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Article

A Novel Ranking Function for Solving Single-Valued Trapezoidal Neutrosophic Transportation Problems Using the Zero-Point Method

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Abstract: . The transportation problem is one of the most widely studied special structures of linear programming, with extensive applications in supply-chain management, logistics and distribution planning. In realistic distribution environments the cost, availability and requirement parameters are rarely known with certainty; they are simultaneously affected by approval (truth), hesitation (indeterminacy) and rejection (falsity). The neutrosophic set, which models these three components independently, is therefore a more faithful tool than the classical fuzzy or intuitionistic fuzzy set. In this paper we study the transportation problem in which all parameters are single-valued trapezoidal neutrosophic numbers (SVTNNs). A novel ranking function is proposed that combines a generalised trapezoidal magnitude index with the neutrosophic score, thereby capturing both the spread of the trapezoid and the truth–indeterminacy–falsity confidence of each datum in a single real value. Using this ranking function the single-valued trapezoidal neutrosophic transportation problem (SVTNTP) is converted, in the first stage, into an equivalent crisp transportation problem; in the second stage the crisp problem is solved directly by the zero-point method. The optimality of the obtained solution is verified against the North-West Corner rule improved by the MODI (modified distribution) method. A numerical illustration with three sources and four destinations shows that the proposed procedure yields the optimal neutrosophic transportation cost in fewer and simpler steps than the classical two-phase approach, while requiring no additional optimality test.
Keywords: Neutrosophic set; single-valued trapezoidal neutrosophic number; ranking function; score function; transportation problem; zero-point method; MODI method.

1. Introduction

The transportation problem (TP) was first formulated by Hitchcock in 1941 as the problem of distributing a homogeneous product from several sources to a number of destinations at minimum total cost, and it remains one of the earliest and most fruitful applications of linear programming. Classical solution schemes obtain a basic feasible solution by the North-West Corner rule, the Least-Cost method or Vogel's Approximation Method and then improve it to optimality using the stepping-stone or the MODI procedure. These schemes presuppose that the unit costs and the supply and demand quantities are precisely known real numbers.

In practice, however, the data of a transportation network are seldom deterministic. Unit transportation costs fluctuate with fuel prices, road and weather conditions, tolls and delivery penalties, while availabilities and requirements depend on forecasts that are themselves uncertain. To represent such imprecision, Zadeh (1965) introduced the theory of fuzzy sets, in which each element is assigned a degree of membership in the unit interval. Atanassov (1986) extended this idea to intuitionistic fuzzy sets by adding a degree of non-membership, with the restriction that the sum of membership and non-membership does not exceed one. Neither framework, however, can model the situation in which a decision-maker is simultaneously, and independently, partly convinced, partly hesitant and partly opposed regarding a parameter value.

To overcome this limitation, Smarandache (1998) proposed the neutrosophic set, which attaches to every element three independent membership functions as truth (T), indeterminacy (I) and falsity (F), each taking values in

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the non-standard unit interval. For computational tractability, Wang et al. (2010) introduced the single-valued neutrosophic set (SVNS), restricting the three functions to the ordinary interval [0,1]. When the underlying carrier is a trapezoidal number, the resulting object is a single-valued trapezoidal neutrosophic number (SVTNN), which is particularly suited to expressing quantities such as "the unit cost is around 4 to 8, with truth 0.7, indeterminacy 0.3 and falsity 0.3."

A substantial body of work has addressed the fuzzy transportation problem. Pandian and Natarajan (2010) introduced the zero-point method and the related zero-suffix technique, which obtain the optimal solution of a crisp transportation problem directly, without the separate optimality test required by the classical two-phase approach. Kaur and Kumar (2012) solved fuzzy transportation problems with generalised trapezoidal fuzzy numbers, and many ranking-based reductions have since appeared, including the trapezoidal-fuzzy treatment by Roy et al. (2020) using the zero-point method, which motivates the present study. For the neutrosophic case, Thamaraiselvi and Santhi (2016) gave an approach for real-life transportation problems in a neutrosophic environment, Abdel-Basset et al. (2019) studied fully neutrosophic linear programs, and Deli and Şubaş (2017) developed a ranking method for single-valued neutrosophic numbers for multi-attribute decision-making. More recently, Dhouib (2021) solved the single-valued trapezoidal neutrosophic transportation problem by combining a score-based defuzzification with the Dhouib-Matrix-TP1 heuristic, and Kalaivani and Kaliyaperumal (2023) proposed a membership-based ranking function for the same problem and compared it with several existing indices. These approaches, however, rely either on a purely score-based defuzzification or on heuristic and classical two-phase solvers; none couples a generalised trapezoidal magnitude index with the neutrosophic score, nor exploits the single-phase zero-point method that attains the optimum without a separate optimality test.

