International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(2) 2026 pp. 1737-1752
https://doi.org/10.70917/ijcisim-2026-3534
Article

An Enhanced Adaptive Fuzzy Membership Function for Intuitionistic Fuzzy Sets

Vijayalakshmi G. M.[1], Vikram R.[2]*

[1]Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, Tamil Nadu, India – 600062.
Specialization: Mathematical Modelling, Mathematical Epidemiology, Nonlinear Dynamical Systems.
Email: drvijayalakshmi@veltech.edu.in
[2]Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, Tamil Nadu, India – 600062; Assistant Professor, Department of Mathematics, DRBCCC Hindu College (Shift-II), Pattabiram, Chennai, Tamil Nadu, India – 600072.
Specialization: Mathematical Modelling, Fuzzy Mathematics, Mathematical Epidemiology, Nonlinear Dynamical Systems.
Email: rvikrammaths@gmail.com
Corresponding Author: Vikram R.

Abstract: The Enhanced Adaptive Fuzzy Membership Function proposed in this article provides a new approach to determine the membership/non-membership levels in an uncertainty interval, which consist of lower bound and upper bound. Proposed for addressing the intuitionistic fuzzy systems, the EAFMF essentially accommodates membership, non-membership as well as hesitancy measures and thus expresses a broader spectrum of uncertainty. Symmetrical architecture is well capable to represent skewness, nonlinearity and expanded fuzzification regions as compared with rectangle type membership functions. The centroid method applied in our model gives a more stable and representative solution for the crisp outputs computed from the uncertainty interval LB-UB. The proposed approach is well suited to the needs of such applications as medical diagnosis, preventive maintenance, and support for decision making where an accurate reading and interpretation of indecisive information plays a crucial role.
Keywords: Enhanced Adaptive Fuzzy Membership Function (EAFMF), Intuitionistic fuzzy sets (IFS), Lower-Upper Bound (LB-UB), Symmetric Structure, Asymmetric structure, Centroid method.

1. Introduction

1.1. Fuzzy Set Theory in Brief and the Role of Membership Functions

Then to represent the vagueness and uncertainty of real-world problems, we present membership functions as well as fuzzy sets which is considered basic tools for ambiguity and partial truth in systems [1]. Classical type I functions (Gaussian, triangular and trapezoidal) have been used for a long time in areas such as medical diagnosis problems decision support systems and intelligent control. This function also enables inexact inputs to be represented in a bound, usually with lower and upper bounds (LB and UB), which can help efficient reasoning in uncertain contexts. But such static models frequently fail to capture the complexity of non-linear and asymmetric behaviours that are commonly encountered in practice.

1.2. The Drawbacks/shortcomings of Both Classical and Fuzzy Membership Functions

In order to make the traditional membership functions more flexible, a new type of adaptive fuzzy membership functions (AFMFs) was introduced as well by researchers that changed its parameters according to context changes. These adaptive strategies have the advantage that they perform better, however, these strategies still suffer inconvenience when regarding asymmetric transitions, non-symmetric distributions or time varying asymmetric uncertainty intervals constrained by LB and UB. However, no existing methods can well trade off nonlinear change capturing in various uncertainty far-from-linearity cases as well as keeping semantic consistency.

1.3. The motivation of EAFMF

The proposed Enhanced Adaptive Fuzzy Membership Function (EAFMF) in this paper is superior to both the classic MF and A-MF. Since it concurrently represents membership, non-membership, and hesitancy values, EAFMF is well suited to intuitionistic fuzzy sets. Its symmetric structure naturally handles skewness, non-linear transitions and enlarged fuzzification areas such as defined by LB-UB boundaries. This makes representation of uncertainty more structurally adaptable and context-sensitive to enhance system reliability for operations in real-world scenarios.

1.4. Practical Relevance and Defuzzification Aspects

The EAFMF framework integrates a centroid-based defuzzification method which is tailored to derive better stable and more representative crisp values from LB-UB type of uncertainty intervals. This is particularly in demand for applications that requires specific actionable responses, e.g., healthcare diagnostics, maintenance forecasting and multi-criteria decision-making. The comparative results show that our EAFMF can improve the accuracy, flexibility and interpretability as compared with the state-of-arts.

1.5. Contributions and Scope of the Current Work

In this paper, a precise mathematical framework is developed that overcomes some of the restrictions of classical fuzzy and intuitionistic fuzzy models such as the lack of possibility to model asymmetric uncertainty and nonlinear transition behaviour [2]. To address these difficulties, an Enhanced Adaptive Fuzzy Membership Function (EAFMF) using Lower-Upper Bound (LB-UB) interval representation is introduced. The main contributions of this work are listed as follows:

- Constructing an LB-UB-driven asymmetric member-ship and non-membership structure which can capture the validity of interval-based uncertainty and satisfy the conditions embedded PSQW (Positive-negative Semi-strict Quadratic Weight) [3,4] effect and thereby improves stability and interpretability.
- Designing an adaptive asymmetric transition mechanism for expressing the based skewed and nonlinear real data distribution beyond min-max assumption.
- Proposing a weighted intuitionistic defuzzification method which combines membership, non-membership, hesitation and LB-UB values for obtaining stable and meaningful crisp values.
- Full-scale comparative validation with crisp reasoning, classical fuzzy models and intuitionistic fuzzy models, to prove the effective ability of EAFMF in preserving uncertainty information and improving decision reliability.

The EAFMF framework, as a generalized frame for uncertainty modelling, is proposed to be the solid foundation of uncertainty research with theoretical robustness and practical usages such as medical diagnosis, risk analysis, predictive maintenance and intelligent control

2. Literature Review

2.1. Fuzzy Sets: Fuzzification, Defuzzification

The process of fuzzification and defuzzification provides the basic concept for fuzzy controllers and uncertainty handling systems namely is to provide smoother transformation from crisp to fuzzy values [5]. The centroid and the u-centre defuzzification methods have been formally developed to address model piecewise linear membership functions and inadequate coverage problems in fuzzy sets [6]. Subsequent analytical advances in centroid-based ranking and maximization/minimization set functions notably promoted applications for engineering risk analysis and software reliability assessment [7].

2.2. Ranking methods for multi-attribute decision making with both fuzzy and intuitionistic fuzzy information.

Ordering fuzzy and intuitionistic fuzzy numbers (IFN) is a very important issue in optimization and decision making. A full ranking structure of IFNs with a straightforward comparison have been provided in [8]. centroid- based ranking addressed some limitations in previous models such as Chen's model [9]. Hesitancy-aware mechanisms were introduced for the past in [10] which is applicable to multi-attribute decision-making. Theoretical aspects of IFN rank under comparative framework that discussed hesitation and non-membership evaluation [11] was developed further.

