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Article

MATHPROGIA: A CONCEPTUAL AND METHODOLOGICAL FRAMEWORK FOR THE EVOLUTION OF MENTAL MODELS IN HYBRID LEARNING ECOSYSTEMS WITH EXPLAINABLE AI

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Abstract: This longitudinal case study evaluated the evolution of the mental models of 17 non-commissioned officers who were students of Topographic Survey Technology at the School of Military Engineers (ESING). The objective was to analyze the evolution of mental models in the domain of variational and statistical-random thinking through the implementation of MathProgIA, a methodological framework developed for the design, implementation and evaluation of hybrid learning ecosystems. Its structure integrates fundamentals from Knowledge Space Theory (KST), knowledge state modeling, competency-based learning, continuous assessment and teacher auditing processes supported by explainable artificial intelligence (XAI). From this perspective, artificial intelligence is not the axis of the training process, but a methodological mediator whose function is to strengthen the construction of knowledge through the critical validation of information and the monitoring of the evolution of mental models. In the implementation developed in this research, explainable artificial intelligence (XAI) was incorporated into the design of the analog and digital learning experiences that made up the hybrid ecosystem. Longitudinal evaluations allowed us to observe the progression of students' mental models from elementary levels to more inclusive levels. The research was carried out over six months with a sample of 15 men and 2 women, using a longitudinal design with a mixed approach, supported by linear regression analysis and multivariate analysis. The results showed a progressive evolution of the mental models and a high explanatory power of the regression model (adjusted $R^2 = 0.983$). An original contribution of the study consists of the identification of the Evaluative Discrepancy Effect, understood as the systematic difference between the answers generated by artificial intelligence tools and the information verified in original sources through teacher auditing processes. This process favored the critical reconstruction of knowledge, metacognition, and the development of critical thinking. Overall, the findings provide empirical evidence that MathProgIA constitutes a conceptual and methodological framework with the potential to promote the evolution of mental models in hybrid learning ecosystems based on explainable artificial intelligence principles.

Keywords: MathProgIA; mental models; hybrid learning ecosystems; explainable artificial intelligence (XAI); Theory of the Knowledge Space (KST); variational thinking; statistical-random thinking; Effect of Evaluative Discrepancy.

1. INTRODUCTION

Training in applied mathematics is considered an essential component in technological programs aimed at preparing professionals for decision-making based on quantitative information. In the Topographic Survey Technology of the School of Military Engineers (ESING), this challenge acquires particular characteristics, due to the need imposed on non-commissioned officers on study commission to integrate concepts of variational and statistical-random thinking with high-precision procedures required for the analysis and representation of the territory. Added to this situation are the demands of military training, which is characterized by hierarchical structures, academic times conditioned by institutional operation, and an organizational culture oriented towards discipline, collaborative work, and compliance with technical standards (Maldonado et al., 2021; Benítez, 2023).

In recent years, learning dynamics in higher education have been significantly transformed by the incorporation of generative artificial intelligence tools. Although these technologies offer opportunities to support problem solving and facilitate access to specialized information, relevant pedagogical challenges also arise when the answers generated are accepted without systematic processes of verification or contrast with the original sources. In these scenarios, the adoption of apparently correct answers can be favored by automation without guaranteeing sufficient conceptual understanding, which limits the development of metacognitive processes and the consolidation of scientifically based mental models (Benítez, 2025; Sevilla et al., 2025).

Although the potential of artificial intelligence for the transformation of educational processes is recognized in the literature, questions persist about how to integrate it into learning ecosystems that promote critical interaction between the student, the teacher, and digital tools. This need is particularly relevant in contexts of military technical training, where conceptual precision and reliability of information are essential requirements for professional performance. Consequently, rather than replacing the student's reasoning, the challenge is defined around the design of pedagogical strategies that turn artificial intelligence into a resource subject to permanent processes of analysis, auditing and validation of knowledge.

In order to respond to this challenge, this research is developed under the MathProgIA framework, through which a hybrid learning ecosystem, the Knowledge Space Theory (KST), a pedagogical perspective of explainable artificial intelligence (XAI) and teaching audit processes from original sources are articulated. From this approach, artificial intelligence is no longer considered only a tool for obtaining answers and is incorporated as a pedagogical resource whose information is contrasted, interpreted and validated through teacher mediation and the analysis of evidence from original documents. In this way, explainability is understood as a process aimed at favoring critical validation, argumentation, and conceptual reconstruction of knowledge by the student, with the purpose of promoting the evolution of their mental models (Benítez, 2023; Benítez, 2025).

In this context, the research was guided by the following question: How is the evolution of mental models favored in the variational and statistical-random thinking of non-commissioned officers of the Technology in Topographic Surveys through the MathProgIA framework in a hybrid learning ecosystem with explainable artificial intelligence? In correspondence with this question, the objective of the research was defined in evaluating this evolution in non-commissioned officers, students of Topographic Survey Technology of the School of Military Engineers.

As a scientific contribution, this study provides empirical evidence on the implementation of the MathProgIA framework in a context of military technical training, documenting how processes of critical reconstruction of knowledge are favored by the integration of a hybrid learning ecosystem, an explainable pedagogical perspective of artificial intelligence and the systematic auditing of original sources. Likewise, the Evaluative Discrepancy Effect is identified as an emerging phenomenon associated with the contrast between the responses generated by exploratory artificial intelligence and the evidence from the original sources, thus providing elements for the design of hybrid learning ecosystems aimed at strengthening scientifically based mental models.