The performance of every ranking-based reduction depends critically on the quality of the ranking function: an inappropriate index can distort the ordering of the parameters and produce a sub-optimal schedule. The contribution of this paper is threefold. First, extending the trapezoidal-fuzzy treatment of Roy et al. (2020) from the fuzzy to the neutrosophic setting, we solve the single-valued trapezoidal neutrosophic transportation problem by the zero-point method, which reaches the optimum in a single phase and requires no auxiliary optimality test—in contrast to the heuristic (Dhouib, 2021) and the classical two-phase schemes used in earlier neutrosophic studies. Second, to enable this reduction we propose a ranking function for SVTNNs that multiplicatively couples a generalised trapezoidal magnitude index, carrying an attitude parameter k, with the neutrosophic score; unlike the score-only or membership-only indices used by Kalaivani and Kaliyaperumal (2023) and others, it lets both the magnitude of a number and its truth–indeterminacy–falsity confidence influence the rank, and it reduces to the classical trapezoidal-fuzzy rank when $w = 1, u = y = 0$. Third, we establish the basic properties of the ranking function and validate the optimality of the resulting schedule against the North-West Corner rule improved by the MODI method, demonstrating through a worked example that the proposed single-phase procedure is computationally lighter than the classical scheme.

The remainder of the paper is organised as follows. Section 2 recalls the necessary preliminaries. Section 3 introduces the proposed ranking function and establishes its basic properties. Section 4 formulates the single-valued trapezoidal neutrosophic transportation problem, and Section 5 presents the zero-point algorithm. Section 6 provides a detailed numerical illustration, Section 7 discusses the results and the comparison, and Section 8 concludes the paper with directions for future research.

2. Preliminaries

This section collects the definitions and operations used throughout the paper.

2.1 Fuzzy set and intuitionistic fuzzy set

Definition 2.1. [17] Let X be a universe of discourse. A fuzzy set A in X is characterised by a membership function $\mu_A: X \to [0,1]$, and is written $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x)$ denotes the degree of membership of x in A .

Definition 2.2. [2] An intuitionistic fuzzy set A in X is given by $\mathrm{A} = \{\langle \mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{A}}(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}$, where $\mu_A(x)$ and $v_A(x)$ are the degrees of membership and non-membership of x, subject to $0 \leq \mu_A(x) + y_A(x) \leq 1$.

2.2 Single-valued neutrosophic set

Definition 2.3. [15] A single-valued neutrosophic set (SVNS) A in X is defined by three independent functions-a truth-membership $\mathrm{T_A}$, an indeterminacy-membership $\mathrm{I_A}$ and a falsitymembership $\mathrm{F_A}$-as

$$A = \{\langle x,\ T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\}, T_A(x), I_A(x), F_A(x) \in [0,1],$$

with the only requirement $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. The three components are mutually independent, which distinguishes the neutrosophic set from the intuitionistic fuzzy set.

2.3 Single-valued trapezoidal neutrosophic number (SVTNN)

Definition 2.4. [4, 14] A single-valued trapezoidal neutrosophic number (SVTNN) is denoted by

$$\tilde{a} = \langle (a_1, a_2, a_3, a_4);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle,$$

where $a_1 \leq a_2 \leq a_3 \leq a_4$ are real numbers, and $w_{\tilde{a}} \in [0,1]$, $u_{\tilde{a}} \in [0,1]$, and $y_{\tilde{a}} \in [0,1]$denote the maximum truth-membership degree, the minimum indeterminacy-membership degree, and the minimum falsity-membership degree, respectively. The truth-, indeterminacy-, and falsity-membership functions are given by

$$T_{\tilde{a}}(x) = \begin{cases} \dfrac{w_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ w_{\tilde{a}}, & a_2 \leq x \leq a_3, \\ \dfrac{w_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise.} \end{cases}$$

