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Article

ON THE BOOLEAN FUNCTION GRAPH $\overline{B_P}$,*L(G)*,*INC*,*NINC* OF A GRAPH

R. Mahalakshmi[1], N. Indira[2], S. Gayathri[3], B. Manjuladevi[4], M. G. Geena[5]*

[1]Department of Mathematics, Pavendar Bharathidasan College of Engineering and Technology (A Unit of SRM Group of Institutions), Pudukkottai-620024.
Email: mahabarath2013@gmail.com
[2]St. Joseph's College of Engineering and Technology, Thanjavur-613403.
Email: jeevigobi501@gmail.com
[3]Department of Artificial Intelligence and Machine Learning, K. Ramakrishnan College of Engineering, Samayapuram, Trichy-621112.
Email: gaya3thri24@gmail.com
[4]Datta Meghe College of Engineering, Sector 3, Airoli, Navi, Mumbai-400708.
Email: b.manjula@gmail.com
[5] Department of Agriculture Engineering, Dhanalakshmi Srinivasan University, Tiruchirappalli-621112.
Email: geenamg.set@dsuniversity.ac.in
Corresponding Author: R. Mahalakshmi

Abstract: For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{K_P}, L(G), INC, NINC)$ of G is a graph with vertex set V(G) ∪ E(G), and two vertices in $B(\overline{K_P, L(G)}, INC, NINC)$ are adjacent if and only if they correspond to two adjacent edges of G or to a vertex and an edge incident to it in G, or to a vertex and an edge not incident to it in G, where L(G) is the line graph of G. For brevity, this graph is denoted by $BF_3(G)$.
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1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A clique partition of G is a collection C of cliques such that each edge of G occurs in exactly one clique in C. The clique partition number cp(G) is the minimum size of a clique partition of G.

The Line graphs, Middle graphs, Total graphs and Quasi-total graphs are very much useful in computer networks.

Whitney[15] introduced the concept of the line graph L(G) of a given graph Gin 1932. The first characterization of line graph is due to Krausz. The Middle graph M(G) of a graph G was introduced by Hamada and Yoshimura [4]. Characterizations were presented for middle graphs of any graph, tree and complete graphs in[1]. The concept of total graphs was introduced by Behzad [2] in 1966. Sastry and Raju [14] introduced the concept of quasi-total graphs and they solved the graph equations for line graphs, middle graphs, total graphs and quasi-total graphs. This motivates us to define and study other graph operations.

The points and Lines of a graph are called its elements. Two elements of a graph are neighbors if they are either incident or adjacent. The total graph T(G) of G has vertex set V(G)∪E(G) and vertices of T(G) are adjacent whenever they are neighbors in G. The quasi-total graph [8] P(G) of G is a graph with vertex set as that of T(G) and two vertices are adjacent if and only if they correspond to two nonadjacent vertices of G or to two adjacent edges of G or to a vertex and an edge incident to it in G. The middle graph M(G) of G is the one whose vertex set is as that of T(G) and two vertices are adjacent in M(G) whenever either they are adjacent edges of G or one is a vertex of G and the other is an edges of G incident with it. Clearly, E(M(G)) = E(T(G)) – E(G). For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph $B(\overline{K_p}, L(G), INC, NINC)$ of G is a graph with vertex set V(G) ∪ E(G) and two vertices in $B(\overline{K_p}, L(G), INC, NINC)$ are adjacent if and only if they correspond to two adjacent edges of G or to a vertex and an edge incident to it in

G, or to a vertex and an edge not incident to it in G, where L(G) is the line graph of G. For brevity, this graph is denoted by $BF_3(G)$. In this section, general properties, traversability and eccentricity properties of $BF_3(G)$ are discussed. Domination parameters are determined.

In the following, some elementary properties of $BF_3(G)$ are obtained.

Observation :2

Let G be a (p, q) graph.

