

Development of an Enhanced Evaluation Model to Assess the Effectiveness of Peer Learning Compared to Traditional Learning: An Improvement of the TPL Model

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Abstract: This study presents the Dynamic Peer Learning Evaluation (DPLE) model, a structured enhancement of the Traditional versus Peer Learning (TPL) evaluation model. The TPL model classifies students into four static compartments which are Skilled from Traditional Learning, Unskilled from Traditional Learning, Skilled from Peer Learning, and Unskilled from Peer Learning. It evaluates learning effectiveness through a linear, curriculum-bounded framework governed by constant transition rates. While conceptually sound, the TPL model does not capture interaction dynamics between student groups, does not incorporate a unified mastery outcome state, and employs parameters whose pedagogical interpretation is insufficiently specified. The DPLE model addresses these limitations through four structural advances: (i) interaction-driven, nonlinear transition rates that model how existing traditional and peer learners recruit unengaged students, mirroring the contact dynamics of the SIR epidemiological framework; (ii) a unified Mastery compartment that consolidates achievement outcomes from both learning pathways; (iii) pedagogically grounded parameter definitions that align directly with observable classroom processes; and (iv) a revised six-step application procedure grounded in longitudinal data collection, finite-difference derivative estimation, and least-squares parameter optimization. Together, these instruments provide a mathematically tractable and practically actionable model for evaluating and optimizing learning strategies in diverse educational settings

Keywords: Peer learning; traditional learning; compartmental model; ordinary differential equations; mastery

1. Introduction

The question of which teaching and learning method best supports student achievement remains a central concern in educational research. Traditional learning, characterized by instructor-led classroom delivery, has long served as the dominant mode of instruction across educational institutions worldwide. However, traditional learning environments continue to exhibit several limitations, including reliance on a one-size-fits-all teaching approach, limited connections to real-world applications, inadequate development of perceptual and higher-order thinking skills, and insufficient opportunities for students to actively participate in the learning process. As a result, conventional classrooms are increasingly unable to meet the diverse needs and expectations of contemporary learners (Hu, 2024). Peer learning, in which students acquire knowledge and skills through structured collaboration and mutual support, has emerged as one of the most promising alternatives, with studies documenting improved performance, enhanced communication, and greater student engagement in peer learning environments (Ahmad & Mohamed, 2018).

Despite the demonstrated benefits of peer learning, there remains no universally applicable method for quantifying its effectiveness relative to traditional learning in a given institutional context. Mazibuko (2026) addressed this gap through the development of the Traditional versus Peer Learning (TPL) evaluation model a mathematical compartmental model that classifies students into four groups (Skilled from Traditional Learning, Unskilled from Traditional Learning, Skilled from Peer Learning, and Unskilled from Peer Learning) and uses ordinary differential equations (ODEs) to track student transitions. A key feature of the TPL model is the basic reproductive ratio (R_0), which determines the relative effectiveness of peer versus traditional learning, and a Specific Threshold (ST) parameter that identifies the minimum proportion of knowledgeable students needed to make peer learning impactful when it is found to be less effective.

While the TPL model represents a significant contribution to the field, it operates within certain



structural constraints that limit its applicability in dynamic classroom environments. Specifically, the TPL model assumes a static, linear progression within a bounded curriculum period, does not explicitly model interactions between students across learning modalities, does not incorporate a direct mastery outcome state, and related parameters are not well defined to align with pedagogical practices. These limitations motivated the development of an enhanced model the Dynamic Peer Learning Evaluation (DPLE) model which is the subject of this paper.

The DPLE model retains the conceptual foundations and philosophical intent of the TPL model whilst introducing structural and mathematical improvements that better reflect the dynamic realities of classroom learning. Specifically, the DPLE model introduces: (i) interaction-driven transition rates that capture how traditional and peer learners influence unengaged students; (ii) a unified mastery compartment that consolidates outcomes from both learning pathways; (iii) redefined pandemic-related parameters to better align with effective pedagogical practices; and (iv) a revised application procedure grounded in longitudinal data collection and parameter fitting using regression optimization.

2. Literature Review

2.1. Evaluation of Peer and Traditional Learning Methods

A substantial body of literature has examined the comparative effectiveness of peer and traditional learning across various disciplines and educational levels. Sabaq, Farouk, and Ismail (2016) compared peer teaching and traditional teaching methods among nursing students in the context of pediatric cardiopulmonary resuscitation, finding significantly higher knowledge and performance scores in the peer teaching group. The authors recommended integrating peer teaching into undergraduate nursing curricula. Similarly, Ahmad and Mohamed (2018) reported that peer learning groups demonstrated higher performance scores and increased peer interaction relative to traditional instruction, with students reporting greater ease of communication with peers than with instructors.