Similarly, the indeterminacy-membership function is

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + u_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ u_{\tilde{a}}, & a_2 \leq x \leq a_3, \\ \dfrac{x - a_3 + u_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 1, & \text{otherwise.} \end{cases}$$

The falsity-membership function is defined by

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + y_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ y_{\tilde{a}}, & a_2 \leq x \leq a_3, \\ \dfrac{x - a_3 + y_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 1, & \text{otherwise.} \end{cases}$$

When

$$w_{\tilde{a}} = 1, u_{\tilde{a}} = 0, y_{\tilde{a}} = 0,$$

the SVTNN reduces to an ordinary trapezoidal fuzzy number. Hence, the trapezoidal fuzzy number is a special case of the single-valued trapezoidal neutrosophic number.

2.4 Arithmetic operations

Definition 2.5. [4] Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4);\ w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two single-valued trapezoidal neutrosophic numbers (SVTNNs), and let $\lambda \geq 0$ be a scalar. Their addition is defined by

$$\tilde{a} \oplus \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle,$$

and the scalar multiplication is defined by

$$\lambda\tilde{a} = \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle, \lambda \geq 0.$$

where $\wedge$ and $\vee$ denote the minimum and maximum operators. Thus, addition is performed component-wise on the trapezoidal parameters, while the resulting truth-membership degree is the minimum of the two truth values, and the resulting indeterminacy- and falsity-membership degrees are the maximum of the corresponding values of the operands

2.5 Score function

Definition 2.6. [4, 16] For a single-valued neutrosophic component with truth-membership degree w, indeterminacy-membership degree u, and falsity-membership degree y, the score (or accuracy of conviction) is defined as

$$S(w,u,y) = \frac{2+w-u-y}{3}, S \in [0,1].$$

The score increases with the truth-membership degree and decreases with the indeterminacy- and falsity-membership degrees. It attains the maximum value of 1for a completely certain datum, that is, $w = 1, u = 0, y = 0$, and the minimum value of 0for a completely rejected datum, that is, $w = 0, u = 1, y = 1$.

3. The Proposed Novel Ranking Function

A ranking function is a mapping $\mathcal{R}: F(\mathbb{R}) \to \mathbb{R}$ that associates a real number with every neutrosophic number so that the resulting real numbers can be compared using the usual order on the real line. The trapezoidal fuzzy literature commonly employs a magnitude index of the generalized form $\frac{w}{4}[k(a_1+a_4)+2(1-k)(a_2+a_3)]$ [8, 12]. However, this index considers only the location and spread of the trapezoidal carrier and ignores the indeterminacy and falsity that are inherent in a neutrosophic datum. Therefore, a new ranking function is proposed by combining the generalized trapezoidal magnitude index with the neutrosophic score, so that a neutrosophic number receives a higher ranking only when it possesses both a larger magnitude and stronger truth support.

Definition 3.1. Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ be a single-valued trapezoidal neutrosophic number (SVTNN), and let $k \in [0,1]$. The proposed ranking function is defined by

$$\mathcal{R}(\tilde{a}) = \frac{1}{4}[k(a_1+a_4)+2(1-k)(a_2+a_3)] \times \frac{2+w_{\tilde{a}}-u_{\tilde{a}}-y_{\tilde{a}}}{3}.$$

The first factor, called the generalized trapezoidal magnitude index, is given by $M(\tilde{a}) = \frac{1}{4}[k(a_1+a_4)+2(1-k)(a_2+a_3)]$.The parameter $k \in [0,1]$reflects the decision-maker's attitude by assigning relative importance to the extreme points $a_1$and $a_4$with respect to the modal points $a_2$and a_3. The second factor is the neutrosophic score defined in Definition 2.6 as $S(w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}) = \frac{2+w_{\tilde{a}}-u_{\tilde{a}}-y_{\tilde{a}}}{3}$. Throughout this paper, the neutral decision-making attitude $k = \frac{1}{2}$ is adopted, giving equal effective importance to the body of the trapezoidal number. Consequently, the proposed ranking function reduces to

$$\mathcal{R}(\tilde{a}) = \left[\frac{1}{8}(a_1+a_4)+\frac{1}{4}(a_2+a_3)\right] \times \frac{2+w_{\tilde{a}}-u_{\tilde{a}}-y_{\tilde{a}}}{3}.$$

3.1 Properties of the ranking function

Proposition 3.1 For every SVTNN $\tilde{a}$, $\mathcal{R}(\tilde{a})$is a uniquely determined real number. If $a_1 \geq 0$, then $\mathcal{R}(\tilde{a}) \geq 0$.

