International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(9s) 2026 pp. 1421-1433
https://doi.org/10.70917/ijcisim-2026-3561
Article

On Product of Total and Range Fuzzy Edge Labeling of Middle graph of Fuzzy Graphs

Geetha D[1], Santhosh Kumar S[2*], Indumathi P [3]

[1]Sri Ramakrishna Mission Vidyalaya College of Arts and Science,Coimbatore, Tamil Nadu, India.

Email: geethamat2010@gmail.com

ORCID ID: 0009-0009-2886-1316

[2]Department of Mathematics, Sri Ramakrishna Mission Vidyalaya College of Arts and Science, Coimbatore - 641020, India.
Email: fuzzysansrmvcas@gmail.com
ORCID ID: 0000-0003-2276-3706
scopus ID: 19640309200

[3]Karpagam College of Engineering, Coimbatore, India
Corresponding Author Email: fuzzysansrmvcas@gmail.com

Abstract: This paper studies the product of total and range fuzzy edge labeling for path, cycle, star, and ladder graphs. Particular emphasis is given to the construction and analysis of their corresponding fuzzy middle graphs. The study is further extended to fuzzy splitting, shadow, line, total, and subdivision graphs. It is established that all these graphs admit the product of total and range fuzzy edge labeling. The results extend fuzzy graph labeling theory and provide a unified framework for analysis.
Keywords: Product of total and range fuzzy edge labelling , Fuzzy path, Fuzzy cycle, Fuzzy middle graph, Fuzzy splitting graph, Fuzzy shadow graph, Fuzzy line graph, Total fuzzy graph, Fuzzy subdivision graph.

1. Introduction

Managing uncertainty is a crucial aspect of many real-world applications. To ad-dress such uncertainty, Lotfi A. Zadeh proposed the concept of fuzzy sets in 1965, which established a mathematical foundation for representing imprecise and vague in-formation. Since then, fuzzy set theory has been extensively utilized in various fields including control systems, communication networks, optimization, medical diagnosis, and decision-making processes.

Fuzzy graph theory generalizes classical graph theory by incorporating uncertainty through membership values assigned to vertices and edges. In this context,[2] A. Nagoor Gani and his collaborators developed the concepts of labeling and magic labeling in fuzzy graphs, which significantly contributed to the growth of fuzzy graph labeling theory. Nevertheless, conventional fuzzy graphs may not be adequate to model more complex systems, leading to the exploration of several derived fuzzy graph structures. Subsequently,[8, 5] numerous researchers have examined different aspects and ex-tensions of fuzzy graphs, including middle graphs, splitting graphs, shadow graphs, line graphs, total graphs, and subdivision graphs. These derived structures are es-sential for analyzing structural characteristics and labeling behaviors under uncertain environments.

Recently, considerable attention has been given to edge labeling methods in fuzzy graphs. [7] In particular, the concept of product of total and range fuzzy edge labeling has been introduced, where edge membership values are assigned based on vertex memberships while satisfying the condition

$$\delta(uv) < min\{\rho(u), \rho(v)\}$$

ensuring consistency between vertex and edge values.

Motivated by these developments, the present work examines the existence of prod-uct of total and range fuzzy edge labeling for fundamental graph classes such as path graphs, cycle graphs, star graphs, and ladder graphs. Special emphasis is placed on the construction and study of the corresponding fuzzy middle graphs. In addition, the analysis is extended to other related fuzzy graph structures, namely fuzzy splitting graphs, fuzzy shadow graphs, fuzzy line graphs, total fuzzy graphs, and fuzzy subdi-vision graphs. It is established that all these graphs admit the product of total and range fuzzy edge labeling under appropriate conditions.

2. Preliminaries

Definition 2.1. (Fuzzy graph [3]) A fuzzy graph $G = (V, \rho, \delta)$ is a pair of functions (ρ, δ), where $\rho : V \to [0, 1]$ is a fuzzy subset of a non-empty set V, and $\delta : V \times V \to [0, 1]$ is a symmetric fuzzy relation on ρ, such that the relation

$$\delta(v_i, v_j) \leq \rho(v_i) \wedge \rho(v_j)$$

is satisfied for all $v_i, v_j \in V$ and $(v_i, v_j) \epsilon E \subseteq V \times V$.

Here, $\rho(v_i)$ denotes the degree of membership of the vertex v_i, and $\delta(v_i, v_j)$ denotes

the degree of membership of the edge $e_{ij} = (v_i, v_j)$ on $V \times V$.

Note: In this paper, we denote

$$\rho(v_i) \wedge \rho(v_j) = min\{\rho(v_i), \rho(v_j)\}$$

Definition 2.2. (Fuzzy path [5]) Let $G = (V, \rho, \delta)$ be a fuzzy graph with underlying crisp graph G^*. A fuzzy path P_n in G is a sequence of distinct vertices $v_0, v_1, \dots\dots, v_n$ such that

$$\delta(v_{i-1}, v_i) > 0, \quad 1 \leq i \leq n.$$

Here, $n \geq 1$ is called the length of the path P_n.

