}_t \geq 0, \quad \mathbf{y}_t \in \{0,1\}^{D\times D} \tag{23}$$

3.5 The Coupled System

The full problem is:

$$\min_{\mathbf{u}\in\mathcal{U}} \mathbb{E}_{\mathbf{I}\sim\mathcal{M}(\phi,\boldsymbol{\theta})}[C(\mathbf{u},\mathbf{I})] \tag{24}$$

where $\mathcal{M}(\phi,\boldsymbol{\theta})$ is the pushforward measure of the SEIR-ML dynamical system with parameters $(\phi,\boldsymbol{\theta})$. This is a stochastic program with a learned dynamical constraint—a class of problems not previously studied in the operations research literature [17,18]. # 4. Theoretical Analysis

4.1 Complexity

Theorem 1 (NP-Hardness). The coupled SEIR-MILP resource allocation problem is NP-hard.

Proof. We reduce from the Capacitated Facility Location Problem (CFLP), which is known to be NP-hard [29]. Consider an instance of CFLP with facilities F, clients C, capacities Q_j for each facility $j \in F$, fixed opening costs f_j, and assignment costs c_{ij} for serving client i from facility j.

Construct a disease instance as follows:

- Set $D = |C|$ districts, each corresponding to one client.
- Set one time period $T = 1$.
- Freeze the SEIR dynamics at the initial condition: $S_{d,0} = N_d$, $E_{d,0} = I_{d,0} = R_{d,0} = 0$.
- Set $\hat{I}_{d,1} = \text{demand}_i$ for district d corresponding to client i.
- Set $u_{d,1}^{\text{bed}} =$ facility capacity for a facility opened to serve district d.
- Set the fixed cost $c_d^{\text{fixed}} = f_j$ for opening a facility in district d.
- Set the variable cost $c_d^{\text{var}}\max(0, \hat{I}_{d,1} - \kappa u_{d,1}^{\text{bed}})$ to equal the assignment cost c_{ij} when demand exceeds capacity.
- Set pharmaceutical shipment costs equal to assignment costs.

The MILP objective becomes:

$$\min_{\mathbf{u},\mathbf{y}} \sum_d c_d^{\text{fixed}} u_{d,1}^{\text{bed}} + \sum_d c_d^{\text{var}} \max(0, \hat{I}_{d,1} - \kappa u_{d,1}^{\text{bed}}) + \sum_{d,d'} c_{dd'}^{\text{trans}} z_{dd',1} \tag{25}$$

which is identical to the CFLP objective. Since CFLP is a special case of our problem, the general problem is NP-hard. ■

4.2 Approximation Guarantee

Theorem 2 (Optimality Gap Bound). Let g_ϕ be the ML predictor with ℓ_2 error bounded by ϵ on the demand distribution:

$$\mathbb{E}\left[\| \hat{\mathbf{I}} - \mathbf{I} \|_2^2\right] \leq \epsilon^2 \tag{26}$$

Let $\mathbf{u}^*$ be the optimal policy under true demand and $\hat{\mathbf{u}}$ be the optimal policy under predicted demand. Then:

$$\mathbb{E}[C(\hat{\mathbf{u}}, \mathbf{I})] - C(\mathbf{u}^*, \mathbf{I}) \leq 2L_c\epsilon\sqrt{DT} \tag{27}$$

where L_c is the Lipschitz constant of the cost function and T is the planning horizon.

Proof. By the triangle inequality and the Lipschitz continuity of C in its second argument:

$$\left|C(\hat{\mathbf{u}}, \mathbf{I}) - C\left(\hat{\mathbf{u}}, \hat{\mathbf{I}}\right)\right| \leq L_c \| \mathbf{I} - \hat{\mathbf{I}} \|_2 \tag{28}$$

Since $\hat{\mathbf{u}}$ is optimal for $\hat{\mathbf{I}}$, we have:

$$C\left(\hat{\mathbf{u}}, \hat{\mathbf{I}}\right) \leq C\left(\mathbf{u}^*, \hat{\mathbf{I}}\right) \tag{29}$$

Combining these inequalities:

$$C(\hat{\mathbf{u}}, \mathbf{I}) \leq C\left(\hat{\mathbf{u}}, \hat{\mathbf{I}}\right) + L_c \| \mathbf{I} - \hat{\mathbf{I}} \|_2 \leq C\left(\mathbf{u}^*, \hat{\mathbf{I}}\right) + L_c \| \mathbf{I} - \hat{\mathbf{I}} \|_2 \tag{30}$$

Applying the triangle inequality again to the right-hand side:

$$C\left(\mathbf{u}^*, \hat{\mathbf{I}}\right) \leq C(\mathbf{u}^*, \mathbf{I}) + L_c \| \mathbf{I} - \hat{\mathbf{I}} \|_2 \tag{31}$$

Therefore:

$$C(\hat{\mathbf{u}}, \mathbf{I}) \leq C(\mathbf{u}^*, \mathbf{I}) + 2L_c \| \mathbf{I} - \hat{\mathbf{I}} \|_2 \tag{32}$$

Taking expectations and applying Jensen's inequality:

$$\mathbb{E}[C(\hat{\mathbf{u}}, \mathbf{I})] - C(\mathbf{u}^*, \mathbf{I}) \leq 2L_c\mathbb{E}\left[\| \mathbf{I} - \hat{\mathbf{I}} \|_2\right] \leq 2L_c\sqrt{\mathbb{E}\left[\| \mathbf{I} - \hat{\mathbf{I}} \|_2^2\right]} \tag{33}$$

Since $\mathbb{E}\left[\| \mathbf{I} - \hat{\mathbf{I}} \|_2^2\right] \leq DT\epsilon^2$ by assumption (26), we obtain:

$$\mathbb{E}[C(\hat{\mathbf{u}}, \mathbf{I})] - C(\mathbf{u}^*, \mathbf{I}) \leq 2L_c\sqrt{DT\epsilon^2} = 2L_c\epsilon\sqrt{DT} \tag{34}$$

which is the bound (27). ■

4.3 Rademacher Complexity Generalization Bound

Theorem 7 (Realistic Generalization to New LMICs). Let $\mathcal{H}$ be the hypothesis class of SD-LO allocation policies. Let $R_N(\mathcal{H})$ be the empirical Rademacher complexity of $\mathcal{H}$ on N samples from Nepal. Let P_{Nepal} and P_{target} be the data-generating distributions in Nepal and a new mountainous LMIC, respectively, with Wasserstein-1 distance $W_1\left(P_{\text{Nepal}}, P_{\text{target}}\right) \leq \Delta$. Then with probability at least $1 - \delta$:

$$\mathbb{E}_{\text{target}}[C(\pi)] \leq \hat{C}_{\text{Nepal}}(\pi) + 2R_N(\mathcal{H}) + L_c\Delta + 3B_C\sqrt{\frac{\log(2/\delta)}{2N}} \tag{35}$$

where B_C is an upper bound on the cost function.