Proof. Both factors are determined by the data of $\tilde{a}$, so $\mathcal{R}(\tilde{a})$is single-valued. The magnitude index $M(\tilde{a}) = \frac{1}{4}[k(a_1 + a_4) + 2(1-k)(a_2 + a_3)]$ is a non-negative combination of $a_1, \ldots, a_4$, hence $M(\tilde{a}) \geq 0$whenever $a_1 \geq 0$; the score $S = \frac{2+w-u-y}{3} \in [0,1]$is also non-negative. The product is therefore non-negative.

Proposition 3.2 For every scalar $\lambda \geq 0$, $\mathcal{R}(\lambda\tilde{a}) = \lambda\mathcal{R}(\tilde{a})$.

Proof. By Definition 2.5, $\lambda\tilde{a} = \langle(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$. The magnitude index is positively homogeneous, $M(\lambda\tilde{a}) = \lambda M(\tilde{a})$, while the score is unchanged because the truth, indeterminacy and falsity degrees are unaffected by positive scaling. Hence, $\mathcal{R}(\lambda\tilde{a}) = \lambda M(\tilde{a}) \cdot S = \lambda\mathcal{R}(\tilde{a})$.

Proposition 3.3 The relation $\tilde{a} \leq \tilde{b} \iff \mathcal{R}(\tilde{a}) \leq \mathcal{R}(\tilde{b})$ is a total preorder on the set of SVTNNs, and it refines the fuzzy ranking of Roy *et al.* (2020): if two numbers share the same truth, indeterminacy and falsity degrees, then $\mathcal{R}$orders them exactly as their trapezoidal magnitude indices order them.

Proof. Reflexivity, antisymmetry up to ties, and transitivity follow from the corresponding properties of $\leq$on $\mathbb{R}$. If two SVTNNs have identical $(w, u, y)$with score $S > 0$, the common score factor is a positive constant, so $\mathcal{R}$preserves the order induced by M, which is the trapezoidal ranking; when $S = 0$, both ranks vanish and the numbers tie.

3.2 Illustrative computation

Consider the SVTNN $\tilde{a} = \langle(4,6,8,10);\ 0.7,0.3,0.3\rangle$ with $k = \frac{1}{2}$. The magnitude index is $M(\tilde{a}) = \frac{1}{8}(4+10) + \frac{1}{4}(6+8) = 1.75 + 3.5 = 5.25$, and the score is $S = \frac{2+0.7-0.3-0.3}{3} = \frac{2.1}{3} = 0.7$. Hence, $\mathcal{R}(\tilde{a}) = 5.25 \times 0.7 = 3.675$.

As a check, for the fully certain number $\langle(0,1,2,3);\ 1,0,0\rangle$, the score is $S = 1$, and $\mathcal{R} = \frac{1}{8}(3) + \frac{1}{4}(3) = 1.125$, which coincides with the classical trapezoidal-fuzzy rank, confirming that the proposed function generalises the fuzzy case.

4. Mathematical Formulation of the SVTN Transportation Problem

Consider a transportation network with $m$sources $O_1, \ldots, O_m$and $n$destinations $D_1, \ldots, D_n$. Let the unit transportation cost from source $i$to destination j, the availability at source i, and the requirement at destination j all be single-valued trapezoidal neutrosophic numbers, denoted by $\tilde{c}_{ij}$, $\tilde{a}_i$, and $\tilde{b}_j$, respectively. If $x_{ij} \geq 0$is the (crisp) number of units shipped on route (i, j), the single-valued trapezoidal neutrosophic transportation problem (SVTNTP) is

$$\min \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i, i = 1,2, \ldots, m,$$
$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j, j = 1,2, \ldots, n,$$
$$x_{ij} \geq 0.$$

The problem is balanced if

$$\sum\mathcal{R}(\tilde{a}_i) = \sum\mathcal{R}(\tilde{b}_j),$$

otherwise a dummy source or destination with zero unit costs is introduced to absorb the difference. The solution is obtained in two stages.

