International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(10s) 2026 pp. 55-71
https://doi.org/10.70917/ijcisim-2026-3566
Article

AN ANALYTICAL STUDY OF STRONGLY REGULAR FERMATEAN NEUTROSOPHIC FUZZY DOMBI OPERATORS

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Abstract: In this study, we propose a new frame work of strongly regular Fermatean neutrosophic fuzzy graphs using Dombi operators. The Dombi operator introduced by József Dombi provides a flexible aggregation mechanism, while Fermatean neutrosophic sets effectively capture truth, indeterminacy, and falsity under uncertainty. By combining these concepts, the notion of strong regularity is extended to a more generalized graph structure. Various fundamental properties, operations, and conditions related to strong regularity are established and analysed. Suitable examples are presented to illustrate the proposed concepts. The developed model is useful for handling complex decision-making and network problems in uncertain and imprecise environments.
Keywords: Fermatean neutrosophic dombi fuzzy graph, fermatean neutrosophic fuzzy edge graph, basic products of fermatean neutrosophic fuzzy dombi graph, regular fermatean neutrosophic fuzzy dombi graph.

1. Introduction:

Fuzzy logic was introduced by Lotfi A. Zadeh[22] in 1965 to handle uncertainty. Tapan Senapati and Ronald R. Yager[20] introduced Fermatean fuzzy sets, where the sum of cubes of membership and non-membership is in [0,2]. This idea was further extended by C. Anthony and Jansi to Fermatean neutrosophic sets with three membership degrees bounded by [0,2]. A graph is formed by nodes and arcs, traditionally based on crisp set theory introduced by Georg Cantor. In 1975, Arnold Kaufmann[11] introduced fuzzy graphs using fuzzy relations. Later, Krassimir T. Atanassov[6] (1983) extended this to intuitionistic fuzzy graphs. Paul and Ghori (2017)[9] proposed neutrosophic graphs. Furthermore, S. Naz and S. Ashraf [5]expanded fuzzy graphs into neutrosophic fuzzy graphs.

The Dombi operator was introduced by József Dombi[8] in 1982 with a flexible parameter. Hamacher[10] emphasised that by taking the solution of associativity oper- ational equality, these operators can be created. Later, he obtained the rational structure of disjunctive and conjunctive operators in accordance with Kuwagaki's[12] results. From that moment on, the researchers working in the field of fuzzy theory proposed a more general form, i.e., triangular norms (t-norms) and triangular conorms (t-conorms).

The concept of Dombi fuzzy graphs was later developed by S. Ashraf et al. (2018). Due to its effectiveness in handling real-life decision-making problems, Mijanur Rahman Seikh and Utpal Mandal (2021)[13] applied Dombi operations to intuitionistic fuzzy graphs in multi-attribute group decision-making . Recently, Akram and Habib[4] discussed regularity of q-rung picture fuzzy graphs with applications. Further advancements include neutrosophic Dombi graphs by Tejindar singh Lakhwani and Karthick[21], and Pythagorean neutrosophic fuzzy graphs proposed by Ajay et al[1]. Recently, Fermatean neutrosophic Dombi fuzzy graphs have been introduced, and D. Sasikala and B. Divya [17,18,19] further developed their properties and fundamental operations with illustrative examples.

The structure of this paper is as follows.

(i) Section 1, outlines the essential definitions and preliminary concepts used throughout the study.
(ii) Section 2, presents the newly proposed strongly regular fermatean neutrosophic dombi fuzzy graph concepts and their associated operations, along with established propositions supported by suitable graphs and illustrative examples.

2. Preliminaries:

Definition : 2.1

Let ϑ_D be a non-empty set. A Neutrosophic set (NS) A^{μ} in $\vartheta_D$is determined by a truth-membership function α_{μ_D}, an indeterminacy-membership function β_{μ_D},and a falsity - membership function γ_{μ_D} is $\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D), \gamma_{\mu_D}(\vartheta_D)$ are real or non-standard subsets of $]\,0^-, 1^+[$on ϑ_D.

i.e.)$\alpha_{\mu_D}(\vartheta_D): \vartheta_D \to]0^-, 1^+[$; $\beta_{\mu_D}(\vartheta_D): \vartheta_D \to]0^-, 1^+[$; $\gamma_{\mu_D}(\vartheta_D): \vartheta_D \to]0^-, 1^+$;

Definition : 2.2

A fuzzy graph of the graph $D^*= (\varphi_D, \varsigma_D)$ is a pair of $D = (\mu_D, \nu_D)$, Where $\mu_D \to [0,1]$is a fuzzy seton φ_D and ν_D:$\varphi_D X \varphi_D \to [0,1]$ is a fuzzy relation on φ_D such that $\nu_D(n,t) \leq \mu_D(n) \wedge \mu_D(t), \forall (n,t) \in \varphi_D X \varphi_D$.

Definition : 2.3

A binary function Ŧ :$[0,1]X[0,1] \to [0,1]$ is referred to as triangular norm (t-norm) if for all n,t,s ∈ [0,1], It fulfilled the requirements listed below.

- (Neutral property or boundary condition) *Ŧ(n,1) = n*
- (commutativity) *Ŧ(n,t) = Ŧ(t,n)*
- (associativity) *Ŧ(n,(t,s)) = Ŧ((n,t),s)*
- (monotonicity) *Ŧ(n,t)≤Ŧ(s,d) if* $n \leq s$ *and* $t \leq d$

Definition : 2.4

A binary function $\mathcal{M}$:$[0,1]X[0,1] \to [0,1]$ is referred to as triangular conorm (t-conorm) if and only if there exists a t-norm *Ŧ* for all *Ŧ(n,t)*∈ [0,1]X[0,1], $\mathcal{M}$(n,t)=1-Ŧ(1-n,1-t).
For t-norms, the preferred possibilities are:

- The minimum operator $\mathcal{M}$ *(n,t)=MIN(n,t)*
- The product operator $\mathcal{P}$*(n,t)=nt*
- The dombi's t-norm,$\frac{1}{1+([(\frac{1-n}{n})]^{\delta}+[(\frac{1-t}{t})]^{\delta})^{\frac{1}{\delta}}}$, $\delta > 0$

The recommended possibilities for correlated t-conorms are:

- The maximum operator $\mathcal{M}^*$*(n,t)=MAX (n,t)*
- The Probabilistic sum Ƥ*(n,t) =n+t-nt*
- The Dombi's t-conorm$\frac{1}{1+([(\frac{1-n}{n})]^{-\delta}+[(\frac{1-t}{t})]^{-\delta})^{-\frac{1}{\delta}}}$, $\delta > 0$

- An additional pair of Ŧ-operators is *Ŧ(n,t)=*$\frac{nt}{n+t-nt}$

$\mathcal{P}$*(n,t)=*$\frac{n+t-2nt}{1-nt}$,It is acquired by replacing $\delta = 1$ in dombi's t-norm and t-conorm. Also $\mathcal{P}$*(n,t)*$\leq \frac{nt}{n+t-nt} \leq \mathcal{M}$*(n,t)* and $\mathcal{M}^*$*(n,t)*$\leq \frac{n+t-2nt}{1-nt} \leq$ Ƥ*(n,t).*

Definition : 2.5

A pair is a dombi fuzzy graph with the elementary set being a countable set φ_D . D=(μ_D, ν_D),where $\mu_D \to [0,1]$ is a symmetric fuzzy on φ_D such that$\varsigma_D(nt) \leq \frac{\varphi_D(n)\varphi_D(t)}{\varphi_D(n)+\varphi_D(t)-\varphi_D(n)\varphi_D(t)}$, $\forall n,t \in \varphi_D$.

Definition : 2.6

Let $D^*= (\varphi_D, \varsigma_D)$ be a crisp undirected graph contain no self loop and parallel edges. Let $\mu_D = (\alpha_{\mu_D,}\beta_{\mu_D,}\gamma_{\mu_D})$such that α_{μ_D} :$\vartheta_D \to [0,1]$,β_{μ_D} :$\vartheta_D \to [0,1]$,γ_{μ_D} :$\vartheta_D \to [0,1]$ α_{ν_D} : $\vartheta_D \to [0,1]$, β_{ν_D} : $\vartheta_D \to [0,1]$,γ_{ν_D} : $\vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D} \to$ The membership function, $\beta_{\mu_D}, \beta_{\nu_D} \to$ The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D} \to$ The falsity function in the neutrosophic dombi fuzzy graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the neutrosophic dombi fuzzy graph, $D = (\varphi_{D,}\mu_D, \nu_D)$

$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}, \forall\, nt \in \varsigma_D$, $\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} \forall\, nt \in \varsigma_D$

$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$, $\forall\ nt \in \varsigma_D$

Definition: 2.7

A fermatean neutrosophic fuzzy dombi graph is defined [its indicated by *FNDFG*] with a finite elementary se $\vartheta_D$of its order pair $D = (\varphi_D, \varsigma_D)$ where $\varphi_D: \vartheta_D \to [0,1]$.Here we consider, $\mu_D = (\alpha_{\varphi_D}, \beta_{\mu_D,} \gamma_{\nu_D})$ such that$\alpha_{\varphi_D}: \vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$,$\gamma_{\nu_D}: \vartheta_D \to [0,1]$, $\alpha_{\mu_D}:\vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$,$\gamma_{\nu_D}: \vartheta_D \to [0,1]$.Here $\alpha_{\mu_D}, \alpha_{\nu_D}$ → The membership function, $\beta_{\mu_D}, \beta_{\nu_D}$ → The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D}$ → The falsity function in the Fermatean neutrosophic fuzzy dombi graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the fermatean neutrosophic fuzzy dombi

$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$,$\forall\ nt \in \varsigma_D$ $\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$,$\forall\ nt \in \varsigma_D$

$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$, $\forall\ nt \in \varsigma_D$

And $0 \leq \alpha_{\nu_D}{}^3(nt)+\beta_{\nu_D}{}^3(nt)+\gamma_{\nu_D}{}^3(nt) \leq 2$; $0 \leq \alpha_{\nu_D}{}^3(nt)+\gamma_{\nu_D}{}^3(nt) \leq 1$; $0 \leq \beta_{\nu_D}{}^3(nt) \leq 1$.

Definition : 2.8

Consider, $\zeta_D = \{nt, \emptyset_{\zeta_D}(nt), \mu_{\zeta_D}(nt), \nu_{\zeta_D}(nt), nt \in \varsigma_D\}$ be a fermatean neutrosophic dombi fuzzy edge set in D:

- The order of O_D is established by $\rho(O_D)$ =[$\sum_{n\in\varphi} \emptyset_\varphi$, $\sum_{n\in\varphi} \mu_\varphi$, $\sum_{n\in\varphi} \nu_\varphi$] from illustration 3.1, order of D, $\rho(O_D)$ = [3.4,4.3,2.9]
- The size of O_{s_D} is symbolized by $\rho_S(O_D)$=[$\sum_{n\in\varphi} \emptyset_\varsigma(nt)$, $\sum_{n\in\varphi} \mu_\varsigma(nt)$, $\sum_{n\in\varphi} \nu_\varsigma(nt)$] from illustration 3.1, order of D,$\rho_S(O_D)$ = [5.33,4.71,6.95]
- The degree of vertex n∈ φ_D is denoted by $ძ_{D_N}$its described by $ძ_{D_N}$=[$ძ_{\emptyset_\varphi}(n)$, $ძ_{\mu_\varphi}(n)$, $ძ_{\nu_\varphi}(n)$], where $ძ_{\emptyset_\varphi}(n)$=$\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt)$ = $\sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$,

  $\sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$,

  $\sum_{n,t\neq n\in\varphi} \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$.