2. Theoretical foundations of the MathProgIA framework

The incorporation of artificial intelligence in educational processes has expanded the possibilities to strengthen learning, facilitate access to knowledge and promote more personalized educational experiences. However, its integration also poses challenges related to conceptual understanding, validation of the information generated, and the development of critical thinking. In particular, it is recognized that generative AI tools can produce rapid and seemingly coherent answers; however, its use in isolation does not guarantee the construction of scientifically grounded knowledge or the transformation of the cognitive structures necessary to understand, explain, and apply disciplinary concepts (Holmes et al., 2022; UNESCO, 2021).

In this context, education mediated by artificial intelligence requires overcoming an instrumental vision of technology as a simple generator of answers. It is proposed to move towards pedagogical proposals where interaction with intelligent systems is accompanied by processes of reflection, critical evaluation and conceptual reconstruction. From this perspective, the educational value of artificial intelligence is acquired when it is integrated into learning ecosystems where students, teachers, technological resources, and knowledge validation strategies interact in an articulated way to promote deep and verifiable learning (Benítez et al., 2024a; Sevilla et al., 2025).

Under this premise, MathProgIA emerges as an integrative framework aimed at explaining and promoting the evolution of mental models through hybrid learning ecosystems mediated by artificial intelligence. In this framework, cognitive, pedagogical, technological, and epistemological foundations are organized that allow understanding the way in which verifiable knowledge is constructed by students through processes of interaction, reflection, validation, and conceptual reconstruction.

To this end, the MathProgIA framework is based on the convergence of four complementary theoretical perspectives. The first corresponds to the evolution of mental models as a process of conceptual transformation; the second addresses hybrid learning ecosystems as dynamic environments where people, technologies and resources converge for the construction of knowledge; the third incorporates the Knowledge Space Theory (KST) as a formal structure to represent learning trajectories; and the fourth integrates Explainable Artificial Intelligence (XAI) as a benchmark for the understandable use of intelligent systems. This last perspective is complemented, within the MathProgIA framework, with pedagogical audit processes of original sources aimed at strengthening the critical validation of knowledge (Falmagne & Doignon, 2011; Holmes et al., 2022).

Consequently, the articulation of these perspectives allows us to conceive MathProgIA as a conceptual and methodological framework where mental models constitute the object of cognitive transformation; hybrid ecosystems represent the space for educational interaction; KST organizes learning trajectories; and Explainable Artificial Intelligence, interpreted from a pedagogical perspective, contributes to promoting processes of auditing, argumentation and validation of knowledge. From these foundations, the theoretical references that support this research are developed.

2.1. Evolution of mental models as the foundation of mathematical learning

The understanding of mathematical learning has evolved from perspectives focused exclusively on the acquisition of procedures to approaches that recognize the importance of cognitive structures through which students interpret phenomena, construct explanations, and solve problems. In this context, mental models are internal representations that allow information to be organized, conceptual relationships to be established, and inferences to be made about specific situations. These representations do not remain static; on the contrary, they evolve through the interaction between prior knowledge, learning experiences and conceptual reflection processes.

From the theory of mental models, human understanding involves the construction of internal representations that allow us to interpret reality and generate explanations about the observed phenomena. Consequently, scientific and mathematical learning can be understood as a process of progressive reorganization of these representations, in which initial conceptions, often intuitive or incomplete, evolve towards conceptual structures more coherent with disciplinary knowledge (Johnson-Laird, 1983; Vosniadou, 1994).

This perspective is especially relevant in applied mathematics education, where different systems of representation must be integrated to solve problems related to professional situations. In programs such as Topographic Survey Technology, variational thinking and statistical-random thinking require the articulation between mathematical concepts, data interpretation, modeling and informed decision-making. Consequently, learning cannot be reduced to the correct execution of algorithms, but must consider the student's ability to understand relationships, justify procedures, and transfer knowledge to new scenarios (Benítez, 2023).

From this perspective, the evolution of mental models is an indicator of learning, since it reflects transformations in the way students represent, explain and use knowledge. Previous research carried out in digital learning ecosystems has shown that interaction with technological resources, accompanied by pedagogical mediation processes, favors progressive transformations in cognitive structures and strengthens conceptual understanding (Benítez, 2025).

Likewise, conceptual evolution is closely related to processes of self-regulation and metacognition, since students continuously evaluate their own representations to identify inconsistencies and adjust their learning strategies. From the sociocognitive perspective, it is proposed that the perception of competence, autonomy and the capacity for self-regulation significantly influence both the construction of knowledge and persistence in the face of complex problems (Bandura, 1997; Zimmerman, 2000).

Within the framework of MathProgIA, mental models are conceived as dynamic cognitive representations through which students interpret, organize and use knowledge to solve problems, explain phenomena and make informed decisions. Consequently, learning is understood as a continuous process of reorganization of these representations, consistent with the approaches of the theory of mental models and conceptual change.

Based on the proposal developed by Benítez (2023), this study adopts a five-level progression of mental models as a methodological reference to analyze the learning trajectories observed during the training process. This classification constitutes an analytical instrument that allows characterizing the degree of cognitive organization achieved by students during problem solving in the domain of variational and statistical-random thinking.

Table 1. Levels of evolution of mental models adapted by MathProgIA (adapted from Benítez, 2023)

Level	Name	General characterization
NMM1	**Elementary mental model**	The student recognizes basic concepts, definitions, and procedures in isolation. Task solving is mainly based on memory or the mechanical application of rules, with little understanding of the relationships between the elements of the knowledge domain.
NMM2	**Basic mental model**	The student understands the fundamental concepts and solves routine situations. Begins to establish relationships between concepts, procedures, and representations, although these are still partial and depend on teacher accompaniment or support resources.
NMM3	**Reconstructive mental model**	The student reorganizes and reconstructs his knowledge by integrating concepts from different learning situations. They are able to explain procedures, identify relationships between variables, and correct conceptual errors based on new evidence.
NMM4	**Prospective mental model**	The student uses knowledge to anticipate results, formulate hypotheses, select solution strategies, and transfer learning to new problems. Demonstrates the ability to argue decisions and establish connections between different representations of knowledge.
NMM5	**Inclusive mental model**	The student critically integrates knowledge from multiple sources, contrasts information, validates evidence, and constructs well-founded explanations to solve complex problems autonomously. In hybrid ecosystems with artificial intelligence, this level incorporates processes of auditing original sources, critical validation of AI-generated responses, and reconstructing knowledge through metacognitive reflection.