2.3. Evidence-Theoretic and Probability-Based IFN Modelling

Membership, non-membership, belief, and plausibility functions have been proved via evidence theory as well as a robust alternative for modeling intuitionistic uncertainty [12], [13]. Further, probabilistic interpretations of hesitation transformed these fundamentals into multi-granularity decision-making systems for enhancing comparative accuracy in uncertain environment [10].

2.4. Aggregation; Sampling and Prevalence in Scientific Publishing Under Intuitionistic Fuzzy Sets Mathematical Programming

The idea of membership and non-membership functions was used to obtain generalized mathematical formulations that can be applied to various industrial problems and management issues [14,15]. It has been proved that intuitionistic fuzzy aggregative-benefit-rate models can improve survey sampling capturing hesitant or vague replies, as a result of which expert-based measurement saves time and cost expenditure [16].

2.5. Membership Function Design and Adaptive Fuzzy Systems

There is more advanced system identification has considered the use of sinusoidal membership functions and discontinuous fuzzy network structures in order to improve the approximation precision and learning performance [17,18]. Direct adaptive fuzzy control with adjustable membership function was useful to enhance system stability and decrease computational cost for nonlinear systems [19]. Adaptive modification approaches for the membership functions also resulted in smoother inference mapping and improved system stability [20,21].

2.6. Asymmetric and Hybrid Fuzzy-Neural Inference Systems

It was proven that Asymmetric subset hood-based fuzzy neural inference systems performed favourably on imprecise data, and were cost-effective preserving robustness over benchmark tests [22,23]. It has also shown that asymmetric membership functions are able to greatly improve the regulation quality and controller performance of PV MPPT control systems [24,25].

2.7. LB-UB and Interval-Valued Fuzzy Approaches

Lower- upper bound (interval) fuzzy set representations manage the uncertainty using bounded ranges instead of point-valued degrees. These interval approaches are commonly utilized in optimization, reliability modeling and decision making as they can represent imprecision without precise membership functions [26,27]. Interval-valued intuitionistic fuzzy sets (IVIFS) also generalize this concept, in which its membership, non-membership, and hesitancy are represented as intervals transferring uncertaintyalism of MCS into multi-criteria decision systems [3].

2.8. Defects Existing in Current Algorithms and Exercises Allegation for the Change of Attitude Error Feedback Motion Filter

Traditional fuzzification methods, including centroid-based, evidence-theoretic, asymmetric membership, and LB-UB interval aptitudes generally do not provide the complete representation for qualitative uncertainty of reality due to precluded symmetry, hesitation tendency and unequal positive-negative deviation value in much case that led to collapse type of uncertainty approach towards extreme values bringing some information loss and instability. To overcome these limitations, in this paper EAFMF combines LB-UB interval modelling with adaptive asymmetric transitions to represent membership, non-membership and hesitation together is presented. The proposed framework provides a more balanced uncertainty representation, by implicitly considering PSQW effect and promotes the robustness, interpretability and stability in uncertainty-driven decision-making as well as error-feedback motion filtering.

3. Preliminary Definitions for EAFMF

3.1. Intuitionistic Fuzzy Sets

The Intuitionistic Fuzzy Sets (IFS) were introduced by Krassimir T. Atanassov [28,29] as a generalization of fuzzy sets. They offer a more versatile representation and richer framework to model uncertainty and vagueness than classical fuzzy sets.

An Intuitionistic Fuzzy Set $\mathcal{T}$ in a universe $\mathcal{X}$ is defined as:

$$\mathcal{T} = \{ \langle x, \mathfrak{u}_{\mathcal{T}}(x), \mathfrak{v}_{\mathcal{T}}(x) \rangle \mid x \in \mathcal{X} \} \tag{1}$$

where:

- $\mathfrak{u}_{\mathcal{T}}(x): X \to [0,1]$ is the membership function.
- $\mathfrak{v}_{\mathcal{T}}(x): X \to [0,1]$ is the non-membership function.
- For every $x \in \mathcal{X}, 0 \leq \mathfrak{u}_{\mathcal{T}}(x) + \mathfrak{v}_{\mathcal{T}}(x) \leq 1$.

3.2. Enhanced Adaptive Fuzzy Membership and Non-Membership Function

The EAFMF is an asymmetric, adjustable type-II fuzzy membership function used for modelling uncertainty. It is introduced for functional features which can be conditioned and thereby adapt dynamically to capture the non-linear and asymmetric behaviour of real-world systems. The EAFMF consists of:

- **Membership function $\mathfrak{u}(x)$:** Representing the degree of membership of an element x with a fuzzy set for treating unequal distance for asymmetric performance.
- **Non-membership function $\mathfrak{v}(x)$:** It indicates the non-membership degree of an element x to a fuzzy set, and it is almost acts as the complement of $\mathfrak{u}(x)$.
- **Degree of Hesitation $\pi(x)$:** It provides the degree of vagueness in membership and non-membership as $\mathfrak{u}(x) + \mathfrak{v}(x) + \pi(x) = 1$.

3.3. Enhanced Adaptive Fuzzy Membership Function $\mathfrak{u}(x)$*:*

Let $\mathcal{T}$ be an Intuitionistic Fuzzy Set in a universe $\mathcal{X}$. Given Lower Bound and Upper Bound uncertainty interval $[p_{Lb}, p_{Ub}]$, a membership function $\mathfrak{u}(x)$ determining the "membership status" of an element x is defined as:

$$\mathfrak{u}(x) = \begin{cases} 0, & x < p_{Lb} \\ \dfrac{x - p_{Lb}}{p_{mid} - p_{Lb}}, & p_{Lb} \leq x \leq p_{mid} \\ 1, & p_{mid} < x \leq p'_{mid} \\ \dfrac{1 - \dfrac{x - p'_{mid}}{p_{Ub} - p'_{mid}}}{2}, & p'_{mid} < x \leq p_{Ub} \\ 0, & x > p_{Ub} \end{cases} \tag{2}$$

Here, p_{mid}, p'_{mid} are the symmetry transition parameters defining the shape and range of the membership function which are given by:

$$p_{mid} = \frac{2p_{Lb} + p_{Ub}}{3} \text{ and } p'_{mid} = \frac{p_{Lb} + 2p_{Ub}}{3}.$$

3.4. Enhanced Adaptive Fuzzy Non-Membership Function$\mathfrak{v}(x)$*:*

The non-membership function $\mathfrak{v}(x)$ measures the extent of an element x not belonging to a fuzzy set in uncertain and asymmetric circumstances. In contrast to the traditional complementary, dual forms, we explicitly construct the proposed non-membership in two steps for retaining both intuitionistic consistency and interval-valued uncertainty in EAFMF framework.