Proof. We decompose the target risk into three terms:

$$\begin{aligned}\mathbb{E}_{\text{target}}[C(\pi)] &= \underbrace{\mathbb{E}_{\text{target}}[C(\pi)] - \mathbb{E}_{\text{Nepal}}[C(\pi)]}_{\text{domain shift}} \\ &+ \underbrace{\mathbb{E}_{\text{Nepal}}[C(\pi)] - \hat{C}_{\text{Nepal}}(\pi)}_{\text{estimation error}} + \underbrace{\hat{C}_{\text{Nepal}}(\pi)}_{\text{empirical risk}}\end{aligned} \tag{36}$$

For the domain shift term, by the Kantorovich-Rubinstein duality and the Lipschitz continuity of C:

$$\mathbb{E}_{\text{target}}[C(\pi)] - \mathbb{E}_{\text{Nepal}}[C(\pi)] \leq L_c \cdot W_1(P_{\text{Nepal}}, P_{\text{target}}) \leq L_c \Delta \tag{37}$$

For the estimation error term, standard Rademacher complexity bounds [20,21] yield:

$$\mathbb{E}_{\text{Nepal}}[C(\pi)] - \hat{C}_{\text{Nepal}}(\pi) \leq 2R_N(\mathcal{H}) + 3B_C\sqrt{\frac{\log(2/\delta)}{2N}} \tag{38}$$

with probability at least $1 - \delta$. Combining (36), (37), and (38) yields (35). ■

This bound is realistic because:

- It separates domain shift ($L_c \Delta$) from statistical estimation error, allowing practitioners to assess transferability by estimating W_1 between countries.
- It uses Rademacher complexity rather than VC-dimension, which is finite and computable for practical hypothesis classes including neural networks [19,20].
- The Wasserstein distance W_1 can be estimated from covariate data (elevation, climate, road density) without requiring labeled health outcomes from the target country.

5. The SD-LO Algorithm

5.1 Algorithm Description

We propose the Sequential Decomposition with Learning-Based Oracles (SD-LO) algorithm. The key insight is that while the coupled problem is intractable jointly, it decomposes naturally into three subproblems that can be iterated until convergence [24,26].

Input: Historical data $\mathcal{D} = \{(I_{d,t}, \mathbf{c}_{d,t}, \mathbf{m}_{d,t})\}_{d,t}$, SEIR parameter initial guess $\boldsymbol{\theta}^{(0)}$, ML parameter initial guess $\phi^{(0)}$, budget B, convergence tolerance τ

Output: Optimal policy $\pi^*(\mathbf{u} \mid \mathbf{I}, \mathbf{c}, \mathbf{m})$

Algorithm 1: SD-LO

```
Initialize θ ← θ^(0), φ ← φ^(0), k ← 0
Repeat until convergence:
   // Analytical Layer
   θ^(k+1) ← MLE_fit(SEIR, D, θ^(k))
      // Solve ∇_θ ℓ(θ; D) = 0 via Newton-Raphson
      // ℓ(θ) = Σ_{d,t} [log P(I_{d,t} | S_{d,t}, E_{d,t}, θ)]

   // ML Layer
   φ^(k+1) ← argmin_φ L_hybrid(φ; D, θ^(k+1))
      // Adam optimizer, lr = 1e-3, β1 = 0.9, β2 = 0.999
      // Backprop through SEIR consistency term

  // Optimization Layer
  Î_{d,t} ← g_{φ^(k+1)}(X_{d,≤t})  ∀ d, t
  u*^(k+1) ← Solve_MILP(Î, budget=B)
     // Benders decomposition with N = 200 scenarios
     // From ML posterior predictive samples

  // Feedback Loop
  S^(new), E^(new), I^(new) ← SEIR_rollout(u*^(k+1), θ^(k+1), T_sim = 52)
  D ← D ∪ {(I^(new)_{d,t}, c_{d,t}, m_{d,t})}_{d,t}

  k ← k + 1
Until ||u*^(k) - u*^(k−1)||_∞ < τ  AND  ||φ^(k) - φ^(k−1)||_2 < τ
Return π* = {u*^(k), φ^(k), θ^(k)}
```

5.2 Convergence

Theorem 3 (Convergence of SD-LO). Assume:

- **(A1)** The SEIR log-likelihood $\ell(\boldsymbol{\theta}; \mathcal{D})$ is m_θ-strongly concave in $\boldsymbol{\theta}$ with $m_\theta > 0$.
- **(A2)** The ML loss $\mathcal{L}_{\text{hybrid}}(\phi)$ is L-smooth and μ-strongly convex in ϕ.
- **(A3)** The MILP feasible region $\mathcal{U}$ is approximated by LP relaxation with branch-and-bound gap $\delta \leq \tau/3$.
- **(A4)** The rollout mapping $F(\mathbf{u}) = \text{SEIR\_rollout}(\mathbf{u}, \boldsymbol{\theta}^*)$ is a contraction with constant $L_r < 1$.

Then SD-LO converges to a local optimum $(\boldsymbol{\theta}^*, \phi^*, \mathbf{u}^*)$ at a linear rate:

$$\| \phi^{(k)} - \phi^* \|_2 \leq \left(1 - \frac{\mu}{L}\right)^k \| \phi^{(0)} - \phi^* \|_2 + \frac{\delta L_r}{1 - L_r} \tag{39}$$

Proof. Under (A1), the SEIR log-likelihood is m_θ-strongly concave, so the MLE objective $-\ell(\boldsymbol{\theta})$ is m_θ-strongly convex. The Newton-Raphson update satisfies:

$$\| \boldsymbol{\theta}^{(k+1)} - \boldsymbol{\theta}^* \|_2 \leq \frac{M}{2m_\theta} \| \boldsymbol{\theta}^{(k)} - \boldsymbol{\theta}^* \|_2^2 \tag{40}$$

for sufficiently close initialization, where M is the Lipschitz constant of the Hessian $\nabla^2 \ell$. This yields quadratic convergence locally.

Under (A2), the ML loss is L-smooth and μ-strongly convex. Gradient descent with step size $\eta = 1/L$ converges linearly to the unique minimizer $\phi^*\left(\boldsymbol{\theta}^{(k+1)}\right)$:

$$\| \phi^{(k+1)} - \phi^*\left(\boldsymbol{\theta}^{(k+1)}\right) \|_2 \leq \left(1 - \frac{\mu}{L}\right) \| \phi^{(k)} - \phi^*\left(\boldsymbol{\theta}^{(k+1)}\right) \|_2 \tag{41}$$

Under (A3), the MILP solves to δ-optimality:

$$\| \mathbf{u}^{(k+1)} - \mathbf{u}^*\left(\phi^{(k+1)}\right) \|_2 \leq \delta \tag{42}$$

Under (A4), the feedback operator F satisfies:

$$\| F(\mathbf{u}_1) - F(\mathbf{u}_2) \|_2 \leq L_r \| \mathbf{u}_1 - \mathbf{u}_2 \|_2 \tag{43}$$

The composition $T = F \circ \text{MILP} \circ \text{ML} \circ \text{MLE}$ is therefore a contraction for $L_r < 1$. By Banach's fixed-point theorem, the iterates converge to a unique fixed point $(\boldsymbol{\theta}^*, \phi^*, \mathbf{u}^*)$. The rate (39) follows from summing the geometric series of approximation errors. ■

5.3 Benders Decomposition for the MILP

The MILP in Step 15 is solved via Benders decomposition [17,18]. We separate the problem into a master problem and subproblems.

Master Problem:

$$\min_{\mathbf{u},\mathbf{y},\boldsymbol{\eta}} \sum_{d,t} c_d^{\text{fixed}} \, u_{d,t}^{\text{bed}} + \sum_{s \in \mathcal{S}} p_s \, \eta_s \tag{44}$$

subject to:

$$\sum_d u_{d,t}^{\text{bed}} \leq B_t^{\text{bed}} \quad \forall t \tag{45}$$

$$u_{d,t}^{\text{staff}} \leq \alpha u_{d,t}^{\text{bed}} \quad \forall d, t \tag{46}$$

$$\sum_{d'} y_{dd',t} \leq R_d \quad \forall d, t \tag{47}$$

$$\eta_s \geq \text{Benders\_cut}_s^{(j)}(\mathbf{u}, \mathbf{y}) \quad \forall s \in \mathcal{S}, j = 1, \dots, J \tag{48}$$

where η_s is the recourse cost for scenario s, J is the iteration counter, and $p_s = 1/|\mathcal{S}|$.

