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Article

A New Nine-Dimensional Dynamical Chaotic System: Construction, Chaotic Behavior, and Stability Analysis Based on Routh-Hurwitz Criterion

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Abstract: Although chaos frequently occurs in nonlinear systems; this is not always the case. Consequently, a great deal of research has been done to determine whether systems, given particular initial conditions or characteristics, are capable of exhibiting chaos. Some systems are exceptionally complex due to their high dimensionality and simple nonlinear terms or parameter values. Thus, the development of higher-dimensional chaotic systems has become an important area of study in chaos theory due to its more complex chaotic features. This paper presents and analyzes a hyper chaotic system built with nonlinear equations in nine dimensions. Fifteen positive factors were employed to create a hyper chaotic system in nine dimensions. Mathematica simulation is used to examine the wave form analysis, Lyapunov proponent, sensitivity reliant on on early condition (SDIC) analysis, chaotic behavior, fractal dimension (), and bifurcation of the proposed system. According to a well recognized definition, a system is considered chaotic if its Lyapunov exponent is positive or if it satisfies SDIC on its domain. And Investigation the Routh-Hurwitz criterion used to examine its stability behavior. Over time, the separation from the initial state will likewise increase. It was understood that even a slight alteration to the initial values would cause the chaotic behavior to become extremely sensitive. Three positive Lyapunov exponents and a non-cyclical time-domain waveform demonstrating more complicated dynamical features highlight the hyper-chaotic nature of the proposed system. In the system, every fixed point is unstable

Keywords: Nine-dimension hyper chaotic, sensitivity dependent on Initial, Lyapunov exponent, Stability, waveform analysis, fractional dimension, Routh-Hurwitz criterion and bifurcation.

1.Introduction

Non-cyclic, dynamically nonlinear and sensitive deterministic models under conditions of positive control and constant start produce small changes [1,2]. In a model of 1-order diff. eq. in (3-D), In 1963, although familiar with the behavior of the climate, the first chaotic attractor was introduced., [3,4]. Numerous messy models [5], were later introduced such as the Rössler, Chen, Qi others. Matters of circuit comprehension and proposal of hyper messy models are well-thought-out [6] seeing the swift expansion of messy theory [7 ,8]. Rössler initially characterized [9] in 1979 hyperactive anarchy [10]. The chaotic dynamics of the model stretches in several directions and that the model is very chaotic. That's why the model is so messy.

[11], and increasingly this is what leads to complex attractions [12,13,14]. Low-D messy models are the ones that are maximum concerned in terms of execution. Notwithstanding this, high-D messy models [15 ,16] are more suited for these types of submissions because of their massive aptitude in tech, cryptanalytics [17, 18], and other spheres [19,20]. Hence, a foremost zone of paper is the messy attractor of high-D messy 1s [21, 22]. In this paper, we generate a new hyper messy model in (9-D) with a Many layers' messy attractor.

2. Related works

The newfangled (9-D) hyper messy model that the researcher established in [23] is expounded by way of followed:

$$\frac{d\mathcal{X}.}{dt} = -\delta_1 * \big(\mathcal{X}(t)\big) + \delta_1 * (\mathcal{Y}(t)) - m(t) + \hbar(t),$$
$$\frac{d\mathcal{Y}.}{dt} = \delta_3 * \big(\mathcal{X}(t)\big) - \mathcal{Y}(t) - \big(\mathcal{X}(t)\big) * (\mathcal{Z}(t)) - \mathcal{U}(t),$$
$$\frac{d\mathcal{Z}.}{dt} = -\delta_2 * \big(\mathcal{Z}(t)\big) + \big(\mathcal{X}(t)\big) * (\mathcal{Y}(t)),$$
$$\frac{d\mathcal{W}.}{dt} = \delta_4 * \big(\mathcal{W}(t)\big) - \big(\mathcal{Y}(t)\big) * (\mathcal{Z}(t)),$$
$$\frac{d\mathcal{U}.}{dt} = \delta_5 * \big(\mathcal{U}(t)\big) + \big(\mathcal{Y}(t)\big) * \big(\mathcal{Z}(t)\big) - \delta_1 * (\mathcal{Y}(t)) \qquad (1)$$
$$\frac{d\wp.}{dt} = \delta_6 * (\wp(t)) + \mathcal{W}(t) - \mathcal{U}(t)$$
$$\frac{d\hbar.}{dt} = \delta_7 * \big(\hbar(t)\big) + \delta_4 * \big(\mathcal{Y}(t)\big) * (\mathcal{W}(t))$$
$$\frac{d\mathcal{V}.}{dt} = \delta_5 * \big(\mathcal{Y}(t)\big) + \delta_6 * (\mathcal{V}(t)),$$
$$\frac{dm.}{dt} = n * (\mathcal{X}(t))$$