Definition 2.3.(Fuzzy cycle [5]) A fuzzy path P_n in which $v_0 = v_n$ and $n \geq 3$ is called a fuzzy cycle C_n of length n.

Definition 2.4. (Fuzzy middle graph [8]) The fuzzy middle graph $M_f\ (G)(V_M\ , \rho_M\ , \delta_M\ )$ of a fuzzy graph $G(V, \rho, \delta)$ is a fuzzy graph with underlying crisp graph $M\ (G)(V_M\ , E_M\ )$, where the vertex set is

$$V_M = V \cup V_{ij},$$

where

$$V = \{v_i \mid v_i \in V\}, \qquad V_{ij} = \{V_{ij} \mid (v_i, v_j) \in E\}.$$

Hence, the number of vertices is

$$\nu M_f(G)) = n + (n + 1) = 2n + 1.$$

The edge set is defined as

$E_M = \{(v_{ij}, v_i), (v_{ij}, v_j)\ \forall i, j\} \cup \{(v_{ij}, v_{rs})$ if the edges (v_i, v_j) and (v_r, v_s) are adjacent in $G\}$.

The membership functions are defined as follows:

$\rho_M(v_i) = \rho(v_i)$,if $v_i \epsilon V$, $\quad 0 \leq i \leq n$,

$\rho_M(v_{ij}) = \delta(v_i, v_j), if\ (v_i, v_j) \in E, \forall i, j$,

$\delta_M(v_{ij}, v_{rs}) = \delta(v_i, v_j) \wedge \delta(v_r, v_s)$,

if the edges $(v_i, v_j)$and(v_r, v_s) are adjacent in G,

$\delta_M(v_i, v_{ij}) = \delta_M(v_j, v_{ij}) = \delta(v_i, v_j)$, $\quad \forall i, j$.

Example 1. The fuzzy middle graph of C_4 is given in Figure1.

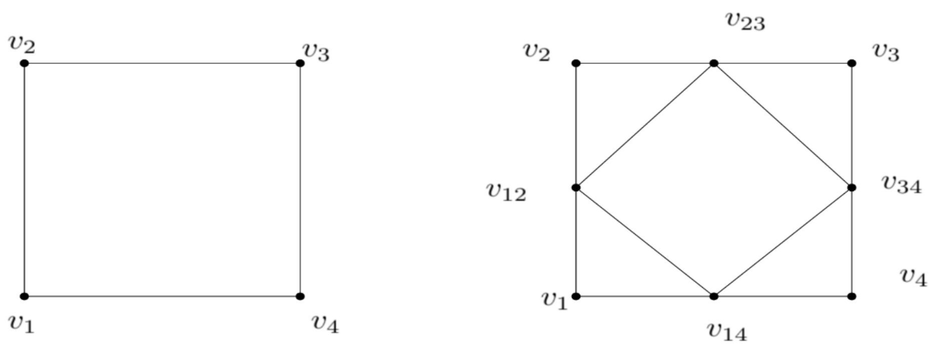

Figure 1:

Definition 2.5. (Fuzzy splitting graph [8]) The fuzzy splitting graph $S_f(G)(V_S, \rho_S, \delta_S)$ of a fuzzy graph $G(V, \rho, \delta)$ is a fuzzy graph with underlying crisp graph $S(G)(V_S, E_S)$, where the vertex set is

Where

$V_S = V \cup V'$,

$V = \{v_i \mid v_i \in V\}, V' = v_i' \mid v_i \in V\}$.

The edge set is defined as

$E_S = \{(v_i, v_j) \mid v_i\ and\ v_j$ are adjacent in $G\} \cup \{(v_i', v_j) \mid v_i' \in V',\ v_j \in V$ and v_j is adjacent to $v_i \in V$

The membership functions are defined as follows:

$$\rho S(v_i) = \rho(v_i), \qquad v_i \in V,$$

$$\rho S(v') = \rho(v_i), \qquad v' \in V',$$

$$(v_i, v_j) = \delta(v_i, v_j), \quad \text{if } v_i \text{ and } v_j \text{ are adjacent in } G,$$
$$\delta S(v', v_j) = \rho S(v') \wedge \rho S(v_j),$$

i ***i***

if $v_i' \in V'$ and $v_j \in V$ are adjacent to $v_i \in V$.

Example 2. The fuzzy splitting graph of C_4 is given in Figure 2.