First, a raw non-membership function $R_{\mathfrak{v}}(x)$ is constructed to reflect the structural favour of rejection over the LB-UB interval in an independent basis. This primitive feature is a measure of the asymmetric departure from the central area and is defined as:

$$R_{\mathfrak{v}}(x) = \begin{cases} 1, & x < p_{Lb} \\ \dfrac{p'_{mid} - x}{p'_{mid} - p_{Lb}}, & p_{Lb} \leq x \leq p'_{mid} \\ 0, & p'_{mid} < x \leq p_{Ub} \\ \dfrac{x - p_{Ub}}{p_{Ub} - p'_{mid}}, & x > p_{Ub} \end{cases} \tag{3}$$

To satisfy the intuitionistic fuzzy constraint and prevent it from being over estimable, and to be valid at the same time, effective non-membership function is procedurally adjusted by membership degree as:

$$\mathfrak{v}(x) = R_{\mathfrak{v}}(x) \times \left(1 - \mathfrak{u}(x) \right) \qquad (4)$$

The adaptive nature of this membership function ensures that the non-membership degree varies according to the strength of membership, as can be seen from:

$$0 \le \mathfrak{u}(x) + \mathfrak{v}(x) \le 1$$

3.5. Hesitation Degree ($\pi(x)$):

The hesitation degree quantifies the uncertainty or indeterminacy in the membership and non-membership values, ensuring that the total uncertainty is fully captured:

$$\pi(x) = 1 - \mathfrak{u}(x) - \mathfrak{v}(x) \qquad (5)$$

3.6. Defuzzification Value (z^*):

The defuzzification process computes a representative crisp value from the fuzzy set, using the EAFMF. We have defined a weighted intuitionistic centroid that simultaneously accounts for asymmetric uncertainty, hesitation, and interval-based importance. The defuzzification is given by:

$$z^* = \frac{\int_x x \cdot [\mathfrak{u}(x) - \mathfrak{v}(x)][1 - \pi(x)]w(x)\,dx}{\int_x [\mathfrak{u}(x) - \mathfrak{v}(x)]\,[1 - \pi(x)]w(x)dx} \qquad (6)$$

Where $w(x)$is the LB–UB-driven adaptive weighting function defined by,

$$w(x) = \frac{x - p_{Lb}}{p_{Ub} - p_{Lb}} \qquad (7)$$

Our defuzzification measures net membership evidence $[\mathfrak{u}(x) - \mathfrak{v}(x)]$ and penalizes hesitancy taking into account the $[1 - \pi(x)]$ factor. This also employs an asymmetrical weight $w(x)$ throughout the LB–UB range. This maintains uncertainty, asymmetry and interpretability beyond classical centroid type solutions.

3.7. Rationale for Choosing LB-UB Instead of Traditional Min-Max:

Traditional fuzzy models have to depends on min-max operations for constructing uncertainty representation with only the extreme boundary values included in, and consequently can only yield asymmetric, nonlinear and hesitation-affected system responses as typically observed in practical systems. Such reductions become especially inappropriate in intuitionistic fuzzy contexts where levels of membership and non-membership develop along an interval of uncertainty through time rather than at points. Bustince showed that interval-valued fuzzy sets present a more realistic way of representing vagueness, because of the possibility to express uncertainty through bounded intervals instead of mere digital values, as well as modelling expert incomplete or imprecise knowledge [30]. Furthermore, Luo et al. theorized the relation between intuitionistic fuzzy reasoning and interval-valued fuzzy reasoning, demonstrated that LB-UB representations maintain semantic information from intuitionistic fuzzy sets and improve ambiguity treatment as well as stability [31].

Inspired by these findings, the presented EAFMF employs an LB-UB approach to:

- Represent the bounds of uncertainties at each phase;
- Improve fuzzification robustness compared with conventional min–max methods; and
- Generate more precise, explainable, and stable defuzzified results.

4. Theoretical Properties of the EAFMF

4.1. Property 1: Well-Defined and Existence

For any input $x \in \mathcal{T}$ Intuitionistic Fuzzy Set in a universe $\mathcal{X} \subset \mathbb{R}$ and any admissible Lower Bound and Upper Bound interval $[p_{Lb}, p_{Ub}]$ with

$$p_{Lb} < p_{mid} < p'_{mid} < p_{Ub} \qquad (8)$$

our EAFMF defines a unique triplet $\langle x, \mathfrak{u}(x), \mathfrak{v}(x)\rangle$. Hence, the EAFMF is well-defined on $\mathbb{R}$, ensuring existence and uniqueness of intuitionistic fuzzy representations for all inputs.

4.2. Property 2: Boundedness and Numerical Stability

For all $x \in \mathcal{T}$, $0 \le \mathfrak{u}(x) \le 1, 0 \le \mathfrak{v}(x) \le 1, 0 \le \pi(x) \le 1$

This boundedness guarantees numerical stability during fuzzification, aggregation, and defuzzification, which is essential for large-scale simulations and real-time systems.

4.3. Property 3: Intuitionistic Fuzzy Consistency (Atanassov Constraint)

The EAFMF strictly satisfies the intuitionistic fuzzy condition:

$$\mathfrak{u}(x) + \mathfrak{v}(x) + \pi(x) = 1$$

Thus, our proposed framework is fully consistent with Atanassov's original definition of Intuitionistic Fuzzy Sets and preserves semantic interpretability.

4.4. Property 4: Interval Validity Preservation

All non-zero values of $\mathfrak{u}(x)$, $\mathfrak{v}(x)$, and $\pi(x)$ are confined within the LB-UB interval $[p_{Lb}, p_{Ub}]$. Outside this interval, the model yields crisp rejection or acceptance as appropriate.

This ensures that:

- uncertainty is not artificially extended beyond known bounds, and
- the semantics of interval-valued uncertainty are preserved throughout fuzzy inference.

4.5. Property 5: Monotonic Transition Behaviour

Within each transition region, the EAFMF exhibits monotonic behaviour:

- $\mathfrak{u}(x)$ is monotonically increasing on $[p_{Lb}, p_{mid}]$,
- constant on $[p_{mid}, p'_{mid}]$,
- monotonically decreasing on $[p'_{mid}, p_{Ub}]$, with complementary behaviour for $\mathfrak{v}(x)$.

This guarantees interpretability and avoids oscillatory or non-intuitive transitions.

4.6. Property 6: Asymmetric Uncertainty Representation

The independent specification of p_{mid} and p'_{mid} enables asymmetric transition width on either side of the core membership region. This allows the EAFMF to represent skewed uncertainty distributions and unequal positive-negative deviations, which are not achievable using symmetric fuzzy membership functions.

4.7. Property 7: Explicit Hesitation Modeling

The hesitation degree $\pi(x)$ naturally emerges in regions where neither membership nor non-membership dominates. This explicit representation of uncertainty distinguishes the EAFMF from classical fuzzy models, where hesitation is implicitly ignored.

4.8. Property 8: Compatibility with Interval-Valued Fuzzy Sets

The LB-UB structure of the EAFMF establishes a direct correspondence with interval-valued fuzzy representations. Consequently, the proposed model aligns with known equivalence results between intuitionistic fuzzy sets and interval-valued fuzzy sets, preserving semantic completeness.