The progression between these five levels is not conceived as a strictly linear process. From the perspective of Knowledge Space Theory (KST), learning can be described by states of knowledge that evolve as students acquire new competencies and satisfy the existing precedence relationships between them. In coherence with this approach, the five levels of evolution of the mental models adopted in MathProgIA are used as a methodological reference to interpret the cognitive trajectories observed during the research, without assuming that all students go through a unique or identical sequence of development

2.2. Hybrid learning ecosystems as environments for the construction of knowledge

The evolution of mental models requires educational environments capable of integrating multiple resources, interactions, and forms of knowledge representation. From this perspective, hybrid learning ecosystems constitute complex environments in which face-to-face and virtual spaces, digital platforms, multimedia resources, computational tools, and pedagogical strategies oriented towards the progressive construction of knowledge converge (Barron, 2006).

Unlike traditional models focused on the transmission of information, in hybrid ecosystems learning is conceived as a distributed process in which various actors and resources participate. Knowledge is built through the interaction between students, teachers, technologies, and learning communities, generating opportunities to explore problems, receive feedback, contrast explanations, and progressively reorganize cognitive structures (Benítez et al., 2024a; Sevilla et al., 2025).

In these environments, technology is not an isolated element, but a mediator that expands the possibilities of interaction, representation and construction of knowledge. However, it is recognized that the availability of digital tools does not in itself guarantee better learning; their impact depends on pedagogical design, teacher mediation, and the student's ability to use these resources in a critical, reflective, and contextualized way (UNESCO, 2021).

In the context of applied mathematics education, hybrid ecosystems are particularly relevant because it allows abstract knowledge to be integrated with professional applications through simulation, modelling, data analysis and visualisation tools. In areas such as topography, where mathematical concepts must be related to technical procedures by students, these environments favor the articulation between theory, practice, and contextualized problem solving (Benítez, 2025).

Various studies on hybrid environments show that these scenarios can strengthen both academic performance and the dimensions associated with professional development, provided that a coherent articulation between technology, teacher mediation, and learning objectives is provided (Arboleda et al., 2025; Sevilla et al., 2025). From the epistemological stance assumed in MathProgIA, the hybrid ecosystem constitutes the space of interaction where the student's mental models, the learning trajectories organized through the Theory of the Knowledge Space and the validation processes supported by artificial intelligence and pedagogical auditing tools converge.

2.3. Knowledge Space Theory (KST) as a support for learning trajectories

The evolution of mental models requires mechanisms that allow the progressive changes in students' states of knowledge to be represented. In this sense, Knowledge Space Theory (KST) provides a mathematical foundation for understanding learning as a dynamic structure composed of different possible states of knowledge and dependency relationships between concepts. From this perspective, learning is not constituted as a linear process in which the same sequence of contents is traversed by all students, but as a variable trajectory through which different cognitive configurations can be traversed by each individual until they reach higher levels of understanding (Falmagne & Doignon, 2011).

In KST it is proposed that a domain of knowledge can be represented by sets of concepts related to each other, where each state reflects a specific combination of knowledge acquired by the student. Through this representation, it is possible to identify which conceptual elements are consolidated and which require strengthening, favoring the construction of adaptive learning trajectories. Consequently, a formal structure is provided by theory to analyze cognitive progression and understand how conceptual structures are reorganized by students during the educational process.

This perspective is especially relevant in mathematics education, where hierarchical relationships and cognitive dependencies presented by concepts condition the resolution of complex problems. The learning of variational, statistical or geometric content requires the progressive integration of previous knowledge, so that the understanding of more complex concepts depends on the consolidation of previous conceptual structures. From this perspective, mathematical learning is interpreted by KST as an organized network of cognitive relationships and not only as a sequential accumulation of content.

Within the MathProgIA framework, the function of organizing learning trajectories and providing a structured representation of the evolution of mental models is fulfilled by KST. Through this integration, it is possible to relate the cognitive changes observed in students with the different levels of conceptual mastery required to solve mathematical problems applied to the professional technical context. In this way, the processes developed within the hybrid ecosystem can be analyzed not only from the final results, but also from the progressive transitions between states of knowledge.

Consequently, the incorporation of KST in MathProgIA allows us to overcome traditional approaches based exclusively on summative assessments, by proposing a vision of learning as a dynamic, observable trajectory that can be analyzed longitudinally. Student evolution is interpreted as a continuous process of transition between cognitive configurations, which is favored by interaction with digital resources, modeling activities, teacher feedback, and metacognitive reflection processes.

2.4. Explainable Artificial Intelligence (XAI) and pedagogical auditing of learning

New possibilities to support processes of information search, problem solving and explanation generation have been opened up in education by the incorporation of generative artificial intelligence. However, the use of these systems poses challenges related to the reliability of the answers produced, the possibility of conceptual errors or simplifications, and the tendency of some students to accept automated results without sufficient critical analysis processes. Consequently, the educational integration of artificial intelligence requires approaches that allow the responses generated by these technological systems to be understood, evaluated, and validated (Holmes et al., 2022).