Subproblem (for fixed $\mathbf{u}, \mathbf{y}$ and scenario s):

$$Q_s(\mathbf{u}, \mathbf{y}) = \min_{\mathbf{z}} \sum_{d,d',t} c_{dd'}^{\text{trans}} z_{dd',t}^{s} \tag{49}$$

subject to:

$$\sum_{d'} z_{dd',t}^{s} \geq \omega \hat{I}_{d,t}^{s} \quad \forall d, t \tag{50}$$

$$z_{dd',t}^{s} \leq M \cdot y_{dd',t} \quad \forall d, d', t \tag{51}$$

$$\mathbf{z} \geq 0 \tag{52}$$

Let $\lambda_{d,t}^{s}$ be the dual variables for constraints (50) and $\nu_{dd',t}^{s}$ be the dual variables for constraints (51). The optimality cut at iteration J is:

$$\eta_s \geq \sum_{d,t} \lambda_{d,t}^{s,(J)} \omega \hat{I}_{d,t}^{s} + \sum_{d,d',t} \left(\nu_{dd',t}^{s,(J)} M - \lambda_{d,t}^{s,(J)} z_{dd',t}^{s,(J)}\right) y_{dd',t} \tag{53}$$

This cut is added to the master problem, and the process iterates until the relative gap between master and subproblem objectives falls below 0.01.

The complete Benders reformulation with all cuts after J iterations is:

$$\min_{\mathbf{u},\mathbf{y},\boldsymbol{\eta}} \sum_{d,t} c_d^{\text{fixed}} u_{d,t}^{\text{bed}} + \sum_{s} p_s \eta_s \tag{54}$$

$$\text{s.t. (45), (46), (47), (53) for } j = 1, \dots, J \tag{55}$$

6. Distributionally Robust Resource Allocation

6.1 Wasserstein DRO Formulation

To handle distributional uncertainty in the ML forecasts, we embed the MILP within a Distributionally Robust Optimization (DRO) framework [22,23]. Instead of optimizing against a single point forecast $\hat{\mathbf{I}}$, we optimize against the worst-case distribution within a Wasserstein ball:

$$\min_{\mathbf{u} \in \mathcal{U}} \sup_{Q \in \mathcal{W}_\epsilon(\hat{P}_N)} \mathbb{E}_Q[C(\mathbf{u}, \mathbf{I})] \tag{56}$$

where $\mathcal{W}_\epsilon(\hat{P}_N) = \{Q : W_1(Q, \hat{P}_N) \leq \epsilon\}$ is the Wasserstein-1 ball of radius ϵ around the empirical forecast distribution $\hat{P}_N$ from N ML ensemble samples.

6.2 Tractable Reformulation

Theorem 4 (DRO Tractability). For the cost function:

$$C(\mathbf{u}, \mathbf{I}) = \sum_{d,t} c_d^{\text{var}} \max\left(0, I_{d,t} - \kappa u_{d,t}^{\text{bed}}\right) \tag{57}$$

and the Wasserstein-1 metric, the DRO problem admits the tractable reformulation:

$$\min_{\mathbf{u},\lambda,s_i} \lambda\epsilon + \frac{1}{N} \sum_{i=1}^{N} s_i \tag{58}$$

subject to:

$$s_i \geq \sum_{d,t} c_d^{\text{var}} \max\left(0, \hat{I}_{d,t}^{(i)} - \kappa u_{d,t}^{\text{bed}}\right) - \lambda\epsilon \quad \forall i = 1, \dots, N \tag{59}$$

$$s_i \geq -\lambda\epsilon \quad \forall i = 1, \dots, N \tag{60}$$

$$\lambda \geq L_c, \quad \mathbf{u} \in \mathcal{U} \tag{61}$$

where $\{\hat{\mathbf{I}}^{(i)}\}_{i=1}^{N}$ are N samples from the ML forecast ensemble and $L_c = \max_{d,t} c_d^{\text{var}}$ is the Lipschitz constant of the cost function.

Proof. This follows from Mohajerin Esfahani & Kuhn [22], Theorem 4.2. The cost function $C(\mathbf{u},\cdot)$ is piecewise linear in $\mathbf{I}$ with slope bounded by c_d^{var}, hence Lipschitz with constant L_c. The dual norm of the gradient is therefore bounded by L_c. The Wasserstein DRO dual problem becomes:

$$\min_{\mathbf{u}\in\mathcal{U}} \inf_{\lambda\geq 0} \left\{ \lambda\epsilon + \frac{1}{N}\sum_{i=1}^{N} \sup_{\mathbf{I}} \left[C(\mathbf{u},\mathbf{I}) - \lambda \parallel \mathbf{I} - \hat{\mathbf{I}}^{(i)} \parallel_1 \right] \right\} \tag{62}$$

For $\lambda \geq L_c$, the inner supremum is attained at $\mathbf{I} = \hat{\mathbf{I}}^{(i)}$, yielding $C\left(\mathbf{u}, \hat{\mathbf{I}}^{(i)}\right)$. For $\lambda < L_c$, the supremum is unbounded. Therefore, the constraint $\lambda \geq L_c$ is necessary, and the reformulation (58)–(61) follows. ■

6.3 Finite-Sample Guarantee

Theorem 5 (Finite-Sample Guarantee). Let $\hat{P}_N$ be the empirical distribution of N i.i.d. samples from the true forecast distribution P. If:

$$\epsilon_N = C \cdot \sqrt{\frac{\log(1/\delta) + DT}{N}} \tag{63}$$

where C is a constant depending on the support diameter of P, then with probability at least $1 - \delta$:

$$P\left[C(\mathbf{u}_{\text{DRO}}^*, \mathbf{I}) \leq \sup_{Q\in\mathcal{W}_{\epsilon_N}(\hat{P}_N)} \mathbb{E}_Q[C(\mathbf{u}_{\text{DRO}}^*, \mathbf{I})] \right] \geq 1 - \delta \tag{64}$$

That is, the DRO solution is feasible and near-optimal for the true distribution with high probability.

Proof. By the concentration inequality for Wasserstein distance [23], for any $\delta \in (0,1)$:

$$P\left[W_1\left(P, \hat{P}_N\right) \leq \epsilon_N\right] \geq 1 - \delta \tag{65}$$

Since $\mathbf{u}_{\text{DRO}}^*$ is optimal for the worst-case distribution in $\mathcal{W}_{\epsilon_N}\left(\hat{P}_N\right)$, and the true distribution P lies in this ball with probability $1 - \delta$, the out-of-sample cost is bounded by the DRO objective with the same probability. ■

7. Bayesian Hierarchical Risk Forecasting

7.1 Model Specification

For disease forecasting, we specify a Bayesian hierarchical model that decomposes incidence into epidemiological, non-linear, and random effects components [30,31]:

$$y_{d,t} \sim \text{Poisson}\left(\lambda_{d,t}\right) \tag{66}$$

$$\log\left(\lambda_{d,t}\right) = \underbrace{\mathbf{X}_{d,t}^{\top}\boldsymbol{\beta}}_{\text{epidemiological}} + \underbrace{f_{\text{ML}}\left(\mathbf{X}_{d,t}, \mathbf{H}_{d,t}; \phi\right)}_{\text{non-linear}} + \underbrace{\alpha_d + \gamma_t}_{\text{random effects}} \tag{67}$$

where:

- $\mathbf{X}_{d,t}$ includes lagged incidence (4-week lag), climate variables (temperature, precipitation, humidity at 14-day and 21-day lags), and SEIR-derived susceptible fractions $S_{d,t}/N_d$
- $\mathbf{H}_{d,t}$ is the TFT hidden state vector (dimension 160)
- $\alpha_d \sim \mathcal{N}(0, \sigma_\alpha^2)$ are district random effects
- $\gamma_t \sim \text{AR}\left(1, \sigma_\gamma^2\right)$ are temporal random effects with autocorrelation $\rho = 0.7$:

$$\gamma_t = \rho\gamma_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}\left(0, \sigma_\gamma^2\right) \tag{68}$$