Stage I (defuzzification by ranking). Apply the proposed ranking function $\mathcal{R}$of Definition 3.1 to every neutrosophic cost, supply and demand. This replaces each SVTNN by a crisp real number and yields the equivalent crisp transportation problem

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \mathcal{R}(\tilde{c}_{ij}) x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = \mathcal{R}(\tilde{a}_i), i = 1,2, \dots, m,$$
$$\sum_{i=1}^{m} x_{ij} = \mathcal{R}(\tilde{b}_j), j = 1,2, \dots, n,$$
$$x_{ij} \geq 0.$$

Stage II (optimisation). Solve the crisp transportation problem by the zero-point method of Section 5. Because $\mathcal{R}$is a linear-ordering, positively homogeneous ranking (Propositions 3.2–3.3), the optimal allocation of the crisp problem is the optimal allocation of the original SVTNTP, and the minimum neutrosophic cost is recovered as

$$\tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \mathcal{R}(\tilde{c}_{ij}) x_{ij}.$$

5. The Zero-Point Algorithm for the Crisp Transportation Problem

The zero-point method obtains the optimal solution of a balanced crisp transportation problem in a single phase, without a separate optimality test. Its steps are as follows.

Step 1. Construct the crisp transportation table from Stage I. If the problem is unbalanced, add a dummy row or column with zero costs so that total supply equals total demand.

Step 2. Row reduction. Subtract the smallest entry of each row from every entry of that row.

Step 3. Column reduction. In the resulting table, subtract the smallest entry of each column from every entry of that column. Every row and column now contains at least one zero. This table is called the reduced (opportunity-cost) table.

Step 4. Feasibility (zero-point) check. Verify that for every column the demand does not exceed the sum of the supplies of the rows having a zero in that column, and that for every row the supply does not exceed the sum of the demands of the columns having a zero in that row.

Step 5. Line covering. If the check fails, cover all the zeros of the reduced table with the minimum possible number of horizontal and vertical lines. Let θ be the smallest entry not covered by any line. Subtract θ from every uncovered entry, add θ to every entry lying at the intersection of two lines, and leave the remaining entries unchanged. Return to Step 4.

Step 6. Allocation. When the check is satisfied, make the maximum feasible allocations to the zero cells, beginning with a zero cell that admits a unique (row- or column-determined) allocation, and continue until all supplies and demands are exhausted. The number of occupied cells equals $m + n - 1$ for a non-degenerate solution.

Step 7. Cost recovery. Compute the optimal crisp cost $Z = \sum \mathcal{R}(\tilde{c}_{ij})\, x_{ij}$, which represents the optimal neutrosophic transportation cost of the original problem.

6. Numerical Illustration

A distributor must ship a product from three plants O_1, O_2, and O_3 to four warehouses D_1, D_2, D_3, and D_4. Owing to volatile fuel prices, traffic, and weather conditions, every unit transportation cost, every plant availability, and every warehouse requirement is estimated as a single-valued trapezoidal neutrosophic number. The data are given in Table 1, where each entry has the form $\langle (a_1, a_2, a_3, a_4);\ w,\ u,\ y \rangle$.

Table 1. Single-valued trapezoidal neutrosophic transportation data

Source	**D_1**	**D_2**	**D_3**	**D_4**	**Supply $\tilde{a}_i$**
O_1	⟨(0,2,2,4); 0.9,0.1,0.1⟩	⟨(1,2,4,5); 0.8,0.2,0.2⟩	⟨(4,6,8,10); 0.7,0.3,0.3⟩	⟨(2,3,4,5); 1.0,0.0,0.0⟩	⟨(4,6,8,10); 0.9,0.1,0.1⟩
O_2	⟨(2,3,5,6); 0.8,0.2,0.2⟩	⟨(3,5,6,8); 0.9,0.1,0.1⟩	⟨(1,3,4,6); 0.7,0.3,0.3⟩	⟨(0,2,4,6); 0.8,0.2,0.2⟩	⟨(2,3,5,6); 0.8,0.2,0.2⟩
O_3	⟨(6,7,9,10); 0.9,0.1,0.1⟩	⟨(8,9,11,12); 0.8,0.2,0.2⟩	⟨(1,2,4,5); 0.7,0.3,0.3⟩	⟨(4,6,8,10); 0.9,0.1,0.1⟩	⟨(4,6,8,10); 0.9,0.1,0.1⟩
Demand $\tilde{b}_j$	⟨(2,4,6,8); 0.9,0.1,0.1⟩	⟨(1,2,4,5); 0.8,0.2,0.2⟩	⟨(2,3,5,6); 0.7,0.3,0.3⟩	⟨(1,3,4,6); 0.9,0.1,0.1⟩	

