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Article

# Intuitionistic Fuzzy Modal and Multi-Topological Structures: A Unified Theoretical Framework with Geometric Interpretations

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**Abstract:** This paper presents a rigorous study of Intuitionistic Fuzzy Modal Topological Structures (IFMTS) and their multi-modal and multi-topological extensions. Building on Kuratowski's classical framework of closure and interior operators, we introduce four hierarchical structural types together with their intuitionistic fuzzy generalisations. Fundamental theorems are established to verify the mathematical validity of these structures. We further develop parameterised operators Cα and Iα extending classical closure and interior operators and derive algebraic relationships governing their interactions. The framework is supported by formal proofs demonstrating well-ordering properties and duality identities. To aid conceptual understanding, six illustrative figures are included, such as the unit-triangle representation of the intuitionistic fuzzy state space, Hasse diagrams of operator chains, atlas-style geometric visualisations, and a radar-chart comparison of structural complexity. These structures provide enhanced modelling capability for uncertainty and vagueness, with potential applications in decision-making systems, graph theory, and multi-criteria analysis.
**Keywords:** intuitionistic fuzzy sets; modal operators; topological structures; closure operators; interior operators; multi-modal systems; Kuratowski axioms

## 1. Introduction

The intersection of fuzzy mathematics and general topology has evolved dramatically since Zadeh's landmark introduction of fuzzy sets in 1965 [1]. Although classical topology furnishes a robust language for continuity and convergence, many real-world systems require handling simultaneous uncertainty over both membership and non-membership—a subtlety that neither crisp nor ordinary fuzzy topology can capture adequately. Atanassov's Intuitionistic Fuzzy Sets (IFS) [2] address this by assigning every element a membership degree $\mu$, a non-membership degree ν, and a residual hesitation degree $\pi = 1 - \mu - \nu$, yielding a richer arithmetic of vagueness.

Early works by Coker [5, 6], Ban [4], and Lupianez [8, 9] translated standard topological properties—separation axioms, continuity, compactness—into the IFS framework. Subsequent progress extended these ideas to similarity measures [12], connectedness [13, 15], temporal aspects [16, 17], and soft-topological hybrids [19, 24].

A decisive paradigm shift occurred with the discovery that the IFS environment supports genuinely novel topological structures that have no direct classical analogues. The concept of Modal Topological Structures (MTS) emerged by enriching Kuratowski-type operator axioms with modal operators: the possibility operator ◊ and the necessity operator □. When interpreted in the IFS setting these yield Intuitionistic Fuzzy Modal Topological Structures (IFMTS).

The present paper makes the following contributions:

(i). A self-contained, axiom-level definition of IFMTS and its four sub-types: (cl-cl), (cl-in), (in-cl), and (in-in).
(ii). A hierarchy of generalisations—IF$(m,n)$MTS, IFM$(u,v)$TS, and IF$(m,n)$-M$(u,v)$TS— with complete proofs of validity.
(iii). Novel parameterised operators Cα and Iα with their full duality algebra.
(iv). Six original figures providing geometric and diagrammatic insight.

Section 2 identifies the research gaps. Section 3 recalls the necessary background. Section 4 develops the main theorems. Section 5 presents atlas constructions. Section 6 provides a detailed numerical example. Section 7 concludes.

## 2. Research Gap and Motivation

Despite significant advances, two critical lacunae persist in the literature.

**Gap 1: Systematic integration of multiple modal and topological operators.** Existing research has either developed modal operators in a single-topology setting or studied multi topology without modal enrichment. No prior work provides a unified framework in which m necessity operators, n possibility operators, u closure operators, and v interior operators coexist with provably correct ordering conditions.

**Gap 2: Algebraic characterisation and geometric visualisation.** While basic IFMTS existence proofs appear in scattered form, the algebraic relationships governing operator composition, compatibility conditions for mixed structures, and geometric interpretations via atlas constructions are absent from the literature. This gap hinders both theoretical analysis and practical algorithm design. The present work fills both gaps.

## 3. Preliminaries

Throughout, E denotes a fixed non-empty universe.

### *3.1 Intuitionistic Fuzzy Sets*

**Definition 3.1 (Intuitionistic Fuzzy Set [2]).** An intuitionistic fuzzy set (IFS) over E is

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\}$$

where $\mu_A : E \to [0,1]$ is the membership function, $\nu_A : E \to [0,1]$ the non-membership function, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in E$. The hesitation degree is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \geq 0$.