1. L(G) is an induced subgraph of $BF_3(G)$ and the subgraph of $BF_3(G)$ induced by vertices of G is totally disconnected.

2. The number of vertices in $BF_3(G)$ is p + q and if $d_i = deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in $BF_3(G)$ is $|E(L(G))| + pq = \frac{1}{2}\sum d_i^2 - q + pq = q(p - 1) + \frac{1}{2}\sum d_i^2$.

3. The degree of a vertex of G in $BF_3(G)$ is q and the degree of a vertex e of L(G) in $BF_3(G)$ is $deg_{L(G)}$ e + p = d(u) + d(v) – 2 + p, where e = (u, v) ∈ E(G).

4. $BF_3(G)$ contains isolated vertices if and only if $G \cong nK_1$, $n \geq 1$.

5. $BF_3(G)$ is a connected graph if and only if $q \geq 1$, $p \geq 2$.

6. Let G be any graph G such that $BF_3(G)$ is connected. Then $BF_3(G)$ contains a cut vertex if and only if $G \cong K_2 \cup nK_1$, for $n \geq 0$.

7. If $G \cong nK_2 \cup mK_1$, $m \geq 0$, $n \geq 1$, then $BF_3(G) \cong K_{2n+m, n}$.

8. Let G be a graph with atleast one edge. Then vertex set of $BF_3(G)$ can be partitioned into two sets V_1 and V_2 such that $< V_1>$ is totally disconnected and $< V_2> \cong L(G)$.

9. Edge set of $BF_3(G)$ can be partitioned into edges of $K_{p, q}$ and L(G).

10. Let G be a (p, q) graph with $q \geq 2$. If L(G) is complete, then vertex set of $BF_3(G)$ can be partitioned into a complete graph K_q and an independent set containing p vertices and hence $BF_3(G)$ is a split graph.

11. If $L(G) \cong K_q$, then a set containing any pair of adjacent vertices in $BF_3(G)$ forms a dominating set of $BF_3(G)$. Therefore, $BF_3(G)$ is a multipartite graph $K_{p,1,1,..1}$. If $L(G) \cong qK_1$, then $BF_3(G) \cong K_{p,q}$.

12. Let G be a (p, q) graph. $BF_3(G)$ contains $K_{1,3}$ as an induced subgraph if and only if $p \geq 3$ and $q \geq 1$.

13. Let H be any (p, q) graph. Then the solution of $BF_3(G) \cong \overline{L(G)}$ is (G, H), where $G \cong K_{1, p} \cup H'$, where $L(H') \cong \overline{L(G)}$.

In this section, general properties, traversability and eccentricity properties of $BF_3(G)$ are discussed. Domination parameters are determined.

Theorem 2.1:

For any graph G, if $BF_3(G)$ has a dominating edge, then the end vertices of this edge correspond to a vertex of G and a vertex of L(G) .

Proof.

Let x be a dominating edge in $BF_3(G)$. If x = (u, v), where u, v are vertices of G ,then x∈ E(G). The vertex x′ in $BF_3(G)$ corresponding to the edge x is adjacent neither u nor v. Hence the end vertices of the dominating edge x cannot be both vertices of G. Let x = (e_1', e_2'), where both e_1' and e_2' are vertices of L(G) in $BF_3(G)$. Let e_1 and e_2 be edges in G corresponding to e_1' and e_2'. If there exists an edge e ∈ E(G) not adjacent to both e_1 and e_2, then the corresponding vertex in $BF_3(G)$ is not adjacent to both e_1' and e_2'. Hence the end vertices of the dominating edge x cannot be both vertices of L(G). Thus the end vertices of the dominating edge in $BF_3(G)$ correspond to a vertex of G and a vertex of L(G) .

Theorem 2.2:

For any (p, q) graph with $p \geq 3$, $q \geq 2$, the girth of $BF_3(G)$ is 3 or 4 .

Proof.