6.1 Stage I — Conversion to a crisp problem

Applying the proposed ranking function with $k = \frac{1}{2}$ to each entry of Table 1 gives a crisp value $\mathcal{R} = M \times S$. For example, for $\tilde{c}_{13} = \langle (4,6,8,10);\ 0.7,0.3,0.3 \rangle$, we obtain

$$M = \frac{1}{8}(14) + \frac{1}{4}(14) = 5.25$$

And $S = 0.7$, so that $\mathcal{R}(\tilde{c}_{13}) = 3.675$.

Table 2 lists the magnitude index, score, and rank of every parameter.

Table 2. Ranking computation for every cost, supply and demand parameter

Parameter	**(a_1,a_2,a_3,a_4)**	**(w, u, y)**	**M (k=½)**	**Score S**	**Rank $\mathscr{R}$**
$\tilde{c}_{11}$	(0,2,2,4)	0.9,0.1,0.1	1.5	0.900	1.350
$\tilde{c}_{12}$	(1,2,4,5)	0.8,0.2,0.2	2.25	0.800	1.800
$\tilde{c}_{13}$	(4,6,8,10)	0.7,0.3,0.3	5.25	0.700	3.675
$\tilde{c}_{14}$	(2,3,4,5)	1.0,0.0,0.0	2.625	1.000	2.625
$\tilde{c}_{21}$	(2,3,5,6)	0.8,0.2,0.2	3.0	0.800	2.400
$\tilde{c}_{22}$	(3,5,6,8)	0.9,0.1,0.1	4.125	0.900	3.7125
$\tilde{c}_{23}$	(1,3,4,6)	0.7,0.3,0.3	2.625	0.700	1.8375
$\tilde{c}_{24}$	(0,2,4,6)	0.8,0.2,0.2	2.25	0.800	1.800
$\tilde{c}_{31}$	(6,7,9,10)	0.9,0.1,0.1	6.0	0.900	5.400
$\tilde{c}_{32}$	(8,9,11,12)	0.8,0.2,0.2	7.5	0.800	6.000
$\tilde{c}_{33}$	(1,2,4,5)	0.7,0.3,0.3	2.25	0.700	1.575
$\tilde{c}_{34}$	(4,6,8,10)	0.9,0.1,0.1	5.25	0.900	4.725
$\tilde{a}_1$	(4,6,8,10)	0.9,0.1,0.1	5.25	0.900	4.725

$\tilde{a}_2$	(2,3,5,6)	0.8,0.2,0.2	3.0	0.800	2.400
$\tilde{a}_3$	(4,6,8,10)	0.9,0.1,0.1	5.25	0.900	4.725
$\tilde{b}_1$	(2,4,6,8)	0.9,0.1,0.1	3.75	0.900	3.375
$\tilde{b}_2$	(1,2,4,5)	0.8,0.2,0.2	2.25	0.800	1.800
$\tilde{b}_3$	(2,3,5,6)	0.7,0.3,0.3	3.0	0.700	2.100
$\tilde{b}_4$	(1,3,4,6)	0.9,0.1,0.1	2.625	0.900	2.3625

Collecting the ranks gives the crisp transportation table (Table 3). The total supply is $\mathcal{R}(\tilde{a}_1) + \mathcal{R}(\tilde{a}_2) + \mathcal{R}(\tilde{a}_3) = 4.725 + 2.400 + 4.725 = 11.850$, whereas the total demand is

$$\mathcal{R}(\tilde{b}_1) + \mathcal{R}(\tilde{b}_2) + \mathcal{R}(\tilde{b}_3) + \mathcal{R}(\tilde{b}_4) = 3.375 + 1.800 + 2.100 + 2.3625 = 9.6375.$$

Since the total supply exceeds the total demand by

$$11.850 - 9.6375 = 2.2125,$$

the transportation problem is unbalanced.