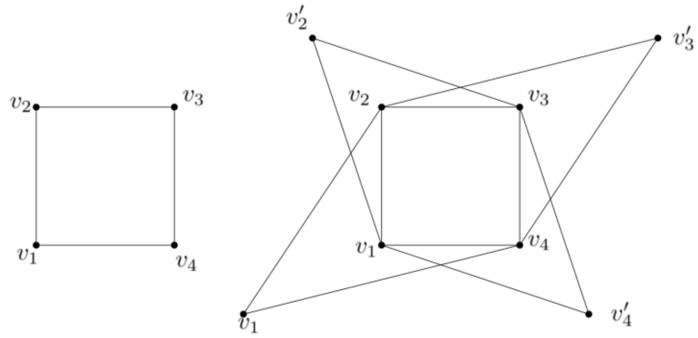

Figure 2:

Definition 2.6. (Fuzzy shadow graph [8]) The fuzzy shadow graph $D_f^2(G)(V_{D_2}, \rho_{D_2}, \delta_{D_2})$ of a fuzzy graph $G(V, \rho, \delta)$ is a fuzzy graph with underlying crisp graph $D_2(G)(V_{D_2}, E_{D_2})$, obtained by taking two copies of G, namely G' and G''.

The vertex set is

$$V_{D_2} = V' \cup V'',$$

where

$$V' = \{v_i' \mid v_i \in V\}, \qquad V'' = \{v_i'' \mid v_i \in V\}.$$

he edge set is defined as

$$E_{D_2} = \{(v_i', v_j'), (v_i'', v_j'') \mid v_i \text{ and } v_j \text{ are adjacent in } G\}$$
$$\cup \{(v_i', v_j'') \mid v_i', \in V', v_j'' \in V'' \text{ and } v_j \text{ is adjacent to } v_i \text{ in } G\}.$$

membership functions are defined as follows:

$$\rho_{D_2}(v') = \rho_{D_2}(v'') = \rho(v), \quad \text{for } v \in V, v' \in V', v'' \in V'',$$
$$\delta_{D_2}(v_i', v_j') = \delta(v_i, v_j), \qquad \delta_{D_2}(v_i'', v_j'') = \delta(v_i, v_j),$$

for $v_i', v_j' \epsilon V'$ and $v_i'', v_j'' \epsilon V''$,

$\delta_{D_2}(v_i', v_j'') = \rho(v_i') \wedge \rho(v_j'')$,

if $v_i' \epsilon V'$ and $v_j'' \epsilon V''$ are adjacent corresponding to $v_i, v_j \epsilon V$ in G.

Example 3. The fuzzy shadow graph of C_4 is given in Figure 3

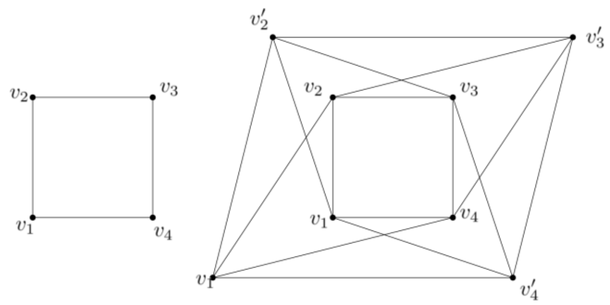

Figure 3:

Definition 2.7. (Fuzzy line graph [8]) The fuzzy line graph $L_f(G)(V_L, \rho_L, \delta_L)$ of a fuzzy graph $G(V, \rho, \delta)$ is a fuzzy graph with underlying crisp graph $L(G)(V_L, E_L)$, where the vertex set is

$$V_L = \{v_{ij} \mid (vi, vj) \in E\},$$

and the edge set is

$E_L = \{(v_{ij}, v_{rs}) \mid$ the edges v_i, v_j and (v_r, v_s) are adjacent in $G\}$.

The membership functions are defined as follows:

$\rho_L(v_{ij}) = \delta(v_i, v_j), if\ v_{ij} \in V_L,$

$\delta_L(v_{ij}, v_{rs}) = \delta(v_i, v_j) \wedge \mu(v_r, v_s),$

if the edges (v_i, v_j) and (v_r, v_s) are adjacent in G.

Example 4 The fuzzy line graph of C_4 is given in Figure 4.

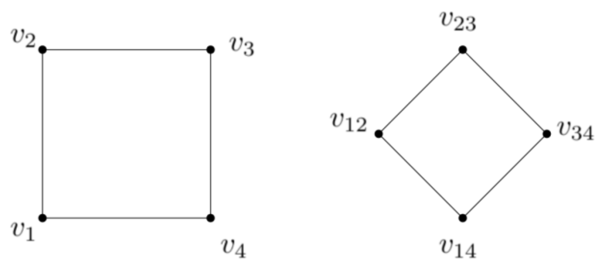

Figure 4:

Definition 2.8. (Total fuzzy graph [8]) The fuzzy total graph $T_f(G)(V_T, \rho_T, \delta_T)$ of a fuzzy graph $G(V,\rho,\delta)$ is a fuzzy graph with underlying crisp graph $T\ (G)(V_T\ ,E_T\ )$, where the vertex set is

where

$$V_T = V \cup V_{ij},$$

$$V = \{v_i \mid v_i \in V\}, V_{ij} = \{v_{ij} \mid (v_i,v_j)\epsilon E\}.$$

The edge set is defined as

$E_T = \{(v_i,v_j) \mid v_i$ and v_j are adjacent in $G\}$

$\cup \{(v_{ij},v_i),(v_{ij},v_j) \mid (v_i,v_j) \in E\}$

$\cup \{(v_{ij},v_{rs}) \mid$ the edges (v_i,v_j) and (v_r,v_s) are adjacent in $G\}$.