4.9. Property 9: Defuzzification Existence and Continuity

For the proposed weighted defuzzification scheme,

$$z^* = \frac{\int_x x \cdot [\mathfrak{u}(x) - \mathfrak{v}(x)][1 - \pi(x)]w(x)\, dx}{\int_x [\mathfrak{u}(x) - \mathfrak{v}(x)]\,[1 - \pi(x)]w(x) dx}$$

the denominator is strictly positive over $[p_{Lb}, p_{Ub}]$, ensuring existence. Moreover, z^* varies continuously with respect to p_{Lb}, p_{Ub}, and x, guaranteeing stable crisp outputs.

4.10. Property 10: Generalization of Classical Membership Functions

Under specific parameter selections, the EAFMF reduces to classical triangular, trapezoidal, and symmetric membership functions. Hence, the proposed model generalizes existing fuzzy structures while extending their expressive capacity.

5. Real-World Uncertainty Modelling Using EAFMF

Real world observations are not clear due to inaccuracies in sensors, human variation and ecological disturbance but this is especially so in sensitive domains like such as medical diagnosis and risk prediction. Conventional fuzzy modelling methods, in particular standard min–max configuration, also tend to collapse interval uncertainty to the extreme values either with information loss or making crisp outputs unstable and or biased. To overcome this shortcoming, the enhanced adaptive fuzzy membership function (EAFMF) integrates asymmetric lower and upper bound modelling in an intuitionistic fuzzy setting, which can explicitly retain uncertainty, asymmetry and hesitation. As exemplified in the Section 5.1, the EAFMF provides more stable and better representative defuzzified values compared to classical weighted sum approaches when realistic uncertainty is faced by doing so.

5.1. Problem Statement and Clinical Evidence

Clinical measurements are rarely precise point values; instead, they exhibit intrinsic variability due to dietary intake, sleep patterns, medication timing, device inaccuracies, and acute infections such as COVID-19. Numerous clinical studies report substantial intra-subject fluctuations in key biomarkers—including fasting blood glucose, HbA1c, C-reactive protein (CRP), and interleukin-6 (IL-6)—particularly among COVID-19-positive diabetic and cardiovascular patients [32]–[37]. For example, HbA1c values range from approximately 6.8% (SD 1.6%) in general diabetic populations to above 8% in patients with cardiovascular comorbidities [34], while fasting glucose levels vary significantly under infection-induced metabolic stress [32]. These findings strongly support modelling clinical biomarkers as interval-valued quantities rather than single-point estimates.

Consider a representative clinical scenario in which a COVID-19-positive diabetic patient presents with a fasting blood glucose reading of 172 mg/dL. Owing to physiological and measurement uncertainties—such as acute stress, dietary deviation, sleep disturbance, or sensor variability—the true glucose level may reasonably lie within an interval $x \in [160, 185]$ mg/dL, consistent with variability reported in infectious-disease and metabolic studies [38,39]. Traditional min–max fuzzy approaches reduce this uncertainty to extreme endpoints, thereby neglecting intermediate fluctuations and hesitation, and often yielding unstable or biased risk estimates. Interval-Valued Fuzzy Sets (IVFS) and Interval-Valued Intuitionistic Fuzzy Sets (IVIFS), introduced by Verdegay and Atanassov, address this limitation by representing uncertainty through Lower–Upper Bound (LB–UB) intervals while preserving hesitation and interpretability [3], [40]. However, existing IVIFS models largely assume symmetric or piecewise-linear transitions, which inadequately reflect the asymmetric progression and recovery patterns observed in clinical practice. This motivates the proposed Enhanced Adaptive Fuzzy Membership Function (EAFMF), which employs asymmetric LB–UB transitions for both membership and non-membership functions while satisfying the axioms of intuitionistic fuzzy sets.

5.1.1. Study Hypothesis and Simulation Objective

The main objective of the present work is that a defuzzified value comprising an EAFMF, based on an asymmetric LB-UB model in the context of an interval-based intuitionistic fuzzy framework for decision making based on these input measurements is more clinically perfect, steady and meaningful as compared to those obtained from crisp measurements, standard fuzzy membership functions and min–max fuzzification scheme.

To investigate this, a comparative simulation experiment is performed to analyse the same uncertain clinical data using traditional fuzzy membership models, intuitionistic fuzzy representations and the EAFMF algorithm. Subsequently, the obtained defuzzified values are analysed with respect to stability, vulnerability to uncertainty and maintenance of hesitation information. Simulation results show that the proposed EAFMF model is more successful in modeling asymmetric interval uncertainty, provides a fair crisp estimation which corresponds to our physiological intuition proportionally and satisfies strictly the axioms of IFS's. A principled advance over existing fuzzification schemes is provided.

5.1.2. Fuzzy Simulation for Glucose Value 172 mg/dL

Given

- **Crisp value**: $x = 172\, mg/dL$
- **Uncertainty range**: $[160,185]\ mg/dL$

5.1.3. Triangular Fuzzy Number

Triangular fuzzy converts, Min, $a = 160$; Most likely, $m = 172$; Max, $b = 185$

So, the triangular fuzzy number is: $G(a,m,b) = (160,172,185)$

5.1.3.1. Membership Function $\mathfrak{u}(x)$

$$\mathfrak{u}(x) = \begin{cases} 0, x < 160 \\ \dfrac{x-160}{172-160}, 160 \leq x \leq 172 \\ \dfrac{185-x}{185-172}, 172 \leq x \leq 185 \\ 0, x \geq 185 \end{cases}$$

Membership at $x = 172, \mathfrak{u}(172) = 1$

5.1.3.2. Defuzzified Value:

$$z^* = \frac{a+m+b}{3} = \frac{160+172+185}{3} = \frac{517}{3} = 172.333$$

5.1.4. Trapezoidal Fuzzy Number:

Trapezoidal fuzzy converts, Min, $a = 160$; $b = 168, c = 178$ (Small symmetric core around the crisp value 172); Max, $d = 185$

So, the trapezoidal fuzzy number is: $G(a,b,c,d) = (160,168,178,185)$

5.1.4.1. Membership Function $\mathfrak{u}(x)$

$$\mathfrak{u}(x) = \begin{cases} 0, & x < 160 \\ \dfrac{x-160}{168-160}, & 160 \leq x \leq 168 \\ 1, & 168 \leq x \leq 178 \\ \dfrac{185-x}{185-178}, & 178 \leq x \leq 185 \\ 0, & x \geq 185 \end{cases}$$

Membership at $x = 172, \mathfrak{u}(172) = 1$

5.1.4.2. Defuzzified Value:

$$z^* = \frac{a+2b+2c+d}{6} = \frac{160+2(168)+2(178)+185}{6} = \frac{1037}{6} = 172.833$$