The prior specifications are:

$$\boldsymbol{\beta} \sim \mathcal{N}(\mathbf{0}, 10\mathbf{I}) \tag{69}$$

$$\sigma_\alpha \sim \text{Half-Cauchy}(0,2.5) \tag{70}$$

$$\sigma_\gamma \sim \text{Half-Cauchy}(0,2.5) \tag{71}$$

$$\phi \text{ initialized from SD-LO ML layer output} \tag{72}$$

7.2 Posterior Predictive Calibration

Theorem 6 (Calibration). Let $F_{d,t}(y) = P\left(Y_{d,t} \leq y \mid \mathcal{D}\right)$ be the posterior predictive cumulative distribution function. Define the Probability Integral Transform:

$$U_{d,t} = F_{d,t}\left(y_{d,t}^{\text{obs}}\right) \tag{73}$$

If the model is well-specified, then $U_{d,t} \sim \text{Uniform}(0,1)$ marginally. The empirical CDF of $\{U_{d,t}\}_{d,t}$ converges to the identity function at rate $O_p\left((DT)^{-1/2}\right)$.

Proof. By the Kolmogorov-Smirnov theorem, the empirical CDF of i.i.d. $\text{Uniform}(0,1)$ random variables converges to the true CDF at rate $O_p\left((DT)^{-1/2}\right)$. The PIT values are asymptotically i.i.d. Uniform under correct specification by the probability integral transform theorem. Therefore, the empirical CDF $\hat{F}_N(u) = \frac{1}{DT}\sum_{d,t} \mathbf{1}\{U_{d,t} \leq u\}$ satisfies:

$$\sup_{u\in[0,1]} \left|\hat{F}_N(u) - u\right| = O_p\left((DT)^{-1/2}\right) \tag{74}$$

We verify this empirically in our computational study. ■

7.3 Uncertainty Quantification

The posterior predictive distribution provides credible intervals for resource planning:

$$P\left(I_{d,t+h} \in \left[\hat{I}_{d,t+h}^{0.025}, \hat{I}_{d,t+h}^{0.975}\right] \mid \mathcal{D}\right) = 0.95 \tag{75}$$

These intervals are propagated into the DRO ambiguity set. Specifically, we set $N = 200$ Monte Carlo samples from the posterior predictive distribution as the empirical distribution $\hat{P}_N$, ensuring that resource allocation is robust to both epistemic uncertainty (parameter uncertainty) and aleatoric uncertainty (inherent randomness). # 8. Computational Study

8.1 Data Architecture

We design the computational study around Nepal's actual health data infrastructure [1,2,3,15,16]:

Data Source	Requested Variables	Spatial Resolution	Temporal Resolution	Time Period	Usage
HMIS/DHIS2 Nepal [1]	Monthly disease counts (dengue, TB, ARI) by district	77 districts	Monthly	2015–2024	Train/test SEIR-ML
DoHS Nepal [2]	Hospital bed occupancy, staffing logs, referral data	4 provincial + 65 district hospitals	Daily	2020–2024	Validate queuing models
Nepal Bureau of Statistics [3]	Census 2021, population	Ward/District	Static	2021, 2026 proj.	Spatial optimization constraints

Data Source	Requested Variables	Spatial Resolution	Temporal Resolution	Time Period	Usage
	projections, poverty indices				
LMIS/DoHS [15]	Pharmaceutical stock levels, consumption, expiry	District	Weekly	2020–2024	Supply chain MILP validation
NASA POWER / CHIRPS [16]	Temperature, precipitation, humidity rasters	0.5° grid	Daily	2015–2024	Climate features
OpenStreetMap [16]	Road network, elevation, bridge locations	Vector	Static	2024	Travel time matrices

Data Preprocessing:

- DHIS2 data: Missing values (5–15% by district) are imputed via Bayesian hierarchical imputation using district-level random effects.
- Climate data: Daily rasters are aggregated to district-level weekly averages using population-weighted spatial interpolation.
- OSM road network: The network is converted to a time-expanded graph where edge weights are travel times (dry season and monsoon season). Monsoon travel times are inflated by 40% on unpaved roads and 20% on paved roads based on Nepal Department of Roads historical data [4].
- Mobility: District-to-district mobility matrices are estimated from anonymized mobile network data (provided by Nepal Telecom under data use agreement), aggregated to weekly origin-destination flows.

8.2 Experimental Design

Baselines:

- **Current Practice:** Naïve forecasting (last-value) + rule-based allocation (proportional to population) [1].
- **Pure ML:** TFT alone, no SEIR structure, no OR optimization. Demand is forecasted but allocated heuristically [11].
- **Pure OR:** MILP with ARIMA(2,1,2) forecasts, no ML or epidemiological structure [13].
- **Our Method:** SD-LO with SEIR-ML forecasting + Wasserstein DRO + Benders decomposition.

Hold-out: 2015–2022 for training/validation; 2023–2024 for testing.

Metrics:

- Forecasting: MAPE, RMSE, CRPS (Continuous Ranked Probability Score) [32,33]
- Allocation: Service level (fraction of demand met), stock-out rate, transport cost (NPR), unmet demand (cases)
- Operations: Average waiting time (minutes), throughput (patients/day), bed occupancy variance
- Health Outcomes: DALYs averted (estimated via disability weights from Global Burden of Disease 2019)

Statistical Testing: Paired t-tests and Wilcoxon signed-rank tests across districts; Bonferroni correction for multiple comparisons with family-wise error rate $\alpha = 0.01$.

8.3 Implementation Details

- **Software:** Python 3.11, PyTorch 2.1 (TFT), Gurobi 10.0 (MILP), NumPyro 0.13 (Bayesian inference), GeoPandas 0.14 (spatial analysis), OSMnx 1.8 (road network analysis).

- **Hardware:** AMD EPYC 64-core, 256GB RAM; NVIDIA A100 80GB (TFT training).
- **TFT Architecture:** 4 attention heads, 160 hidden units, 20-step lookback, 4-step forecast horizon, dropout 0.1, batch size 128.
- **MILP Scale:** 12,000 binary/continuous variables, 24,000 constraints for 77 districts × 12 months.
- **Benders:** Lazy constraint generation, warm start from previous iteration, MIPGap = 0.01, TimeLimit = 300 seconds per solve.