Definition:2.9

A fermatean neutrosophic dombi fuzzy graph [*FNDFG*] with a finite elementary set $\vartheta_D$of its order pair $D = (\varphi_D, \varsigma_D)$ where $\varphi_D: \vartheta_D \to [0,1]$.Here we consider, $\varphi_D = (\alpha_{\varphi_D}, \beta_{\mu_D,} \gamma_{\nu_D})$ such that$\alpha_{\varphi_D}: \vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$,$\gamma_{\nu_D}: \vartheta_D \to [0,1]$ $\alpha_{\mu_D}:\vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$,$\gamma_{\nu_D}: \vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D}$ → The membership function, $\beta_{\mu_D}, \beta_{\nu_D}$ → The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D}$ → The falsity function is defined as the regular of degree $(\Re_{\varphi_D,}, \Re_{\mu_D}, \Re_{\nu_D})$. If

$ძ_{\emptyset_\varphi}(n)$=$\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt)$ = $\sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)} = \Re_{\varphi_D,}$,

$ძ_{\emptyset_\mu}(n)$=$\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt)$ = $\sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} = \Re_{\mu_D}$,

$ძ_{\emptyset_\nu}(n)$=$\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt)$ = $\sum_{n,t\neq n\in\varphi} \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)} = \Re_{\nu_D}$.For all $n \in \varphi$

Definition : 2.10

A fermatean neutrosophic dombi fuzzy graph $D = (\emptyset_D, \mu_{D,} \nu_D)$ on the crisp graphs D*= (φ_D, ς_D) is defined as the totally regular of degree $(\Im_{\varphi_D,}, \Im_{\mu_D}, \Im_{\nu_D})$. If

- $Ŧ_{\varphi_D}(n)$=$\sum_{n,t\neq n\in\varphi} \varsigma_{\emptyset_D}(nt) + \emptyset_{\varphi_D}(n)$

$$=\sum_{n,t\neq n\in\varphi}\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}+\emptyset_{\varphi_D}(n)\ =\Im_{\varphi_{D,}}$$

$$\text{Ŧ}_{\mu_D}(n)=\sum_{n,t\neq n\in\varphi}\varsigma_{\mu_D}(nt)+\emptyset_{\mu_D}(n)$$

$$=\sum_{n,t\neq n\in\varphi}\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}+\emptyset_{\varphi\mu_D}(n)=\Im_{\mu_D}$$

$$\text{Ŧ}_{\nu_D}(n)=\sum_{n,t\neq n\in\varphi}\varsigma_{\nu_D}(nt)+\emptyset_{\nu_D}(n)$$

$$=\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}+\emptyset_{\nu_D}(n)=\Im_{\nu_D}\text{ for all } n\in\varphi.$$

3. Strongly regular fermatean neutrosophic fuzzy dombi operators

Definition : 3.1

A fermatean neutrosophic dombi fuzzy graph [$FNDFG$] $D=(\varphi_D,\varsigma_D)$ on n vertices is said to be strongly regular if the following properties are satisfied:

(i) D is regular of degree $\mathfrak{R}=(\mathfrak{R}_{\varphi_{D,}},\mathfrak{R}_{\mu_D},\mathfrak{R}_{\nu_D})$

(ii) The sum of the membership and non-membership grades of the common neighbouring vertices of any pair of adjoining vertices of D are equal and represented as $\Theta=(\Theta_1,\Theta_2,\Theta_3)$.

(iii) The sum of the membership and non-membership grades of the common neighbouring vertices of any pair of non-adjoining vertices of D are equal and represented as $£=(£_1,£_2,£_3)$.

A Strongly fermatean neutrosophic fuzzy Dombi operators $D=(\varphi_D,\varsigma_D)$ is represented as $D=$ (Ŋ, $\mathfrak{R},\Theta,£$)

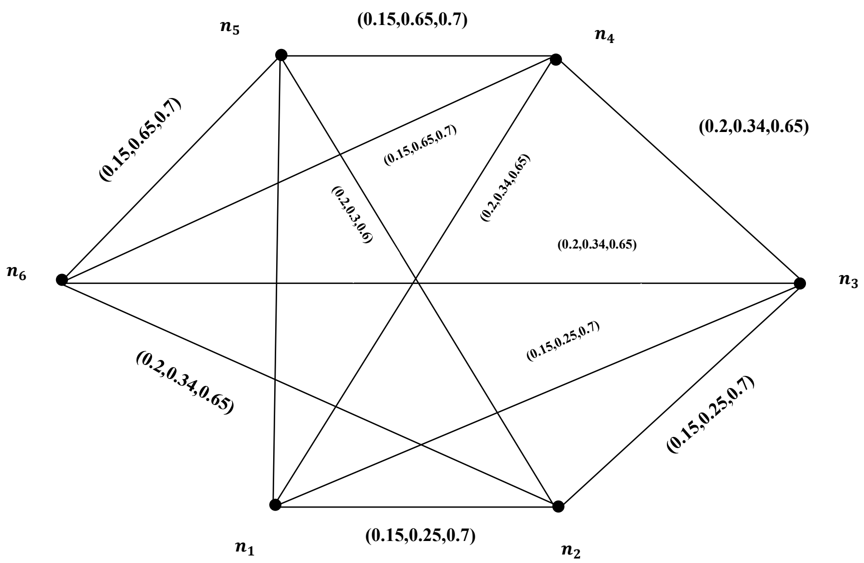
Example: 3.1

Consider a graph $D^*=(\varphi_D,\varsigma_D)$, where $\varphi_D=\{n_1,n_2,n_{3,}n_{4,}n_5,n_6\}$ and $\varsigma_{D=}\{n_1n_{2,}n_1n_{3,}n_1n_{4,}n_1n_{5,}n_2n_{3,}n_2n_5,n_2n_{6,}n_3n_{4,}n_3n_6,n_4n_{5,}n_4n_6,n_5n_6\}$. Let $\alpha_{\varphi_D},\beta_{\mu_D},\gamma_{\nu_D}$ be fermatean Dombi fuzzy vertex set and fermatean Dombi fuzzy edge set defined on φ_D *and* ς_D respectively.

$$\varphi_D=\begin{bmatrix}\frac{n_1}{0.3}&\frac{n_2}{0.3}&\frac{n_3}{0.3}&\frac{n_4}{0.5}&\frac{n_5}{0.5}&\frac{n_6}{0.5}\\ \frac{n_1}{0.4}&\frac{n_2}{0.4}&\frac{n_3}{0.4}&\frac{n_4}{0.7}&\frac{n_5}{0.7}&\frac{n_6}{0.7}\\ \frac{n_1}{0.9}&\frac{n_2}{0.9}&\frac{n_3}{0.9}&\frac{n_4}{0.6}&\frac{n_5}{0.6}&\frac{n_6}{0.6}\end{bmatrix};$$

$$\varsigma_D=\begin{bmatrix}\frac{n_1n_2}{0.15}&\frac{n_1n_3}{0.15}&\frac{n_1n_4}{0.2}&\frac{n_1n_5}{0.2}&\frac{n_2n_3}{0.15}&\frac{n_2n_5}{0.2}&\frac{n_2n_6}{0.2}&\frac{n_3n_4}{0.2}&\frac{n_3n_6}{0.2}&\frac{n_4n_5}{0.15}&\frac{n_4n_6}{0.15}&\frac{n_5n_6}{0.15}\\ \frac{n_1n_2}{0.25}&\frac{n_1n_3}{0.25}&\frac{n_1n_4}{0.34}&\frac{n_1n_5}{0.34}&\frac{n_2n_3}{0.25}&\frac{n_2n_5}{0.34}&\frac{n_2n_6}{0.34}&\frac{n_3n_4}{0.34}&\frac{n_3n_6}{0.34}&\frac{n_4n_5}{0.53}&\frac{n_4n_6}{0.53}&\frac{n_5n_6}{0.53}\\ \frac{n_1n_2}{0.7}&\frac{n_1n_3}{0.7}&\frac{n_1n_4}{0.65}&\frac{n_1n_5}{0.65}&\frac{n_2n_3}{0.7}&\frac{n_2n_5}{0.65}&\frac{n_2n_6}{0.65}&\frac{n_3n_4}{0.65}&\frac{n_3n_6}{0.65}&\frac{n_4n_5}{0.7}&\frac{n_4n_6}{0.7}&\frac{n_5n_6}{0.7}\end{bmatrix}$$

$\text{ɗ}_{D_N}=[\text{ɗ}_{\emptyset_\varphi}(n_i),\text{ɗ}_{\mu_\varphi}(n_i),\text{ɗ}_{\nu_\varphi}(n_i)]$ = (0.7,1.18,2.7) for all i = 1,2,3,4,5,6, D is a (0.7,1.18,2.7) regular FNDFG. Meanwhile, the sum of the membership, indeterminacy, non-membership grades of the common neighboring vertices of each pair of the adjoining and non-adjoining vertices of D are equal. i.e) $\Theta=(\Theta_1,\Theta_2,\Theta_3)$ = (0.75,0.8,2.7) and $£=(£_1,£_2,£_3)$ = (1.6,2,3.2). Hence $D=$ (Ŋ, $\mathfrak{R},\Theta,£$) is a strongly regular FNDFG.



Strongly regular FNDFG

Definition:3.2 A FNDFG $D = (\varphi_D, \varsigma_D)$ is known as bipartite if $\sigma = \{ t_1, t_2, \ldots \ldots, t_n\}$ can be partioned into two non-empty disjoint sets $\sigma_1 and\ \sigma_2$ such that $\emptyset_{\zeta_D}(n_i, n_j) = 0,\ \ \mu_{\zeta_D}(n_i, n_j) = 0\ \ and\ v_{\zeta_D}(n_i, n_j) = 0$. If $n_i, n_j \in \sigma_1$ or $n_i, n_j \in \sigma_2$. Furthermore, if

$$\alpha_{\varsigma_D}(n_i n_j) \leq \frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(n_j)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}, \qquad \beta_{\varsigma_D}(n_i n_j) \leq \frac{\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}{\beta_{\mu_D}(n_i)+\beta_{\mu_D}(n_j)-\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}$$

$$\gamma_{\varsigma_D}(n_i n_j) \leq \frac{\gamma_{v_D}(n_i)+\gamma_{v_D}(n_j)-2\gamma_{v_D}(n_i)\gamma_{v_D}(n_j)}{1-\gamma_{v_D}(n_i)\gamma_{v_D}(n_j)}$$

$\forall\ n_i \in \sigma_1$ and $n_j \in \sigma_2$.Then $D = (\varphi_D, \varsigma_D)$ is known as complete bipartite, represented by ¥ $\varrho_1, \varrho_2, \varrho_3$, where $\varrho_1, \varrho_2, \varrho_3$ are the restrictions of ¥ to σ_1 and ¥ to σ_2 , respectively.