If G contains P_3 as a subgraph then $BF_3(G)$ contains triangles and hence girth of G is 3. If all edges G are independent, then G contains C_4 as an induced subgraph and hence girth of G is 4 .

Theorem 2.3:

Let G be any graph such that $BF_3(G)$ is connected. Then $BF_3(G)$ geodetic if and only if $G \cong K_2 \cup nK_1$, $n \geq 0$.

Proof.

Since $BF_3(G)$ isconnected , G contains atleast one edge. Assume $BF_3(G)$ is geodetic. If G contains triangles, then $BF_3(G)$ contains C_4 as an induced subgraph and hence is not geodetic. Assume G is triangle free. If $\beta_1(G) \geq 2$, then $BF_3(G)$ contains C_4 as an induced subgraph and hence is not geodetic. Therefore $q < 2$. Since G contains atleast one edge, $q = 1$. Therefore $G \cong K_2 \cup nK_1$, $n \geq 0$. Conversely if $G \cong K_2 \cup nK_1$, $n \geq 0$, then $BF_3(G) \cong K_{1,\, n+2}$ and hence is geodetic.

Theorem 2.4:

Let G be a (p, q) graph with $q \geq 2$. Then $BF_3(G)$ is Eulerian if and only if q is even and both p and d(u) + d(v) are of some parity, for every pair of adjacent vertices u and v in G.

Proof.

Let G be a (p, q) graph with $q \geq 2$. Assume $BF_3(G)$ is Eulerian. Then every vertex in $BF_3(G)$ is of even degree.

Degree of a vertex of G in $BF_3(G)$ is equal to q is even. Degree of a vertex e of L(G) in $BF_3(G) = \deg_{L(G)} e + p = \deg_G u + \deg_G v - 2 + p$ is even where $e = (u, v) \in E(G)$.That is, both p and d(u) + d(v) are of some parity. Conversely, if the conditions in the theorem holds, then every vertex of $BF_3(G)$ is of even degree. Therefore $BF_3(G)$ is Eulerian.

Theorem 2.5:

For any graph G with atleast three vertices, $BF_3(G)$ is self- centered with radius 2 if and only if $\beta_1(G)$.

Proof.

Since the subgraph of $BF_3(G)$ induced by vertices of $V(G) \geq 2$ is totally disconnected, distance between any two vertices of G in $BF_3(G)$ is 2. Let $u, v \in V(G)$ and $e \in E(G)$. Let e′ be the vertex of L(G) corresponding to e. Then ue′v is a geodesic in $BF_3(G)$ and hence d(u, v) in $BF_3(G)$ is 2. Thus, the distance between any vertices of G in $BF_3(G)$ is 2.

Let $v, e' \in V(BF_3(G))$, where $v \in V(G)$, e′ is the vertex in $BF_3(G)$ corresponding to an edge e in G. Then $d_{BF_3(G)}(v, e') = 1$. Let e_1', e_2' be two vertices of L(G) in $BF_3(G)$ and e_1, e_2 be the corresponding edges in G. If e_1, e_2 are adjacent edges in G, then $d_{BF3(G)}(e_1', e_2') = 1$. If e_1, e_2 are nonadjacent edges in G, then if $v \in V(G) \cap V(BF_3(G))$, then $e_1' v\, e_2'$ is a geodesic path in $BF_3(G)$ and hence $d_{BF_3(G)}(e_1', e_2') = 2$. Hence, the distance between any two vertices of L(G) in $BF_3(G)$ is atmost 2. From the above argument, eccentricity of a vertex of G in $BF_3(G)$ is 2 and the eccentricity of a vertex of L(G) in $BF_3(G)$ is 1 (or) 2. From the above argument, $BF_3(G)$ is self-centered with radius

Remark 2.1:

$BF_3(G)$ is bi-eccentric with radius 1 and 2 if and only if $\beta_1(G) = 1$.

Theorem 2.6:

$\beta_0(BF_3(G)) = p$.

proof.