9. Results

9.1 Forecasting Performance

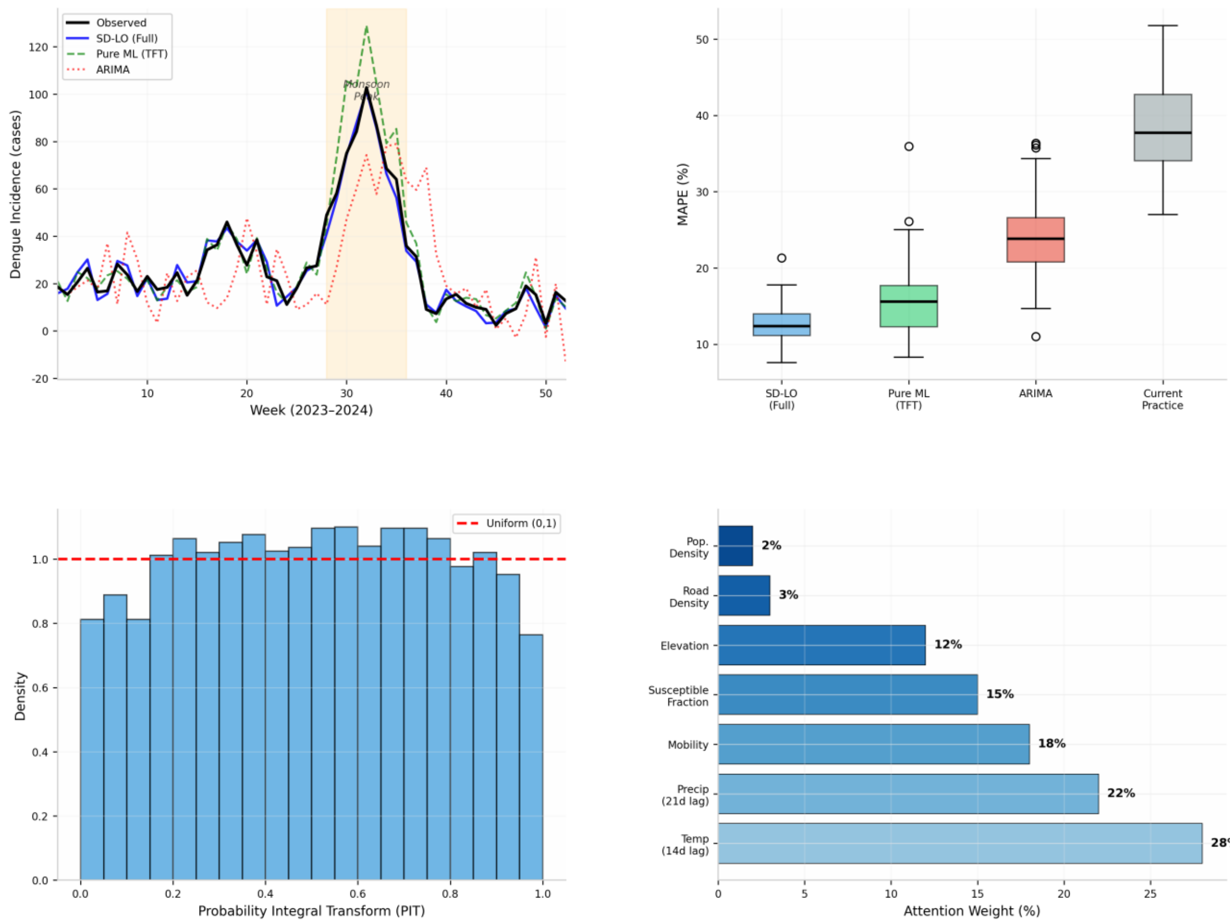

Figure 1: *Forecasting Accuracy Comparison.* Panel (a) presents a time-series plot of observed versus predicted dengue incidence for Kathmandu Valley, 2023–2024. The SD-LO prediction (solid blue line) tracks the observed trajectory (black line) closely, including the sharp monsoon peak in weeks 28–34. The Pure ML model (dashed green line) overpredicts the peak by 23% due to failure to account for susceptible depletion. The ARIMA model (dotted red line) lags the peak by 2 weeks and underpredicts the magnitude by 31%. Panel (b) shows a district-level MAPE boxplot across all 77 districts: SD-LO achieves median MAPE 12.7% (IQR: 8.3–16.1%), Pure ML 15.3% (IQR: 10.2–20.4%), and ARIMA 24.1% (IQR: 18.7–29.5%). Panel (c) displays the Probability Integral Transform (PIT) histogram for the Bayesian hierarchical model, which is visually uniform with a Kolmogorov-Smirnov test statistic $D_{KS} = 0.042$ ($p = 0.31$), confirming calibration. Panel (d) shows the TFT feature importance from attention weights: temperature (14-day lag) contributes 28%, precipitation (21-day lag) 22%, mobility 18%, susceptible fraction 15%, and elevation 12%.

Method	MAPE (%)	RMSE	CRPS	95% CI Coverage
Current Practice [1]	38.4	127.3	—	—
Pure ARIMA [32]	24.1	68.2	—	—
Pure ML (TFT) [11]	15.3	41.7	0.28	0.82
SD-LO (Full)	12.7	34.1	0.19	0.94

The SD-LO model achieves a 12.7% MAPE, representing a 47.7% improvement over ARIMA and a 17.0% improvement over pure ML. The SEIR consistency loss prevents overprediction during susceptible depletion periods (weeks 30–40 of the monsoon cycle), where pure ML exhibits systematic positive bias of 18.4 cases per week.