Definition: 3.3

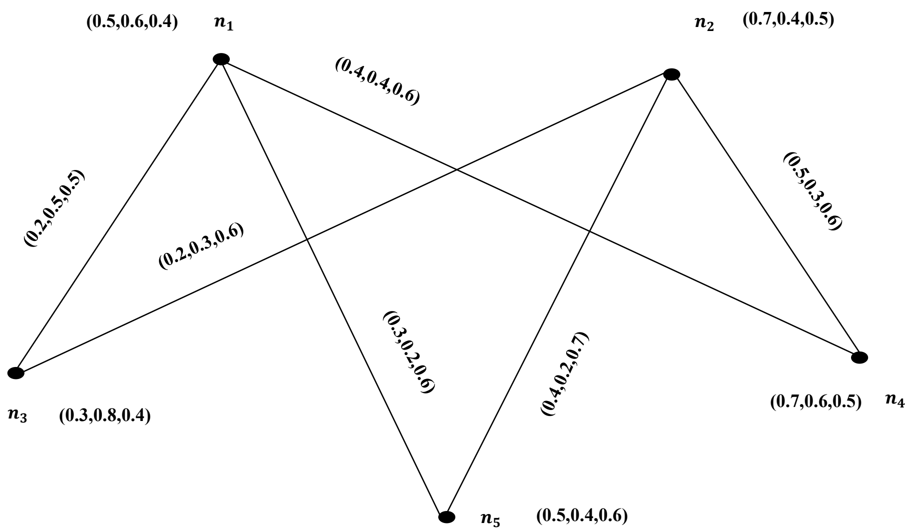
A bipartite FNDFG $D = (\varphi_D, \varsigma_D)$ is known as biregular if each vertex in $\sigma_1 and\ \sigma_2$ have equal degree $\chi = (\chi_1, \chi_2, \chi_3)$ and $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ respectively, where χ and Ω are constants. Further, a biregular FNDFG is represented by $D = (n, \chi, \Omega)$.

Example: 3.2

Consider a graph $D^* = (\varphi_D, \varsigma_D)$,where $\varphi_D = \{n_1, n_2, n_3, n_4, n_5\}$ is partitioned into two non-empty disjoint sets $\varphi_{D_1} = \{n_1, n_2\}$ and $\varphi_{D_2} = \{n_3, n_4, n_5\}$ such that $\varsigma_D = \{n_1n_3, n_1n_4, n_1n_5, n_2n_3, n_2n_4, n_2n_5\}$. Since each vertex in φ_{D_1} and φ_{D_2} have equal degree $\chi = (\chi_1, \chi_2, \chi_3) = (0.8, 1.2, 1.7)$ and $\Omega = (\Omega_1, \Omega_2, \Omega_3) = (1.3, 0.9, 1.9)$ respectively , $D = (n, \chi, \Omega)$ is a biregular FNDFG.

$$\varphi_{D_1} = \{n_1, n_2\} = \begin{bmatrix} \frac{n_1}{0.5}, & \frac{n_2}{0.7} \\ \frac{n_1}{0.6}, & \frac{n_2}{0.4} \\ \frac{n_1}{0.4}, & \frac{n_2}{0.5} \end{bmatrix} \qquad \varphi_{D_2} = \{n_3, n_4, n_5\} = \begin{bmatrix} \frac{n_3}{0.3}, & \frac{n_4}{0.7}, & \frac{n_5}{0.5} \\ \frac{n_3}{0.8}, & \frac{n_4}{0.6}, & \frac{n_4}{0.4} \\ \frac{n_3}{0.4}, & \frac{n_4}{0.5}, & \frac{n_4}{0.6} \end{bmatrix}$$

$$\varsigma_D = \{n_1n_2, n_2n_3, n_3n_4, n_4n_1\} = \begin{bmatrix} \frac{n_1n_3}{0.2}, & \frac{n_1n_4}{0.4}, & \frac{n_1n_5}{0.3}, & \frac{n_2n_3}{0.2}, & \frac{n_2n_4}{0.5}, & \frac{n_2n_5}{0.4} \\ \frac{n_1n_3}{0.5}, & \frac{n_1n_4}{0.4}, & \frac{n_1n_5}{0.2}, & \frac{n_2n_3}{0.3}, & \frac{n_2n_4}{0.3}, & \frac{n_2n_5}{0.2} \\ \frac{n_1n_3}{0.5}, & \frac{n_1n_4}{0.6}, & \frac{n_1n_5}{0.6}, & \frac{n_2n_3}{0.6}, & \frac{n_2n_4}{0.6}, & \frac{n_2n_5}{0.7} \end{bmatrix}$$



Biregular FNDFG

Theorem: 3.1

Let $D = (\varphi_D, \varsigma_D)$ be a FNDFG .If D is complete with $\alpha_{\varphi_D,}\beta_{\mu_{D,}}\gamma_{\nu_D}, \emptyset_{\zeta_D}, \mu_{\zeta_D}, \nu_{\zeta_D}$ as constant functions, then $D = (\varphi_D, \varsigma_D)$ is strongly regular FNDFG.

Proof:

Assume that $D = (\varphi_D, \varsigma_D)$ is a complete FNDFG with $\sigma = \{ t_1, t_2, \ldots \ldots, t_n\}$. As $\alpha_{\varphi_D,}\beta_{\mu_{D,}}\gamma_{\nu_D}, \emptyset_{\zeta_D}$, $\mu_{\zeta_D}, \nu_{\zeta_D}$ are constant functions, $\alpha_{\varphi_D} = C_1, \beta_{\mu_D} = C_2, \gamma_{\nu_D} = C_3, \emptyset_{\zeta_D} = C_4, \mu_{\zeta_D} = C_5, \nu_{\zeta_D} = C_6\, n_i, n_j \in \zeta_D$ .To prove that $D = (\varphi_D, \varsigma_D)$ is strongly regular FNDFG, we must show that D is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ regular FNDFG. Further, the adjoining and non-adjoining vertices have equal common neighborhood $\Theta = (\Theta_1, \Theta_2, \Theta_3)$ and £$=($£$_1,$ £$_2,$ £$_3)$ respectively.

As D is complete FNDFG, we must have

- $ძ_{D_N} = [ძ_{\emptyset_\varphi}(n_i), ძ_{\mu_\varphi}(n_i), ძ_{\nu_\varphi}(n_i)]$, where

$$ძ_{\emptyset_\varphi}(n) = \left[\sum_{n_i n_j \in \zeta_D} \alpha_{\varphi_{D_\varsigma}}(n_i n_j), \sum_{n_i n_j \in \zeta_D} \mu_{\varphi_{D_\varsigma}}(n_i n_j), \sum_{n_i n_j \in \zeta_D} \gamma_{\varphi_{D_\varsigma}}(n_i n_j)\right]$$

$$= \sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i) + \alpha_{\varphi_D}(n_j) - \alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)},$$

$$\sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}{\beta_{\mu_D}(n_i) + \beta_{\mu_D}(n_j) - \beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)},$$

$$\sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\gamma_{\nu_D}(n_i) + \gamma_{\nu_D}(n_j) - 2\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}{1 - \gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}.$$

$$= [(n-1)C_4, (n-1)C_5, (n-1)C_6]$$

Hence D is a $[(n-1)C_4, (n-1)C_5, (n-1)C_6]$ regular FNDFG. Further, the sum of the membership, indeterminacy and non-membership grades of the common neighboring vertices of any pair of adjoining vertices $\Theta = (\Theta_1, \Theta_2, \Theta_3) = [(n-2)C_1, (n-2)C_2, (n-2)C_3]$ are equal. As D is complete, the sum of the membership, indeterminacy and non-membership grades of the common neighboring vertices of any pair of non-adjoining vertices £$=($£$_1,$ £$_2,$ £$_3) = [0,0]$ are equal. Since all the properties are satisfied, we conclude that $D = (\varphi_D, \varsigma_D)$ is strongly regular FNDFG.

Remark:1

If $D = (\varphi_D, \varsigma_D)$ is a strongly regular disconnected FNDFG, then the sum of the membership, indeterminacy and non-membership grades of the common neighboring vertices of any pair of non-adjoining vertices is £$=($£$_1,$ £$_2,$ £$_3) = [0,0]$

Remark:2

If $D = (\varphi_D, \varsigma_D)$ is a strongly regular complete bipartite FNDFG with equal bipartition, then the sum of the membership, indeterminacy and non – membership grades of the common neighboring vertices of any pair of non – adjoining vertices £=$(£_1, £_2, £_3)$ = [0,0].

Theorem:3.2

Let $D = (\varphi_D, \varsigma_D)$ be a FNDFG. If D is strongly regular and strong, then $\overline{D}$ is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG.

Proof:

Assume that D is strongly regular FNDFG; then by definition D is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG. Further, as D is strong, then $\overline{D}$ is also strong. Therefore, we must have

$$\overline{\alpha_{\varsigma_D}(n_i n_j)} = \begin{cases} \frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(n_j)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)} & if\ \alpha_{\varsigma_D}(n_i n_j) = 0, \\ 0 & if\ 0 < \alpha_{\varsigma_D}(n_i n_j) \leq 1 \end{cases}$$

$$\overline{\beta_{\varsigma_D}(n_i n_j)} = \begin{cases} \frac{\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}{\beta_{\mu_D}(n_i)+\beta_{\mu_D}(n_j)-\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)} & if\ \beta_{\varsigma_D}(n_i n_j) = 0, \\ 0 & if\ 0 < \beta_{\varsigma_D}(n_i n_j) \leq 1 \end{cases}$$

$$\overline{\gamma_{\varsigma_D}(n_i n_j)} = \begin{cases} \frac{\gamma_{\nu_D}(n_i)+\gamma_{\nu_D}(n_j)-2\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}{1-\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)} & if\gamma_{\varsigma_D}(n_i n_j) = 0, \\ 0 & if\ 0 < \gamma_{\varsigma_D}(n_i n_j) \leq 1 \end{cases}$$

In $\overline{D}$, the degree of a vertex n_i is defined by

$$ძ_{\emptyset_{\varphi\,\overline{D}}}(n) = [ძ_{\overline{\varphi_D}}(n_i), ძ_{\overline{\mu_\varphi}}(n_i), ძ_{\overline{\nu_\varphi}}(n_i)]$$
$$= \left[\sum_{n_i n_j \in \zeta_D} \overline{\alpha_{\varsigma_D}(n_i n_j)}, \sum_{n_i n_j \in \zeta_D} \overline{\beta_{\varsigma_D}(n_i n_j)}, \sum_{n_i n_j \in \zeta_D} \overline{\gamma_{\varsigma_D}(n_i n_j)}\right]$$

$$ძ_{\emptyset_\varphi}(n) = \sum_{n_i, n_j \neq n_i \in \varsigma_D} \emptyset_\varsigma(nt) = \sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(n_j)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)},$$
$$\sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}{\beta_{\mu_D}(n_i)+\beta_{\mu_D}(n_j)-\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)},$$
$$\sum_{n_i, n_j \neq n_i \in \varsigma_D} \frac{\gamma_{\nu_D}(n_i)+\gamma_{\nu_D}(n_j)-2\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}{1-\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)} = (\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$$

As $ძ_{\emptyset_{\varphi\,\overline{D}}}(n) = (\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$, $\overline{D}$ is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG.