In this scenario, Explainable Artificial Intelligence (XAI) emerges as a perspective aimed at increasing transparency and understanding of the processes developed by algorithms. Although initially its development was located in areas such as medicine, engineering and automated decision systems, its incorporation into education raises the need for intelligent tools not only to be used by students, but also for their limitations to be understood, their results to be interpreted critically and criteria to be developed to assess the quality of the information obtained.

However, it is established that in educational contexts the technical explainability of the algorithm, by itself, is not enough to guarantee deep learning. A system can offer explanations about the way in which a response is produced, but this does not necessarily imply that the concept involved is understood by the student or that the relevance of said response can be assessed within a specific disciplinary context. From this perspective, pedagogical auditing is incorporated by MathProgIA as a complementary component, understood as a systematic process of contrast between the answers generated by artificial intelligence tools and the evidence from verifiable original sources.

An epistemological dimension is introduced by pedagogical auditing in the educational use of artificial intelligence by transforming the interaction with technology into an opportunity to develop critical thinking, metacognition and conceptual reconstruction. Instead of assuming the answer generated by the AI as a definitive product, it must be analyzed and contrasted with original documents by the student, identifying possible inconsistencies and justifying the validity of the knowledge constructed.

This process becomes especially important when apparently correct answers, but conceptually incomplete or insufficiently substantiated, are produced by generative tools. Cognitive conflict is favored by the confrontation between the automated response and documentary evidence, promoting the reorganization of mental models and strengthening the processes of conceptual validation. In this sense, pedagogical auditing becomes a mechanism for regulating learning, through which artificial intelligence ceases to constitute a final source of knowledge to assume the role of a mediating resource within a process of critical construction of knowledge.

Consequently, within the MathProgIA framework, a balance between technological innovation and academic rigor is established by the relationship between Explainable Artificial Intelligence and pedagogical auditing. In this work, Explainable Artificial Intelligence is interpreted from a pedagogical perspective, in which explainability is not restricted to algorithmic transparency, but processes of auditing of original sources, critical validation, argumentation and conceptual reconstruction developed by the student are incorporated. Under this conception, the possibilities of exploration and feedback are expanded by artificial intelligence, while the epistemological reliability of the constructed knowledge is guaranteed by teacher mediation.

2.5. MathProgIA: an integrative framework for the evolution of mental models through hybrid learning ecosystems with Explainable Artificial Intelligence

From the theoretical perspectives developed in the previous sections, it is possible to understand that the evolution of learning in contexts mediated by artificial intelligence cannot be explained from a single dimension. The cognitive transformation of the student is interpreted through mental models; the environment where this transformation occurs is described by hybrid ecosystems; a formal structure to represent learning trajectories is provided by the Theory of the Knowledge Space; and the mechanisms to favor the critical validation of the knowledge constructed are provided by Explainable Artificial Intelligence, articulated with pedagogical auditing.

From this integration, MathProgIA emerges as a conceptual framework that articulates cognitive, pedagogical, technological, and epistemological dimensions of mathematical learning. Rather than incorporating artificial intelligence tools into the educational process, a pedagogical architecture is proposed by MathProgIA, in which the evolution of mental models constitutes the central axis of learning and technology acts as a mediator to favor processes of understanding, reflection and conceptual validation.

In this framework, the expected outcome of learning is represented by the evolution of mental models; the space of interaction where students, teachers and technologies converge is constituted by hybrid ecosystems; trajectories are organized by the Theory of Knowledge Space through progressive states of conceptual domain; and the processes of analysis, argumentation and validation of knowledge are favored by Explainable Artificial Intelligence —interpreted from a pedagogical perspective— together with the audit of original sources.

Consequently, artificial intelligence is not conceived by MathProgIA as a substitute for human reasoning or as an autonomous teaching agent, but as a cognitive mediator whose effectiveness depends on intentionally designed pedagogical strategies. Opportunities to explore information, receive feedback, and contrast explanations are offered by technology; however, the construction of knowledge continues to be an essentially human process based on interpretation, argumentation and critical validation.

In the context of military technical training, this framework acquires special relevance due to the need to integrate technical precision, intellectual autonomy and reliability in decision-making. Learning environments where students not only solve problems, but also develop the ability to critically substantiate their professional decisions, are built thanks to the articulation between mathematical knowledge, digital tools and pedagogical auditing.

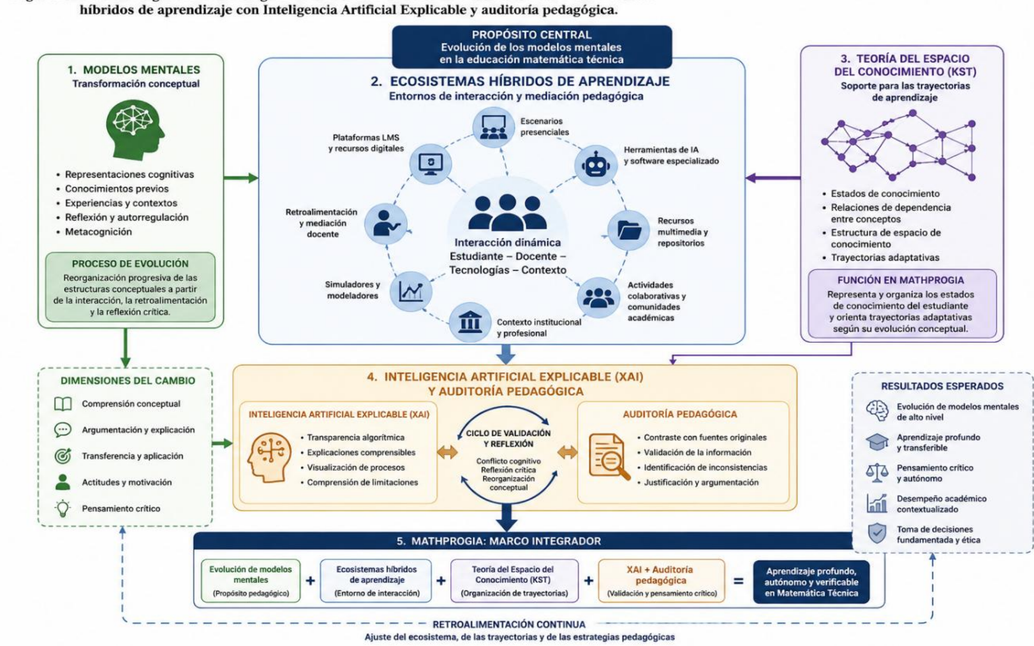
Consequently, MathProgIA constitutes a theoretical and methodological proposal aimed at understanding how hybrid learning ecosystems mediated by artificial intelligence can favor the evolution of mental models. Its main contribution is to integrate cognitive, technological and epistemological foundations within a coherent framework to analyze learning processes mediated by artificial intelligence from a pedagogical perspective based on the critical validation of knowledge.