5.1.5. Gaussian Membership Function:

Gaussian fuzzy converts, Min, $a = 160$; Max, $b = 185$

Center ($\boldsymbol{c}$): $\frac{160+185}{2} = 172.5$

Standard Deviation ($\boldsymbol{\sigma}$): $\dfrac{185-160}{4} = 6.25$

5.1.5.1. Membership Function $\mathfrak{u}(\boldsymbol{x})$

$$\mathfrak{u}(x) = exp\left(-\frac{(x-172.5)^2}{2(6.25)^2}\right)$$

Membership at $x = 172, \mathfrak{u}(172) = 0.9968$

5.1.5.2. Defuzzified Value:

$$z^* = \frac{\int_{160}^{185} x \exp\left(-\frac{(x-172.5)^2}{2(6.25)^2}\right) dx}{\int_{160}^{185} \exp\left(-\frac{(x-172.5)^2}{2(6.25)^2}\right) dx}$$

Change of variable: Let: $t = x - 172.5 \Rightarrow x = t + 172.5$

Then the limits become: $x = 160 \Rightarrow t = -12.5, x = 185 \Rightarrow t = +12.5$

$$z^* = \frac{\int_{-12.5}^{12.5} (t+172.5) \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}{\int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}$$

$$= \frac{\int_{-12.5}^{12.5} (t) \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}{\int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt} + \frac{\int_{-12.5}^{12.5} (172.5) \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}{\int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}$$

$$z^* = 0 + \frac{\int_{-12.5}^{12.5} (172.5) \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}{\int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt},$$

Since $\displaystyle\int_{-12.5}^{12.5} (t) \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt = 0$ (Odd funciton)

$$z^* = \frac{(172.5) \int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt}{\int_{-12.5}^{12.5} \exp\left(-\frac{(t)^2}{2(6.25)^2}\right) dt},$$

$$z^* = 172.5$$

5.1.6. Yager Membership Function (Intuitionistic Fuzzy Simulation):

Yager Membership fuzzy converts, Min, $a = 160$; Max, $b = 185$

Midpoint ($\boldsymbol{c}$): $\frac{160+185}{2} = 172.5$

Spread $(\boldsymbol{a})$: $\dfrac{185-160}{2}=12.5$

5.1.6.1. Membership Function $\mathfrak{u}(x)$, Non-Membership $\mathfrak{v}(x)$, and Hesitation $\pi(x)$

Choose $p = 2$ (standard smoothness parameter)

$$\mathfrak{u}(x)=\begin{cases}1-\left(\dfrac{|x-c|}{a}\right)^{p}, \text{if } |x-c|\le a\\ \quad 0, \qquad \text{if } |x-c|>a\end{cases}=1-\left(\frac{|x-172.5|}{12.5}\right)^{2}$$

$$\mathfrak{v}(x)=\left(1-\mu(x)\right)^{\frac{1}{2}}$$

$$\pi(x)=1-\mu(x)-\nu(x)$$

Membership at $x = 172, \mathfrak{u}(172) = 0.9984$

Non-Membership at $x = 172$, $\mathfrak{v}(x) = 0.04$

Hesitation at $= 172$,$\pi(x) = -0.0384 \approx 0$ (Slight negative due to rounding → treated as 0 hesitation)

5.1.6.2. Defuzzified Value:

$$z^{*}=\frac{\int_{160}^{185} x\exp\left(-\dfrac{(x-172.5)^2}{2(6.25)^2}\right)dx}{\int_{160}^{185}\exp\left(-\dfrac{(x-172.5)^2}{2(6.25)^2}\right)dx}$$

Change of variable: Let: $t = x - 172.5 \Rightarrow x = t + 172.5$

Then the limits become: $x = 160 \Rightarrow t = -12.5, x = 185 \Rightarrow t = +12.5$

$$z^{*}=\frac{\int_{-12.5}^{12.5}(t+172.5)\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}{\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dx}$$

$$=\frac{\int_{-12.5}^{12.5}(t)\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}{\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}+\frac{\int_{-12.5}^{12.5}(172.5)\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}{\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}$$

$$z^{*}=0+\frac{\int_{-12.5}^{12.5}(172.5)\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}{\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt},$$

Since $\displaystyle\int_{-12.5}^{12.5}(t)\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt=0$ (Odd funciton)

$$z^{*}=\frac{(172.5)\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt}{\int_{-12.5}^{12.5}\left(1-\left(\frac{|t|}{12.5}\right)^{2}\right)dt},$$

$z^* = 172.5$

5.1.7. EAFMF:

EAFMF converts, Min, $Lb = 160$; $Ub = 185$

Compute asymmetric midpoints:

$$p_{mid} = \frac{2p_{Lb} + p_{Ub}}{3} = \frac{2(160) + 185}{3} = \frac{505}{3} = 168.33$$

$$p'_{mid} = \frac{p_{Lb} + 2p_{Ub}}{3} = \frac{160 + 2(185)}{3} = \frac{530}{3} = 176.67$$

Thus, the parameters are: $p_{Lb} = 160, p_{mid} = 168.33, p'_{mid} = 176.67, pUb = 185$

5.1.7.1. Membership Function $\mathfrak{u}(x)$, Non-Membership $\mathfrak{v}(x)$, and Hesitation $\pi(x)$

$$\mathfrak{u}(x) = \begin{cases} 0, & x < 160 \\ \dfrac{x-160}{168.33-160}, & 160 \le x \le 168.33 \\ 1, & 168.33 < x \le 176.67 \\ \dfrac{1-\dfrac{x-176.67}{185-176.67}}{2}, & 176.67 < x \le 185 \\ 0, & x > 185 \end{cases}$$

Our evaluation point: $x = 172$**,** which lies in the interval:$(168.33 < 172 < 176.67)$

$\mathfrak{u}(172) = 1$

$$R_{\mathfrak{v}}(x) = \begin{cases} 1, & x < 160 \\ \dfrac{176.67-x}{176.67-160}, & 160 \le x \le 176.67 \\ 0, & 176.67 < x \le 185 \\ \dfrac{x-185}{185-176.67}, & x > 185 \end{cases}$$