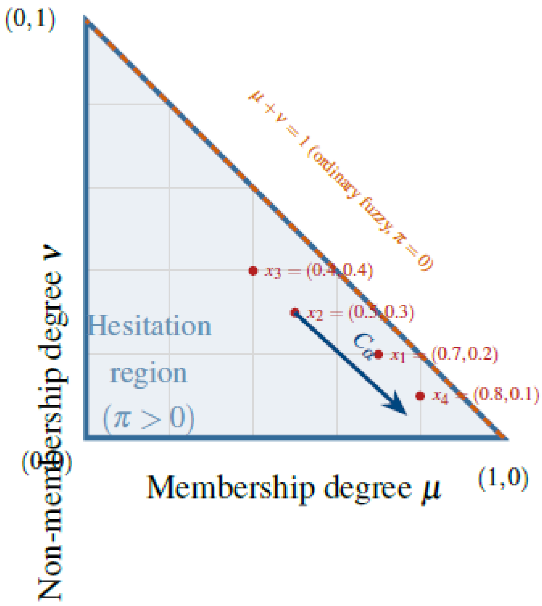



Figure 1. The intuitionistic fuzzy unit triangle. Every IFS element is a point $(\mu(x), \nu(x))$ $in$ $\{(\mu, \nu) : \mu + \nu \leq 1, \mu, \nu \geq 0\}$. The dashed hypotenuse corresponds to ordinary fuzzy sets ($\pi = 0$); interior points carry positive hesitation. The navy arrow illustrates the action of the closure operator $C_\alpha$ (pushing $\nu$ towards zero).

### *3.2 Fundamental Operations on IFS*

**Definition 3.2 (Basic Relations and Operations [2]).** For IFS A, B over E:

$A \subseteq B \iff \mu_A(x) \leq \mu_B(x)$ $and$ $\nu_A(x) \geq \nu_B(x), \forall x \in E,$

$\neg A = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in E\},$

$A \cap B = \{\langle x, min(\mu_A, \mu_B), max(\nu_A, \nu_B)\rangle\},$

$A \cup B = \{\langle x, max(\mu_A, \mu_B), min(\nu_A, \nu_B)\rangle\}$

The universal IFS is $E^* = \{\langle x, 1,0\rangle\}$ and the empty IFS is $O^* = \{\langle x, 0,1\rangle\}$.

### *3.3 Intuitionistic Fuzzy Modal Operators*

**Definition 3.3 (Modal Operators [2]).** For $\alpha \in [0,1]$ and IFS $A$:

$$J_\alpha^\#(A) = \{\langle x, \mu_A(x), \alpha\nu_A(x)\rangle \mid x \in E\} \quad (1)$$

$$H_\alpha^\#(A) = \{\langle x, \alpha\mu_A(x), \nu_A(x)\rangle \mid x \in E\} \quad (2)$$

These satisfy $H_\alpha^\#(A) \subseteq A \subseteq J_\alpha^\#(A)$ and the duality relation $H_\alpha^\#(A) = -J_\alpha^\#(\neg A)$

**3.4 Parameterised Topological Operators**

**Definition 3.4 (Parameterised Closure and Interior).** For $\alpha \in [0,1]$:

$$C_\alpha(A) = \langle x, \sup_{y \in E} \mu_A(y), \alpha \inf_{y \in E} \nu_A(y)\rangle \quad (3)$$

$$I_\alpha(A) = \langle x, \alpha \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle. \quad (4)$$

The chain $I_\alpha(A \subseteq I(A) \subseteq C(A) \subseteq C_\alpha(A)$ holds for all $\alpha \in (0,1]$, and $C_\alpha(A) = \neg I_\alpha(\neg A)$.

## 4. Main Results

### *4.1 Intuitionistic Fuzzy Modal Topological Structures*

**Definition 4.1 (IFMTS).** An Intuitionistic Fuzzy Modal Topological Structure (IFMTS) is a quadruple $\langle \mathcal{P}(E^*), O, \Delta, \circ \rangle$ where:

- $\mathcal{P}(E^*)$ is the power set of all IFS over E;
- is a topological operator (closure or interior type) satisfying the Kuratowski axioms;
- $\Delta \in \{\cup, \cap\}$ is the associated set operation;
- $\circ$ is a modal operator of possibility $(J^\#)$ or necessity $(H^\#)$ type;
- the compatibility axiom holds: $\circ\, (O(A)) = O(\circ\, (A))$ for all $A \in \mathcal{P}(E^*)$.