Finally, the fundamentals developed allow us to understand that the evolution of learning in contexts mediated by artificial intelligence emerges from the interaction between the transformation of mental models, the learning trajectories represented by the Theory of the Knowledge Space, the mediation of hybrid ecosystems and the validation processes promoted by Explainable Artificial Intelligence and pedagogical auditing. In this sense, a new theory of learning or a redefinition of Explainable Artificial Intelligence by MathProgIA is not proposed; An integrative framework is proposed that articulates these foundations within a coherent pedagogical architecture to explain how students construct verifiable knowledge through processes of interaction, reflection, argumentation, and validation.

The integration of these foundations is synthesized in Figure 1, which represents the conceptual architecture of the MathProgIA framework and the relationships between the evolution of mental models, hybrid learning ecosystems, the Theory of the Knowledge Space and Explainable Artificial Intelligence interpreted from a pedagogical perspective. This representation constitutes the conceptual foundation on which the methodology is developed and the results of this research are interpreted (Benítez, 2023, 2025, 2026).

Figure 1. MathProgIA Conceptual Framework for the Evolution of Mental Models by

Figura 1. **Marco integrador MathProgIA: Evolución de modelos mentales mediante ecosistemas híbridos de aprendizaje con Inteligencia Artificial Explicable y auditoría pedagógica.**



Nota. La figura representa la integración de los cuatro fundamentos teóricos que estructuran el marco MathProgIA y su relación con la evolución de los modelos mentales en educación matemática técnica.

3. Methodological design

The research was developed through a longitudinal case study with a mixed approach, aimed at analyzing the evolution of mental models associated with variational and statistical-random thinking in a context of military technical training. The design was based on the systematic follow-up of the same academic cohort for six months, integrating quantitative and qualitative evidence obtained at different times of the training process.

Given the size of the cohort and the institutional characteristics of the participating population, the study did not seek to establish generalizable causal relationships. Instead, a longitudinal analysis strategy typical of the case study was adopted, aimed at understanding learning trajectories and transformations of mental models based on the convergence of multiple sources of evidence.

From this perspective, the research was developed under a high-data density approach, understood as the systematic integration of records obtained through various instruments, scenarios and observation moments. In coherence with the nature of a hybrid ecosystem, it was assumed that all the components integrated by the teacher in the educational space —both in virtual environments and in face-to-face classroom interactions— were a constituent part of the environment and had a strictly assessable character. The evidence analyzed comprised eighteen workshops implemented over six months, interaction records on the Moodle platform, activities developed in two interactive classrooms, evaluations organized by performance levels, more than 2000 user interaction records, semi-structured interviews, and products derived from both digital and analog learning experiences.

The methodological implementation was structured according to the MathProgIA framework, integrating four articulated components: the evolution of mental models as an object of analysis, hybrid learning ecosystems as an interaction environment, the Knowledge Space Theory (KST) as a reference to interpret learning trajectories and Explainable Artificial Intelligence (XAI), understood from the pedagogical perspective proposed in this study. through processes of auditing original sources and critical validation of knowledge.

3.1 Participants and context

The study was carried out with a cohort of seventeen non-commissioned officers of the Technology in Topographic Surveys of the School of Military Engineers (ESING), made up of fifteen men and two women (five sergeants and twelve corporals). The selection responded to a non-probabilistic sampling for convenience, determined by the institutional makeup of the academic group during the implementation period.

All participants took simultaneously the subjects related to variational and statistical-random thinking, developed within the technology curriculum. The longitudinal nature of the design allowed continuous monitoring of the students' evolution for six months, maintaining homogeneous conditions of training, evaluation and teacher accompaniment.

3.2 Implementing the hybrid learning ecosystem

The implementation of the hybrid ecosystem was developed through eighteen workshops organized on the Moodle platform and integrated with face-to-face activities developed in two interactive classrooms. The workshops addressed content related to variational thinking, descriptive and inferential statistics, data analysis and mathematical modeling applied to the context of topographic surveys.

The activities were designed to favor progressive processes of conceptual construction through the resolution of contextualized problems. Initially, the situations were solved by the students using their own procedures, mainly through written work, Excel spreadsheets and other conventional mathematical tools.

Subsequently, when it was considered relevant, generative artificial intelligence tools were incorporated to support exploring alternative solutions, expanding explanations or contrasting procedures. Within the MathProgIA ecosystem, these tools were not conceived as substitutes for mathematical reasoning, but as exploration resources whose information had to be critically analyzed.