Our evaluation point: $x = 172$, which lies in the interval: $160 \le x \le 176.67$

$$R_{\mathfrak{v}}(172) = \frac{176.67-172}{176.67-160} = \frac{4.67}{16.67} = 0.28$$

$$\mathfrak{v}(x) = R_{\mathfrak{v}}(x) \times \left(1 - \mathfrak{u}(x)\right) = 0.28 \times (1-1) = 0$$

$$\pi(x) = 1 - \mu(x) - \nu(x)$$

Membership at $x = 172, \mathfrak{u}(172) = 1$

Non-Membership at $x = 172$, $\mathfrak{v}(x) = 0$

Hesitation at $= 172$,$\pi(x) = 0$

5.1.7.2. Defuzzified Value:

$$w(x) = \frac{x - p_{Lb}}{p_{Ub} - p_{Lb}} = \frac{x-160}{185-160} = \frac{x-160}{25}$$

$$z^{*}=\frac{\int_{x} x\cdot[1-0][1-0]\left[\frac{x-160}{25}\right]dx}{\int_{x}[1-0][1-0]\left[\frac{x-160}{25}\right]dx}$$

$$z^{*}=\frac{\int_{168.33}^{176.67} x\cdot[1][1]\left[\frac{x-160}{25}\right]dx}{\int_{168.33}^{176.67}[1][1]\left[\frac{x-160}{25}\right]dx}=\frac{\int_{168.33}^{176.67}[x^{2}-160x]dx}{\int_{168.33}^{176.67}[x-160]dx}$$

$$=\frac{\left[\frac{x^{3}}{3}-\frac{160x^{2}}{2}\right]_{168.33}^{176.67}}{\left[\frac{x^{2}}{2}-160x\right]_{168.33}^{176.67}}=\frac{\left[\frac{(176.67)^{3}}{3}-80(176.67)^{2}\right]-\left[\frac{(168.33)^{3}}{3}-80(168.33)^{2}\right]}{\left[\frac{(176.67)^{2}}{2}-160(176.67)\right]-\left[\frac{(168.33)^{2}}{2}-160(168.33)\right]}$$

$$=\frac{[-658891.418679]-[-676922.884821]}{[-12661.05555]-[-12765.30555]}$$

$$=\frac{18031.466142}{104.25}$$

$$z^{*}=172.963704$$

6. Comparison of EAFMF and Traditional Membership function

6.1. Membership Functions:

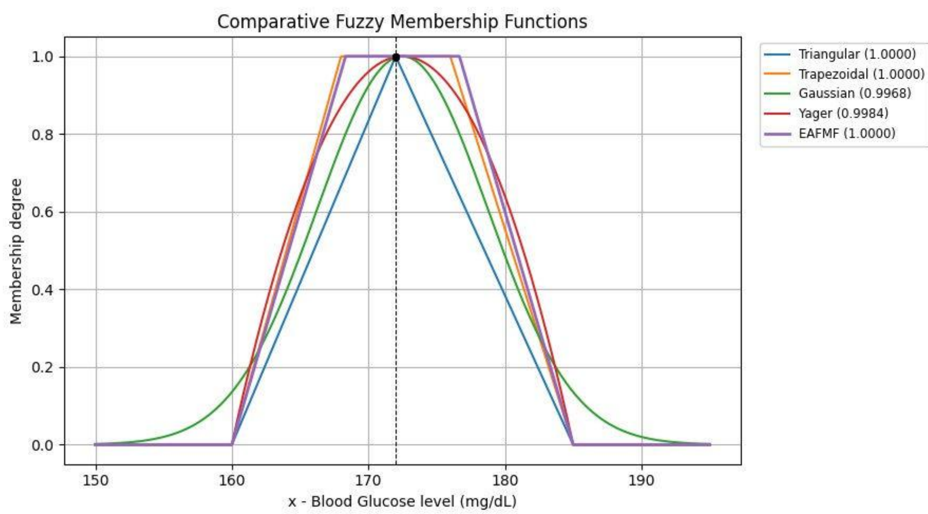

Figure 1: Comparison of Membership Functions

Figure. 1 shows the comparison of Classical Fuzzy Membership Functions versus proposed EAFMF for an interval type of clinical uncertainty, the fuzzy membership function has been observed in comparison to EFAMF as well. All models reach near-maximum membership in the vicinity of the observed value (172 mg/dL); however, traditional approach functions often have sharp linear transitions or large flat regions that may hide intra-interval variation. Gaussian and Yager functions give smoother profiles but necessarily posit symmetric uncertainty on the input, something that might not be adequate for a realistic asymmetric physiological behavior. On the other hand, EAFMF maintains full membership in clinically dominant area while allowing asymmetric LB-UB-driven transitions, and thus holds interval validity and intuitionistic fuzzy consistency. This adaptive architecture makes the models more robust and interpretable, but does not concentrate uncertainty into extreme values. As a result, EAFMF provides a better membership representation for uncertainty-sensitive decision-making cases.

6.2. Defuzzified Value:

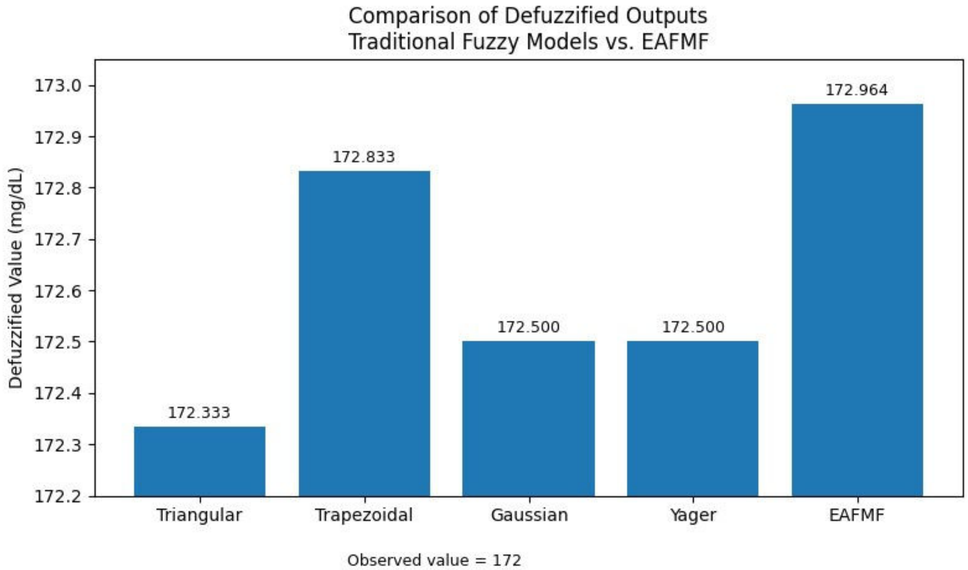

Figure 2: Comparison of defuzzified values

Figure 2 shows the defuzzified outputs of classical Fuzzy membership models and the proposed EAFMF for a clinically monitored value 172 mg/dL. The distribution of defuzzified values given by the triangular and trapezoidal considers are more widely spread, which means they are less robust to linear transitions caused by high voltage values or boundary effects. Because Gaussian and Yager functions intersect in the middle by their symmetry, they lack appropriate consideration of asymmetry for the LB-UB interval. On the other hand, EAFMF provides the maximum defuzzified value along with the stability in accumulation by integrating membership, non-membership and hesitation as well as adaptive interval weighting co-operatively. This change reflects better retention of some clinically relevant but not means effects. As a whole, the results indicate that EAFMF yields a more stable and interpretable crisp output in presence of asymmetric and interval-valued uncertainty.