The membership functions are defined as follows:

$$\rho_T(v_i) = \rho(v_i), \quad v_i \in V,$$
$$\rho_T\ (v_{ij}) = \delta(v_i,v_j), \quad \text{if } (v_i,v_j) \in E,$$

$\delta_T\ (v_i,v_j) = \delta(v_i,v_j)$, if v_i and v_j are adjacent in G,

$\delta_T\ (v_{ij},v_{rs}) = \delta(v_i,v_j) \wedge \delta(v_r,v_s)$,

if the edges (v_i,v_j) and (v_r,v_s) are adjacent in G,

$\delta_T\ (v_i,v_{ij}) = \delta_T\ (v_i,v_{ij}) = \delta(v_i,v_j), \forall i,j.$

Example 5. The total fuzzy graph of C_4 is given in Figure 5.

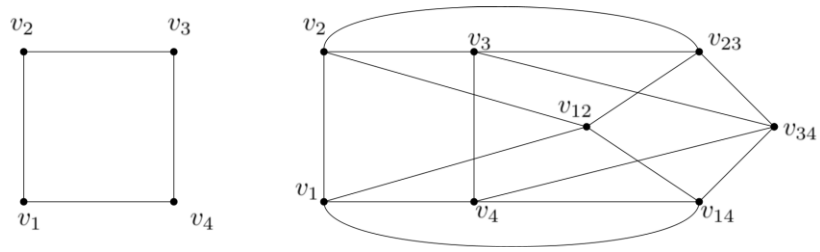

Figure 5:

Definition 2.9. (Fuzzy subdivision graph [8]) The fuzzy subdivision graph $S_{d_f}\ (G)(V_{S_d},\ \rho_{S_d},\ \mu_{S_d})$ of a fuzzy graph $G(V,\rho,\mu)$ is a fuzzy graph with underlying crisp graph $S_d(G)(V_{S_d},E_{S_d})$,

where the vertex set is

where

$V_{S_d} = V \cup V_{ij}$,

$V = \{v_i \mid v_i \in V\}, \qquad Vij = \{v_{ij} \mid (v_i, v_j) \in E\}.$

The edge set is defined as

$$E_{S_d} = \{(v_{ij}, v_i), (v_{ij}, v_j) \mid (v_i, v_j) \in E\}.$$

The membership functions are defined as follows:

$$\rho_{S_d}(v_i) = \rho(v_i), \qquad v_i \in V,$$

$$\rho_{S_d}(v_{ij}) = \delta(v_i, v_j), \quad \text{if } (v_i, v_j) \in E,$$

$$\delta_{S_d}(v_i, v_{ij}) = \delta_{S_d}(v_j, v_{ij}) = \delta(v_i, v_j), \forall i, j.$$

Example 6 The fuzzy subdivision graph of C_4 is given in Figure 6.

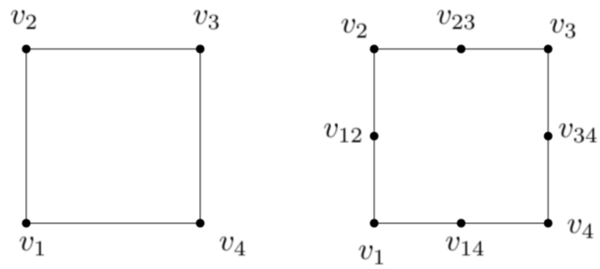

Figure 6:

Definition 2.10. A graph $G = (V, E)$ admits a product of total and range fuzzy edge labeling if there exist bijections $\rho(v) : V \to [0, 1]$ and $\delta(uv) : E \to [0, 1]$ satisfying

$$\delta(uv) < min\{\rho(u), \rho(v)\}$$

and

for all $uv \in E$.

$$\delta(uv) = \rho(u)^2 - \rho(v)^2$$

3. Computation of Fuzzy Labelling under Uncer-tainty using the Product of Total and Range Method

Theorem 3.1. Let P_n be a fuzzy graph. Then the middle graph of fuzzy path $M_f(P_n)$ admits a product of total and range fuzzy edge labeling for $n \geq 2$.