In order to validate the results obtained, the teacher had reference solutions previously developed using verified mathematical procedures and supported by computational tools such as R and Python. These solutions were the benchmark for comparing the procedures developed by the students and the responses generated by the artificial intelligence systems.

3.3 Pedagogical audit and validation of knowledge

One of the central components of the methodology corresponded to the pedagogical audit process incorporated within the MathProgIA framework. This phase consisted of systematically contrasting three types of evidence: the mathematical procedures developed by the students, the answers generated by artificial intelligence tools, and the solutions verified by the teacher based on mathematical procedures and original sources.

Through the audit, it was possible to identify coincidences, differences, simplifications and inconsistencies between the different answers, generating spaces for analysis aimed at strengthening mathematical argumentation and critical validation of knowledge. From the pedagogical perspective adopted in this study, explainability was not limited to understanding the functioning of the algorithm, but was materialized through processes of information verification, analysis of original sources and conceptual reconstruction supported by teacher mediation.

During this stage, situations were observed in which the students were expected to automatically confirm their answers by intelligent systems; however, the contrast with verifiable mathematical evidence promoted processes of questioning, conceptual revision and metacognitive reflection on the reliability of the information obtained.

3.4 Information collection and analysis

The information used for the longitudinal analysis integrated multiple sources of evidence obtained throughout the implementation of the hybrid ecosystem, under the principle that any pedagogical mediation and didactic resource incorporated in the classroom is considered an assessable source of data. These included the eighteen workshops developed in Moodle, progressive evaluations organized by performance levels, digital interaction records, interviews, activities developed in interactive classrooms and academic products developed by the participants in both analog and digital dynamics.

The integration of this evidence allowed the construction of a longitudinal matrix used to analyze the evolution of the mental models during the six months of implementation. From this matrix, linear regression analyses and multivariate procedures were developed to identify trends of change, relationships between variables and learning trajectories.

Through the triangulation between the different sources of information, it was possible to interpret the transformations observed in the forms of representation, argumentation and mathematical problem solving, relating them to the evolution of mental models within the hybrid learning ecosystem proposed by MathProgIA.

4. Results

In this section, the findings derived from the implementation of the MathProgIA framework in Topographic Survey Technology of the School of Military Engineers are presented and analyzed. The analysis is structured through a mixed approach, where the quantitative trends obtained from the longitudinal statistical monitoring are complemented and enriched by the qualitative perspectives emerging from the voices of the participants. Through this articulation, it seeks to understand both the behavior of the learning model and the cognitive and epistemological transformations experienced by students.

4.1.Longitudinal evolution of mental models

The quantitative analysis was developed from the longitudinal matrix built with the evidence obtained during six months of implementation of the hybrid learning ecosystem. The information corresponded to the 17 participating students, who developed 18 workshops with an intensity of nine hours per week. The integration of progressive assessments, academic products, performance records, and interactions with digital resources made it possible to analyze the evolution of mental models as a dynamic trajectory of conceptual construction and not as an isolated measurement of learning.

The study variable corresponded to the evolution of the mental model in variational and statistical-random thinking. For its analysis, five progressive levels of conceptual construction were established—elementary, basic, reconstructive, prospective, and inclusive—organized according to the logic of Knowledge Space Theory (KST). This organization made it possible to represent learning trajectories as displacements between states of knowledge, considering not only the obtaining of correct answers, but also the ability to interpret, relate, reconstruct and transfer mathematical concepts to contextualized situations.

The longitudinal follow-up showed a progressive evolution of the participants between the different levels of the knowledge space. In the early stages, fragmented procedures, dependence on previously solved examples, and difficulties in integrating graphical representations, statistical analysis, and mathematical modeling in problems associated with topographic surveys predominated. These limitations were especially evident in activities that required interpreting data, analyzing variability, and justifying mathematical decisions.

As the training process progressed, progressive changes were observed towards higher levels of conceptual construction. Students developed greater ability to argue procedures, establish relationships between different mathematical representations, interpret results, and apply knowledge in contextualized technical situations. Overall, the evidence indicates a gradual reorganization of mental models towards more integrated conceptual structures, characterized by greater interpretative autonomy and transference capacity.

4.2. Performance of the longitudinal model

Based on the records generated during the 18 workshops, a multivariate statistical model was estimated to analyze the association between the variables of the hybrid learning ecosystem and the evolution of conceptual performance. Explanatory variables included interaction with digital resources, time spent on activities, quality of questions asked by students, feedback processes, integration of digital and analog experiences, resources used, teacher mediation, and progressive organization of content through KST.

The model presented an *adjusted R2* of 0.983, indicating a high explanatory capacity for the set of longitudinal observations analyzed. This result shows consistent associations between the characteristics of the training process and the observed evolution of the mental models during the study period.

However, due to the methodological design based on a case study, the small number of participants and the absence of a control group, these results should be interpreted as associative evidence limited to the context investigated and not as evidence of generalizable causal relationships.

The greatest transformations were observed in activities related to the modeling of topographical errors, applied statistical analysis and the resolution of contextualized problems. The interaction sustained for six months, continuous feedback and the articulation between face-to-face and digital activities were associated with progressive improvements in the interpretation of data, mathematical argumentation and the application of procedures in scenarios typical of military technical training.

4.3. Findings of the training process

In addition to the quantitative evolution observed, longitudinal monitoring allowed the identification of relevant patterns associated with the functioning of the hybrid learning ecosystem. One of them was the progressive transformation of students' interaction with artificial intelligence tools. In the early stages, some participants tended to accept the automatically generated responses without undergoing sufficient conceptual verification. However, as the implementation of the pedagogical audit progressed, the answers produced by artificial intelligence began to be used as inputs for comparison with mathematical procedures, original sources, and validations carried out by the teacher.