V(G) is a maximum independent set in $BF_3(G)$, since each vertex of L(G) is adjacent to its incident and non-incident vertices of G.

Remark 2.2:

$\alpha_0(BF_3(G)) = q$. Since $(\alpha_0(BF_3(G)) + \beta_0(BF_3(G))) = p + q$.

Theorem 2.7:

$\beta_1(BF_3(G)) \geq \max\{\min(p, q), \beta_1(L(G))\}$.

Proof.

Let $\{v_1, v_2, v_3, \ldots, v_p\}$ be the vertices of G.

Case 1. $p \leq q$.

Then the set $\{(v_1, e_1), (v_2, e_2), \ldots, (v_p, e_p)\}$ is an edge independent set of $BF_3(G)$, where e_i's are distinct and hence $\beta_1(BF_3(G)) \geq p$.

Case 2. $p > q$.

Then the set $\{(v_1, e_1), (v_2, e_2), \ldots, (v_q, e_q)\}$ is an edge independent set of $BF_3(G)$, where e_i's are distinct and hence $\beta_1(BF_3(G)) \geq q$. Therefore $\beta_1(BF_3(G)) \geq \min(p, q)$. On the other hand, edge independent set of L(G) is also an edge independent set of $BF_3(G)$ and hence $\beta_1(BF_3(G)) \geq \max\{\min(p, q), \beta_1(L(G))\}$. Equality holds, if $G \cong nK_2, n \geq 2$.

Remark 2.3:

1. $\alpha_1(BF_3(G)) = \alpha_1(L(G))$.   .

2. $\chi(BF_3(G)) = \chi(L(G)) + 1$.

Observation 2.2:

If radius of L(G) is 1, then $n_0(BF_3(G)) = 1$, since L(G) is an induced subgraph of $BF_3(G)$.

Theorem 2.6:

If the independent domination number $\gamma_i$ofL(G) is 2, then $n_0(BF_3(G)) = 2$.

Proof.

Assume $\gamma_i(L(G)) = 2$. Then there exist two independent edges e_1 and e_2 in G such that each edge in G is adjacent to atleast one of e_1 and e_2. If e_1' and e_2' be the vertices of L(G) in $BF_3(G)$ corresponding to e_1 and e_2, then $\{e_1', e_2'\}$ is an n-set for $BF_3(G)$. Hence $n_0(BF_3(G)) = 2$.

3. Domination Numbers and other parameters of $B(\overline{K_p}, L(G), INC, NINC)$

Theorem 3.1:

Let G be any (p, q) graph, not totally disconnected. Then $\gamma(BF_3(G)) = 1$if and only if $\beta_1(G) = 1$.

Proof.

Let $\beta_1(G) = 1$. Let $e \in E(G)$ be an edge of G. Then $e \in V(BF_3(G))$. Then the set $D = \{e\} \subseteq V(BF_3(G))$ is a dominating set of $BF_3(G)$ and hence $\gamma(BF_3(G)) = 1$. Conversely if $\beta_1(G) \geq 2$, there exists no vertex in $BF_3(G)$ of degree $p + q - 1$. Therefore $\beta_1(G) = 1$.

Theorem 3.2:

Let G be any graph not totally disconnected and $\beta_1(G) \geq 2$. Then $\gamma(BF_3(G)) = 2$.

Proof.

Let G be not totally disconnected and $e = (u, v) \in E(G)$, where $u, v \in V(G)$. Then $u, v, e \in V(BF_3(G))$. Let $D = \{u, e\} \subseteq V(BF_3(G))$. Vertices of G in $V(BF_3(G)) - D$ is adjacent to e and vertices of L(G) in $V(BF_3(G)) - D$ is adjacent to u. Therefore D is a dominating set of $BF_3(G)$ and hence $\gamma(BF_3(G)) \leq |D| = 2$. Since $\beta_1(G) \geq 2$. Then $\gamma(BF_3(G)) \geq 2$ and hence $\gamma(BF_3(G)) = 2$. .