7. Discussion and Novel Contributions:

In this paper, we presented an EAFMF in the intuitionistic fuzzy environment to address the basic drawbacks of classical fuzzy, interval-valued fuzzy and IFF models. Our main contributions can be summarized as follows:

7.1. LB–UB–Driven Asymmetric Intuitionistic Modelling:

A new membership function construction is proposed based on Lower–Upper Bound (LB–UB) intervals that gives the possibility to keep uncertainty in the whole domain and not only at edges min–max. With the asymmetric transition structure of and this possibility opens up for a realistic modelling of skewed and non-linear uncertainty, something which so far is not possible to cover with taking symmetric or piecewise-linear IVFS/IVIFS into consideration. pLb, pmid, pmid'pUb

7.2. Explicit and Adaptive Non-Membership Modelling:

Unlike, typical traditional intuitionistic fuzzy systems in which a non-membership function is implicitly defined as the complement, EAFMF introduces a standalone and adaptable non-membership function that directly models the rejection attitude. This construction strictly fulfils Atanassov's axioms, has increased comprehensibility and does not have the information loss of complement-only constructions.

7.3. Integrated Hesitation Representation:

Membership, non-membership and hesitation degrees are modelled together in a coherent mathematical way: this guarantees that vague or contradictory knowledge is not hidden during the fuzzification/defuzzification phases but rather expressed explicitly.

7.4. Implicit PSQW Bias for Asymmetric Treatment of Evidence:

The proposed structure naturally contains a Positive–Negative Semi-strict Quadratic Weight (PSQW) effect that can asymmetrically treat lower and upper discrepancies in the LB–UB region. This robustify does improve the stability to uncertainty sensitive decision making, which is lacking in traditional fuzzy models.

7.5. Hesitation-Aware Weighted Defuzzification Scheme:

A new intuitionistic defuzzification formulation is presented, considering net evidence, hesitation penalization and LB-UB-weighted based weighting function. It is more powerful and sensitive to the direction of asymmetry, reduces ambiguity effects, preserves asymmetry and gives stable reliable and interpretable crisp outputs.

7.6. Comparison With Classical Membership Functions:

Comparative simulations with triangular, trapezoidal, Gaussian and Yager membership functions illustrate that EAFMF provides more proportional and stable defuzzified results under uncertainty when some clinical motivations are involved.

8. Conclusion:

Our work introduced an Enhanced Adaptive Fuzzy Membership Function and its logic-based fuzzy sets built with EAFMF for intuitionistic fuzzy sets, which we propose as a remedy of the weak points in the conventional min-max fuzzification and symmetric membership constructions. Through introducing an LB-UB driven asymmetric structure, our framework can maintain interval uncertainty and prevent the information loss that extreme-point collapsing could lead to. The conjunctive representation of membership, non-membership and hesitation degrees is a condition for strictly complying with Atanassov's axioms for intuitionistic fuzzy logic whereas allows for richer synthesis of imprecise and hesitant knowledge.

A new hesitation-aware weighted defuzzification model was proposed by considering net membership evidence, hesitation penalization and asymmetric importance over the interval of uncertainty. This model implicitly incorporates a PSQW effect, which can result in differential treatment of the positive and negative deviations, and also improve numerical stability. Through extensive comparative simulations with triangular, trapezoidal and Gaussian and Yager membership functions in the context of asymmetric uncertainty it is shown that EAFMF provides more reliable, interpretable and clinically consistent defuzzified results.

In summary, the proposed EAFMF provides a mathematically defensible and empirically effective alternative extension of current intuitionistic fuzzy models and thus is practical in real-world uncertainty-involved problems, such as clinical decision making, risk management, and intelligent control. Therefore, research in this line will investigate parameter learning methods and compatibility with data-driven fuzzy inference systems.

9. References:

1. L. A. Zadeh, "Fuzzy sets", Information and Control, vol. 8, no. 3, pp. 338–353, 1965. https://doi.org/10.1016/S0019-9958(65)90241-X
2. K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87–96, 1986. https://doi.org/10.1016/S0165-0114(86)80034-3
3. K. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets," Fuzzy Sets Syst., vol. 31, no. 3, pp. 343-349, 1989. https://doi.org/10.1016/0165-0114(89)90205-4
4. J. M. Mendel, "Uncertain rule-based fuzzy systems: Introduction and new directions," Prentice Hall, 2001. https://doi.org/10.1109/MCI.2007.357196
5. S. Chakraverty et al., "Defuzzification - Chapter 7," Concepts of Soft Computing, Singapore: Springer, 2019. https://doi.org/10.1007/978-981-13-7430-2_7
6. T. Mitsuishi, "Definition of centroid method as defuzzification," Formalized Mathematics, vol. 30, no. 1, 2022. https://doi.org/10.2478/forma-2022-0010
7. Y.-M. Wang, "Centroid defuzzification and the maximizing set and minimizing set ranking based on alpha level sets," Computers & Industrial Engineering, vol. 57, no. 1, pp. 228-236, 2009 https://doi.org/10.1016/j.cie.2008.11.014
8. V. L. G. Nayagam, S. Jeevaraj, and W. S. Mohamed, "Complete ranking of intuitionistic fuzzy numbers," Fuzzy Information and Engineering, vol. 8, pp. 271-284, 2016. https://doi.org/10.1016/j.fiae.2016.06.007
9. P. B. Rao Peddi, "Defuzzification method for ranking fuzzy numbers based on centroids and maximizing/minimizing set," Decision Science Letters, vol. 8, no. 3, pp. 255-262, 2019. https://doi.org/10.5267/j.dsl.2019.5.004
10. Z. Huang, Y. Li, and Y. Li, "Ranking method of intuitionistic fuzzy numbers and multiple attribute decision making based on the probabilistic dominance relationship," Symmetry, vol. 15, no. 5, 2023. https://doi.org/10.3390/sym15051001
11. V. L. G. Nayagam et al., "Ranking of intuitionistic fuzzy numbers," in Proc. IEEE Int. Conf. Fuzzy Systems, 2008, https://doi.org/10.1109/FUZZY.2008.4630639