This change was accompanied by a strengthening of the processes of argumentation, conceptual review and informed decision-making, evidencing a progressively more critical use of artificial intelligence within the learning ecosystem.

Likewise, the analysis allowed us to recognize the influence of factors specific to the institutional context. Initially, the military hierarchy conditioned some dynamics of participation and collaboration; however, as the training process progressed, interaction tended to be organized around mathematical argumentation, problem solving and the quality of academic evidence rather than the hierarchical position of the participants. Although the presence of only two female students does not allow comparisons by gender, the observations made suggest that the hybrid ecosystem favored participation scenarios focused on academic performance and technical argumentation

4.4. Student perception of the use of artificial intelligence and knowledge validation processes

In order to complement the quantitative results, a qualitative analysis of the participants' open-ended responses to five questions was carried out to explore the evaluation of the workshops, the use of artificial intelligence tools, the identification of hallucinations, and the perceived differences between the evaluation carried out by artificial intelligence and the teacher evaluation. The thematic analysis allowed the identification of six emerging categories:

evaluation of the workshops, benefits of the use of artificial intelligence, difficulties during its use, identification of hallucinations, validation strategies and perception of the differences between automated assessment and human assessment.

The participants positively valued the didactic sequence implemented, highlighting the progressive organization of the contents and their application to situations close to the professional context. Although a significant part of the students considered that the length of the workshops was high, this perception did not translate into a negative assessment of the learning experience, but was interpreted as a consequence of the level of deepening required to develop competencies in mathematical and statistical reasoning.

In relation to the use of artificial intelligence, most students described it as a support tool to understand procedures, resolve doubts and verify results, rather than as a mechanism to replace the learning process. However, they also recognized limitations associated with the formulation of queries, the generation of incorrect answers and the appearance of interpretation errors, especially in problems that required mathematical reasoning or database management.

A particularly relevant finding was that all participants stated that they had identified at least one hallucination generated by artificial intelligence during the development of the activities. The strategies used to overcome these inconsistencies included comparing different artificial intelligence systems, manually verifying procedures, consulting other sources of information and, mainly, contrasting with the concepts worked on during the face-to-face sessions. These results show that the interaction with artificial intelligence did not lead to an uncritical acceptance of the answers generated, but rather promoted verification and evaluation processes based on previously constructed disciplinary knowledge.

Finally, the students established a clear difference between the evaluation carried out by artificial intelligence and that developed by the teacher. While the technological tool tended to privilege the final result and to grant more favorable grades, teacher evaluation was perceived as a process focused on the quality of the procedure, mathematical argumentation and the application of previously defined technical criteria. This perception is a finding of special interest, since it supports one of the central principles of the MathProgIA framework, demonstrating that artificial intelligence acquires educational value when its answers are subjected to systematic pedagogical audit processes guided by the teacher. The emerging categories identified from this qualitative analysis and their implications are summarized in Table 2.

Table 2. Thematic analysis of student perception of the use of artificial intelligence.

Category	**Main finding**	**Involvement for MathProgIA**
Valuation of the hybrid ecosystem	Positive acceptance of the didactic sequence and observations on the length of the tasks.	The relevance of knowledge organization through KST for progressive design is confirmed.
Benefits of AI	Use for the explanation of procedures, resolution of doubts and verification of results.	The technological resource is validated as a cognitive support for learning.
Difficulties experienced	Limitations associated with the formulation of queries, incorrect answers and interpretative errors.	The need to incorporate literacy processes in the critical use of technology is evident.
Identifying hallucinations	Recurrent detection of miscalculations, reasoning failures, and misinterpretations.	The methodological requirement to submit AI responses to validation processes is ratified.
Validation strategies	Execution of contrasts with alternative sources, manual developments and acquired knowledge.	The development of critical thinking and self-regulation of the student is promoted.
Evaluative discrepancy	AI's preference for the final result over the teaching requirement of procedure and argumentation.	The pedagogical audit is supported as an essential and irreplaceable

		component of the proposed framework.

5. Discussion

The results obtained show that the implementation of the MathProgIA framework favored processes of evolution of mental models in students of military technical training through the articulation between hybrid learning ecosystems, progressive organization of knowledge and pedagogical mediation supported by artificial intelligence. In this sense, the findings are consistent with the theoretical approach developed in this study, according to which conceptual evolution does not depend exclusively on access to technological tools, but on the interaction between cognitive processes, pedagogical strategies and knowledge validation mechanisms.

The high adjustment observed in the longitudinal model (adjusted R2 = 0.983) reflects the consistency between the variables analyzed and the conceptual evolution recorded during the six months of implementation. However, given the methodological characteristics of the study, this result should be interpreted as contextual evidence of association and not as a demonstration of generalizable causal relationships.

One of the most relevant findings corresponds to the pattern of conceptual reconstruction identified from the analysis of the semi-structured interviews, complemented with the evidence obtained during the longitudinal follow-up of the training process. The students' testimonies showed that the contrast between the answers produced by artificial intelligence systems, the original sources, the mathematical procedures and the pedagogical audit progressively modified their way of interacting with technology and with disciplinary knowledge.

In particular, the contrast between the answers generated by an exploratory artificial intelligence (Gemini) and the pedagogical audit carried out by the teacher, supported by NotebookLM configured exclusively with original and unaltered sources, promoted systematic processes of questioning, contrast of evidence, argumentation and conceptual review. The convergent analysis of this evidence allowed us to identify a recurrent mechanism of knowledge reconstruction that, as an emerging finding of this research, is called the Evaluative Discrepancy Effect.