Not all comparative studies favour peer learning. Schwerdt and Wuppermann (2009) found that traditional teaching was associated with higher student achievement in a within-student, between-subject design. Tularam (2018) reviewed mathematics and engineering education studies, noting that while non-traditional methods are generally preferred, many studies report only marginal or non-significant improvements in learning outcomes. Wang (2022) similarly concluded that although modern methods offer broader advantages, the choice of teaching method must be tailored to the specific instructional context, student needs, and learning objectives.

These conflicting findings reinforce the position that no single teaching method is universally superior, and that institutions require context-specific evaluation tools to determine which approach best serves their students. The TPL model (Mazibuko, 2026) was designed precisely to meet this need, and the DPLE model extends this work into more dynamic classroom contexts.

2.2. Mathematical Compartmental Models in Education

Mathematical compartmental modelling in education is rooted in epidemiological theory, particularly the Susceptible-Infected-Recovered (SIR) model developed by Kermack and McKendrick (1927). Under this model, a population is divided into three distinct groups according to disease status: those who are susceptible (S), those who are currently infected (I), and those who have since recovered (R). The model is governed by two key parameters: β , the transmission rate, and γ , the recovery rate, as depicted in Figure 1.

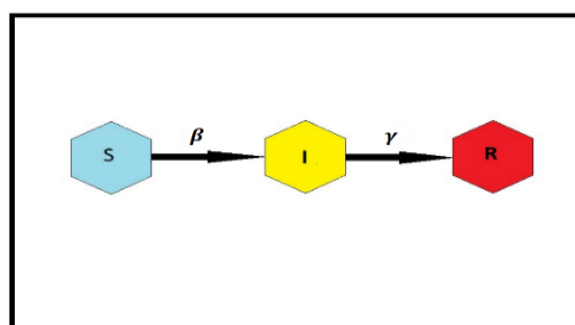


Figure 1. Susceptible, Infected and Recovered (SIR) Model (Mazibuko and Maharaj, 2024a)

Here is a short description of each compartment in the SIR model,

- **Susceptible:** Individuals who have not yet encountered the infection and remain vulnerable to

contracting it. Once they acquire the disease, they shift into the Infected group.

- **Infected:** Individuals who currently carry the disease and are capable of transmitting it to those who are susceptible. They stay in this category throughout their contagious phase before moving into the Recovered group.
- **Recovered:** Individuals who have overcome the illness and are considered to have lasting immunity, meaning they no longer return to the susceptible state.

The equations of the SIR model are expressed as follows (Kermack and McKendrick, 1927):

$$\left. \begin{aligned} \frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned} \right\} \quad (1)$$

Mazibuko and Maharaj (2024a) adapted this framework to evaluate higher-order thinking skills (HOTS) in mathematics curricula through the SVHIR model, which extended the SIR model to five compartments. A subsequent extension by Mazibuko and Maharaj (2024b) produced the RCUSP model, broadening the scope to general curriculum evaluation across multiple skill types.

Building on this lineage, the original TPL model (Mazibuko, 2026) applied compartmental modelling principles to the direct comparison of peer and traditional learning. The TPL model uses four compartments and linear ODEs to track skilled and unskilled populations within each learning modality. While this approach is conceptually elegant and mathematically tractable, it does not incorporate interaction dynamics between student groups a feature that the DPLE model is designed to address.

3. Methodology

3.1. Design Principles of the DPLE Model

The DPLE model was designed according to the following principles:

- i) The model must capture interaction driven transitions. Specifically, the influence of existing traditional and peer learners on unengaged students.
- ii) The model must include a mastery compartment that consolidates skilled outcomes from both pathways.
- iii) The model must retain analytical tractability to permit parameter estimation from empirical classroom data.
- iv) The model must include a revised Knowledge Reproduction Ratio that directly compares the mastery-production rates of the two learning pathways.
- v) The model must be accompanied by a clear, step-by-step application procedure suitable for practitioners without advanced mathematical training.

3.2. Analogy with the SIR Model

As with the TPL model, the DPLE model draws its structural inspiration from the SIR epidemiological model. In the SIR model, susceptible individuals are recruited into the infected compartment through contact with existing infected individuals at a rate proportional to both populations (βSI). This interaction term is key: it ensures that the spread of infection accelerates when the infected population is large and decelerates as the susceptible pool is exhausted.