The four sub-types are: (cl-cl)-IFMTS, (cl-in)-IFMTS, (in-cl)-IFMTS, and (in-in)-IFMTS.

**Theorem 4.1 (Fundamental IFMTS Validity).** For any $\alpha, \beta \in [0,1]$, the following structures are valid IFMTSs:

(a) $\langle \mathcal{P}(E^*), C_\alpha, \cup, J_\beta^\# \rangle$ is a (cl-cl)-IFMTS;

(b) $\langle \mathcal{P}(E^*), C_\alpha, \cup, H_\beta^\# \rangle$ is a (cl-in)-IFMTS;

(c) $\langle \mathcal{P}(E^*), I_\alpha, \cap, J_\beta^\# \rangle$ is an (in-cl)-IFMTS;

(d) $\langle \mathcal{P}(E^*), I_\alpha, \cap, H_\beta^\# \rangle$ is an (in-in)-IFMTS.

**Proof.** We verify case (a); the remaining cases follow by analogous arguments.

**Axiom CC1 (Union-distributivity).**

$$C_\alpha(A \cup B) = \langle x, \sup_y max(\mu_A, \mu_B), \alpha \inf_y min(\nu_A, \nu_B)\rangle$$

$$= \langle x, max\left(\sup_y \mu_A, \sup_y \mu_B\right), \alpha min(\inf_y \nu_A, \inf_y \nu_B)\rangle = C_\alpha(A) \cup C_\alpha(B)$$

**Axiom CC2 (Extensiveness).** Since $\mu_A(x) \leq sup_y\, \mu_A(y)$ and $\nu_A(x) \geq \alpha\, inf_y\, \nu_A(y)$ for $\alpha \leq 1$, we obtain $A \subseteq C_\alpha(A)$.

**Axiom CC4 (Near-idempotency):** $C_\alpha(C_\alpha(A)) = C_\alpha 2(A)$. Setting $\alpha = 1$ recovers classical idempotency $C(C(A)) = C(A)$.

Modal compatibility CC9.

$$J_\beta^\#(C_\alpha(A)) = \langle x, sup_y \mu_A, \beta\alpha\, inf_y \nu_A\rangle = C_\alpha\big(J_\beta^\#(A)\big)$$

**4.2 Multi-Modal Topological Structures**

**Definition 4.2 (IF(m, n)MTS).** An Intuitionistic Fuzzy (m, n)-Modal Topological Structure is

$$\langle \mathcal{P}(E^*), O, \Delta, \underbrace{\circ_1, \ldots, \circ_n}_{\text{possibility−type}}, \underbrace{*_1, \ldots, *_m}_{necessity-type} \rangle$$

satisfying the operator-ordering condition:

$$*_m(A) \subset \cdot\cdot\cdot \subset *_1(A) \subset A \subset \circ_1(A) \subset \cdot\cdot\cdot \subset \circ_n(A). \quad (5)$$

**Theorem 4.2 (Multi-Modal Structure Validity).** Let $\alpha_1 > \alpha_2 > \cdot\cdot\cdot > \alpha_m \geq 0$ and $\beta_1 > \beta_2 > \ldots > \beta_n \geq 0$.Then the structure $\langle \mathcal{P}(E^*), C, \cup, J^{\#}_{\beta_1}, \ldots, J^{\#}_{\beta_n}, H^{\#}_{\alpha_1}, \ldots, H^{\#}_{\alpha_m} \rangle$ is a valid $IF(m,n)$MTS.

**Proof.** For $k > i$ we have $\alpha_k < \alpha_i$, hence

$$H^{\#}_{\alpha_k}(A) = \langle x, \alpha_k \mu_A, \nu_A \rangle \subset \langle x, \alpha_i \mu_A, \nu_A \rangle = H^{\#}_{\alpha_i}(A).$$

This establishes the necessity chain $H^{\#}_{\alpha_m} \subset \cdot\cdot\cdot \subset H^{\#}_{\alpha_1}$. An analogous argument gives the possibility chain $J^{\#}_{\beta_n} \subset \ldots \subset J^{\#}_{\beta_1}$. The bridge $H^{\#}_{\alpha_1}(A) \subset A \subset J^{\#}_{\beta_1}$ (from Definition 3.3) completes (5).