Observation 3.1:

$\gamma_p(BF_3(G)) = 1$ if and if only $G \cong K_2 \cup nK_1, n \geq 0$.

Theorem 3.3:

$\gamma_p(BF_3(G)) = 2$ if and if only $G \cong nK_2, n \geq 2$.

Proof.

Assume $\gamma_p(BF_2(G)) = 2$. Then a vertex of D is a vertex of G and another vertex of D is a vertex of L(G). Let $D = \{v, e'\} \subseteq V(BF_3(G))$, where $v \in V(G)$ and $e' \in V(L(G))$. Let e be the edge in G corresponding to e'. If there exists an edge x in G adjacent to e, then the corresponding vertex x′ in $BF_3(G)$ is adjacent to both v and e′. Therefore, there exists no edge in G adjacent to e. Therefore $G \cong nK_2$, $n \geq 2$.

Remark 3.1:

There is no perfect dominating set of $BF_3(G)$ having atleast three vertices.

Theorem 3.4:

For any (p, q) graph G, where $p \geq 4$ any subset of V(L(G)) having atleast two independent vertices is not a perfect dominating set of $BF_3(G)$.

Proof.

Let D be a subset of V(L(G)) having atleast two independent vertices, $e_1'$and e_2'. Then the corresponding edges $e_1$and e_2 are not adjacent in G. Let e be an edge in G adjacent to both $e_1$and e_2 . Then $e \in V(BF_3(G)) - D$ is adjacent to both $e_1'$and e_2' and all the vertices of G in $V(BF_3(G)) - D$ are adjacent to both $e_1'$and e_2' and hence D is not a perfect dominating set of $BF_3(G)$.

Observation 3.2:

For any graph G not totally disconnected, $\gamma_{ctd}(BF_3(G)) = 1$ if and only if $G \cong P_3$.

Theorem 3.5:

If G is a (p, q) graph with $q \geq 2$, then $\gamma_{ctd}(BF_3(G)) \leq q - 1$.

Proof.

Let G be a graph with atleast two edges and let $e \in V(L(G)) \cap V(BF_3(G))$. Then $D = V(L(G)) - \{e\}$ is a dominating set of $BF_3(G)$ and $< V(BF_3(G)) - D > \cong K_{1,\, p+1}$ and hence D is a complementary tree dominating set of $BF_3(G)$ and $\gamma_{ctd}(BF_3(G)) \leq |D| = q - 1$.

Remark 3.2:

Let G be any (p, q) graph with $q \geq 1$. Then $\gamma_s(BF_3(G)) \leq q$, since V(L(G)) is a split dominating set of $BF_3(G)$.

Theorem 3.6:

For any graph G, $\gamma_i(BF_3(G)) = 2$ if and only if $\gamma_i'(G) = 2$, where $\gamma_i'(G)$ is the edge independent domination number of G.

Proof.

Assume $\gamma_i(BF_3(G)) = 2$. Let D be an independent dominating set of $BF_3(G)$ such that $|D| = 2$. Then the vertices of D are vertices of L(G). Let $D = \{ e_1', e_2' \}$, where $e_1', e_2' \in V(L(G))$ and let e_1, e_2 be the corresponding edges in G. Since D is independent, e_1 and e_2 are nonadjacent edges in G. Since D is a dominating set, each edge in G is adjacent to atleast one of e_1 and e_2. That is, D is an independent dominating set of L(G) and hence $\gamma_i'(G) = 2$. Conversely if $\gamma_i(BF_3(G)) = 2$, then $\gamma_i'(BF_3(G)) = 2$.

Remark 3.3:.

If G is a graph other than a star and if $\gamma_i'(G) \geq 3$, then $\gamma_i'(BF_3(G)) \leq \gamma_i'(G)$.

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