Proof. Let P_n be a fuzzy path. Then by the middle graph of fuzzy path the vertex set of P_n is define as ,

$$V(P_n) = \{v_i \mid v_i \in V\}$$

and the edge set is defined as,

$V_j = \{v_{ij} \forall (v_i, v_j) \epsilon E\}$

Clearly, and

$$\rho(M_f(P_n)) = n + 1 + n = 2n + 1$$

$$E_M = \begin{Bmatrix} (u_{ij}, u_i), (u_{ij}, u_j)\ \forall i, j, \\ (u_{ij}, u_{rs})\ if\ the\ edges\ (u_i, u_j)\ and\ (u_r, u_s) are\ adjacent\ in\ G \end{Bmatrix}$$

The membership function for the vertices,

$$\rho(v) : V \to [0, 1]$$

is defined as

$$\rho(u_i) = \frac{n+i}{2n},\ 1 \leq i \leq n$$

and

$$\rho\big(u_{i(i+1)}\big) = \frac{2n+2i+1}{4n}$$

The membership function for the edges,

$$\delta(uv): E(P_n) \to [0,1],$$

is defined by

$$\delta(e_i) = |\rho(u_i)^2 - \rho(u_{i+1})^2|, 1 \leq i \leq n-1$$

$$\delta(f_i) = \rho\big(u_i\big)^2 - \rho(u_{i(i+1)})\big|$$

Hence, the middle graph of fuzzy path $M_f\ (P_n)$ produces a product of total and range fuzzy path. The membership value of each edge satisfies

$$\delta(uv) < min\{\rho(u), \rho(v)\}.$$

Remark: Similarly, splitting graph, shadow graph, line graph, total graph, subdivision graph of fuzzy path also admits the products of total and range fuzzy edge labeling.

Example: Consider a fuzzy path graph P_4

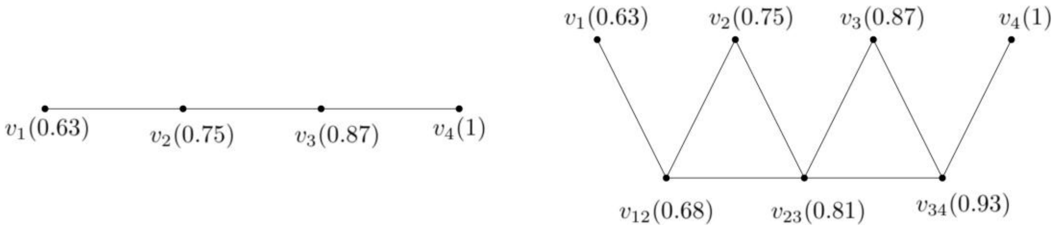

Figure 7: Fuzzy path P_4 and its middle graph $M_f\ (P_n)$.

Theorem 3.2. Let C_n be a fuzzy cycle. Then the middle graph of fuzzy cycle $M_f\ (C_n)$admits a product of total and range fuzzy edge labelling graph for $n \geq 3$.

Proof. Let C_n be a fuzzy cycle . Then, the vertex set of the middle graph of fuzzy cycle C_n is defined as,

$$V\ (C_n) = \{u_1, u_2, \ldots\ldots u_n\}$$

$$V(M_f\ (C_n)) = C_n\ \oplus\ C_{2n}$$

and the edge set

$E(C_n) = \{u_i u_{i+1} \mid 1 \le i \le n-1\} \cup \{u_n u_1\}.$

$V(M_f\ (C_n)) = V\ (C_n) \cup V_{ij}$

Where

$$V\ (C_n) = \{v_1, v_2, v_3, \dots, v_n\}$$

$$V_{ij} = \{v_1\ v_{12}\ v_2\ v_{23} \dots v_n\ v_{n1}\ v_1\}$$

The vertex membership function for the fuzzy cycle,

$$\rho(v) : V\ (C_n) \to [0,1]$$

Case (i): n is even.

$$\rho(u_i) \begin{cases} 1, & if\ i = 1, \\ \frac{2n-i}{2n}, & if\ i = 2,4,\dots,n-2, \\ \frac{2n-i+2}{2n}, & if\ i = 3,5,\dots.n-1, \\ \frac{n+3}{2n+1}, & if\ i = n. \end{cases}$$

Case (ii): n is odd.

$$\rho(u_i) \begin{cases} 1, & if\ i = 1, \\ \frac{2n-i}{2n}, & if\ i = 2,4,\dots,n-2, \\ \frac{2n-i+2}{2n}, & if\ i = 3,5,\dots.n-1, \\ \frac{n+3}{2n+1}, & if\ i = n. \end{cases}$$

The Vertex membership function for the middle graph of fuzzy cycle,

$$\rho_M\ (u_i) = \rho(u_i) - \frac{1}{100}$$

The defined the edge membership function for fuzzy cycle and middle graph of fuzzy cycle,