Table 3. Crisp (ranked) transportation table — unbalanced

Source	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
O_1	1.350	1.800	3.675	2.625	4.725
O_2	2.400	3.7125	1.8375	1.800	2.400
O_3	5.400	6.000	1.575	4.725	4.725
Demand	**3.375**	**1.800**	**2.100**	**2.3625**	

A dummy destination D_5 with zero unit costs and demand 2.2125 is introduced to balance the problem (Table 4).

Table 4. Balanced crisp transportation table with dummy destination D_5

Source	**D_1**	**D_2**	**D_3**	**D_4**	**D_5 (dummy)**	**Supply**
O_1	1.350	1.800	3.675	2.625	0	4.725
O_2	2.400	3.7125	1.8375	1.800	0	2.400
O_3	5.400	6.000	1.575	4.725	0	4.725
Demand	**3.375**	**1.800**	**2.100**	**2.3625**	**2.2125**	**11.850**

6.2 Stage II — Zero-point method

Step 2 (row reduction). Each row already contains a zero in the dummy column, so the row minima are all 0 and the table is unchanged. Step 3 (column reduction). The column minima are 1.350, 1.800, 1.575, 1.800 and 0 for D_1–D_5; subtracting them gives the reduced opportunity-cost table (Table 5).

Table 5. Reduced table after row and column reduction

Source	**D_1**	**D_2**	**D_3**	**D_4**	**D_5**
O_1	0	0	2.100	0.825	0
O_2	1.050	1.9125	0.2625	0	0
O_3	4.050	4.200	0	2.925	0

Step 4 (feasibility check). The zero cells of Table 5 cannot serve all requirements: columns D_1 and D_2 both carry a zero only in row O_1, yet their combined demand is $3.375 + 1.800 = 5.175$, which exceeds the supply 4.725 of O_1. The zero-cell allocation is therefore infeasible, so a line-covering iteration is required.

Step 5 (iteration 1). Cover all zeros of Table 5 with the minimum number of lines, leaving the cells that violate the check uncovered: draw one line through row $O_1$and one line through each of columns D_3, D_4, and D_5. The smallest uncovered entry is $\theta = 1.050$ (cell O_2D_1). Subtracting $\theta$from every uncovered entry and adding it to the entries at the intersections of two lines gives the revised reduced table (Table 6).

Table 6. Reduced table after the first line-covering iteration

Source	D_1	D_2	D_3	D_4	D_5
O_1	0	0	3.150	1.875	1.050
O_2	0	0.8625	0.2625	0	0
O_3	3.000	3.150	0	2.925	0

Step 4 (re-check). Table 6 is still infeasible on its zero cells: the only zero cells in row O_3 are D_3, D_5, whose combined demand 2.100 + 2.2125 = 4.3125 is less than the supply 4.725 of O_3, so 0.4125 units of O_3 cannot be despatched through zero cells. A second iteration is performed.

Step 5 (iteration 2). Cover the zeros of Table 6 by lines through rows O_1, O_2 and columns D_3, D_5. The smallest uncovered entry is θ = 2.925 (cell O_3D_4). Repeating the subtract-and-add operation yields the optimal opportunity table (Table 7), in which every row and every column can now be fully served through zero cells.

Table 7. Optimal opportunity table after the second line-covering iteration

Source	D_1	D_2	D_3	D_4	D_5
O_1	0	0	6.075	1.875	3.975
O_2	0	0.8625	3.1875	0	2.925
O_3	0.075	0.225	0	0	0

Step 6 (allocation). Allocating to the zero cells of Table 7 in feasible order yields x_{11} = 2.925, x_{12} = 1.800, x_{21} = 0.450, x_{24} = 1.950, x_{33} = 2.100, x_{34} = 0.4125 and the dummy allocation x_{35} = 2.2125. The seven occupied cells equal $m + n - 1 = 3 + 5 - 1 = 7$, so the solution is non-degenerate. The allotment is displayed in Table 8.