Corollary:1

Let $D = (\varphi_D, \varsigma_D)$ be a FNDFG. If $\overline{D}$ is strongly regular and strong, then D is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG.

Theorem:3.3

Consider D = (φ_D, ς_D) be a strong FNDFG. Then D is a strongly regular FNDFG if and only if $\overline{D}$ is a strongly regular FNDFG.

Proof:

Suppose that D is a strongly regular FNDFG. Then by definition, D is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG. Further, the adjoining and non-adjoining vertices have equal common neighborhood $\Theta = (\Theta_1, \Theta_2, \Theta_3)$ and £=$(£_1, £_2, £_3)$, respectively. To prove that $\overline{D}$ is a strongly regular FNDFG. We must show that $\overline{D}$ is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Since D is strongly regular and strong, then by theorem 3.2, $\overline{D}$ is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Moreover, assume that $\mathcal{N}_1 = \{(n_i, n_j)/(n_i, n_j) \in \zeta_D\}$ and $\mathcal{N}_2 = \{(n_i, n_j)/(n_i, n_j) \notin \zeta_D\}$ be the sets of all adjoining and non- adjoining vertices of D, where $n_i and\ n_j$ have equal common neighborhood $\Theta = (\Theta_1, \Theta_2, \Theta_3)$ and £=$(£_1, £_2, £_3)$, respectively. Then $\overline{\mathcal{N}_1} = \{(n_i, n_j)/(n_i, n_j) \in \overline{\zeta_D}\}$ and $\overline{\mathcal{N}_2} = \{(n_i, n_j)/(n_i, n_j) \notin \overline{\zeta_D}\}$, where $n_i\ and\ n_j$ have equal common neighborhood $\Theta = (\Theta_1, \Theta_2, \Theta_3)$ and £=$(£_1, £_2, £_3)$, respectively. Hence $\overline{\mathrm{D}}$ is strongly regular FNDFG.

Conversely, $\overline{D}$ is a strongly regular FNDFG; then by definition, $\overline{D}$ is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Further, the adjoining and non-adjoining vertices have equal common neighborhood. To prove that D is a strongly regular FNDFG. we must show that D is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Since $\overline{D}$ is a strongly regular and strong, then by corollary 1, D is $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Furthermore, assume that $\overline{\mathcal{N}_1} = \{(n_i, n_j)/(n_i, n_j) \in \overline{\zeta_D}\}$ and

$\overline{\mathcal{N}_2}$ $=\{(n_i,n_j)/(n_i,n_j)\notin\overline{\zeta_D}\}$be the sets of all adjoining and non-adjoining vertices of $\overline{D}$, where n_i and n_j have equal common neighborhood $\Theta=(\Theta_1,\Theta_2,\Theta_3)$ and $£=(£_1,£_2,£_3)$, respectively. Hence D is a strongly regular FNDFG.

Theorem:3.4

Consider D = (φ_D,ς_D) be a FNDFG such that the underlying graph $D^*=(\varphi_D,\varsigma_D)$ is strongly regular graph; then D is strongly regular FNDFG if $\alpha_{\varphi_D},\beta_{\mu_D,}\gamma_{\nu_D},\emptyset_{\zeta_D},\mu_{\zeta_D},\nu_{\zeta_D}$ are constant functions.

Proof:

Assume that $D^*=(\text{Ŋ},\Re,\Theta,£)$ is a strongly regular graph. Also, assume that $\alpha_{\varphi_D}=C_1,\beta_{\mu_D}=C_2,\gamma_{\nu_D}=C_3$ for all $n\in\varphi$ and $\emptyset_{\zeta_D}=C_4,\mu_{\zeta_D}=C_5,\nu_{\zeta_D}=C_6\, nt\in\zeta_D$ .Then the degree of the vertex n is given by

$$ძ_{\emptyset_\varphi}(n)=\sum_{n,t\neq n\in\varphi}\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$$

$$=\sum_{n,t\neq n\in\varphi}C_4=ძ_{D^*\varphi}(n)C_4=\Re_{\varphi_D}C_4\qquad,$$

$$ძ_{\emptyset_\mu}(n)=\sum_{n,t\neq n\in\varphi}\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$$

$$=\sum_{n,t\neq n\in\varphi}C_5=ძ_{D^*\mu}(n)C_5=\Re_{\mu_D}C_5,$$

$$ძ_{\emptyset_\nu}(n)=\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$$

$$=\sum_{n,t\neq n\in\varphi}C_6=ძ_{D^*\nu}(n)C_6=\Re_{\nu_D}C_6.$$

This implies that $ძ_{D_N}=[ძ_{\emptyset_\varphi}(n),ძ_{\mu_\varphi}(n),ძ_{\nu_\varphi}(n)]=(\Re_{\varphi_D}C_4,\Re_{\mu_D}C_5,\Re_{\nu_D}C_6)$. Therefore, D is a $(\Re_{\varphi_D}C_4,\Re_{\mu_D}C_5,\Re_{\nu_D}C_6)$ – regular FNDFG. Likewise, since $\alpha_{\varphi_D,}\beta_{\mu_D,}\gamma_{\nu_D}$ are constant functions, the parameters are $(\Theta C_1,\Theta C_2,\Theta C_3)$ and $(£C_1,£C_2,£C_3)$.Hence D is a strongly regular FNDFG with parameters $(\Re_{\varphi_D}C_4,\Re_{\mu_D}C_5,\Re_{\nu_D}C_6)$, $(\Theta C_1,\Theta C_2,\Theta C_3)$ and $(£C_1,£C_2,£C_3)$.

Remark: 3

Converse of the theorem 3.4 need not be true as seen in the example given below:

Definition: 3.4

Let $D=(\varphi_D,\varsigma_D)$ be a FNDFG and $\left(ძ_{D_\varphi}\right)_1,\left(ძ_{D_\varphi}\right)_2\dots\dots,\left(ძ_{D_\varphi}\right)_r$ be the degree of the vertices of D. Then the degree sequence is represented by $\left(ძ_{D_\varphi}\right)_1,\left(ძ_{D_\varphi}\right)_2\dots\dots,\left(ძ_{D_\varphi}\right)_r=$

$$\left[\left(ძ_{\emptyset_\varphi}\right)_1,\left(ძ_{\emptyset_\varphi}\right)_2\dots\dots,\left(ძ_{\emptyset_\varphi}\right)_r,\left(ძ_{\mu_\varphi}\right)_1,\left(ძ_{\mu_\varphi}\right)_2\dots\dots,\left(ძ_{\mu_\varphi}\right)_r,\left(ძ_{\nu_\varphi}\right)_1,\left(ძ_{\nu_\varphi}\right)_2\dots\dots,\left(ძ_{\nu_\varphi}\right)_r\right]$$

Where $\begin{bmatrix}\left(ძ_{\emptyset_\varphi}\right)_1\geq\left(ძ_{\emptyset_\varphi}\right)_2\geq\cdots\dots\geq\left(ძ_{\emptyset_\varphi}\right)_r,\\ \left(ძ_{\mu_\varphi}\right)_1\geq\left(ძ_{\mu_\varphi}\right)_2\geq\cdots\dots\geq\left(ძ_{\mu_\varphi}\right)_r\ and\\ \left(ძ_{\nu_\varphi}\right)_1\geq\left(ძ_{\nu_\varphi}\right)_2\geq\cdots\dots\geq\left(ძ_{\nu_\varphi}\right)_r\end{bmatrix}$

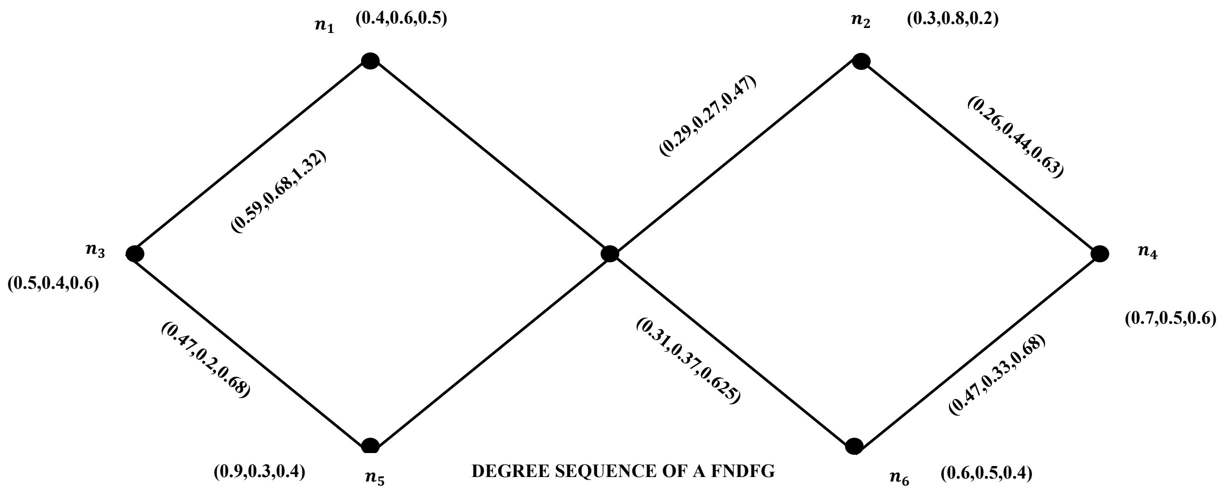
Definition: 3.5

The set of different positive real values arising in the degree sequence of a FNDFG is known as degree set.