$\delta(uv): E(C_n) \to [0,1]$ and $E_M(C_n) \to [0,1]$

$\delta(e_i) = |\rho(u_i)^2 - \rho(u_{i+1})^2|.$

$\delta(e_i) = |\rho(u_i)^2 - \rho(u'_i)^2|.$

$\delta(e_i) = |\rho(u'_i)^2 - \rho(u_{i+1})|$

$\delta(e_i) = |\rho(u'_i)^2 - \rho(u_{i+1})|$

Hence, the middle graph of the fuzzy cycle graph $M_f\ (C_n)$ produces a product of total and range fuzzy graph. The membership value of each edge satisfies

$$\delta(uv) < min\{\rho(u), \rho(v)\}.$$

Remark Similarly, splitting graph, shadow graph,line graph,total graph,subdivision graph of fuzzy cycle also admits the products of total and fuzzy edge labeling

Example

Consider the fuzzy cycle C_4.

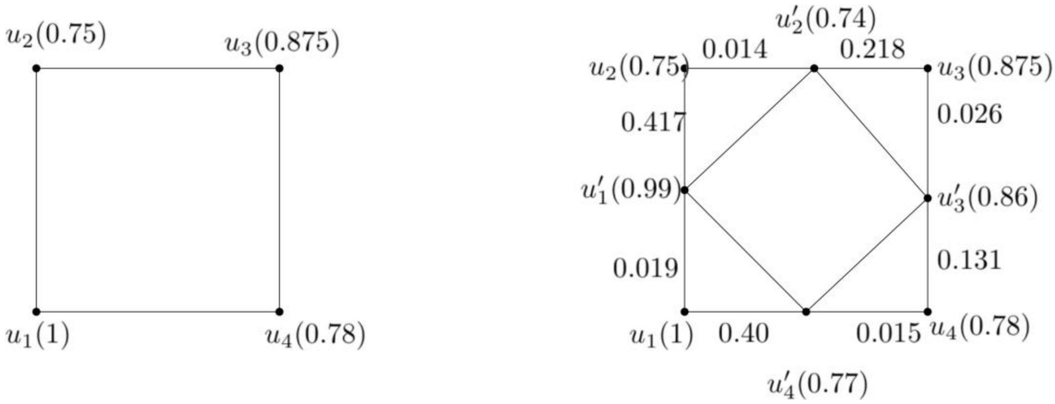

Figure 8: Fuzzy cycle C_4 and its fuzzy middle graph $M_f(C_4)$.

Theorem 3.3. Let S_n be a fuzzy graph. Then the middle graph of fuzzy star $M_f\ (S_n)$ admits a product of total and range fuzzy edge labeling.

Proof. Let S_n be a star graph with vertex set

$V\,(S_n) = \{v_0, v_1, v_2, \dots, v_n\}$

and edge set

$$E\,(S_n) = \{(v_0, v_i) | 1 \le i \le n\}.$$

The middle graph $M_f\ (S_n)$ is obtained by introducing a new vertex v'_{0i} corresponding to each edge (v_0, v_i).

Hence, the vertex set of $M_f\ (S_n)$ is

$$\mathrm{V}(M_f\ (S_n))=\{v_0, v_1, v_2, \dots, v_n\} \cup \{v''{}_0, v'_1, v'_2, \dots, v'_n\}.$$

The edge set of $M_f\ (S_n)$ is

$$\mathrm{E}(M_f\ (S_n))=\{(v_0, v'_i) | 1 \le i \le n\} \cup \{(v_{0i}, v_{0j}) | i \ne j\}$$

Define the vertex labeling function

$$\rho :\ \mathrm{V}(M_f\ (S_n))\ \rightarrow\ [0,1]$$

by

$$\rho(v_0) = \frac{n+1}{n+2},$$

$$\rho(v_i) = \frac{2n-i}{2n}, 1 \le i \le n,$$

$$\rho(v'_i) = \frac{2n-i+2}{2n+2}, 1 \le i \le n.$$

Define the edge labeling function

$$\delta(uv) = |\rho(u)^2 - \rho(v)^2|.$$

Now we verify the condition

$$\delta(uv) < \min\{\rho(u), \rho(v)\}.$$

For edges (v_0, v'_i),

$$\delta(v_0, v'_i) = |\rho(v_0)^2 - \rho(v'_i)^2| < \min\{\rho(v_0), \rho(v'_i)\}.$$

For edges (v'_i, v_i),

$$\delta(v'_i, v_i) = |\rho(v'_i)^2 - \rho(v_i)^2| < \min\{\rho(v'_i), \rho(v_i)\}.$$

For edges (v'_i, v'_j),

$$\delta(v'_i, v'_j) = |\rho(v'_i)^2 - \rho(v'_j)^2| < \min\{\rho(v'_i), \rho(v'_j)\}.$$

Thus, all edges satisfy the condition

$$\delta(uv) < min\{\rho(u), \rho(v)\}.$$

Hence, the middle graph of the fuzzy star graph $M_f(S_n)$ admits a product of total and range fuzzy edge labeling.