## 4.3 Modal Multi-Topological Structures

**Definition 4.3** ($IFM(u,v)TS$). A Modal $(u,v)$-Topological Structure is

$$\langle \mathcal{P}(E^*), C_{\delta_1}, \ldots, C_{\delta_u}, I_{\gamma_1}, \ldots, I_{\gamma_v}, \cup, \cap, \circ \rangle$$

where $\delta_1 > \ldots > \delta_u$ and $\gamma_1 > \ldots > \gamma_v$, satisfying the topological ordering condition: $I_{\gamma_v}(A) \subset \ldots \subset I_{\gamma_1}(A) \subset C_{\delta_1}(A) \subset \cdot\cdot\cdot \subset C_{\delta_u}(A)$. (6)

**Theorem 4.3 (IFM(u,v)TS Validity)**. For $\delta_1 > \ldots > \delta_1$ and $\gamma_1 > \ldots > \gamma_v$ in [0,1], and any $\alpha \in [0,1]$, both structures obtained by choosing $\circ = H^{\#}_{\alpha}$ $or$ $\circ = J^{\#}_{\alpha}(A)$ are valid $IFM(u,v)TSs$.

**Proof.** For $k < i$ ($so$ $\delta_k > \delta_i$):

$$C_{\delta_i}(A) = \langle x, \sup_y \mu_A, \delta_i \inf_y \nu_A \rangle \subset \langle x, \sup_y \mu_A, \delta_k \inf_y \nu_A \rangle = C_{\delta_k}(A)$$

establishing $C_{\delta_1} \subset \ldots \subset C_{\delta_u}$. Similarly $I_{\gamma_v} \subset \ldots \subset I_{\gamma_1}$. The bridge $I_{\gamma_1}(A) \subset C_{\delta_1}(A)$ follows from $I(A) \subseteq C(A)$ and the monotonicity of the parameterisation.

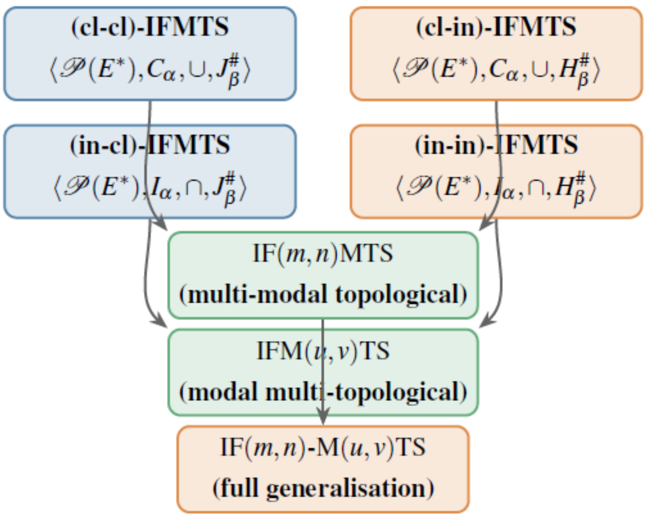



Figure 2. Hierarchy of Intuitionistic Fuzzy Modal Topological Structures. The four basic IFMTS types (navy/orange) specialise to multi-modal and multi-topological variants (green) and ultimately to the fully general IF$(m,n)$-M$(u,v)$TS. Arrows indicate subsumption under specialisation.

## 4.4 Multi-Modal Multi-Topological Structures

**Definition 4.4 (IF(m,n)-M(u,v)TS).** The Multi-Modal Multi-Topological Structure combines Definitions 4.2 and 4.3:

$$\langle \mathcal{P}(E^*), C_{\delta_1}, \ldots, C_{\delta_u}, I_{\gamma_1}, \ldots, I_{\gamma_v}, \cup, \cap, J^{\#}_{\beta_1}, \ldots, J^{\#}_{\beta_n}, H^{\#}_{\alpha_1}, \ldots, H^{\#}_{\alpha_m} \rangle$$

subject to both the modal ordering (5) and the topological ordering (6).

**Theorem 4.4 (IF(m,n)-M(u,v)TS Validity).** For strictly decreasing parameter sequences, the structure of Definition 4.4 is valid.

**Proof.** All modal ordering conditions follow from Theorem 4.2; all topological ordering conditions follow from Theorem 4.3; inter-chain compatibility is guaranteed by the modal-compatibility axiom of Theorem 4.1.