Example: 3.3

Consider a graph $D^*=(\varphi_D,\varsigma_D)$, where $\varphi_D=\{n_1,n_2,n_3,n_4,n_5,n_6\}$ and $\varsigma_D=\{n_1n_3,n_1n_6,n_2n_5,n_2n_4,n_3n_5,n_4n_6\}$. Let $\alpha_{\varphi_D},\beta_{\mu_D},\gamma_{\nu_D}$ be fermatean Dombi fuzzy vertex set and fermatean Dombi fuzzy edge set defined on φ_D and ς_D respectively.

$$\varphi_D=\begin{bmatrix} \frac{n_1}{0.4} & \frac{n_2}{0.3} & \frac{n_3}{0.5} & \frac{n_4}{0.7} & \frac{n_5}{0.9} & \frac{n_6}{0.6} \\ \frac{n_1}{0.6} & \frac{n_2}{0.8} & \frac{n_3}{0.4} & \frac{n_4}{0.5} & \frac{n_5}{0.3} & \frac{n_6}{0.5} \\ \frac{n_1}{0.5} & \frac{n_2}{0.2} & \frac{n_3}{0.6} & \frac{n_4}{0.6} & \frac{n_5}{0.4} & \frac{n_6}{0.4} \end{bmatrix};\ \varsigma_D=\begin{bmatrix} \frac{n_1n_3}{0.28} & \frac{n_1n_6}{0.31} & \frac{n_2n_5}{0.29} & \frac{n_2n_4}{0.26} & \frac{n_3n_5}{0.47} & \frac{n_4n_6}{0.47} \\ \frac{n_1n_3}{0.31} & \frac{n_1n_6}{0.37} & \frac{n_2n_5}{0.27} & \frac{n_2n_4}{0.44} & \frac{n_3n_5}{0.2} & \frac{n_4n_6}{0.33} \\ \frac{n_1n_3}{0.7} & \frac{n_1n_6}{0.62} & \frac{n_2n_5}{0.47} & \frac{n_2n_4}{0.63} & \frac{n_3n_5}{0.68} & \frac{n_4n_6}{0.68} \end{bmatrix}$$



DEGREE SEQUENCE OF A FNDFG

The degree of the vertices are $ꝺ_{D_N}(n_1)$ =(0.59, 0.68, 1.32) ,$ꝺ_{D_N}(n_2)$ =(0.55,0.71,1.1), $ꝺ_{D_N}(n_3)$ =(0.75,0.51,1.38),$ꝺ_{D_N}(n_4)$ =(073,0.77,1.31),$ꝺ_{D_N}(n_5)$ =(0.76,0.47,1.15) ,$ꝺ_{D_N}(n_6)$ =(0.78,0.7,1.30).Hence the degree sequence of the membership grades, indeterminacy grades, non-membership grades are (0.78,0.76,0.75,0.73,0.59,0.55) , (0.77,0.71,0.7,0.68,0.51,0.47) and (1.38,1.32,1.31,1.30,1.15,1.11) whereas the corresponding degree sets are (0.78,0.76,0.75,0.73,0.59,0.55),(0.77,0.71,0.7,0.68,0.51,0.47)and (1.38,1.32,1.31,1.30,1.15,1.11)

Theorem: 3.5

Let D be a $(\text{Ŋ}, \Re, \Theta, £)$ strongly regular FNDFG. Then the degree sequence of n elements of D is a constant sequence $(\Re_{\varphi_{D,}}, \Re_{\varphi_{D,}} \dots\dots\dots, \Re_{\varphi_{D,}};\ \Re_{\mu_D}, \Re_{\mu_D}, \dots\dots, \Re_{\mu_D}; \Re_{\nu_D}, \Re_{\nu_D}, \dots\dots, \Re_{\nu_D})$.

Proof:

Assume that D is a $(\text{Ŋ}, \Re, \Theta, £)$ strongly regular FNDFG; then by definition, $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Thus, all the vertices have same degree $ꝺ_{D_N}(n_i) = (\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$, where i= 1, 2,……., n. Hence we conclude that the degree sequence of D is a constant sequence $(\Re_{\varphi_{D,}}, \Re_{\varphi_{D,}} \dots\dots\dots, \Re_{\varphi_{D,}};\ \Re_{\mu_D}, \Re_{\mu_D}, \dots\dots, \Re_{\mu_D}; \Re_{\nu_D}, \Re_{\nu_D}, \dots\dots, \Re_{\nu_D})$.

Theorem: 3.6

Let $D = (\text{Ŋ}, \Re, \Theta, £)$ strongly regular FNDFG. Then the degree set of membership and non-membership grades of D is a singleton set $\{\Re_{\varphi_{D,}}\}, \{\Re_{\mu_D}\}$ and $\{\Re_{\nu_D}\}$, respectively.

Proof:

Assume that D is a $(\text{Ŋ}, \Re, \Theta, £)$ strongly regular FNDFG; then by definition, D is a $(\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular. Thus, all the vertices have same degree $ꝺ_{D_N}(n_i) = (\Re_{\varphi_{D,}}, \Re_{\mu_D}, \Re_{\nu_D})$, where i= 1, 2, ……., n. As the degree sequence of D is a constant sequence $(\Re_{\varphi_{D,}}, \Re_{\varphi_{D,}} \dots\dots\dots, \Re_{\varphi_{D,}};\ \Re_{\mu_D}, \Re_{\mu_D}, \dots\dots, \Re_{\mu_D}; \Re_{\nu_D}, \Re_{\nu_D}, \dots\dots, \Re_{\nu_D})$, then the corresponding membership and non-membership degree set is $\{\Re_{\varphi_{D,}}\}, \{\Re_{\mu_D}\}$ and $\{\Re_{\nu_D}\}$, respectively.

Definition : 3.6

Let $\zeta_D = \{nt, \emptyset_{\zeta_D}(nt), \mu_{\zeta_D}(nt), \nu_{\zeta_D}(nt), nt \in \varsigma_D\}$ be a fermatean Dombi fuzzy edge set in FNDFG D. then

- The degree of an edge set nt $\in \varsigma_D$ is symbolized by

  $ꝺ_{D_N}(nt) = [ꝺ_{\emptyset_\varphi}(nt), ꝺ_{\mu_\varphi}(nt), ꝺ_{\nu_\varphi}(nt)]$, where

$$ძ_{\emptyset_\varphi}(nt) = \sum_{nq\in\varsigma_D, q\neq t}\emptyset_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\emptyset_{\zeta_D}(qt)$$
$$= ძ_{\emptyset_\varphi}(n) + ძ_{\emptyset_\varphi}(t) - 2ძ_{\emptyset_\varphi}(nt)$$
$$= ძ_{\emptyset_\varphi}(n) + ძ_{\emptyset_\varphi}(t) - 2\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) \quad ;$$
$$ძ_{\mu_\varphi}(nt) = \sum_{nq\in\varsigma_D, q\neq t}\mu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\mu_{\zeta_D}(qt)$$
$$= ძ_{\mu_\varphi}(n) + ძ_{\mu_\varphi}(t) - 2ძ_{\mu_\varphi}(nt)$$
$$= ძ_{\mu_\varphi}(n) + ძ_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) \quad ;$$

$$ძ_{\nu_\varphi}(nt) = \sum_{nq\in\varsigma_D, q\neq t}\nu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\nu_{\zeta_D}(qt)$$
$$= ძ_{\nu_\varphi}(n) + ძ_{\nu_\varphi}(t) - 2ძ_{\nu_\varphi}(nt)$$
$$= ძ_{\nu_\varphi}(n) + ძ_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right)$$

The total degree of an edge nt $\in \varsigma_D$ is symbolises by

the totally regular of degree $(\Im_{\varphi_D}, \Im_{\mu_D}, \Im_{\nu_D})$. If

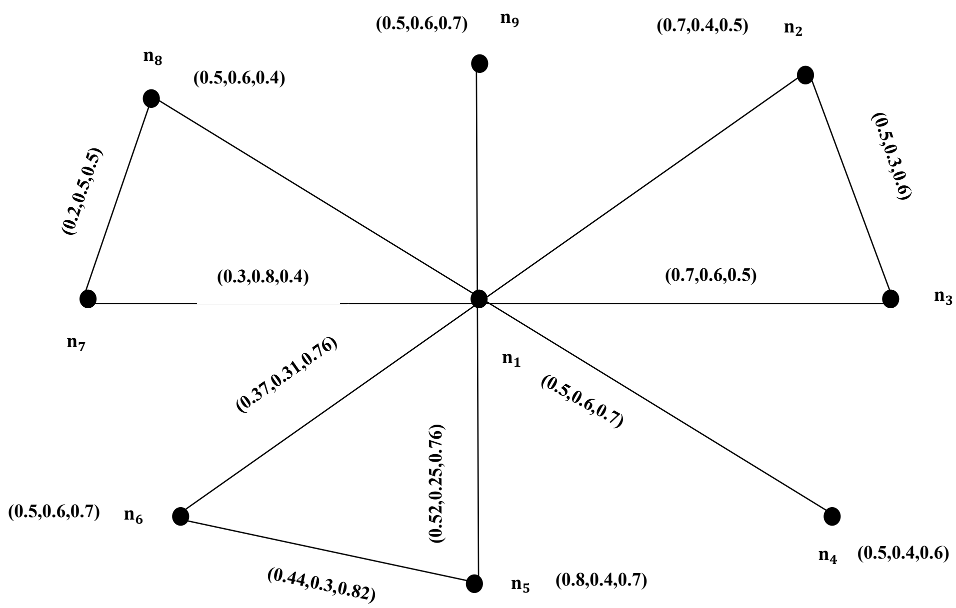
$$Ŧ_{\varphi_D}(nt) = \sum_{nq\in\varsigma_D, q\neq t}\emptyset_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\emptyset_{\zeta_D}(qt) + \emptyset_{\zeta_D}(nq)$$
$$= ძ_{\emptyset_\varphi}(n) + ძ_{\emptyset_\varphi}(t) - ძ_{\emptyset_\varphi}(nt)$$
$$= ძ_{\emptyset_\varphi}(n) + ძ_{\emptyset_\varphi}(t) - \left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) \quad ;$$
$$Ŧ_{\mu_D}(nt) = \sum_{nq\in\varsigma_D, q\neq t}\mu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\mu_{\zeta_D}(qt) + \mu_{\zeta_D}(nq)$$
$$= ძ_{\mu_\varphi}(n) + ძ_{\mu_\varphi}(t) - ძ_{\mu_\varphi}(nt)$$
$$= ძ_{\mu_\varphi}(n) + ძ_{\mu_\varphi}(t) - \left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) \quad ;$$
$$Ŧ_{\nu_D}(n) = \sum_{nq\in\varsigma_D, q\neq t}\nu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t}\nu_{\zeta_D}(qt) + + \nu_{\zeta_D}(nq)$$
$$= ძ_{\nu_\varphi}(n) + ძ_{\nu_\varphi}(t) - ძ_{\nu_\varphi}(nt)$$
$$= ძ_{\nu_\varphi}(n) + ძ_{\nu_\varphi}(t) - \left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right)$$

Example: 3.4

Consider a FNDFG over $\varphi_D=\{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. Then (i) the degree of an edge set $n_5 n_6$ is ,$ძ_{D_N}(n_5 n_6) = [ძ_{\emptyset_\varphi}(nt), ძ_{\mu_\varphi}(nt), ძ_{\nu_\varphi}(nt)]$;

$ძ_{\emptyset_\varphi}(n_5 n_6) = ძ_{\emptyset_\varphi}(n_5) + ძ_{\emptyset_\varphi}(n_6) - 2ძ_{\emptyset_\varphi}(n_5 n_6) = 0.89$
$ძ_{\mu_\varphi}(n_5 n_6) = ძ_{\mu_\varphi}(n_5) + ძ_{\mu_\varphi}(n_6) - 2\, ძ_{\mu_\varphi}(n_5 n_6) = 0.56$ $ძ_{\nu_\varphi}(n_5 n_6) = ძ_{\nu_\varphi}(n_5) + ძ_{\nu_\varphi}(n_6) - 2\, ძ_{\nu_\varphi}(n_5 n_6) = 1.52$

(i) the total degree of an edge set $n_5 n_6$ is , $Ŧძ_{D_N}(n_5 n_6) = [Ŧ_{\varphi_D}(n_5 n_6), Ŧ_{\mu_D}(n_5 n_6), Ŧ_{\mu_D}(n_5 n_6)]$; $Ŧ_{\varphi_D}(n_5 n_6) = ძ_{\emptyset_\varphi}(n_5) + ძ_{\emptyset_\varphi}(n_6) - ძ_{\emptyset_\varphi}(n_5 n_6) = 1.33$ $Ŧ_{\mu_D}(n_5 n_6) = ძ_{\mu_\varphi}(n_5) + ძ_{\mu_\varphi}(n_6) - ძ_{\mu_\varphi}(n_5 n_6) = 0.87$
$Ŧ_{\nu_D}(n_5 n_6) = ძ_{\nu_\varphi}(n_5) + ძ_{\nu_\varphi}(n_6) - ძ_{\nu_\varphi}(n_5 n_6) = 2.34$ i.e)) $ძ_{D_N}(n_5 n_6) = [0.89, 0.56, 1.52]$ and $Ŧძ_{D_N}(n_5 n_6) = [1.33, 0.87, 2.34]$



FERMATEANDOMBI FUZZY EDGE SET IN FNDFG

Theorem 3.7:

Let (φ_D, ς_D) be a strongly regular FNDFG with $\emptyset_{\zeta_D}$ (nt), μ_{ζ_D} (nt),ν_{ζ_D} (nt) as constant functions. then the edge degree sequence and edge degree set are constant sequence and singleton set, respectively.