In model (1), the constant positive tagged { $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6$ and δ_7 }, while the model variable are the states $\mathcal{X}(t), \mathcal{Y}(t), \mathcal{Z}(t), \mathcal{W}(t), \mathcal{U}(t), \wp(t), \hbar(t), \mathcal{V}(t)$ and $m(t)$. When the coefficient is = {$(1\,0).$, $(8.3), (2\,8.\ ), (-0\,.5), (\,1.\ )\,,)(-2\,0.\ ), (-\,6\,1.)$ } correspondingly, with the early values {$x_{0.}((t)\,, z_{0.}(t), w_{0.}(t), u_{0.}(t), p_{0.}(t), h_{0.}(t), v_{0.}(t), m_{0.}(t)$ } are occupied as; {(. ①.), (. ①.), (. ①.), (. ①.), (. ①.), (. ①.), (. ①.), (. ①.), (. ①.)}

and (ŋ = ❸) (linear constant) the model behaves in a hyper messy way. The Lyap unov proponents as said by the overhead values are: $((\mathcal{LE}))_{1.} = 0.47\,19i95, \big((\mathcal{LE})\big)_{2.} = 0.29\,53\,5i4, ((\mathcal{LE}))_{3.} = 0.04\,8i89\,1, ((\mathcal{LE}))_{4.} = 0.0\,00\,06i9, \big((\mathcal{LE})\big)_{5.} = -0.0\,5\,57\,4i2, ((\mathcal{LE}))_{6.} =$

$-14.0\,02\,58i5\,, \big((\mathcal{LE})\big)_{7.} = -1\ \ 9.93\,351i7, \big((\mathcal{LE})\big)_{8.} = -20.0\,00\,0i11\,, \big((\mathcal{LE})\big)_{9.} = -60.98i8190$

The case is very chaotic because there are more than positive exponents, so the sum of all the exponents is less than zero.

Hu in 2009 [24]. Obtainable set of non-linear calculations in actual framework with the (5-D) by counting (2 states) contribution control $\mathcal{U}(t), \mathcal{V}(t)$ *we get of a eq.* tangible a Lorenz's:

$$\grave{\mathcal{X}}(t) = -(\beta_{1.}) * \big(\mathcal{X}(t)\big) + ((\beta_{1.}) * (\mathcal{Y}(t))) + \mathcal{U}(t),$$
$$\grave{\mathcal{Y}}(t) = (\beta_{2}.) * \big(\mathcal{X}(t)\big) - \mathcal{Y}(t) - \big(\mathcal{X}(t)\big) * (\mathcal{Z}(t)) - \mathcal{V}(t),$$
$$\grave{\mathcal{Z}}(t) = -(\beta_{3}.) * \big(\mathcal{Z}(t)\big) + ((\mathcal{X}(t)) * (\mathcal{Y}(t))) \qquad (2)$$
$$\grave{\mathcal{U}}(t) = -\big(\mathcal{X}(t)\big) * \big(\mathcal{Z}(t)\big) + ((\mathcal{K}_1) * \big(\mathcal{U}(t)\big)),$$
$$\grave{\mathcal{V}}(t) = (\mathcal{K}_2) * (\mathcal{Y}(t)),$$

where {$x.(t)\,, \mathcal{Y}.(t)\,, z.(t)\,, \mathcal{U}.(t)$ and $\mathcal{V}.(t)$ } are real states, similarly the positive constants {β_1, β_2 and β_3} are the elementary positive constants and ($(\mathcal{K})_{1.}\,, (\mathcal{K})_{2.}$) are the controller positive constants. A equations (2) have a hyper-chaotic demeanor through (3) positive Lyapunov the foundations. The principal aim is in order to study Equations (2)in a complex way, we generalize this study and the dynamics of the previous equations. The equations (2) A composite figure depicting Miley (O D E s):