The DPLE model applies the same logic to the learning context: unengaged students (U) are recruited into traditional learning (T) through interactions with existing traditional approach at rate α , and into peer learning (P) through interactions with existing peer learners at rate θ . This produces nonlinear interaction terms (αUT and θUP) that more faithfully represent how learning behaviors spread through a classroom community, more details about the model will be found in the next section.

4. Results and Discussion

4.1 The DPLE Model Structure

The DPLE model partitions the student population into four compartments, as illustrated in Figure 2:

- Unengaged students (U): Students who have not yet been recruited into either learning pathway. On the first day of the course, all students are assumed to begin in this compartment.
- Students progressing through traditional learning (T): Students who are actively engaged in the curriculum via traditional instruction (lectures, textbooks, instructor-led activities).
- Students progressing through peer learning (P): Students who are actively engaged in the curriculum primarily through peer interaction, collaborative activities, and shared knowledge construction.
- Students who have mastered the content (M): Students who have achieved the predetermined curriculum objectives, irrespective of the learning pathway through which mastery was attained.

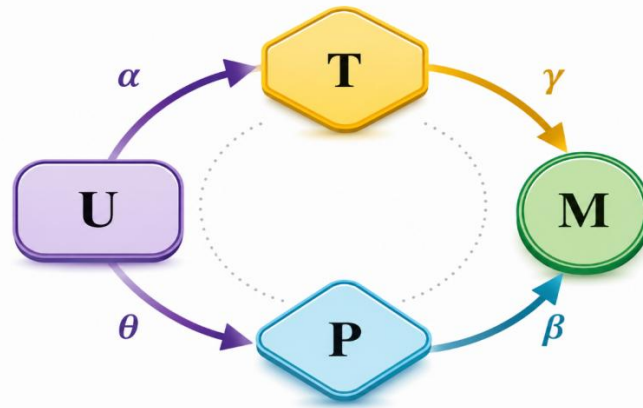


Figure 2. The Dynamic Peer Learning Evaluation (DPLE) Model. U = Unengaged; T =Traditional learners; P = Peer learners; M = Mastery.

4.2 Model Parameters

The DPLE model is governed by four parameters:

- *Alpha* (α): The rate at which traditional learners (T) recruit unengaged students (U) into traditional learning. It measures how strongly traditional teaching converts students into active learners.
- *Theta* (θ): The rate at which peer learners (P) recruit unengaged students (U) into peer learning. It measures how strongly peer learning converts students into active learners.
- *Gamma* (γ): The transition rate (speed) at which traditional learners move from active engagement to mastery (M). Reflects the efficiency of traditional instruction in producing content mastery.
- *Beta* (β): The transition rate (speed) at which peer learners move from active engagement to mastery (M). Reflects the efficiency of peer learning in producing content mastery.

4.3 Mathematical Formulation

The DPLE model is described by the following system of ordinary differential equations (ODEs)

$$\left. \begin{aligned} \frac{dU}{dt} &= -\alpha UT - \theta UP & (a) \\ \frac{dT}{dt} &= \alpha UT - \gamma T & (b) \\ \frac{dP}{dt} &= \theta UP - \beta P & (c) \\ \frac{dM}{dt} &= \gamma T + \beta P & (d) \end{aligned} \right\} \quad (2)$$

Equation 2(a) – describes the change of unengaged students' compartment over time $\left(\frac{dU}{dt}\right)$

- $-\alpha UT$: Traditional learners recruit unengaged students. The term is negative because it reduces the number of unengaged students.
- $-\theta UP$: Peer learners recruit unengaged students. Again, negative because unengaged students leave this compartment.

In summary, equation 2(a) shows that the unengaged pool decreases as both traditional and peer learners bring them into active learning.

Equation 2(b) – describes the change of traditional learners' compartment over time $\left(\frac{dT}{dt}\right)$

- αUT : Growth term, unengaged students recruited by traditional learners increase the number of traditional learners.
- $-\gamma T$: Loss term, traditional learners eventually achieve mastery and leave this compartment.

In summary, equation 2(b) balances recruitment into traditional learning with the transition out to mastery.

Equation 2(c) – describes the change of peer learners' compartment over time $\left(\frac{dP}{dt}\right)$

- θUP : Growth term, unengaged students recruited by peer learners increase the peer learner population.
- $-\beta P$: Loss term, peer learners eventually achieve mastery and leave this compartment.

In summary, equation 2(c) mirrors the traditional learner dynamics but for peer learning.

Equation 2(d) – describes the change of mastery' compartment over time $\left(\frac{dM}{dt}\right)$

- γT : Traditional learners who reach mastery add to this compartment.
- βP : Peer learners who reach mastery also add here.