Proof:

Assume that $\emptyset_{\zeta_D}$ (nt) = C_1, μ_{ζ_D} (nt) = C_2,ν_{ζ_D} (nt)= C_3. Also, suppose that D is a strongly regular FNDFG. Then by definition, D is $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ -regular such that

$$d_{\emptyset_\varphi}(n) = \sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)} = \Re_{\varphi_D},$$

$$d_{\emptyset_\mu}(n) = \sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} = \Re_{\mu_D},$$

$$d_{\emptyset_\nu}(n) = \sum_{n,t\neq n\in\varphi} \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)} = \Re_{\mu_D}$$

Therefore, the degree of edge is $d_{D_N}(nt)$= $[d_{\emptyset_\varphi}(nt), d_{\mu_\varphi}(nt), d_{\nu_\varphi}(nt)]$, where as

$d_{\emptyset_\varphi}(nt) = d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - d_{\emptyset_\varphi}(nt) =$
$d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - 2\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right)$;

$= \Re_{\varphi_D} + \Re_{\varphi_D} - 2C_1 = 2(\Re_{\varphi_D} - C_1) = \Re_{\varphi_D}$

$d_{\mu_\varphi}(nt) = \sum_{nq\in\varsigma_D, q\neq t} \mu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t} \mu_{\zeta_D}(qt) + \mu_{\zeta_D}(nq)$

$= d_{\mu_\varphi}(n) + d_{\mu_\varphi}(t) - d_{\mu_\varphi}(nt) = d_{\mu_\varphi}(n)$
$+ d_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right)$

$= \Re_{\mu_D} + \Re_{\mu_D} - 2C_2 = 2(\Re_{\mu_D} - C_2) = \Re_{\mu_D}$

$d_{\nu_\varphi}(nt) = \sum_{nq\in\varsigma_D, q\neq t} \nu_{\zeta_D}(nq) + \sum_{nq\in\varsigma_D, q\neq t} \nu_{\zeta_D}(qt) + + \nu_{\zeta_D}(nq)$

$= d_{\nu_\varphi}(n) + d_{\nu_\varphi}(t) - d_{\nu_\varphi}(nt) = d_{\nu_\varphi}(n)$
$+ d_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right)$

$= \Re_{\nu_{\varphi_D}} + \Re_{\nu_{\varphi_D}} - 2C_3 = 2\,(\Re_{\nu_{\varphi_D}} - 2C_3) = \Re_{\nu_{\varphi_D}}$

Hence the edge degree sequence of membership , indeterminacy and non-membership grades are constant sequence and its corresponding edge degree set $\{2\,(\Re_{\varphi_D} - C_1)\}$, $\{2\,(\Re_{\mu_D} - C_2)\}$ and $\{2\,(\Re_{\nu_{\varphi_D}} - 2C_3)\}$ are singleton sets.

Definition: 3.7

A FNDFG $D = (\text{Ŋ}, \Re, \Theta, \pounds)$ on crisp graph $D^* = (\varphi_D, \varsigma_D)$ is said to be edge regular of degree $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ or $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ – edge regular. If

$$d_{\emptyset_\varphi}(nt) = d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - 2d_{\emptyset_\varphi}(nt)$$

$$= d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - 2\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) = \Re_{\varphi_D}$$

$$d_{\mu_\varphi}(nt) = d_{\mu_\varphi}(n) + d_{\mu_\varphi}(t) - 2\, d_{\mu_\varphi}(nt)$$

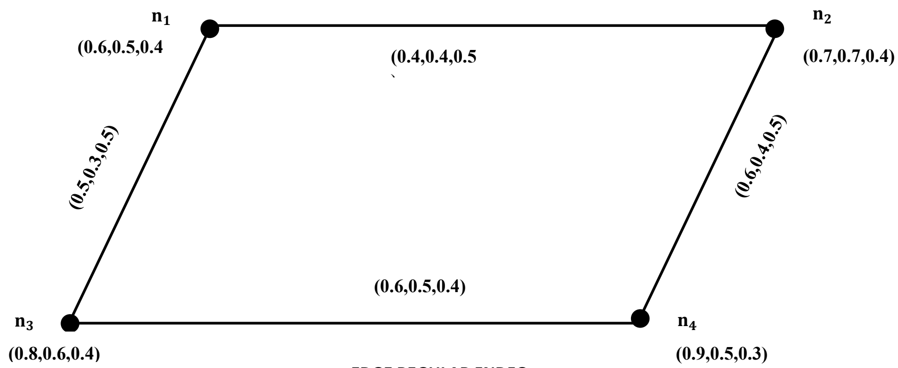
$$= d_{\mu_\varphi}(n) + d_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) = \Re_{\mu_D}$$

$$d_{\mu_\varphi}(nt) = d_{\nu_\varphi}(n) + d_{\nu_\varphi}(t) - 2\, d_{\nu_\varphi}(nt)$$

$$= d_{\nu_\varphi}(n) + d_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) = \Re_{\nu_{\varphi_D}}$$

for all nt ∈ ς_D E**xample: 3.5**

Consider a graph $D^* = (\varphi_D, \varsigma_D)$ where where $\varphi_D = \{n_1, n_2, n_3, n_4\}$ and $\varsigma_D = \{n_1n_2, n_1n_3, n_3n_4, n_2n_4\}$. Let $\alpha_{\varphi_D}, \beta_{\mu_D}, \gamma_{\nu_D}$ be fermatean Dombi fuzzy vertex set and fermatean Dombi fuzzy edge set defined on φ_D *and* ς_D respectively.



EDGE REGULAR FNDFG

$$\varphi_D = \begin{bmatrix} \frac{n_1}{0.6} & \frac{n_2}{0.7} & \frac{n_3}{0.8} & \frac{n_4}{0.9} \\ \frac{n_1}{0.5} & \frac{n_2}{0.7} & \frac{n_3}{0.6} & \frac{n_4}{0.5} \\ \frac{n_1}{0.4} & \frac{n_2}{0.4} & \frac{n_3}{0.3} & \frac{n_4}{0.2} \end{bmatrix}; \ \varsigma_D = \begin{bmatrix} \frac{n_1n_2}{0.47} & \frac{n_1n_3}{0.52} & \frac{n_3n_4}{0.73} & \frac{n_2n_4}{0.64} \\ \frac{n_1n_3}{0.41} & \frac{n_1n_3}{0.37} & \frac{n_3n_4}{0.37} & \frac{n_2n_4}{0.41} \\ \frac{n_1n_3}{0.57} & \frac{n_1n_3}{0.52} & \frac{n_3n_4}{0.40} & \frac{n_2n_4}{0.15} \end{bmatrix}$$

Since the degree of each edge $d_{D_N}(nt) = [d_{\emptyset_\varphi}(nt), d_{\mu_\varphi}(nt), d_{\nu_\varphi}(nt)]$ =(1.1,0.7,1) for all nt $= 1,2,3,4$ in D^* is (1.1,0.7,1) edge regular FNDFG.

Definition : 3.8

A FNDFG $D = (\text{Ŋ}, \Re, \Theta, \pounds)$ on crisp graph $D^* = (\varphi_D, \varsigma_D)$ is said to be totally edge regular of degree $(\Im_{\varphi_D}, \Im_{\mu_D}, \Im_{\nu_D})$. or $(\Im_{\varphi_D}, \Im_{\mu_D}, \Im_{\nu_D})$ totally edge regular. If

$$\text{Ŧ}_{\varphi_D}(nt) = d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - d_{\emptyset_\varphi}(nt)$$

$$= d_{\emptyset_\varphi}(n) + d_{\emptyset_\varphi}(t) - \left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) = \Im_{\varphi_D,} \quad ;$$

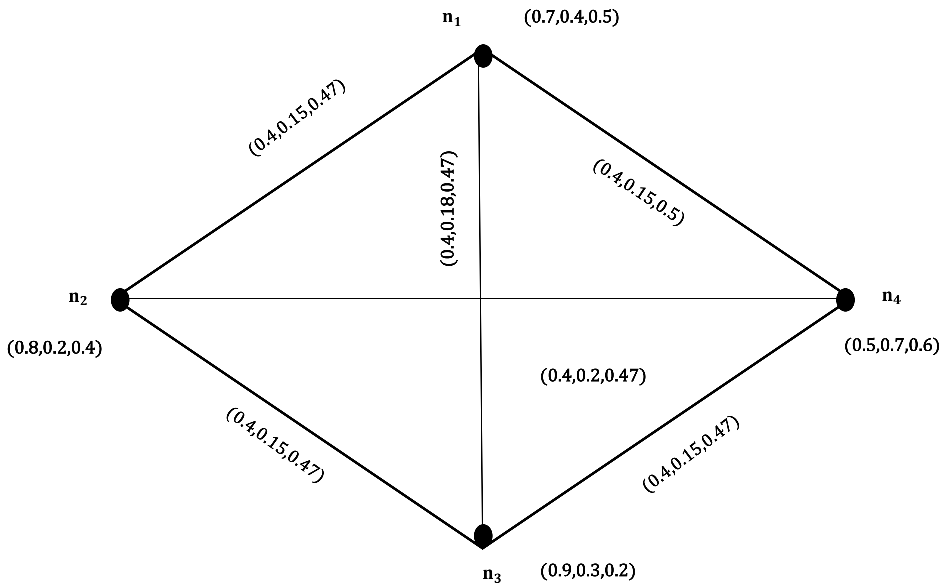
$$\text{Ŧ}_{\mu_D}(nt) = d_{\mu_\varphi}(n) + d_{\mu_\varphi}(t) - d_{\mu_\varphi}(nt)$$

$$= d_{\mu_\varphi}(n) + d_{\mu_\varphi}(t) - \left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) = \Im_{\mu_\varphi} \quad ;$$

$\text{Ŧ}_{\nu_D}(n) = \text{ժ}_{\nu_\varphi}(n) + \text{ժ}_{\nu_\varphi}(t) - \text{ժ}_{\nu_\varphi}(nt)$

$= \text{ժ}_{\nu_\varphi}(n) + \text{ժ}_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) = \Im_{\nu_D}$ for all nt $\in \varsigma_D$. **Example: 3.6**

Consider a graph $D^* = (\varphi_D, \varsigma_D)$ where where $\varphi_D = \{n_1, n_2, n_3, n_4\}$ and $\varsigma_D = \{n_1n_2, n_1n_3, n_1n_4, n_2n_3, n_2n_4, n_3n_4\}$. Let $\alpha_{\varphi_D}, \beta_{\mu_D}, \gamma_{\nu_D}$ be fermatean Dombi fuzzy vertex set and fermatean Dombi fuzzy edge set defined on φ_D and ς_D respectively.