$$\grave{\mathcal{X}}(t) = -(\beta_{1.})\mathcal{X}(t)) + (\beta_{1.}) * (y(t)) + \mathcal{U}(t),$$
$$\grave{\mathcal{Y}}(t) = (\beta_{2.}) * \big(\mathcal{X}(t)\big) - \mathcal{Y}(t) - \big(\mathcal{X}(t)\big) * (\mathcal{Z}(t) - \mathcal{V}(t),$$

$$\grave{\mathcal{Z}}(t) = -(\beta_{3.}) * (\mathcal{Z}(t)) + \frac{1}{2}(\bar{\mathcal{X}}(t)) * (\mathcal{Y}(t)) + \frac{1}{2}(\mathcal{X}(t)) * (\bar{\mathcal{Y}}(t)) \qquad (3)$$

$$\grave{\mathcal{U}}(t) = -(\mathcal{X}(t) * (\mathcal{Z}(t)) + (\mathcal{K}_1) * (\mathcal{U}(t)),$$

$$\grave{\mathcal{V}}(t) = (\mathcal{K}_{2.}) * (\mathcal{Y}(t)),$$

where $\{x(t), y(t), u(t)$ and$v(t)\}$ are complex states,

i.e. $(\mathcal{X}(t) = \mathcal{X}_{\zeta.} + j\mathcal{X}_{\eta.}), . (\mathcal{Y}(t) = \mathcal{Y}_{\zeta.} + j\mathcal{Y}_{\eta.}),\ (\mathcal{U}(t) = \mathcal{U}_{\zeta.} + j\mathcal{U}_{\eta.})$, and $. (\mathcal{V}(t) = \mathcal{V}_{\zeta.} + j\mathcal{V}_{\eta.})$,

$(\mathcal{Z}.(t))$ it's the actual states.

Herein domain, nearly properties of the equations (3) They are being explored [24]. For the sake of that, bifurcation diagrams and Lyapunov exponents are using to determine the elements of the hyper-chaotic equation. and then unraveling equating in system (3) into real portions with fantasy portions we get on (4):

$$\grave{\mathcal{X}}_\zeta(t) = -(\beta_1) * (\mathcal{X}_{\zeta.}(t)) + (\beta_1) * (\mathcal{Y}_{\zeta.}(t)) + \mathcal{U}_{\zeta.}(t),$$

$$\grave{\mathcal{X}}_\eta(t) = -(\beta_1) * (\mathcal{X}_{\eta.}(t)) + (\beta_1) * (\mathcal{Y}_{\eta.}(t)) + (\mathcal{U}_{\eta.}(t),$$

$$\grave{\mathcal{Y}}_\zeta(t) = (\beta_2) * (\mathcal{X}_\zeta(t)) - (\mathcal{Y}_\zeta(t)) - (\mathcal{X}_\zeta(t) * (\mathcal{Z}(t) - \mathcal{V}_\zeta(t),$$

$$\grave{\mathcal{Y}}_\eta(t) = (\beta_2) * (\mathcal{Y}_\eta(t)) - \mathcal{Y}_\eta(t) - (\mathcal{X}_\eta(t)) * (\mathcal{Z}(t) - \mathcal{V}_{\eta.}(t),$$

$$\grave{\mathcal{Z}}(t) = -(\beta_3.) * (\mathcal{Z}(t)) + (\mathcal{X}_{\zeta.}(t)) * (\mathcal{Y}_\zeta(t)) + (\mathcal{X}_{\eta.}(t)) * (\mathcal{Y}_{\eta.}(t)) \qquad (4)$$

$$\grave{\mathcal{U}}_\zeta(t) = -(\mathcal{X}_\zeta(t) * (\mathcal{Z}(t)) + (\mathcal{K}_1) * (\mathcal{U}_{\zeta.}(t)),$$

$$\grave{\mathcal{U}}_\eta(t) = -(\mathcal{X}_{\eta.}(t) * (\mathcal{Z}(t)) + (\mathcal{K}_1) * (\mathcal{U}_{\eta.}(t)),$$

$$\grave{\mathcal{V}}_\zeta(t) = (\mathcal{K}_2) * (\mathcal{Y}_{\zeta.}(t)),$$

$$\grave{\mathcal{V}}_\eta(t) = (\mathcal{K}_2) * (\mathcal{Y}_\eta(t)),$$

As for the situation $\{\beta_{(1).} = ('1\,0\,.)\,, \beta_{(2).} = ('3\,.7)\,, \beta_{(3).} = ('5\,5\,.)\,, \mathcal{K}_{1.} = -(5\,0\,.), \mathcal{K}_{(2).} =$ $(1\,.5)$ with the Early circumstances $. \{\mathcal{X}_\zeta(0.) =$