The Evaluative Discrepancy Effect, proposed in this study as an emergent construct, is understood as the process by which the discrepancy between an AI-generated response and verifiable disciplinary evidence triggers processes of metacognition, critical validation, and reorganization of mental models. This interpretation emerges from the triangulation between the semi-structured interviews, the learning records and the performance evidence collected during the study, so it does not constitute an effect attributable to a specific artificial intelligence tool, but to the pedagogical design implemented in the MathProgIA framework. From this perspective, the Evaluative Discrepancy Effect is configured as a pedagogical and epistemological phenomenon that favors argumentation, conceptual reconstruction and the consolidation of scientifically based mental models.

Consequently, the results suggest that artificial intelligence acquires educational value when it is incorporated into a learning ecosystem where interaction with the teacher, mathematical argumentation and critical validation guide its use. From this perspective, artificial intelligence does not replace human reasoning, but acts as a mediating resource whose contribution depends on the quality of the pedagogical design and the reflection processes that accompany its use.

6. Conclusions

The present research demonstrated that the evolution of mental models in variational and statistical-random thinking can be understood as a dynamic process of construction and reconstruction of knowledge when learning is developed in a hybrid ecosystem articulated through the MathProgIA framework. The longitudinal follow-up showed that the conceptual transformations observed in the students were not only associated with access to digital tools or artificial intelligence, but also with the permanent interaction between teacher mediation, contextualized problem solving, the progressive organization of knowledge and the processes of critical validation of information. From this perspective, learning is configured as a progressive epistemological construction in which the student reorganizes his or her mental models through contrast, argumentation, and reflection on his or her own cognitive process.

The results support the theoretical position that guides MathProgIA when considering that artificial intelligence is not an autonomous teaching agent, but a mediator whose contribution depends on the pedagogical architecture in which it is integrated. Consequently, the educational value of these technologies does not lie in their ability to generate answers, but in favoring processes of interrogation, analysis, explanation, and validation that lead to the construction of scientifically grounded knowledge. In this sense, Explainable Artificial Intelligence acquires a pedagogical and epistemological interpretation in which explainability is materialized through processes of source auditing, evidence contrast and conceptual reconstruction, overcoming a vision restricted to algorithmic transparency.

The main scientific contribution of this research is to consolidate the MathProgIA framework as an integrative proposal that articulates cognitive, technological, pedagogical and epistemological foundations to explain the evolution of mental models in educational contexts mediated by artificial intelligence. The framework integrates the theory of mental models as the foundation of conceptual transformation, hybrid ecosystems as spaces of interaction for the construction of knowledge, the Theory of the Knowledge Space as a structure to represent learning trajectories and Explainable Artificial Intelligence, complemented by pedagogical audit processes, as a mechanism to strengthen the reliability of the knowledge constructed. Rather than proposing a new theory of learning or redefining Explainable Artificial Intelligence, MathProgIA offers a conceptual architecture that allows us to understand the interaction between these perspectives within a coherent educational process.

Likewise, the research allowed us to identify a recurrent pattern of conceptual reconstruction associated with the contrast between the responses generated by artificial intelligence and verifiable disciplinary evidence. This behavior, called in the context of this study the Evaluative Discrepancy Effect, constitutes an emergent construct derived from the qualitative analysis of the interviews and the triangulation with the other empirical evidence. The finding describes the process by which the contrast between the responses generated by an exploratory artificial intelligence and verifiable disciplinary evidence triggers processes of cognitive destabilization, followed by active verification, argumentation and reorganization of mental models. Due to the methodological scope of the research, this effect should be understood as a contextual finding whose general validation will require new studies in different educational settings.

The results obtained also contribute to consolidating a line of research aimed at studying the evolution of mental models through hybrid learning ecosystems with artificial intelligence. The research carried out progressively in recent years has made it possible to move from the characterization of mathematical learning processes to the formulation of a conceptual and methodological framework that integrates empirical evidence, theoretical foundations and educational implementation strategies. In this sense, this work represents a point of convergence within a research agenda that seeks to understand how knowledge is constructed, validated, and transformed in educational scenarios mediated by smart technologies.

However, the findings should be interpreted considering the limitations of the study. The longitudinal case study design, the small sample size, the absence of a comparison group and the development of the research in a specific context of military technical training limit the generalizability of the results. Consequently, the identified associations describe the behavior of the implemented learning ecosystem and offer contextual evidence on the observed processes, without pretending to establish universal causal relationships.

As a projection, the need to expand the empirical validation of the MathProgIA framework in other educational levels, disciplines and institutional contexts is raised, as well as to deepen the study of the Evaluative Discrepancy Effect to accurately establish its conditions of appearance, its underlying cognitive mechanisms and its direct relationship with the evolution of mental models. Likewise, future research may integrate learning analytics, adaptive models based on the Theory of the Knowledge Space and new pedagogical audit strategies that strengthen the construction of verifiable knowledge in environments mediated by artificial intelligence technologies.

Overall, this research provides evidence that the construction of knowledge in educational settings mediated by artificial intelligence constitutes an essentially cognitive, social and epistemological process. Technology, by itself, does not transform learning; It is the interaction between the student, the teacher, disciplinary knowledge and the systematic processes of validation that favors the evolution of mental models. From this perspective, MathProgIA is configured as a conceptual and methodological framework that guides the understanding of these processes and provides a conceptual basis for future research on the construction of knowledge in hybrid learning ecosystems mediated by artificial intelligence. The results obtained suggest that the methodological principles that support this framework have the potential for transfer to other educational contexts characterized by cultural diversity, social vulnerability and technical training. However, this projection requires empirical validation through multicenter and intercultural studies. In this sense, MathProgIA offers a conceptually solid basis for future research aimed at evaluating its transferability, its adaptation to different educational contexts and its contribution to the design of hybrid learning ecosystems based on principles of explainable artificial intelligence and educational equity.

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