TOTALLY EDGE REGULAR FNDFG

$$\varphi_D = \begin{bmatrix} \frac{n_1}{0.7} & \frac{n_2}{0.8} & \frac{n_3}{0.9} & \frac{n_4}{0.5} \\ \frac{n_1}{0.4} & \frac{n_2}{0.2} & \frac{n_3}{0.3} & \frac{n_4}{0.7} \\ \frac{n_1}{0.5} & \frac{n_2}{0.4} & \frac{n_3}{0.2} & \frac{n_4}{0.6} \end{bmatrix}; \varsigma_D = \begin{bmatrix} \frac{n_1n_2}{0.4} & \frac{n_1n_3}{0.4} & \frac{n_1n_4}{0.40} & \frac{n_2n_3}{0.40} & \frac{n_2n_4}{0.4} & \frac{n_3n_4}{0.4} \\ \frac{n_1n_3}{0.15} & \frac{n_1n_3}{0.2} & \frac{n_1n_4}{0.15} & \frac{n_2n_3}{0.15} & \frac{n_2n_4}{0.18} & \frac{n_3n_4}{0.2} \\ \frac{n_1n_3}{0.47} & \frac{n_1n_3}{0.47} & \frac{n_1n_4}{0.50} & \frac{n_2n_3}{0.47} & \frac{n_2n_4}{0.47} & \frac{n_3n_4}{0.47} \end{bmatrix}$$

Since the total degree of each edge $\text{Ŧ}\text{ժ}_{D_N}(nt) = [\text{ժ}_{\emptyset_\varphi}(nt), \text{ժ}_{\mu_\varphi}(nt), \text{ժ}_{\nu_\varphi}(nt)] = (2,0.85,2.35)$ for all nt $= 1,2,3,4$ in D^* is (1.1,0.7,1) totally edge regular FNDFG.

Theorem: 3.8

Let $D = (\varphi_D, \varsigma_D)$ be a $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$- regular FNDFG such that $\text{ժ}_{\emptyset_\varphi}(nt), \text{ժ}_{\mu_\varphi}(nt), \text{ժ}_{\nu_\varphi}(nt)]$ are constant functions then D is $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ – edge regular FNDFG.

Proof:

Assume that D is a $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG such that

$$\text{ժ}_{\emptyset_\varphi}(n) = \sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)} = \Re_{\varphi_D},$$

$$\text{ժ}_{\emptyset_\mu}(n) = \sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)} = \Re_{\mu_D},$$

$$\text{ժ}_{\emptyset_\nu}(n) = \sum_{n,t\neq n\in\varphi} \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)} = \Re_{\mu_D}$$

As $\emptyset_{\zeta_D}$ (nt), μ_{ζ_D} (nt), ν_{ζ_D} (nt) are constant functions, $\emptyset_{\zeta_D}$ (nt) = C_1, μ_{ζ_D} (nt) = C_2, ν_{ζ_D} (nt) = C_3 for all nt $\in \varsigma_D$. By using the definition of edge degree $\text{ժ}_{D_N} = [\text{ժ}_{\emptyset_\varphi}(nt), \text{ժ}_{\mu_\varphi}(nt), \text{ժ}_{\nu_\varphi}(nt)]$, where as

$$\text{ժ}_{\emptyset_\varphi}(nt) = \text{ժ}_{\emptyset_\varphi}(n) + \text{ժ}_{\emptyset_\varphi}(t) - \text{ժ}_{\emptyset_\varphi}(nt)$$

$$= \text{ժ}_{\emptyset_\varphi}(n) + \text{ժ}_{\emptyset_\varphi}(t) - 2\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right);$$

$= \Re_{\varphi_D} + \Re_{\varphi_D} - 2C_1 = 2\,(\Re_{\varphi_D} - C_1) = \Re_{\varphi_D}$

$ɗ_{\mu_\varphi}(nt) = ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - ɗ_{\mu_\varphi}(nt)$

$= ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right)$;

$= \Re_{\mu_D} + \Re_{\mu_D} - 2C_2 = 2\,(\Re_{\mu_D} - C_2) = \Re_{\mu_D}$

$ɗ_{\nu_\varphi}(nt) = ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - ɗ_{\nu_\varphi}(nt)$

$= ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right)$

$= \Re_{\nu_{\varphi_D}} + \Re_{\nu_{\varphi_D}} - 2C_3 = 2\,(\Re_{\nu_{\varphi_D}} - 2C_3) = \Re_{\nu_{\varphi_D}}$

Hence, we conclude that D is a $\left(\Re_{\varphi_D,}, \Re_{\mu_D}, \Re_{\nu_D}\right)$ – edge regular FNDFG.

Theorem:3.9

Let $D = (\varphi_D, \varsigma_D)$ be a $\left(\Re_{\varphi_D,}, \Re_{\mu_D}, \Re_{\nu_D}\right)$ – edge regular $(\Im_{\varphi_D,}, \Im_{\mu_D}, \Im_{\nu_D})$ – totally edge regular FNDFG then $\emptyset_{\zeta_D}$ (nt), μ_{ζ_D} (nt) and ν_{ζ_D} (nt) are constant functions.

Proof:

Assume that D is a $\left(\Re_{\varphi_D,}, \Re_{\mu_D}, \Re_{\nu_D}\right)$ – edge regular and $(\Im_{\varphi_D,}, \Im_{\mu_D}, \Im_{\nu_D})$ – totally edge regular FNDFG. Then the degree of edge is

$ɗ_{\emptyset_\varphi}(nt) = ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - ɗ_{\emptyset_\varphi}(nt)$

$= ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - 2\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) = \Re_{\varphi_D}$

$ɗ_{\mu_\varphi}(nt) = ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - ɗ_{\mu_\varphi}(nt)$

$= ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) = \Re_{\mu_D}$

$ɗ_{\nu_\varphi}(nt) = ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - ɗ_{\nu_\varphi}(nt)$

$= ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) = \Re_{\nu_{\varphi_D}}$

And the total degree of edge is

$Ŧ_{\varphi_D}(nt) = ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - ɗ_{\emptyset_\varphi}(nt)$

$= ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - \left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) = \Im_{\varphi_D,}$

$Ŧ_{\mu_D}(nt) = ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - ɗ_{\mu_\varphi}(nt)$

$= ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - \left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) = \Im_{\mu_D}$

$Ŧ_{\nu_D}$ (nt) $= ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - ɗ_{\nu_\varphi}(nt)$

$= ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - \left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) = \Im_{\nu_D}$

Further it follows that (i) for membership grade,

$Ŧ_{\varphi_D}(nt) = \Im_{\varphi_D}$

$= ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - ɗ_{\emptyset_\varphi}(nt)$

$ɗ_{\emptyset_\varphi}(n) + ɗ_{\emptyset_\varphi}(t) - \left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right) = \Im_{\varphi_D}$

$ɗ_{\emptyset_\varphi}(n)$+$ɗ_{\emptyset_\varphi}(t)$-2 $\left(\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}\right)$+$ɗ_{\emptyset_\varphi}(nt)$ =$\Im_{\varphi_D}$ $Ŧ_{\varphi_D}(nt) = \Im_{\varphi_D} - \Re_{\varphi_D}$

(ii) for indeterminacy grade,

$Ŧ_{\mu_D}(nt) = \Im_{\mu_D}$

$= ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - ɗ_{\mu_\varphi}(nt)$

$ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - \left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) = \Im_{\mu_D}$

$ɗ_{\mu_\varphi}(n) + ɗ_{\mu_\varphi}(t) - 2\left(\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}\right) + ɗ_{\mu_\varphi}(nt) = \Im_{\mu_D}$

$Ŧ_{\mu_D}(nt) = \Im_{\mu_D} - \Re_{\mu_D}$

(iii) for non-membership grade,

$Ŧ_{\nu_D}$ (nt) = $\Im_{\nu_D}$

$= ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - ɗ_{\nu_\varphi}(nt)$

$ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - \left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) = \Im_{\nu_D}$

$ɗ_{\nu_\varphi}(n) + ɗ_{\nu_\varphi}(t) - 2\left(\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}\right) + ɗ_{\nu_\varphi}(nt) = \Im_{\nu_D}$

$Ŧ_{\nu_D}(nt) = \Im_{\nu_D} - \Re_{\nu_{\varphi_D}}$

Hence we conclude that $Ŧ_{\varphi_D}(nt) = \Im_{\varphi_D} - \Re_{\varphi_D}$, $Ŧ_{\mu_D}(nt) = \Im_{\mu_D} - \Re_{\mu_D}$ and $Ŧ_{\nu_D}(nt) = \Im_{\nu_D} - \Re_{\nu_{\varphi_D}}$ are constant functions.

Theorem :3.10

Let $D = (\varphi_D, \varsigma_D)$ be a FNDFG on a regular crisp graph $D^* = (\varphi_D, \varsigma_D)$ then $\emptyset_{\zeta_D}(n_in_j)$, $\mu_{\zeta_D}(n_in_j)$ and $\nu_{\zeta_D}(n_in_j)$ are constant functions if and only if $D = (\varphi_D, \varsigma_D)$ is both a $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ – edge regular $(\Im_{\varphi_D}, \Im_{\mu_D}, \Im_{\nu_D})$ – totally edge regular FNDFG.