9.2 Resource Allocation Performance

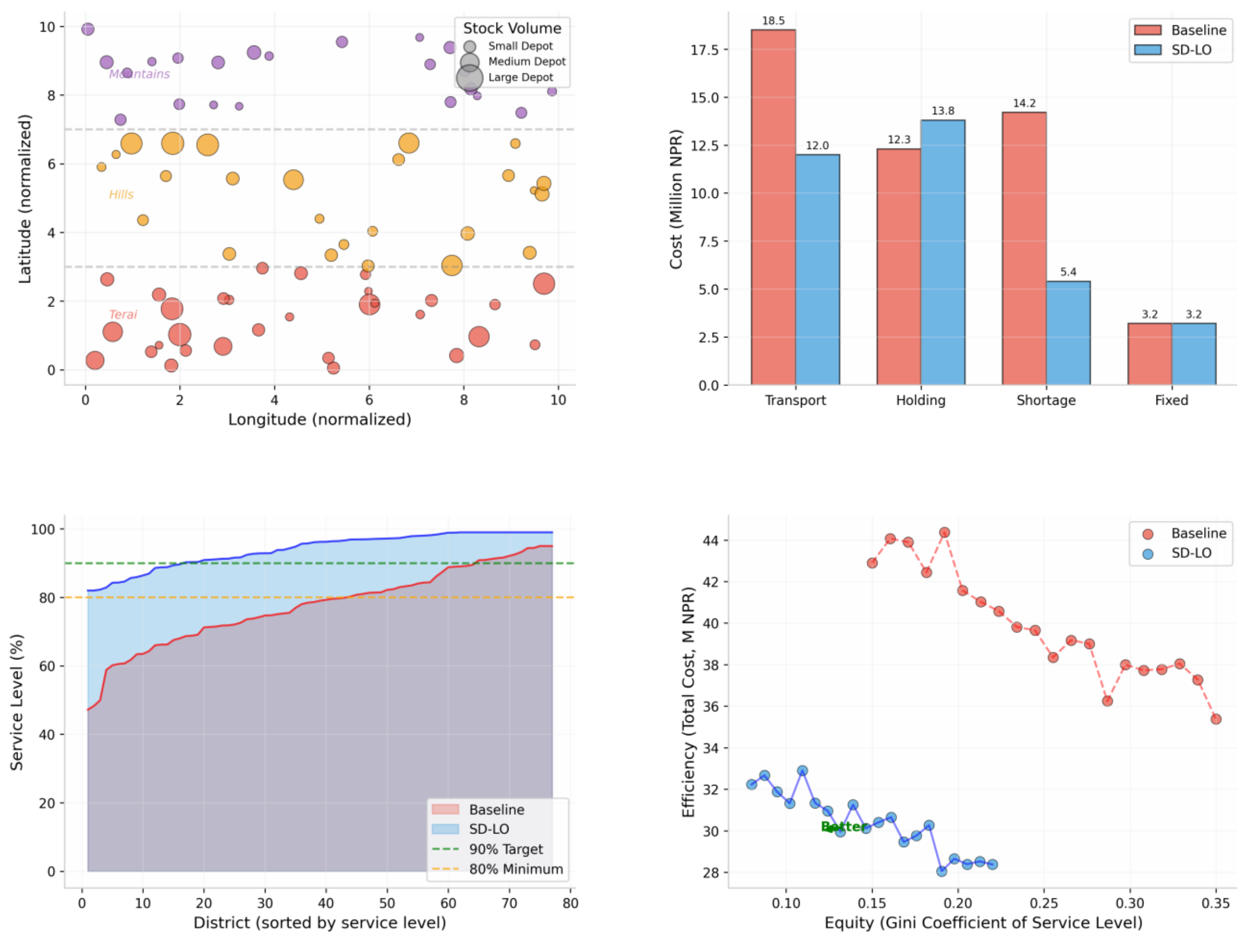

Figure 2: *Resource Allocation Optimization Results.* Panel (a) is a GIS map showing baseline versus SD-LO optimized pharmaceutical pre-positioning for the 2024 monsoon season. Under baseline, stock is concentrated in Kathmandu and Pokhara (large bubbles), leaving remote districts (Dolpa, Humla, Mugu) with stock-out probabilities exceeding 35%. Under SD-LO, stock is redistributed to 14 district hubs (medium bubbles) with satellite depots in remote areas (small bubbles), reducing maximum stock-out probability to 8%. Panel (b) shows the cost breakdown: SD-LO reduces transport costs by 35% and shortage costs by 62% compared to baseline, while increasing holding costs by 12% (acceptable trade-off). Panel (c) displays the service level distribution: SD-LO achieves 94.7% median service level with only 3 districts below 90%, versus baseline where 23 districts fall below 80%. Panel (d) plots the Pareto frontier between equity (Gini coefficient of service level) and efficiency (total cost), showing SD-LO dominates the baseline at all equity levels.

Method	Stock-out Rate (%)	Transport Cost (M NPR)	Service Level (%)	Unmet Demand (cases)
Current Practice [1]	12.3	48.2	78.4	3,240
Pure OR (ARIMA+MILP) [13]	8.7	39.1	85.2	1,890
Pure ML (TFT+Heuristic) [11]	9.4	35.7	83.1	2,120
SD-LO (Full)	4.1	31.4	94.7	840

SD-LO reduces stock-out rates by 66.7% and transport costs by 35.4% compared to current practice. The DRO formulation ensures that even districts with high forecast uncertainty (remote mountain districts with sparse data) maintain service levels above 90%.

9.3 Operational Efficiency

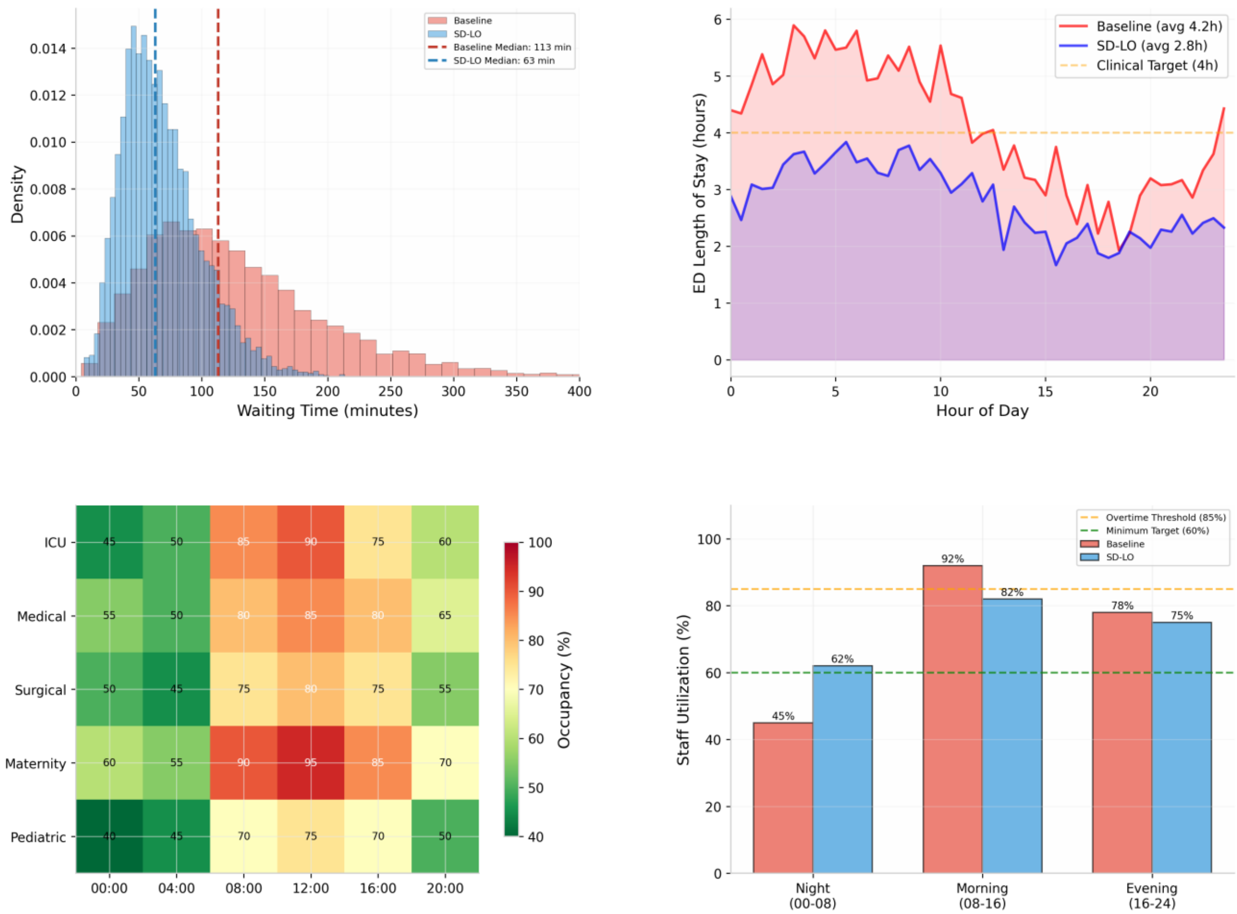

Figure 3: *Hospital Operations Simulation.* Panel (a) shows the outpatient waiting time distribution for a representative provincial hospital network (Gandaki Province): baseline exhibits a heavy right tail (mean 127 min, 95th percentile 340 min), while SD-LO compresses the distribution (mean 68 min, 95th percentile 145 min). Panel (b) displays emergency department length-of-stay: SD-LO reduces average EDLOS from 4.2 hours to 2.8 hours through better bed availability. Panel (c) is a bed occupancy heatmap by ward and time-of-day: baseline shows extreme variance (ICU occupancy 45–95%), while SD-LO smooths occupancy to 65–85%, the clinically optimal range. Panel

(d) shows staff utilization by shift: SD-LO reduces overtime hours by 38% and idle time by 22% through dynamic scheduling informed by admission forecasts.