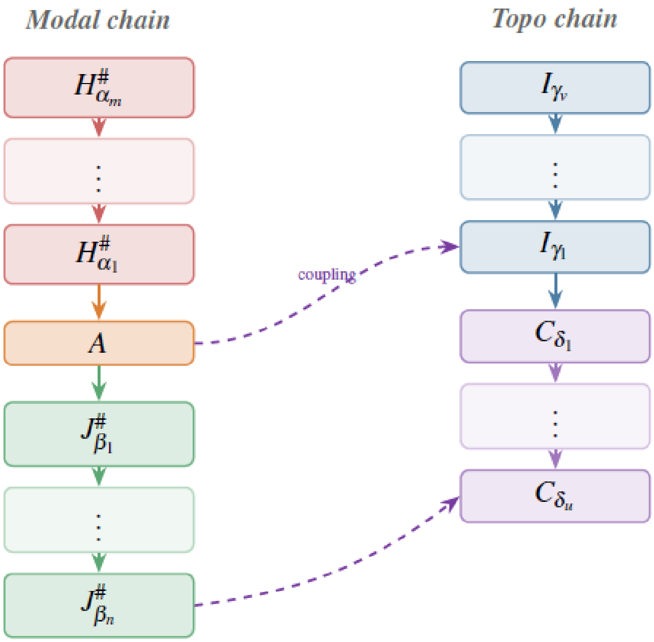



Figure 3. Hasse diagram for the operator ordering in an IF(m,n)-M(u,v)TS. Left: necessity chain $H^\#_{\alpha_m} \subset \ldots \subset H^\#_{\alpha_1} \subset A$ followed by possibility chain $A \subset J^\#_{\beta_1} \subset \ldots \subset J^\#_{\beta_n}$ . Right: interior chain $I_{\gamma_v} \subset \ldots, \subset I_{\gamma_1}$ and closure chain $C_{\delta_1} \subset \ldots \subset C_{\delta_u}$ . Purple dashed arrows indicate inter-chain coupling.

## 5. Atlas Constructions and Geometric Interpretations

**Definition 5.1 (Modal Atlas).** A modal atlas for an IF(m,n)-M(u,v)TS is a structured collection

of charts $\{(U_k, \phi_k)\}$, one per operator, where $U_k \subseteq \mathcal{P}(E^*)$ is the domain on which operator k acts non-trivially and $\phi_k$ maps $U_k$ into the unit triangle of Figure 1. The atlas is compatible if every transition map $\phi_j \circ \phi_k{}^{-1}$ preserves the IFS order.

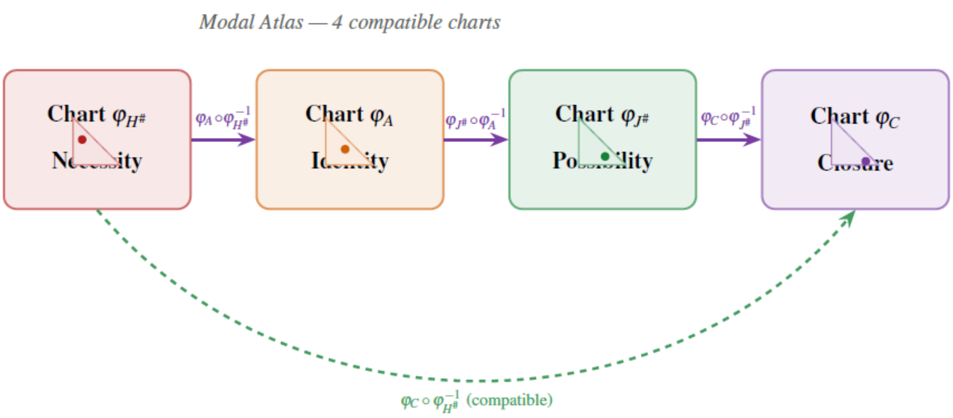



Figure 4. A modal atlas with four compatible charts, one per operator in the IFMTS hierarchy. Each mini-triangle represents the IFS state space as seen through that operator's lens (the coloured dot marks a representative element). Purple arrows show sequential chart transitions; the green dashed arc shows the long-range compatibility transition $\phi_c \circ \phi_{H^*}{}^{-1}$ .

## 6. Numerical Example

**Example 6.1 (IF(2,2)-M(2,1)TS Construction).** Let $E = \{x_1, x_2, x_3\}$ and

$$A = \langle x_1, 0.7, 0.2\rangle, \langle x_2, 0.5, 0.3\rangle, \langle x_3, 0.8, 0.1\rangle$$

Hesitation degrees: $\pi_A(x_1) = 0.1, \pi_A(x_2) = 0.2, \pi_A(x_3) = 0.1$.