Proof:

Assume that $D = (\varphi_D, \varsigma_D)$ is a FNDFG on a regular crisp graph D^*. Also suppose that $\emptyset_{\zeta_D}(n_in_j)$, $\mu_{\zeta_D}(n_in_j)$ and $\nu_{\zeta_D}(n_in_j)$ are constant functions $\emptyset_{\zeta_D}(n_in_j) = C_1$, $\mu_{\zeta_D}(n_in_j) = C_2$, $\nu_{\zeta_D}(n_in_j) = C_3$ for all $n_in_j \in \varsigma_D$. By using the definition of vertex degree $ɗ_{\emptyset_\varphi}(n_in_j), ɗ_{\mu_\varphi}(n_in_j), ɗ_{\nu_\varphi}(n_in_j)$, we must have

$$ɗ_{\emptyset_\varphi}(n_j) = \sum_{n_i,n_j \neq n \in \varphi} \frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(n_j)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}$$

$$=\sum_{n_i,n_j \in \zeta_D} \alpha_{\varphi_{D_\varsigma}}(n_in_j) = \sum_{n_i,n_j \in \zeta_D} \alpha_{\varphi_{D_\varsigma}}(C_1) = \Re\, C_1 = \Re_{\varphi_D}$$

$$ɗ_{\emptyset_\mu}(n_j) = \sum_{n_i,n_j \neq n \in \varphi} \frac{\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}{\beta_{\mu_D}(n_i)+\beta_{\mu_D}(n_j)-\beta_{\mu_D}(n_i)\beta_{\mu_D}(n_j)}$$

$$= \sum_{n_i,n_j \in \zeta_D} \mu_{\varphi_{D_\varsigma}}(n_in_j) = \sum_{n_i,n_j \in \zeta_D} \beta_{\mu_{D_\varsigma}}(C_2) = \Re\, C_2 = \Re_{\mu_D}$$

$$ɗ_{\emptyset_\nu}(n_j) = \sum_{n_i,n_j \neq n \in \varphi} \frac{\gamma_{\nu_D}(n_i)+\gamma_{\nu_D}(n_j)-2\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}{1-\gamma_{\nu_D}(n_i)\gamma_{\nu_D}(n_j)}$$

$$= \sum_{n_i,n_j \in \zeta_D} \gamma_{\varphi_{D_\varsigma}}(n_in_j) = \sum_{n_i,n_j \in \zeta_D} \gamma_{\varphi_{D_\varsigma}}(C_3) = \Re\, C_3 == \Re_{\nu_{\varphi_D}}$$

Therefore, $D^* = (\varphi_D, \varsigma_D)$ is a $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ – regular FNDFG. Further , by definition of total degree of edge, we have

$$Ŧ_{\varphi_D}(n_in_j) = \sum_{n_in_l \in \varsigma_D, i \neq l} \emptyset_{\zeta_D}(n_in_l) + \sum_{n_ln_j \in \varsigma_D, l \neq j} j_{\zeta_D}(n_ln_j) + \emptyset_{\zeta_D}(n_in_j)$$

$=\sum_{n_i n_l \in \varsigma_D}(C_1)+\sum_{n_l n_j \in \varsigma_D, l\neq j}(C_1)+(C_1)$

$= C_1(\mathrm{R}-1) + C_1(\mathrm{R}-1) + C_1 = C_1\,(2\mathrm{R}-1) = \Im_{\varphi_D}$ For all $n_i n_j \in \varsigma_D$

In the similar case , we can easily prove that $Ŧ_{\mu_D}(n_i n_j) = \Im_{\mu_D}$ For all $n_i n_j \in \varsigma_D$ and $Ŧ_{\nu_D}(n_i n_j) = \Im_{\nu_D}$ For all $n_i n_j \in \varsigma_D$. Hence we conclude that $D^*=(\varphi_D,\varsigma_D)$ is both $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ – regular and $(\Im_{\varphi_D},\Im_{\mu_D},\Im_{\nu_D})$ – totally edge regular FNDFG.

Conversely, assume that D^* is both $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ – regular and $(\Im_{\varphi_D},\Im_{\mu_D},\Im_{\nu_D})$ – totally edge regular FNDFG. To prove that $\emptyset_{\zeta_D}(n_i n_j)$, $\mu_{\zeta_D}(n_i n_j)$ and $\nu_{\zeta_D}(n_i n_j)$ are constant functions, consider the definition of total degree of edge, we have

$$Ŧ_{\varphi_D}(n_i n_j) = ɗ_{\emptyset_\varphi}(n_i) + ɗ_{\emptyset_\varphi}(n_j) - ɗ_{\emptyset_\varphi}(n_i n_j)$$

$$= ɗ_{\emptyset_\varphi}(n_i) + ɗ_{\emptyset_\varphi}(n_j) - \left(\frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}\right)$$

$$\Im_{\varphi_D} = \Re_{\varphi_D} + \Re_{\varphi_D} - \left(\frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}\right)$$

$Ŧ_{\varphi_D}(n_i n_j) = 2\,\Re_{\varphi_D} - \Im_{\varphi_D}$ For all $n_i n_j \in \varsigma_D$

In the similar case , we can easily prove that $Ŧ_{\mu_D}(n_i n_j) = 2\,\Re_{\mu_D} - \Im_{\mu_D}$ For all $n_i n_j \in \varsigma_D$ and $Ŧ_{\nu_D}(n_i n_j) = 2\Re_{\nu_{\varphi_D}} - \Im_{\nu_D}$ For all $n_i n_j \in \varsigma_D$. Hence, we conclude that $\emptyset_{\zeta_D}(n_i n_j)$, $\mu_{\zeta_D}(n_i n_j)$ and $\nu_{\zeta_D}(n_i n_j)$ are constant functions of FNDFG.

Theorem: 3.11

Let $D = (\varphi_D,\varsigma_D)$ be a FNDFG on a crisp graph $D^*=(\varphi_D,\varsigma_D)$ .If $\emptyset_{\zeta_D}(n_i n_j)$, $\mu_{\zeta_D}(n_i n_j)$ and $\nu_{\zeta_D}(n_i n_j)$ are constant functions, then $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ – edge regular if and only if D^* is an edge regular graph**.**

Proof:

Suppose that $\emptyset_{\zeta_D}(n_i n_j)$, $\mu_{\zeta_D}(n_i n_j)$ and $\nu_{\zeta_D}(n_i n_j)$ are constant functions such that $\emptyset_{\zeta_D}(n_i n_j) = C_1$, $\mu_{\zeta_D}(n_i n_j) = C_2$, $\nu_{\zeta_D}(n_i n_j) = C_3$ for all $n_i n_j \in \varsigma_D$. Assume that D^* is $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ – edge regular. To prove that D^* is an edge regular graph, we suppose on contrary that D^* is not an edge regular graph. i.e) $D^*(n_i n_j) \neq)$ $D^*(n_k n_l)$ for atleast one pair of $n_i n_j$, $n_k n_l \in \varsigma_D$.By definition of degree of edge of FNDFG , we have

$$ɗ_{\emptyset_\varphi}(n_i n_j) = ɗ_{\emptyset_\varphi}(n_i) + ɗ_{\emptyset_\varphi}(n_j) - ɗ_{\emptyset_\varphi}(n_i n_j)$$

$$= ɗ_{\emptyset_\varphi}(n_i) + ɗ_{\emptyset_\varphi}(n_j) - 2\left(\frac{\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}{\alpha_{\varphi_D}(n_i)+\alpha_{\varphi_D}(n_j)-\alpha_{\varphi_D}(n_i)\alpha_{\varphi_D}(n_j)}\right)$$

$$= \sum_{n_i n_l \in \varsigma_D, i\neq l}\emptyset_{\zeta_D}(n_i n_l) + \sum_{n_l n_j \in \varsigma_D, l\neq j} j_{\zeta_D}(n_l n_j) - 2\,\emptyset_{\zeta_D}(n_i n_j)$$

$$= \sum_{n_i n_l \in \varsigma_D}(C_1) + \sum_{n_l n_j \in \varsigma_D, l\neq j}(C_1) - 2\,(C_1)$$

$$= ɗ_{D^*\emptyset_\varphi}(n_i)C_1 + ɗ_{D^*\emptyset_\varphi}(n_j)C_1 - 2C_1$$

$= C_1(ɗ_{D^*\emptyset_\varphi}(n_i) + ɗ_{D^*\emptyset_\varphi}(n_j) - 2) = C_1 ɗ_{D^*\emptyset_\varphi}(n_i n_j)$.

Likewise, we can easily prove that $ɗ_{\mu_D}(n_i n_j) = C_2 ɗ_{D^*\mu_D}(n_i n_j)$ and $ɗ_{\nu_D}(n_i n_j) = C_3 ɗ_{D^*\nu_D}(n_i n_j)$ For all $n_i n_j \in \varsigma_D$.Therefore, $ɗD_{\emptyset_\varphi}(n_i n_j) =[\, C_1 ɗ_{D^*\emptyset_\varphi}(n_i n_j),\, C_2 ɗ_{D^*\mu_D}(n_i n_j),\, C_3 ɗ_{D^*\nu_D}(n_i n_j)\,]$ and $ɗD_{\emptyset_\varphi}(n_k n_l) =[\, C_1 ɗ_{D^*\emptyset_\varphi}(n_k n_l),\, C_2 ɗ_{D^*\mu_D}(n_k n_l),\, C_3 ɗ_{D^*\nu_D}(n_k n_l)\,]$ .As $ɗD^*_{\emptyset_\varphi}(n_i n_j) \neq ɗD^*_{\emptyset_\varphi}(n_k n_l)$, thus $ɗD_{\emptyset_\varphi}(n_i n_j) \neq ɗD_{\emptyset_\varphi}(n_k n_l)$. Therefore , D is not $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ edge regular FNDFG, a contradiction. Hence we conclude that D^* is an edge regular graph.

Conversely, assume that G^* is an edge regular graph. To prove that D is a $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ edge regular FNDFG. we suppose on the contrary that D is not $(\Re_{\varphi_D},\Re_{\mu_D},\Re_{\nu_D})$ edge regular FNDFG. i.e) $ɗD_{\emptyset_\varphi}(n_i n_j) \neq ɗD_{\emptyset_\varphi}(n_k n_l)$ for atleast one pair of $n_i n_j, n_k n_l \in \varsigma_D$, $[\, ɗ_{D^*\emptyset_\varphi}(n_i n_j),\, ɗ_{D^*\mu_D}(n_i n_j),\, ɗ_{D^*\nu_D}(n_i n_j)\,] \neq [\, ɗ_{D^*\emptyset_\varphi}(n_k n_l),\, ɗ_{D^*\mu_D}(n_k n_l),\, ɗ_{D^*\nu_D}(n_k n_l)\,]$. Now $ɗD_{\emptyset_\varphi}(n_i n_j) \neq ɗD_{\emptyset_\varphi}(n_k n_l)$ implies that

$[\sum_{n_i n_m \in \varsigma_D, i\neq m} \emptyset_{\zeta_D}(n_i n_m) + \sum_{n_m n_j \in \varsigma_D, m\neq j} j_{\zeta_D}(n_m n_j)] \neq [\sum_{n_o n_k \in \varsigma_D, o\neq k} \emptyset_{\zeta_D}(n_o n_k) + \sum_{n_o n_l \in \varsigma_D, o\neq l} j_{\zeta_D}(n_o n_l)$
]

$\emptyset_{\zeta_D}(n_i n_j)$ is a constant function, we have $ძD^*_{\emptyset_\varphi}(n_i n_j) \neq ძD^*_{\emptyset_\varphi}(n_k n_l)$ a contradiction. Hence we conclude that D is a $(\Re_{\varphi_D}, \Re_{\mu_D}, \Re_{\nu_D})$ edge regular FNDFG.

4. Conclusion:

In this work, a new class of strongly regular fermatean neutrosophic fuzzy dombi operators has been developed to improve aggregation under uncertainty. The operators were shown to satisfy important regularity properties, guaranteeing reliable and consistent results. Their adaptability makes them suitable for complex decision-making problems. The proposed approach offers a meaningful extension to existing models and opens avenues for further research in advanced neutrosophic frame works.

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