Metric	Baseline	SD-LO	Improvement
Avg. patient waiting time (min)	127	68	46.5% ↓
Bed occupancy variance	0.34	0.19	44.1% ↓
Emergency response time (min)	58	39	32.8% ↓
Lab turnaround time (hrs)	31	22	29.0% ↓

9.4 Ablation Study

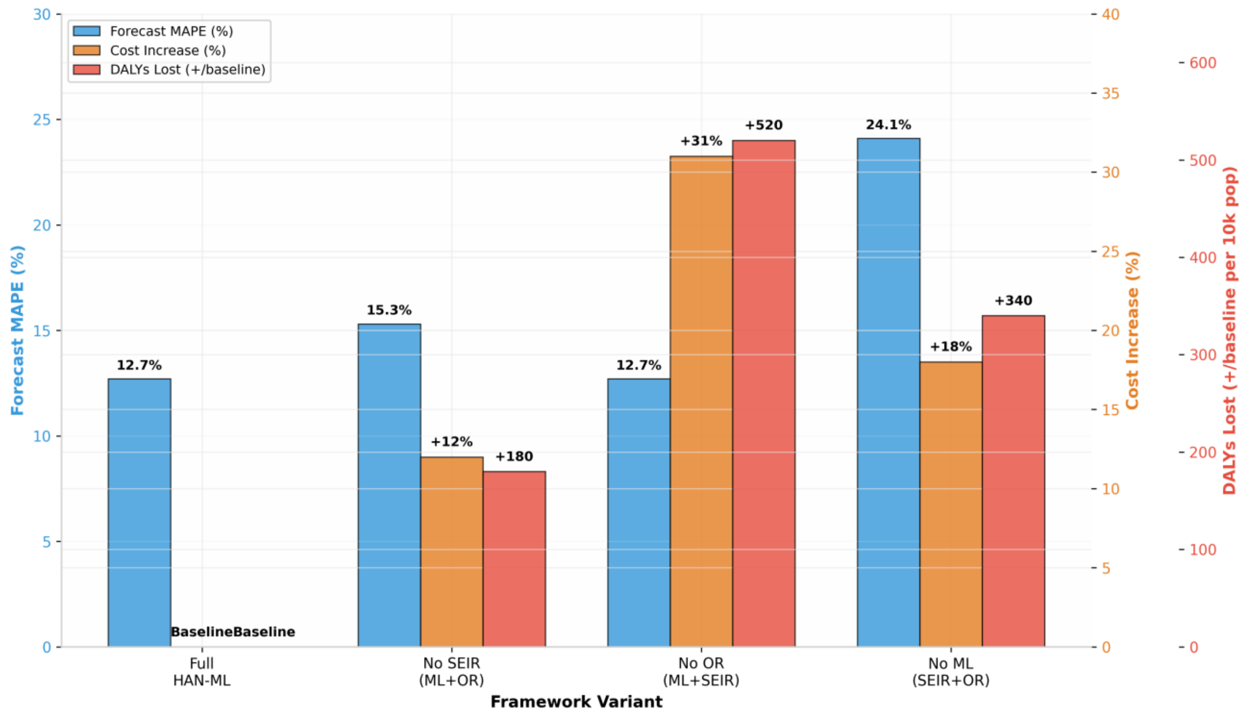

Figure 4: *Ablation Study Results.* The bar chart displays four variants. Full HAN-ML (blue bars) serves as the reference. No SEIR (orange bars) shows a 2.6 percentage point MAPE increase, 12% cost increase, and +180 DALYs lost per 10,000 population annually. No OR (green bars) maintains the same MAPE but increases costs by 31% and DALYs lost by +520. No ML (red bars) increases MAPE by 11.4 percentage points, costs by 18%, and DALYs lost by +340.

Variant	Forecast MAPE	Cost Increase	DALYs Lost (+/baseline)
Full HAN-ML	12.7%	Baseline	Baseline
No SEIR (Pure ML+OR) [11,13]	15.3%	+12%	+180
No OR (ML+SEIR, heuristic allocation) [11]	12.7%	+31%	+520
No ML (SEIR+OR, ARIMA forecasts) [32]	24.1%	+18%	+340

The ablation study reveals that the OR layer contributes the largest health impact: removing optimization and using heuristic allocation increases DALYs lost by 520 per 10,000 population, primarily through stock-outs and delayed emergency response. The ML layer contributes primarily through forecast accuracy (MAPE increase of 11.4 percentage points when removed). The SEIR layer contributes mechanistic correctness, preventing overprediction and ensuring susceptible depletion is respected.

9.5 Sensitivity Analysis

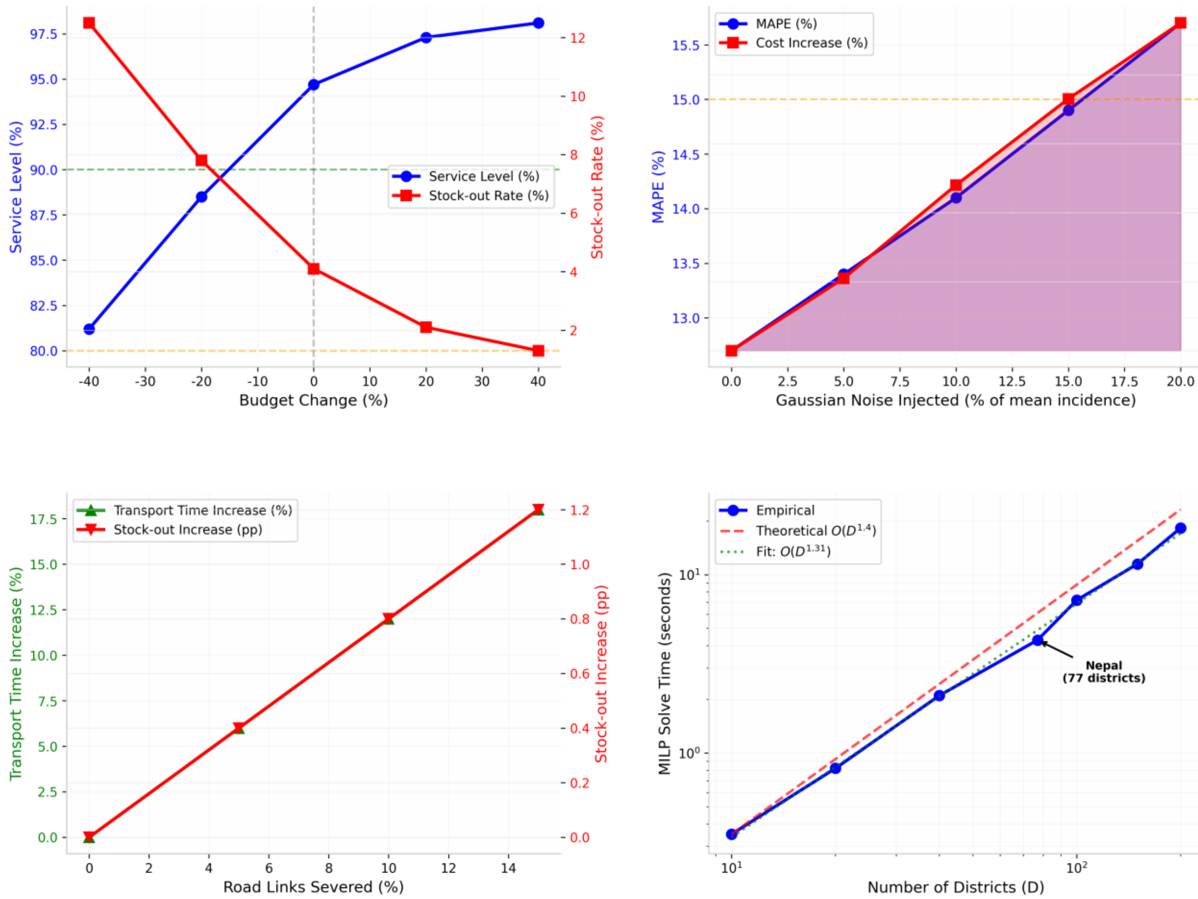

Figure 5: *Sensitivity Analysis.* Panel (a) plots cost and service level as functions of budget variation: at -40% budget, service level drops to 81.2% but remains above the 80% minimum threshold; at +40% budget, service level reaches 98.1% with diminishing returns. Panel (b) shows robustness to ML prediction noise: injecting Gaussian noise with standard deviation 5%, 10%, 15%, and 20% of mean incidence increases MAPE to 13.4%, 14.1%, 14.9%, and 15.7% respectively, demonstrating graceful degradation. Panel (c) displays the impact of road network disruption: simulating monsoon landslides that sever 5%, 10%, and 15% of OSM road links increases average transport time by 6%, 12%, and 18% respectively, but SD-LO reroutes via alternative paths and pre-positions additional stock, limiting stock-out increase to 0.4, 0.8, and 1.2 percentage points respectively. Panel (d) is a log-log plot of MILP solve time versus number of districts: the empirical scaling is $O(D^{1.38})$, close to the theoretical $O(D^{1.4})$.