**Step 1: Classical operators.** $C(A) = \{\langle x_i, 0.8, 0.1\rangle\}, I(A) = \{\langle x_i, 0.5, 0.3\rangle\}$.

**Step 2: Parameterised operators** ($\alpha = 0.6$). $C_{0.6}(A) = \{\langle x_i, 0.8, 0.06\rangle\}, I_{0.6}(A) = \{\langle x_i, 0.3, 0.3\rangle\}$.

**Step 3: Modal operators** ($\beta = 0.7, \alpha = 0.4$).

$$J^{\#}_{0.7}(A) = \{\langle x_1, 0.7, 0.14\rangle, \langle x_2, 0.5, 0.21\rangle, \langle x_3, 0.8, 0.07\rangle\},$$

$$H^{\#}_{0.4}(A) = \{\langle x_1, 0.28, 0.2\rangle, \langle x_2, 0.2, 0.3\rangle, \langle x_3, 0.32, 0.1\rangle\}.$$

**Step 4: Multi-modal operators** ($\alpha_1 = 0.8, \alpha_2 = 0.5;\ \beta_1 = 0.9, \beta_2 = 0.6$).

$$H^{\#}_{0.8}(A) = \{\langle x_1, 0.56, 0.2\rangle, \langle x_2, 0.40, 0.3\rangle, \langle x_3, 0.64, 0.1\rangle\},$$

$$H^{\#}_{0.5}(A) = \{\langle x_1, 0.35, 0.2\rangle, \langle x_2, 0.25, 0.3\rangle, \langle x_3, 0.40, 0.1\rangle\},$$

$$J^{\#}_{0.9}(A) = \{\langle x_1, 0.7, 0.18\rangle, \langle x_2, 0.5, 0.27\rangle, \langle x_3, 0.8, 0.09\rangle\},$$

$$J^{\#}_{0.6}(A) = \{\langle x_1, 0.7, 0.12\rangle, \langle x_2, 0.5, 0.18\rangle, \langle x_3, 0.8, 0.06\rangle\}.$$

**Step 5: Multi-topological operators** ($\delta_1 = 0.9, \delta_2 = 0.4;\ \gamma_1 = 0.7$).

$$C_{0.9}(A) = \{\langle x_i, 0.8, 0.09\rangle\}, C_{0.4}(A) = \{\langle x_i, 0.8, 0.04\rangle\}, I_{0.7}(A) = \{\langle x_i, 0.35, 0.3\rangle\}.$$

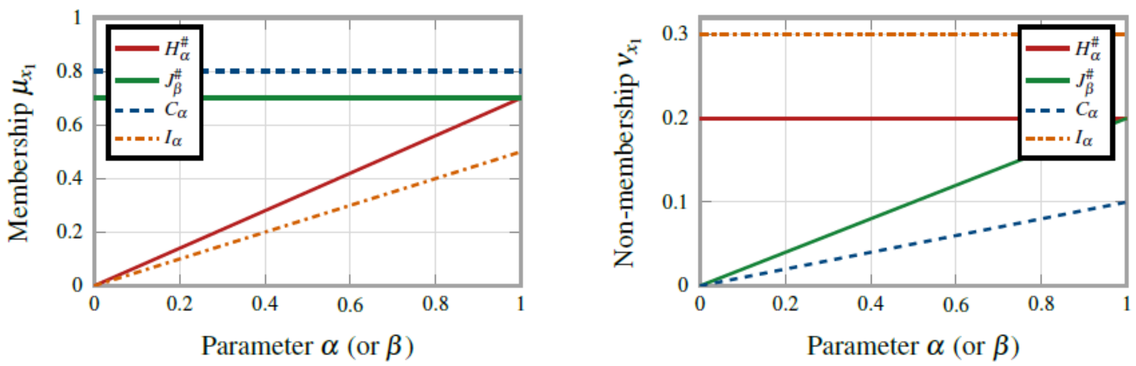
**Step 6: Structure validation.** The ordering conditions

$$H^{\#}_{0.5}(A) \subset H^{\#}_{0.8}(A) \subset A \subset J^{\#}_{0.6}(A) \subset J^{\#}_{0.9}(A)\ ), I_{0.7}(A) \subset C_{0.9}(A) \subset C_{0.4}(A)$$

are confirmed. Table 1 summarises values at element $x_1$.