$1., \mathcal{X}_{\eta.}(0.) = ('2.), \mathcal{Y}_{\zeta.}(0.) = ('3.), \mathcal{Y}_{\eta.}('0.) = ('4.), \mathcal{Z}(0.) = ('5.), \mathcal{U}_{\zeta.}(0.) = ('6.), \mathcal{U}_{\eta.}(0.) =$ $(7.), \mathcal{V}_{\zeta.}(0.) = ('8.)\,, \mathcal{V}_{\eta.}(0.) = ('9.)\,,\}$.

We reckon the Lyap unov the foundations the follows:

$\{(\mathcal{LE})_{1.} = (1.0\,1), (\mathcal{LE})_{2.} = (0.74\,1), (\mathcal{LE})_{3.} = (0.4\,1), (\mathcal{LE})_{4.} = (0.14\,1), (\mathcal{LE})_{5.} = (0.)\,, (\mathcal{LE})_{6.} = -(17.94\text{i}51), (\mathcal{LE})_{7.} =$

$-(23.91\,08)\,, (\mathcal{LE})_{8.} = -(70.27\,53)\ \,, (\mathcal{LE})_{9.} = -970.31\,2\text{i}7)\,\}$.

Then this is illustrating these equations (4) with this choosing of $\{\beta_1, \beta_2, \beta_3, \kappa_1$ and $\kappa_2\}$ is a overexcited messy model with (+4) Lyapun ov the foundations (P $\mathcal{LE}$ s). In previous research in the literature, we note the hyper-chaotic Equations shew up with (+2) Lyap unov the foundations. But, in these equations the hyper-chaotic demeanor emerges with (+4) Lyapu nov the foundations because of higher-D in these equations.

A new mathematical model of the (9-D). IN 8-order messy model was engendered by the Jianliang Zhu at el. [25] with the follow:

$$\frac{dx.}{dt} = -\gamma_1 x(t) + \gamma_2 v(t) - (y(t) * z(t) * w(t) * v(t) * u(t) * m(t) * f(t) * p(t))$$

$$\frac{dy.}{dt} = \gamma_3 y(t) - \gamma_2 v(t) + (x(t) * z(t) * w(t) * v(t) * u(t) * m(t) * f(t) * p(t))$$

$$\frac{dz.}{dt} = -\gamma_4 z(t) + \gamma_1 p(t) - (x(t) * y(t) * w(t) * v(t) * u(t) * m(t) * f(t) * p(t))$$

$$\frac{dw.}{dt} = -\gamma_5 w(t) + \gamma_4 v(t) + (x(t) * y(t) * z(t) * v(t) * u(t) * m(t) * f(t) * p(t))$$

$$\frac{dv.}{dt} = -\gamma_1 v(t) + \gamma_1 p(t) - (x(t) * y(t) * z(t) * w(t) * u(t) * m(t) * f(t) * p(t)) \quad (5),$$

$$\frac{du.}{dt} = -\gamma_1 u(t) + \gamma_5 w(t) + (x(t) * y(t) * z(t) * w(t) * v(t) * m(t) * f(t) * p(t))$$

$$\frac{dm.}{dt} = -\gamma_1 m(t) + \gamma_2 z(t) - (x(t) * y(t) * z(t) * w(t) * v(t) * u(t) * f(t) * p(t))$$

$$\frac{df.}{dt} = -\gamma_4 f(t) + \gamma_1 m(t) + (x(t) * y(t) * z(t) * w(t) * v(t) * u(t) * m(t) * p(t))$$

$$\frac{dp.}{dt} = -\gamma_4 p(t) + \gamma_6 y(t) - (x(t) * y(t) * z(t) * w(t) * v(t) * u(t) * m(t) * f(t))$$

Where $\{\gamma_1 = (1\,5.), \gamma_2 = (3\,0.), \gamma_3 = (\,2.), \gamma_4 = (5.)\ , \gamma_5 = (1\,0.)\ and\ \gamma_6 = (4.)\}$