In summary, equation 2(d) accumulates all students who have completed the curriculum, regardless of pathway.

The DPLE model is founded on the three set of underlying conditions. These conditions shape the design and application of the model. They are essential to understanding how the model operates and are as follows:

- 1) The model assumes that effectiveness is measured solely by the rate of transition to mastery, independent of participation size.
- 2) The model is closed, meaning we do not consider dropouts, non-participants and external inflow/outflow therefore:

$$U + P + T + M = N \quad (\text{where } N \text{ is the total population}) \quad (3)$$

- 3) Initial conditions are considered to be at $t=0$.

4.4. The Mastery Efficiency Ratio (MER)

Drawing on the concept of the basic reproductive ratio (R_0) from epidemiology and from the TPL model, the DPLE model redefines it as Mastery Efficiency Ratio (MER) as the ratio of the mastery transition rate for traditional learning to that for peer learning expressed as:

$$MER = \frac{\beta}{\gamma} \quad (4)$$

It quantifies the relative effectiveness of instructional methods by comparing their rates of transition from participation to mastery. Therefore, MER measures how efficiently each method converts learners into knowledgeable individuals. The value of MER determines the effectiveness of the peer learning as follows:

Case 1: If $MER > 1$, means the rate at which peer learners master the content is higher compare of the traditional learners. Hence peer learning is effective.

Case 2: If $MER < 1$, means the rate at which peer learners master the content is lower compare of the traditional learners. Hence peer learning is ineffective.

Case 3: If $MER = 1$, means the rates at which peer learners and traditional learners master the content are equal. Hence both learning approaches are equally effective.

4.5. The Threshold Value

When Case 2 is observed, that is when traditional learning currently outperforms peer learning. The key institutional question becomes: how many knowledgeable students must be present in the system to make peer learning impactful? Drawing on the herd immunity threshold concept from epidemiology (Topley & Wilson, 1923; Smith, 2010), the DPLE model introduces the Threshold Value (TV):

$$TV = \left(1 - \frac{1}{MER}\right) \times 100\% \quad (5)$$

The TV formula determines the actual percentage of knowledgeable students required in the system to make peer learning impactful. This provides actionable guidance: if the required threshold is low, the institution can invest directly in peer learning infrastructure; if it is high, alternative strategies should be prioritized.

4.6. Comparison with the Original TPL Model

Table 1 provides a structured comparison of the original TPL model (Mazibuko, 2026) and the enhanced DPLE model.

Table 1. Comparison of the original TPL model (Mazibuko, 2026) and the enhanced DPLE model.

Feature	Original TPL Model (Mazibuko, 2026)	Enhanced DPLE Model (This Study)
Transition mechanism	Linear, constant rates ($\gamma_T, \gamma_P, \beta_T, \beta_P$)	Interaction-driven (αUT and θUP); nonlinear ODEs
Mastery outcome	Separate skilled compartments (S_T and S_P); no unified mastery state	Unified mastery compartment M ; aggregates from both pathways
Parameter Descriptors	- Reproductive Ratio: $R_0 = \frac{\gamma_P}{\gamma_T}$ - Specific Threshold: $ST = \left(1 - \frac{1}{R_0}\right) \times 100\%$	- Mastery Efficiency Ratio $MER = \frac{\beta}{\gamma}$ - Threshold Value: $TV = \left(1 - \frac{1}{MER}\right) \times 100\%$
Application procedure	4 steps; data collection, model validation, ratio computation, conclusion	6 steps; adds derivative estimation and regression fitting for parameter estimation

4.7. Application Procedure

The DPLE model is applied through a six-step procedure designed to be accessible to educational practitioners with limited mathematical training. Each step is described below.

Step 1: Collect Longitudinal Classroom Data

The evaluator must track students at regular intervals (weekly or per lesson) and classify each student into one of the four DPLE compartments: U (unengaged/disengaged), T (actively learning through traditional instruction), P (actively learning through peer learning), or M (mastered the content). A suitable data collection instrument is questionnaire, observation checklist, or assessment. It should be designed and validated for the specific course context.

Step 2: Observed Rate of Change from the Data

Once longitudinal data have been collected, the rates of change for each compartment must be

estimated using finite differences. For a time, interval Δt , the derivative is approximated as:

$$\left. \begin{aligned} \frac{dU}{dt} &= \frac{U_{t+1} - U_t}{\Delta t} & (a) \\ \frac{dT}{dt} &= \frac{T_{t+1} - T_t}{\Delta t} & (b) \\ \frac{dP}{dt} &= \frac{P_{t+1} - P_t}{\Delta t} & (c) \\ \frac{dM}{dt} &= \frac{M_{t+1} - M_t}{\Delta t} & (d) \end{aligned} \right\} \quad (6)$$

This procedure is repeated across all consecutive time intervals, producing a dataset of observed derivatives that will be used in the next step.