□

Remark: Similarly, splitting graph, shadow graph, line graph, total graph, subdivision graph of fuzzy star also admits the products of total and fuzzy edge labeling

Example

Consider the fuzzy star S_4.

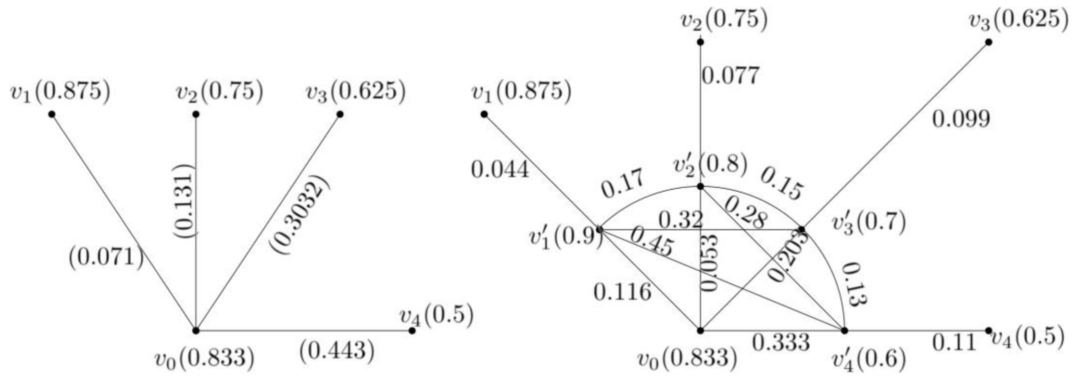

Figure 9: Fuzzy star S_4 and its fuzzy middle graph $M_f(S_4)$.

Theorem 3.4. Let L_n be a fuzzy graph. Then the middle graph of fuzzy ladder $M_f(L_n)$ admits a product of total and range fuzzy edge labeling.

Proof. LetL_n be a fuzzy ladder graph consisting of two parallel paths

$$u_1, u_2, \dots, u_n$$

and

$$v_1, v_2, \dots, v_n$$

with edges $(u_i, u_{i+1}), (v_i, v_{i+1})$ for $1 \leq i \leq n-1$ and rungs (u_i, v_i) for $1 \leq i \leq n$.

The middle graph $M_f(L_n)$ is obtained by introducing a new vertex for each edge of L_n

Hence, the vertex set of $M_f(L_n)$ is

$$V(M_f(L_n)) = V(L_n) \cup V_{ij},$$

where $\qquad V(L_n) = \{u_i, v_i \mid 1 \leq i \leq n\}$

and $\qquad V_{ij} = \{u_{i(i+1)}, v_{i(i+1)}, w_i \mid 1 \leq i \leq n-1\}.$

The edge set of $M_f(L_n)$ is

$$E(M_f(L_n)) = \{(u_i, u_{i(i+1)}), (u_{i(i+1)}, u_{i+1}\}$$
$$\cup \{(v_i, v_{i(i+1)}), (v_{i(i+1)}, v_{i+1})\}$$
$$\cup \{(u_i, w_i), (w_i, v_i)\}$$
$$\cup \{(u_{i(i+1)}, v_{i(i+1)})\}.$$

Define the vertex labeling function

$$\rho : V(M_f(L_n)) \to [0,1]$$

$$\rho(u_i) = \frac{i}{(5n)^2}, \qquad \rho(v_i) = \frac{n+i}{(5n)^2},$$

$$\rho(u_{i(i+1)}) = \frac{2n+i}{(5n)^3}, \qquad \rho(v_{i(i+1)}) = \frac{3n+i}{(5n)^3},$$

$$\rho(w_i) = \frac{2n-i}{(5n)^3}.$$

Define the edge labelling function

$$\delta(uv) = |\rho(u)^2 - \rho(v)^2|.$$

and verify the condition

$$\delta(uv) < min\{\rho(u), \rho(v)\}.$$

Thus all edges satisfy

$$\delta(uv) < min\{\rho(u), \rho(v)\}.$$

Hence, the ladder middle graph $M_f(L_n)$ admits a product of total and range fuzzy edge labeling.

Example

$\square$

Consider the fuzzy ladder L_3.