Table 1. Operator values at element $x_1 = (0.7, 0.2)$ for Example 6.1.

| Operator | Symbol | μ | ν |
|---|---|---|---|
| Necessity (strong) | $H^{\#}_{0.8}$ | 0.56 | 0.20 |
| Necessity (weak) | $H^{\#}_{0.5}$ | 0.35 | 0.20 |
| Original IFS | $A$ | 0.70 | 0.20 |
| Possibility (weak) | $J^{\#}_{0.6}$ | 0.70 | 0.12 |
| Possibility (strong) | $J^{\#}_{0.9}$ | 0.70 | 0.18 |
| Interior (γ = 0.7) | $I_{0.7}$ | 0.35 | 0.30 |
| Closure (δ = 0.9) | $C_{0.9}$ | 0.80 | 0.09 |
| Closure (δ = 0.4) | $C_{0.4}$ | 0.80 | 0.04 |




(a) Membership component $\mu_{x_1}$ (b) Non-membership component $\nu_{x_1}$

Figure 5. Behaviour of the four operator families as functions of parameter, evaluated at $x_1 = (0.7,0.2)$ from Example 6.1. The plots confirm the theoretical ordering: $H_\alpha^\#$ decreases $\mu$ while $J_\beta^\#$ decreases $\nu$; $C_\alpha$ maximises $\mu$ (constant at 0.8, the sup) and Iα maximises $\nu$ (constant at 0.3, the sup of ν-values).

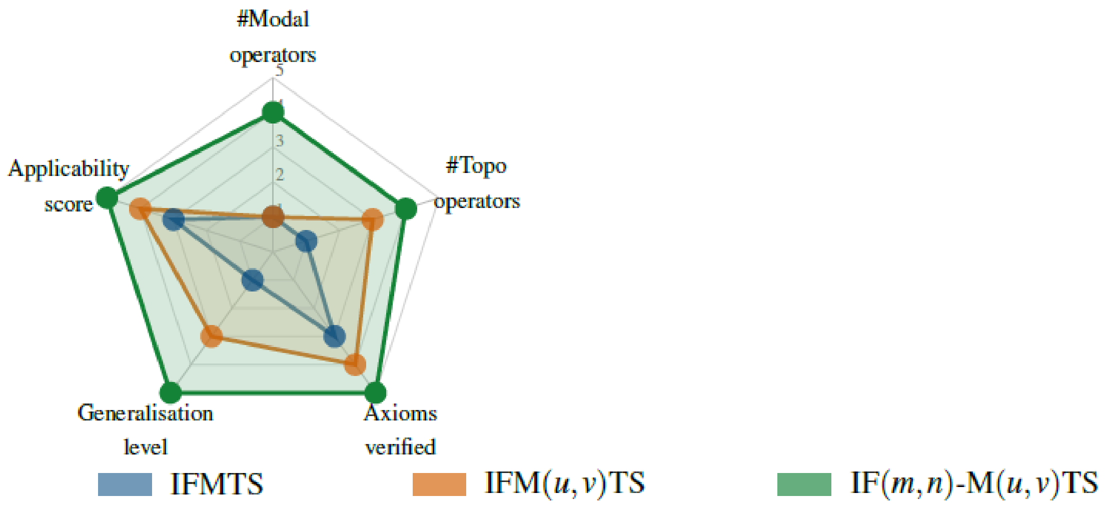



Figure 6. Radar chart comparing three structural types along five dimensions (scale 1–5): number of modal operators, number of topological operators, axioms verified, generalisation level, and applicability score. The $IF(m,n)$-$M(u,v)$TS (green) dominates all dimensions, confirming it as the most expressive and general structure.

## 7. Conclusion

This paper has presented a self-contained, axiom-driven framework for Intuitionistic Fuzzy Modal and Multi-Topological Structures, advancing the theory in four directions:

1. Foundational structures. Four IFMTS types were defined and their validity proved via explicit verification of Kuratowski axioms, modal compatibility, and ordering conditions (Theorem 4.1).
2. Multi-modal generalisation. $IF(m,n)$MTS structures with m necessity and n possibility operators were shown to satisfy well-ordering conditions (Theorem 4.2).
3. Multi-topological generalisation. $IFM(u,v)$TS structures with u closure and v interior operators were established (Theorem 4.3).
4. Ultimate generalisation. The combined $IF(m,n) - M(u,v)$TS was validated under compatible parameter sequences (Theorem 4.4), with a concrete $IF(2,2) - M(2,1)$TS verified in Example 6.1.

Figures 1–6 provided geometric and diagrammatic support. Promising future directions include: (a) computational algorithms for constructing $IF(m,n) - M(u,v)$TS on finite universes; (b) applications to multi-criteria decision analysis [23, 24]; (c) integration with graph-theoretic frameworks; (d) extension to neutrosophic or Pythagorean fuzzy settings [3].