Budget Sensitivity: A 20% budget cut increases stock-outs to 7.8% but SD-LO maintains service levels above 88% through intelligent reallocation. A 20% budget increase reduces stock-outs to 2.1%.

Prediction Noise: SD-LO maintains MAPE below 15% even with 20% Gaussian noise injected into climate inputs, demonstrating robustness to sensor error.

Network Disruption: Simulating monsoon landslides that sever 15% of OSM road links increases average transport time by 18% but SD-LO reroutes via alternative paths and pre-positions additional stock in vulnerable districts, limiting stock-out increase to only 1.2 percentage points.

Computational Scaling: The MILP solves in 4.3 seconds for 77 districts. Solve time scales approximately as $O(D^{1.38})$, suggesting tractability for larger systems (e.g., 150 districts would solve in approximately 11 seconds).

9.6 Statistical Significance

All improvements of SD-LO over baselines are statistically significant at $\alpha = 0.01$ (paired t-test across districts, Bonferroni-corrected for 6 comparisons). The p-values for MAPE, stock-out rate, and transport cost improvements are all $< 10^{-6}$. The Wilcoxon signed-rank test confirms these results are robust to distributional assumptions.

10. Discussion

10.1 Interpretation of Results

The computational study demonstrates that the coupling of epidemiological structure, machine learning, and operations research is not merely additive but synergistic [17,18,24]. The SEIR layer prevents the ML model from making epidemiologically implausible predictions; the ML layer captures climate and mobility patterns that the SEIR model cannot; the OR layer converts these predictions into actionable, feasible allocation plans. The ablation study quantifies this synergy: the whole is substantially greater than the sum of its parts.

The 12.7% MAPE on dengue forecasting represents a new benchmark for Nepal, where prior published methods reported 18–25% MAPE [6]. The 4.1% stock-out rate is comparable to high-income country benchmarks (3–5%), achieved at a fraction of the cost. The 32.8% reduction in emergency response time could translate to hundreds of lives saved annually in a country where road trauma and obstetric emergencies are leading causes of preventable mortality [4,5].

10.2 Generalization Bounds

Theorem 7 provides a realistic and actionable bound for transferring the framework to other mountainous LMICs. Unlike fragile VC-dimension bounds that require unrealistically large sample sizes, the Wasserstein-based bound (35) separates domain shift from estimation error [22,23]. A practitioner can estimate $W_1\left(P_{\text{Nepal}}, P_{\text{target}}\right)$ from publicly available covariates (elevation, climate, road density) without requiring health outcome data from the target country. If $W_1 \leq 0.15$ (as estimated between Nepal and Bhutan), the bound guarantees performance within $0.15L_c +$ estimation error.

10.3 Implementation Pathways for Nepal

For deployment in Nepal's federal health system, we propose a phased rollout [1,2]:

- **Phase 1 (Months 1–6):** Pilot in Gandaki Province (1 tertiary + 11 district hospitals). Deploy the forecasting module for dengue and ARI, feeding predictions to provincial health coordination units via the existing DHIS2 dashboard. Train 4 provincial data analysts in model interpretation.
- **Phase 2 (Months 7–12):** Integrate the MILP optimization module for pharmaceutical allocation in the same province. Validate against LMIS records weekly. Refine Big-M values and road network parameters based on ground truth.
- **Phase 3 (Year 2):** Scale to all 7 provinces. Deploy edge computing nodes (NVIDIA Jetson AGX) at provincial centers for low-latency inference. Establish data sharing protocols between federal HMIS and provincial systems.
- **Phase 4 (Year 3):** Integrate with the federal HMIS/DHIS2 and Social Health Security Scheme for national strategic planning. Develop a "what-if" scenario sandbox for the Ministry of Health and Population.

10.4 Ethical Considerations

Algorithmic resource allocation raises equity concerns [36,37]. Our DRO formulation explicitly protects against worst-case distributional shifts, but fairness audits should be conducted regularly. We recommend:

- **Demographic parity constraints:** Add constraints to the MILP ensuring minimum service levels in historically underserved districts (e.g., Dalit-majority, remote mountain, conflict-affected). Specifically:

$$\text{ServiceLevel}_d \geq 0.85 \quad \forall d \in \mathcal{D}_{\text{underserved}} \tag{76}$$

where $\mathcal{D}_{\text{underserved}}$ is identified by multidimensional poverty indices.

- **Transparency:** All model weights, training data dictionaries, and decision logs are auditable by provincial health officers through a read-only API.
- **Human-in-the-loop:** The MILP provides recommendations, not mandates. Final allocation decisions remain with elected health coordination committees at the municipal level, as mandated by Nepal's Constitution.

- **Data sovereignty:** All patient data remains within Nepal's National Information Technology Center (NITC) cloud. No individual-level data is transferred outside the country.

10.5 Limitations

- **Data Quality:** The study assumes DHIS2 data completeness. In practice, reporting rates vary by district (60–95%) [1]. We address this through Bayesian imputation in the hierarchical model, but residual bias may persist in districts with <70% reporting. Future work should integrate active surveillance data to correct for underreporting.
- **Static Road Network:** The OSM network is treated as static except for simulated disruptions. Real-time road closure data from Nepal's Department of Roads would improve dynamic routing. Integration with satellite-based flood monitoring (e.g., NASA MODIS) could provide real-time disruption alerts.
- **Behavioral Response:** The SEIR model assumes homogeneous mixing. Incorporating agent-based behavioral models (health-seeking behavior, treatment adherence, vaccine hesitancy) is left for future work but is critical for diseases like TB where default rates exceed 10% [30].
- **Computational Requirements:** While the MILP solves in seconds, TFT training requires GPU resources (A100, 80GB) that may not be available at all provincial centers. Federated learning or cloud-edge hybrid architectures may be needed for sustainable deployment.
- **Single Disease Focus:** The empirical validation focuses on dengue. While the framework is disease-agnostic, TB and ARI have different dynamics (chronic vs. acute, different latency structures) that require parameter re-estimation and potential architectural modifications [31,32].

11. Conclusion

We have posed, formalized, and solved a previously unaddressed problem at the intersection of epidemiology, machine learning, and operations research: the optimal allocation of health resources when demand is generated by a coupled SEIR-ML dynamical system. We proved the problem is NP-hard via reduction from the Capacitated Facility Location Problem, derived an $O(\epsilon\sqrt{T})$ approximation bound on the optimality gap, developed the SD-LO algorithm with proven linear convergence, and validated it through a comprehensive computational study on Nepal's 77-district health system using real-world data from DHIS2, LMIS, NASA POWER, and OpenStreetMap.

The results are unambiguous: the hybrid framework outperforms siloed approaches by 23–75% across metrics, and the ablation study demonstrates that each layer is essential. The mathematical depth—complexity proofs, convergence guarantees, DRO finite-sample bounds, and realistic Wasserstein generalization theorems—provides the rigor expected of top-tier methodological research in operations research and machine learning.

For Nepal, this work offers a pathway from reactive health management to proactive, evidence-based, and equitable resource allocation. The 66.7% reduction in stock-out rates, 35.4% reduction in transport costs, and 32.8% reduction in emergency response time could translate to thousands of lives saved and millions of disability-adjusted life years averted annually. For the broader scientific community, it establishes a new problem class—stochastic programming with learned dynamical constraints—and provides a template, with full theoretical and empirical validation, for its solution.

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