Table 8. Optimal allotment obtained by the zero-point method (units x_{ij})

Source	D_1	D_2	D_3	D_4	D_5	Supply
O_1	2.925	1.800	–	–	–	4.725
O_2	0.450	–	–	1.950	–	2.400
O_3	–	–	2.100	0.4125	2.2125	4.725
Demand	**3.375**	**1.800**	**2.100**	**2.3625**	**2.2125**	**11.850**

Step 7 (cost). The optimal crisp transportation cost is

$$Z = 1.350(2.925) + 1.800(1.800) + 2.400(0.450) + 1.800(1.950) + 1.575(2.100) + 4.725(0.4125) + 0(2.2125)$$

$$= 3.94875 + 3.24 + 1.08 + 3.51 + 3.3075 + 1.9490625 + 0 = \mathbf{17.0353}.$$

Hence the minimum total neutrosophic transportation cost is $\tilde{Z} \approx 17.0353$ units. (The 2.2125 units allotted to the dummy destination represent the surplus capacity of the plants that is not despatched.)

7. Results and Comparative Discussion

To validate the optimality of the zero-point solution, the same balanced crisp problem (Table 4) was solved by the classical two-phase scheme. An initial basic feasible solution obtained by the North-West Corner (NWC) rule is $x_{11} = 3.375, x_{12} = 1.350, x_{22} = 0.450, x_{23} = 1.950, x_{33} = 0.150, x_{34} = 2.3625, x_{35} = 2.2125$, with cost

$$Z_{\text{NWC}} = 1.350(3.375) + 1.800(1.350) + 3.7125(0.450) + 1.8375(1.950) + 1.575(0.150) + 4.725(2.3625) + 0(2.2125) = \mathbf{23.6391}.$$

Applying the MODI (modified distribution) optimality test to this initial solution and iterating until all net evaluations are non-negative reproduces the same optimal cost 17.0353 as the zero-point allocation of Table 8. Thus both routes lead to the same optimum, confirming the correctness of the zero-point solution. It should be noted that, unlike the two-phase scheme, the proposed procedure does not generate a separate initial basic feasible solution that is later improved; the reduction iterations of the zero-point method lead directly to the optimal allocation of Table 8, so no intermediate (non-optimal) cost is reported for it. Table 9 summarises the comparison.

Table 9. Comparison of the proposed method with the classical two-phase approach

Aspect	North-West Corner + MODI	Proposed: ranking + zero-point
Solution phases	Two (construction, then improvement)	One (direct)
Initial basic feasible cost	23.6391	Not generated (optimum reached directly)
Separate optimality test	Required (MODI iterations)	Not required
Optimal transportation cost	17.0353	**17.0353**

Two observations follow. First, the zero-point method reaches the optimum directly: it does not pass through an intermediate basic feasible solution that must subsequently be improved, and it requires no auxiliary optimality test such as MODI or the stepping-stone procedure. This shortens the computation and removes the bookkeeping of dual potentials. Second, the proposed ranking function preserves the information content of the neutrosophic data more faithfully than a purely magnitude-based index: because the score factor down-weights costs that carry high indeterminacy or falsity, routes whose costs are uncertain are penalised relative to equally large but well-supported costs, which is precisely the behaviour a cautious planner expects. The reduction to a crisp problem is exact in the sense of Propositions 3.2–3.3, so no optimality is lost in the defuzzification.

8. Conclusion

This paper has proposed a novel ranking function for single-valued trapezoidal neutrosophic numbers that combines a generalised trapezoidal magnitude index with the neutrosophic score, and has used it to solve the single-valued trapezoidal neutrosophic transportation problem in two stages: defuzzification by ranking, followed by the zero-point method. A three-source, four-destination example was solved in full, and the optimal neutrosophic transportation cost of approximately 17.0353 units was obtained. The result was independently verified by the North-West Corner rule improved with the MODI method, which produced an identical optimum. The proposed procedure is therefore exact, yet it is simpler and computationally lighter than the classical two-phase approach because it reaches optimality in a single phase and needs no separate optimality test.

Several extensions are natural. The ranking function and the zero-point procedure can be applied to unbalanced, multi-objective, multi-index and solid transportation problems in the neutrosophic setting; to interval-valued, bipolar and Pythagorean neutrosophic numbers; and to related network problems such as the assignment, transshipment and shortest-path problems. A comparative study of the proposed ranking function against the score-and-accuracy and value-and-ambiguity rankings on benchmark data, together with a sensitivity analysis in the attitude parameter k, would further establish its robustness.

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