Step 3: Solve for Parameters Using the Model Equations

For each time point, the observed compartment values obtained in Step 1 and the observed derivatives computed in Step 2 are substituted into the DPLE model equations. Repeating this process across all time intervals generates a system of equations involving the unknown parameters α , θ , γ and β . Since the number of equations will typically exceeds the number of unknown parameters, an over-determined system is obtained, which is subsequently resolved in Step 4 using the least-squares fitting method.

Step 4: Estimate Parameters Using Regression Optimization

The four parameters (α , θ , γ and β) are estimated by minimizing the total squared error between observed and model-predicted values obtained from Equation (2):

$$\text{Minimize } \sum (\text{Observed} - \text{Predicted})^2 \quad (7)$$

This optimization can be implemented using standard computational tools: Python, MATLAB, R, or Excel Solver for smaller datasets.

The primary objective of this step is to identify the parameter values that provide the best fit to the observed data from among the range of possible parameter combinations. Consequently, this stage is concerned with parameter estimation rather than model validation. Although it determines the most appropriate values for the model parameters, it does not evaluate the statistical significance of the differences between observed and predicted values, nor does it provide evidence regarding the overall validity of the model.

Step 5: DPLE model validation

The DPLE model is formulated in a general framework and does not automatically apply to a specific dataset. Consequently, it is necessary to calibrate the model using the available data and validate its performance before drawing any conclusions. This validation process involves solving compartments in equations 2(a) – 2(d), then substitute the rates of change and parameters obtained in Steps 2 – 4 into to generate predicted values for each compartment. The predicted values are then compared with the corresponding observed values. If the discrepancies between the observed and predicted compartment values are found to be statistically insignificant, the model may be regarded as valid for the dataset under consideration. Various statistical and error-based measures can be employed to assess the significance of these differences; therefore, the choice is left to the discretion of the researcher.

Step 6: Compute the Mastery Efficiency Ratio and Threshold Value; Draw Conclusions

Using the obtained parameters in step 4, compute:

$$MER = \frac{\beta}{\gamma}$$

Hence, determine the case in the following for conclusion.

Case 1: If $MER > 1$, means the rate at which peer learners master the content is higher compare of the traditional learners. Hence peer learning is effective.

Case 2: If $MER < 1$, means the rate at which peer learners master the content is lower compare of the traditional learners. Hence peer learning is ineffective.

Case 3: If $MER = 1$, means the rates at which peer learners and traditional learners master the content are equal. Hence both learning approaches are equally effective.

For Case 2, the Threshold Value (TV) defined in equation (5) should be calculated to establish the precise percentage of knowledgeable students required within the system for peer learning to become impactful. This computation provides actionable guidance by quantifying the minimum critical mass of informed learners necessary to shift peer learning from ineffective to effective.

5. Conclusions

This paper presents the Dynamic Peer Learning Evaluation (DPLE) model as a structured advancement of the Traditional versus Peer Learning (TPL) model developed by Mazibuko (2026). The DPLE model responds to several limitations inherent in the TPL framework, notably its assumption of a static, linear progression within a bounded curriculum period, its limited representation of student interactions across diverse learning modalities, and the absence of a distinct mastery outcome state to capture consolidated achievement. In addition, the parameters within the TPL model are not sufficiently specified to ensure alignment with effective pedagogical practices.

To address these concerns, the DPLE model incorporates nonlinear ordinary differential equation (ODE) based interaction terms, introduces a consolidated mastery compartment, and refines key parameter definitions to enhance their pedagogical relevance, specifically Ratio and Threshold. It is further supported by a six-step application procedure grounded in longitudinal data collection and parameter optimization. Together, these developments offer a more dynamic, realistic, and practically applicable framework for evaluating peer learning.

As with the original TPL model, the DPLE model has not yet been empirically validated in a real classroom study. The next phase of research will focus on applying the model to longitudinal classroom data to assess its goodness-of-fit, refine parameter estimation procedures, and evaluate its practical utility as an institutional decision-support tool. The authors acknowledge that further refinements to the model structure may emerge during this empirical phase.

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Declaration of Interest

In the current study, AI-assisted tools were used for grammar correction purposes only. All mathematical content, model development, and intellectual contributions are solely those of the author.

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