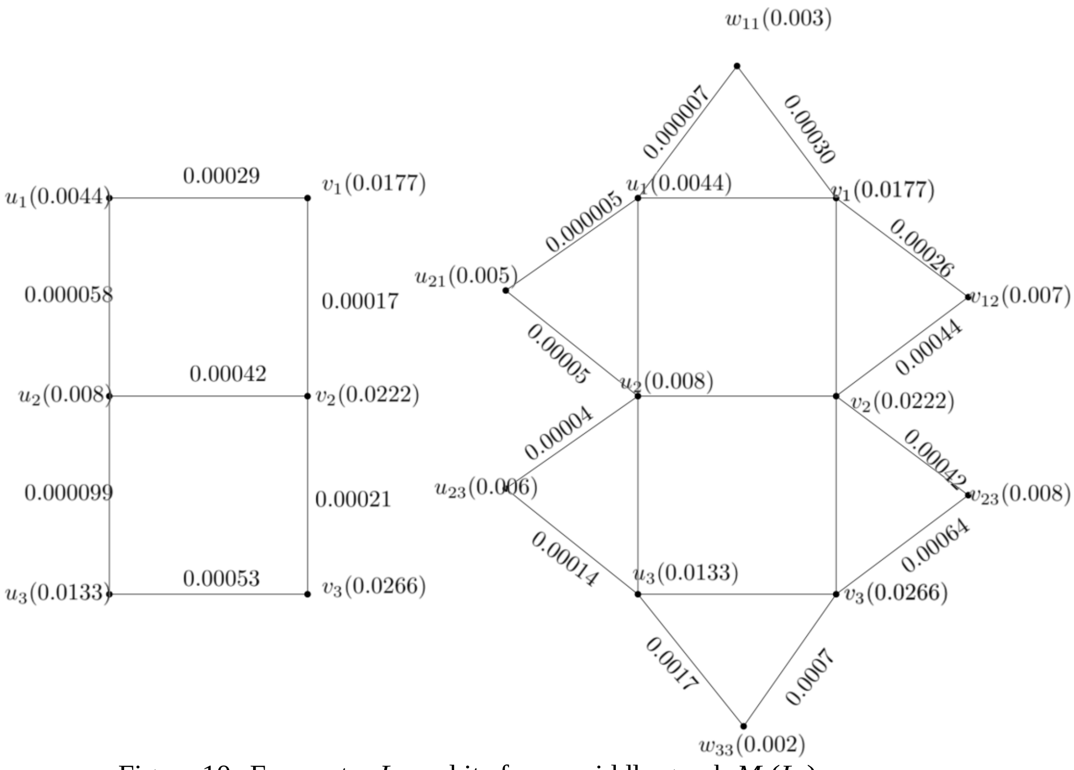

Figure 10: Fuzzy star L_3 and its fuzzy middle graph $M_f(L_3)$.

Remark: Similarly, fuzzy ladder of splitting graph , shadow graph, line graph, total graph, subdivision graph also admits the products of total and fuzzy edge labeling

4. Conclusion

In this paper, we studied the product of total and range fuzzy edge labeling for path, cycle, star, and ladder graphs, with special focus on their fuzzy middle graphs. The study was extended to fuzzy splitting, shadow, line, total, and subdivision graphs. It is established that all these graphs admit the proposed labeling. The results strengthen fuzzy graph labeling theory and provide a basis for further research.

References

1. J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 18 (2011), DS6.
2. A. Nagoor Gani, D. Rajalaxmi and Subahash, A note on fuzzy labeling, Interna-tional Journal of Fuzzy Mathematical Archive, Vol. 4, No. 2 (2014), pp. 88–95.
3. S. Mathew, J. N. Mordeson, and D. S. Malik, Fuzzy Graph Theory, Springer International Publishing, Berlin, Germany, 2018.
4. M. Pal, S. Samanta, and G. Ghorai, “Fuzzy Cut Vertices and Fuzzy Trees,” in Modern Trends in Fuzzy Graph Theory, Springer, Berlin, 2020, ch. 3, pp. 115–124.
5. M. G. Karunambigai and J. A. Mathew, “The Chromatic Number of Fuzzy Paths and Its Related Graphs,” Int. J. Environ. Sci., vol. 11, no. 18s, pp. 1858–1869, 2025.
6. K. Ameenal Bibi and M. Devi, “Bi-Magic labeling of interval valued fuzzy graph,” Advances in Fuzzy Mathematics, Vol. 12, No. 3 (2017), pp. 645–656. ISSN: 0973–533X. Published by Research India Publications
7. Geetha D. and S. Santhosh Kumar, Product of total and range fuzzy edge labeling for flower related graphs, International Journal of Applied Mathematics, Vol. 38, No. 11s (2025).
8. M. G. Karunambigai andJ. A. Mathew , The Chromatic Number Of Fuzzy Cycle And Its Related Graphs, International Journal of Environmental Sciences, vol.11, no. 20s, p.3697, 2025. ISSN: 2229-7359.
9. A. Pradeepa, O. V. Shanmuga Sundaram and N. Pushpalatha, Graphical image of trisomy ultrascan related total edge magic labelling, EAI Endorsed Transactions on Pervasive Health and Technology, Vol. 10 (2024), pp. 1–7, E-ISSN: 2411–7145.