In model (5), the constant positive considered $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ and $\gamma_6\}$, while the model variable are the states $\{\mathcal{X}(t), \mathcal{Y}(t), \mathcal{Z}(t), \mathcal{W}(t), \mathcal{V}(t), \mathcal{U}(t), m(t), f(t) and p(t)\}$.The Lyapunov proponents rendering to the overhead values are:$\{((\mathcal{L}\mathcal{E}))_{1.} = 1\,.3\,4\,7, ((\mathcal{L}\mathcal{E}))_{2.} = 0\,.\,, ((\mathcal{L}\mathcal{E}))_{3.} = -2\,.0\,33\,7i4, ((\mathcal{L}\mathcal{E}))_{4.} = -6\,.\,7\,2i\,9, ((\mathcal{L}\mathcal{E}))_{5.} =$

$-1\,0.5\,0i4, ((\mathcal{L}\mathcal{E}))_{6.} = -1\,3.53\,9i8, ((\mathcal{L}\mathcal{E}))_{7.} = -1\,5\,.3\,5\,1, ((\mathcal{L}\mathcal{E}))_{8.} = -1\,6.56i3$

$, ((\mathcal{L}\mathcal{E}))_{9.} = -1\,9\,.56\,7i3\,\}$.

The new (9-D) hyperactive messy model that the researches established in [26] is clarified as shadows.:

$$\begin{aligned}
\frac{d\mathcal{X}.}{dt} &= (\mu * (\mathcal{W}(t))) - \mu * (\chi(t)),\\
\frac{d\mathcal{Y}.}{dt} &= (\mu * (\mathcal{M}(t))) - \mu * (\mathcal{Y}(t)),\\
\frac{d\mathcal{Z}.}{dt} &= (\mu * (\mathcal{M}(t))) - (\mu) * (\mathcal{Z}(t)),\\
\frac{d\mathcal{W}.}{dt} &= ((\mu) * (\mathcal{F}(t))) - (\mu) * (\mathcal{W}(t)),\\
\frac{d\mathcal{V}.}{dt} &= ((\phi) * (\mathcal{X}\ (t\ ))) - \mathcal{V}(t\ \ ) - (\ \mathcal{X}(t\ )\ ) * \ (p\ (t\ )), \qquad (6)\\
\frac{d\mathcal{U}.}{dt} &= (\phi) * (\mathcal{Y}(t)) - \mathcal{U}(t\ ) - (\mathcal{Y}(t\ )) * (p(t\ )),\\
\frac{d\mathcal{M}.}{dt} &= \left((\phi) * (Z(t))\right) - \mathcal{M}(t) - (\mathcal{Z}(t\ )) * (p(t\ )),\\
\frac{d\mathcal{F}.}{dt} &= ((\phi) * (\mathcal{W}(t))) - \mathcal{F}(\,t) - (W(t)) * (\,p(t)),\\
\frac{dp.}{dt} &= ((\psi) * (p(t))) + (\mathcal{X}(t)) * (\mathcal{V}(t)) + (\mathcal{Y}\ (t)) * (\mathcal{U}\ (t))\\
&\quad + \left((\mathcal{Z}(t)) * (\mathcal{M}(t))\right) + \left((\mathcal{W}(t)) * (\mathcal{F}(t))\right)
\end{aligned}$$

For $\{\,\mu = 2\,5.\,, \phi = 4\,8. and\ \psi = 5.\}$. Additionally, early conditions $\{\mathcal{X}_{(0)}.('t), \mathcal{Y}_{(0).}('t),$

$\mathcal{Z}_{(0).}('t), \mathcal{W}_{(0).}(''t), \mathcal{V}_{(0).}('t), \mathcal{U}_{(0).}('t), \mathcal{M}_{(0).}('t), \mathcal{F}_{(0).}('t)\ and\ p_{9(0).}('t)\,\},$

the Lyapunov proponents remain calculated

As: $\{((\mathcal{L}\,\mathfrak{E}\,))_{(1).} = (2\,.4\,5)\,, (\,(\mathcal{L}\,\mathfrak{E})\,)_{(2).} = (0.), ((\mathcal{L}\,\mathfrak{E}))_{(3).} = (0.), (\,(\mathcal{L}\,\mathfrak{E})\,)_{(4).} =$

$(0.)\,, (\,(\mathcal{L}\,\mathfrak{E}))_{(5).} = (0.)\ \ , (\mathcal{L}\mathfrak{E})_{(6).} = (-3\,7\,.5)\,, (\,(\mathcal{L}\mathcal{E})\,)_{(7).} = (-3\,7.5\,), \qquad ((\mathcal{L}\mathfrak{E}))_{(8).}$

$= (-3\,7.5\,), ((\mathcal{L}\mathfrak{E})\,)_{(9).} = (-\ 4\,7\,.1)\,.\}$.