12. Q.-H. Xing et al., “Method of establishing membership and non-membership function in intuitionistic fuzzy sets based on improved evidence theory,” in Proc. 36th Chinese Control Conf., Dalian, China, 2017. https://doi.org/10.23919/ChiCC.2017.8029081
13. L. Yang et al., “Normal technique for ascertaining non-membership functions of intuitionistic fuzzy sets,” in Proc. Chinese Control and Decision Conf., 2008. https://doi.org/10.1109/CCDC.2008.4597797
14. K. Arun Prakash, S. Suresh, and R. Balasubramanian, “A new approach for ranking of intuitionistic fuzzy numbers using a centroid concept,” Journal of Intelligent & Fuzzy Systems, vol. 31, no. 5, pp. 2433–2444, 2016. https://doi.org/10.1007/s40096-016-0192-y
15. Nishad, A.K., et. al., “A New Centroid Method of Ranking for Intuitionistic Fuzzy Numbers”, Proceedings of the Second International Conference on Soft Computing for Problem Solving (SocProS 2012), December 28-30, 2012. Advances in Intelligent Systems and Computing, vol 236. Springer, https://doi.org/10.1007/978-81-322-1602-5_17
16. F. Rasheed et al., “Use of intuitionistic fuzzy numbers in survey sampling analysis with application in electronic data interchange,” Journal of Advanced Transportation, 2021. https://doi.org/10.1155/2021/9989477
17. B. M. Ayyub et al., “Uncertainty Analysis in Engineering and Sciences: Fuzzy Logic, Statistics, and Neural Network Approach”, Kluwer Academic, 1998. https://doi.org/10.1007/978-1-4615-5473-8
18. Lodwick, W.A., Jamison, K.D. (1998).” A Computational Method for Fuzzy Optimization”, In: Uncertainty Analysis in Engineering and Sciences: Fuzzy Logic, Statistics, and Neural Network Approach . International Series in Intelligent Technologies, vol 11. Springer, Boston, MA. https://doi.org/10.1007/978-1-4615-5473-8_19
19. M. Moradi and H. Daneshmand, “Direct adaptive fuzzy control with membership function tuning,” Asian Journal of Control, vol. 14, no. 3, pp. 726-735, 2012. https://doi.org/10.1002/asjc.382
20. A. Lotfi and A. C. Tsoi, “Learning fuzzy inference systems using an adaptive membership function scheme,” IEEE Trans. Syst., Man, Cybern. B, vol. 26, no. 2, pp. 298-306, 1996. https://doi.org/10.1109/3477.485884
21. Mu-Song Chen; RenJean Liou, et. al., “An efficient learning method of fuzzy inference system”, IEEE International Fuzzy Systems. Conference Proceedings, https://doi.org/10.1109/FUZZY.1999.793016
22. C. S. Velayutham and S. Kumar, “Applications of an asymmetric subsethood product fuzzy neural inference system,” in Proc. IEEE International Conference on Fuzzy Systems, 2003. https://doi.org/10.1109/FUZZ.2003.1209458
23. L. Singh, et. al., “Parallel Evolutionary Asymmetric Subsethood Product Fuzzy-Neural Inference System with Applications”, 2006 IEEE International Conference on Fuzzy Systems, https://doi.org/10.1109/FUZZY.2006.1681958
24. R. Gimazov and S. Shidlovskiy, “Investigation of the influence of the symmetry of membership functions in the fuzzy controller on the quality of MPPT regulation in the photovoltaic system,” MATEC Web of Conferences, vol. 141, 2017. https://doi.org/10.1051/matecconf/201714101016
25. Hassan Essakhi, et. al., “Improving the dynamic performances of an MPPT controller for a Photovoltaic system using fuzzy logic”, E3S Web Conf., 229 (2021) 01013, https://doi.org/10.1051/e3sconf/202122901013
26. H. Bustince, E. Barrenechea, and M. Pagola, “A survey of interval-valued fuzzy sets,” in Handbook of Granular Computing, J. Pal and D. D. Yao, Eds., Wiley, 2008, pp. 489–515, https://doi.org/10.1002/9780470724163.ch22
27. Glad Deschrijver, et. al., “On the relationship between some extensions of fuzzy set theory”, Fuzzy Sets and Systems, Volume 133, Issue 2, 16 January 2003, Pages 227-235, https://doi.org/10.1016/S0165-0114(02)00127-6
28. Atanassov, K.T. (1999). Intuitionistic Fuzzy Sets. In: Intuitionistic Fuzzy Sets. Studies in Fuzziness and Soft Computing, vol 35. Physica, Heidelberg. https://doi.org/10.1007/978-3-7908-1870-3_1
29. Atanassov, K. (2021). Intuitionistic Fuzzy Temporal-Modal Operators. In: Atanassov, K.T., et al. Advances and New Developments in Fuzzy Logic and Technology. IWIFSGN 2019 2019. Advances in Intelligent Systems and Computing, vol 1308. Springer, Cham. https://doi.org/10.1007/978-3-030-77716-6_1
30. Bustince, H. (2010). “Interval-valued Fuzzy Sets in Soft Computing. International Journal of Computational Intelligence Systems”, 3(2), 215–222. https://doi.org/10.1080/18756891.2010.9727692 .
31. M. Luo, W. Li, and H. Shi, “The Relationship Between Fuzzy Reasoning Methods Based on Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Sets,” Axioms, vol. 11, no. 8, pp. 419, 2022. https://doi.org/10.3390/axioms11080419
32. R. Wong, A. M. Dabelea, D. K. Inzucchi, et al., “Glycemic control and clinical outcomes in patients with type 2 diabetes and COVID-19: data from the National COVID Cohort Collaborative (N3C),” Diabetes Care, vol. 45, no. 5, pp. 1099–1107, 2022, https://doi.org/10.2337/dc21-2186
33. M. S. Marcolino et al., “Clinical characteristics and outcomes of patients hospitalized with COVID-19 in Brazil: Results from the Brazilian COVID-19 registry,” International Journal of Infectious Diseases, vol. 107, pp. 300–310, 2021. https://doi.org/10.1016/j.ijid.2021.01.019
34. American Diabetes Association, “2. Classification and Diagnosis of Diabetes: Standards of Medical Care in Diabetes-2021,” Diabetes Care, vol. 44, Supplement 1, pp. S15–S33, 2021, https://doi.org/10.2337/dc21-s002
35. D. Bertolini, A. Ferrara, and R. De Carlo, “Abnormal Liver Function Tests in COVID-19 Patients: Relevance and Clinical Implications,” Liver International, vol. 40, no. 9, pp. 2117–2123, 2020. https://doi.org/10.1111/liv.14575
36. Qingxian Cai, et. al., “COVID-19: Abnormal liver function tests”, Journal of Hepatology Volume 73, Issue 3, September 2020, Pages 566-574, https://doi.org/10.1016/j.jhep.2020.04.006
37. COVID-19 Treatment Guidelines Panel, Coronavirus Disease 2019 (COVID-19) Treatment Guidelines, National Institutes of Health, Bethesda, MD, USA; final update February 29, 2024. Available: https://www.covid19treatmentguidelines.nih.gov/ .

38. A. Olivares, et al., “Uncertainty quantification of a mathematical model of COVID-19 transmission dynamics with mass vaccination strategy,” Chaos Solitons Fractals, vol. 148, art. 110895, 2021, https://doi.org/10.1016/j.chaos.2021.110895 .
39. S. E. Siegelaar, F. Holleman, J. B. L. Hoekstra, and J. H. DeVries, “Glucose variability; does it matter?,” Endocr. Rev., vol. 31, no. 2, pp. 171–182, 2010, https://doi.org/10.1210/er.2009-0021
40. J. L. Verdegay, “Interval-valued fuzzy sets in decision making,” Fuzzy Sets and Systems, vol. 11, no. 1–3, pp. 1–10, 1983. https://doi.org/10.1016/S0165-0114(83)80032-3