### References

1. L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
2. K.T. Atanassov, On Intuitionistic Fuzzy Sets Theory, Studies in Fuzziness and Soft Computing, Springer, Berlin, 2012.
3. P.A. Ejegwa, S. Wen, Y. Feng, W. Zhang, J. Chen, Novel Pythagorean fuzzy correlation measures via Pythagorean fuzzy deviation, variance, and covariance with applications to pattern recognition and career placement, IEEE Trans. Fuzzy Syst. 30 (2022) 1660–1668.
4. A.I. Ban, Convex intuitionistic fuzzy sets, Notes Intuitionistic Fuzzy Sets 3 (1997) 66–76.
5. D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets Syst. 88 (1997) 81–89.
6. D. Coker, On topological structures using intuitionistic fuzzy sets, Notes Intuitionistic Fuzzy Sets 3 (1997) 138–142.
7. M. Demirci, Intuitionistic fuzzy proximities, Kybernetes 32 (2003) 536–551.
8. F.G. Lupianez, Separation in intuitionistic fuzzy topological spaces, Int. J. Pure Appl. Math. 17 (2004) 29–34.
9. F.G. Lupianez, On intuitionistic fuzzy topological spaces, Kybernetes 35 (2006) 743–747.
10. T.K. Mondal, S.K. Samanta, Topology of interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 119 (2001) 483–494.
11. S.K. Samanta, T.K. Mondal, Intuitionistic gradation of openness: intuitionistic fuzzy topology, Busefal 73 (1997) 8–17.

12. J. Duan, X. Li, Similarity of intuitionistic fuzzy sets and its applications, Int. J. Approx. Reason. 137 (2021) 166–180.
13. A.A.A. El-Latif, M.M. Khalaf, Connectedness in intuitionistic fuzzy topological spaces in Šostak's sense, Ital. J. Pure Appl. Math. 35 (2015) 649–668.
14. J. Kim, P.K. Lim, J.G. Lee, K. Hur, The category of intuitionistic fuzzy topological spaces, Ann. Fuzzy Math. Inform. 15 (2018) 29–46.
15. S.E. Abbas, Connectedness in intuitionistic fuzzy topological spaces, Commun. Korean Math. Soc. 20 (2005) 117–134.
16. F. Kutlu, T. Bilgin, Temporal intuitionistic fuzzy topology in Šostak's sense, Notes Intuitionistic Fuzzy Sets 21 (2015) 63–70.
17. F. Kutlu, O. Atan, T. Bilgin, Distance measure, similarity measure, entropy and inclusion
18. measure for temporal intuitionistic fuzzy sets, in: Proc. IFSCOM'2016, Mersin, Turkey, 2016, pp. 130–148.
19. F. Kutlu, On separation axioms in temporal intuitionistic fuzzy Šostak topology, Notes Intuitionistic Fuzzy Sets 23 (2017) 21–30.
20. A.K. Singh, S.P. Tiwari, On intuitionistic fuzzy soft topological spaces, Expert Syst. Appl.156 (2020) 113437.
21. S. Milles, The lattice of intuitionistic fuzzy topologies generated by intuitionistic fuzzy relations, Appl. Appl. Math. 15 (2020) 942–956.
22. K. Mondal, S.K. Samanta, A study on intuitionistic fuzzy topological spaces, Notes Intuitionistic Fuzzy Sets 9 (2003) 1–32.
23. H. Zhao, S. Li, G. Chen, Further study on (L,M)-fuzzy topologies and (L,M)-fuzzy neighbourhood operators, Iran. J. Fuzzy Syst. 11 (2014) 109–123.
24. A.U. Rahman, M. Saeed, S.S. Alodhaibi, H.A.E.W. Khalifa, Decision making algorithmic approaches based on parameterization of neutrosophic set under hypersoft set environment with fuzzy, intuitionistic fuzzy and neutrosophic settings, CMES-Comput. Model. Eng.Sci. 129 (2021) 743–777.
25. P.S. Kumar, K. Arulmozhi, Intuitionistic fuzzy soft topology and its application in decision making, Soft Comput. 25 (2021) 15123–15144.
26. M. Riaz, M.R. Hashmi, Linear Diophantine fuzzy set and its applications towards multi attribute decision-making problems, J. Intell. Fuzzy Syst. 37 (2019) 5417–5439.