Below this excellent of μ, ϕ with ψ, these consequences demonstration that (6) has one positive Lyapunov proponent and hence messy conduct.

3. The New Nine-Dimensional Hyper-Chaotic

Designing a mathematical equation of a new- chaotic cryptographer system is the primary purpose. The following is how the new 9-D independent system is shaped:

$$\dot{x}(\mathrm{t}) = -((a_1) * (\mathcal{X}(\mathrm{t}))) - ((a_1) * (p(\mathrm{t}))) + ((a_2) * (z(\mathrm{t})) * (v(\mathrm{t}))) + ((a_3) * (v(\mathrm{t})) * (u(\mathrm{t}))),$$

$$\dot{y}(\mathrm{t}) = -(\left((a_4) * \left(y(\mathrm{t})\right)\right)) + ((a_5) * \left(x(\mathrm{t})\right)) + ((a_6) * \left(f(\mathrm{t})\right)) - ((a_7) * \left(x(\mathrm{t})\right) * \left(p(\mathrm{t})\right)),$$

$$\dot{z}(\mathrm{t}) = -(a_8) * \left(z(\mathrm{t})\right) + (a_9) * \left(f(\mathrm{t})\right) + (a_{10}) * \left(x(\mathrm{t})\right) * \left(m(\mathrm{t})\right) + \left(y(t)\right) * \left(v(\mathrm{t})\right),$$

$$\dot{w}(\mathrm{t}) = -(((a_{11}) * (w(\mathrm{t})))) + ((a_{12}) * (f(\mathrm{t}))) - ((a_4) * (p(\mathrm{t}))) - ((a_6) * (v(\mathrm{t})) * (m(\mathrm{t}))) \quad \ldots\ldots\ldots \textbf{(7)},$$

$$\dot{v}(\mathrm{t}) = (-(a_{13}) * \left(v(\mathrm{t})\right)) + ((a_{10}) * \left(u(\mathrm{t})\right)) - ((a_6) * \left(y(\mathrm{t})\right) * \left(m(\mathrm{t})\right)) - ((a_{10}) * \left(z(\mathrm{t})\right) * (u(t))),$$

$$\dot{u}(\mathrm{t}) = (-(a_1) * \left(u(\mathrm{t})\right)) - (((a_6) * \left(m(\mathrm{t})\right) + \left(f(\mathrm{t})\right) - ((a_{10}) * \left(z(\mathrm{t})\right) * \left(w(\mathrm{t})\right)),$$

$$\dot{m}(\mathrm{t}) = -((a_6) * \left(m(\mathrm{t})\right)) - ((a_6) * \left(v(\mathrm{t})\right)) + (\left(x(\mathrm{t})\right) * \left(y(\mathrm{t})\right)) + ((a_{12}) * \left(x(\mathrm{t})\right) * \left(v(\mathrm{t})\right),$$

$$\dot{f}(\mathrm{t}) = -(a_{14}) * \left(f(\mathrm{t})\right) + (a_1) * \left(y(\mathrm{t})\right) - (a_2) * \left(w(\mathrm{t})\right) + \left(u(\mathrm{t})\right) * \left(m(\mathrm{t})\right),$$

$$\dot{p}(\mathrm{t}) = -(a_{15}) * (p(\mathrm{t})) + (a_1) * (y(\mathrm{t})) - (a_7) * (f(\mathrm{t})) - (a_8) * (x(\mathrm{t})) * (f(\mathrm{t})),$$

where $\{x\ (\mathrm{t})\ , y\ (\mathrm{t}), z\ (\mathrm{t}), w\ (\mathrm{t}), v\ (\mathrm{t}), u\ (\mathrm{t}), m\ (\mathrm{t}), f\ (\mathrm{t}), p\ (\mathrm{t})\} \in \mathfrak{R}$ are the variables of the model, while $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_1, a_{13}, a_{14}\ and\ a_{15}\ \}$are the system's positive constants. The following positive constants of systems values cause the new 9-D Model (5) to indication a messy attraction:

$\{\ a_1 = 11., a_2 = 4., a_3 = 2., a_4 = 6., a_5 = 35., a_6 = 5., a_7 = 3.3\ , a_8 = 8.5\ , a_9 = 1.2, a_{10} = 3., a_{11} = 8., a_{12} = 2.5, a_{13} = 0.1\ , a_{14} = 10.\ and\ \ a_{15} = 1.5.$ **(7.a)**

therefore, we adopted initiatory conditions as the following $\{\ \mathcal{X}\ (0.) \equiv (0\ .6), \mathcal{Y}\ (\ 0.) \equiv (0\ \ .5\ ), \mathcal{Z}(0.) \equiv (3.)\ , \mathcal{W}(0.) \equiv (0.6\ )\ ,\ \ \mathcal{V}\ (\ 0.) \equiv (0.4)\ , \mathcal{U}\ (0.) \equiv (8.)\ , m\ (0.\ \ ) \equiv (2\ .\ \ 5)\ , f\ (0.) \equiv (0\ .5)\ and\ \wp\ (\ 0.) \equiv (0\ .2)\ \}$. **(7.b)**

4.Lyapunov exponent and Kaplan-Yorke

The sensitivity dynamic of systems with predictability up to variations in its early conditions are gauged by the Lyapunov exponent. It satisfied as a quantitative display of the sensitive reliance on early conditions. Both for a trajectory beginning with (x_0) and for a detached model, $x_{n+1} = f(x_n)$. Equation (7) [8] is used to compute the Lyapunov proponent.

$$(\mathcal{LE}).(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{\infty} ln \left| f'(x_i) \right| \ . \qquad (8)$$

where (f') is the derived of the function (f) and $(\mathcal{LE})$. it's the **Lyapu nov** foundations. The following are three distinct dynamical cases for the $(\mathcal{LE})$:

• A model is considered non-chaotic if the Lyapu nov proponent $(\mathcal{LE})$ is (negative).
•In the steady variable method, the model is neutrally stable if the Lyap unov exponent$(\mathcal{LE})$ is (zero).

•A model is considered messy if the Lyapu nov exponent $(\mathcal{LE})$. is (positive).

As per the nonlinear dynamical theory, the Lyapunov exponent can be calculated as a quantitative measure of the sensitive dependency on the initial conditions. It is the mean speed at which two neighboring trajectories diverge (or converge). Additionally, the following are the nine Lyapunov exponents with parameters for the nonlinear dynamical system (7):

$\{(a_1) = ('11.), (a_2) = ('4.), (a_3) = ('2.), (\ a_4) = ('6.), (\ a_5) = ('3\ 5.), (a_6) = (5.)\ , (\ a_7) = 3.3\ , (a_8) = (8.5)\ , (a_9) = ('1\ .2), (a_{10}) = ('3.), (a_{11}) = ('8.), (a_{12}) = ('2.5), (a_{13}) = ('0\ .\ \ 1),$

$(a_{14}) = ('1\ 0.)\ and\ (\ a_{1\ 5}) = ('1.\ 5)\}$.

With an early circumstance:

$\{\mathcal{X}\ (0\ ) \equiv ('0\ .6), \mathcal{Y}\ \ (0\ .) \equiv ('0\ .\ 5), \mathcal{Z}(0.) \equiv ('3\ .)\ , \mathcal{W}\ (0.) \equiv ('0\ .6)\ , \mathcal{V}\ (\ 0.) \equiv ('0\ .4),$

$\mathcal{U}\ (\ 0) \equiv ('8\ .) m\ (0.) \equiv ('2\ .5), \mathscr{f}\ (\ 0.) \equiv ('0\ .5)\ and\ \mathscr{p}\ (0.) \equiv ('0\ .2)\}.$

we obtained as follows:

$((\mathcal{LE}))_{1.} = (10.10\ 59), \big((\mathcal{LE})\ \big)_{2.} = (\ 5.59\ 14i4), \big((\mathcal{LE})\big)_{3.} = (5.3\ 4\ 9\ 8\ 5), \big((\mathcal{LE})\big)_{4.} = -0.74\ 74i43, (\mathcal{LE})_5 = -2.98\ 74i7,$

$((\mathcal{LE}))_{6.} = (-3\ .32\ 67i1), \big((\mathcal{LE})\big)_{7.} = (-6.5\ 5\ 8\ 5\ 8), \ \big((\mathcal{LE})\big)_{8.} = (-1\ 1.0\ 8\ 38), \big((\mathcal{LE})\big)_{9.} = -12.44\ 01.$

The fact that the large Lyapu nov proponent is positive indicates a model has messy features. While $((\mathcal{LE}))_{1.}, ((\mathcal{LE}))_{2.},$ and $((\mathcal{LE}))_{3.}$ are **positive** Lyapunov exponents, the other six are all negative. As a result, the system is incredibly chaotic. The fractal dimension is another characteristic of chaos that is established by Lyapuvnov exponents from the $\mathcal{K}$aplan-Yorke dimension, and $(\mathcal{D}_{\mathcal{K}\mathcal{Y}})$ can be written as follows:[4]

$$(\ \mathcal{D}_{\mathcal{K}\mathcal{Y}}) = j + \frac{1}{|((\mathcal{LE}))_{j+1}|} \sum_{i=1}^{j} ((\mathcal{LE}))_i \tag{9}$$

Where j is the highest value of i in the time is Investigates$\{\ \sum_{i=1}^{j} ((\mathcal{LE}))_{i.} > 0\}$ and $\{\sum_{i=1}^{j+1} \big((\mathcal{LE})\big)_{i.} < 0\}$, it indicates that the first (j) Lyapu nov foundations is non-negative. The Lyapunov the foundations categorization. places $((\mathcal{LE}))_{i.}$ in downhill Arrangement the categorization. $(\mathcal{D}_{\mathcal{K}\mathcal{Y}})$. is the maximum value from a model information's length. Regarding the model in this study, by looking at the

For the following reasons, it is possible to cross the Kaplan York distance $\{\big((\mathcal{L}\mathfrak{E})\big)_{1.} + ((\mathcal{L}\mathfrak{E}))_{2.} \oplus \big((\mathcal{L}\mathfrak{E})\ \big)_{3.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{4.} \oplus ((\mathcal{L}\mathfrak{E}))_{5.} \oplus ((\mathcal{L}\mathfrak{E}))_{6.} \oplus ((\mathcal{L}\mathfrak{E}))_{7.}\}>0$ and $\{\big((\mathcal{L}\mathfrak{E})\big)_{1.} + \big((\mathcal{L}\mathfrak{E})\big)_{2.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{3.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{4.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{5.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{6.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{7.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{8.}\}<0$ as a result of the ten Lyapunov exponent values in the above, where j is **seven**. In the newfangled chaotic- system, the Lyapunov dim. was

$$\big(\mathcal{D}_{\mathcal{K}\mathcal{Y}}\big). = j + \frac{1}{\big|((\mathcal{L}\mathfrak{E})\ )_{(j+1).}\big|} \sum_{i=1}^{j} (\ (\mathcal{L}\mathfrak{E}))_{i.}$$

$$(\mathcal{D}_{\mathcal{K}\ \mathcal{Y}}). = 7 + \frac{1}{\big|((\mathcal{L}\mathfrak{E}))_{(j+1).}\big|} \sum_{i=1}^{j} ((\mathcal{L}\mathfrak{E}))_{\ i}.$$

$$= 7 + \frac{\{\big((\mathcal{L}\mathfrak{E})\big)_{1.} \oplus \big((\mathcal{L}\mathfrak{E})\big)_{2.} \oplus \ \big((\mathcal{L}\mathcal{E}\mathfrak{E})\ \big)_{3.} \oplus (\ (\mathcal{L}\mathfrak{E})\ )_{4.} \oplus (\ (\mathcal{L}\mathfrak{E})\ )_{5.} \oplus \big(\ (\mathcal{L}\mathfrak{E})\big)_{6.} \oplus (\ (\mathcal{L}\mathfrak{E})\ )_{7.}\}}{|((\mathcal{LE}))_{8.}|}$$

$$= 7 + \frac{10.1059+5.5914+5.3498-0.7474-2.9874-3.3267-6.5585}{11.0838} == \quad 7.67009$$

Consequently, System (7)'s Lyapunov dimension is Fractal. A model's orbits are Aperiodic due to it's fractality character; also, its proximity paths depart. Thus, chaos is indeed present in this nonlinear model.

Dynamical Analysis of the Proposed 9D Hyper Chaotic System

5. Dis sipat ivity

The suggested model (7) can be definite in trajectory representation under:

$$
\boldsymbol{f} = \begin{cases} f1 = -((a_1) * (x(\mathrm{t})) - ((a_1) * (p(\mathrm{t})) + ((a_2) * (z(\mathrm{t})) * (v(\mathrm{t})) + ((a) * \big(v(\mathrm{t})\big) * \big(u(\mathrm{t})\big)) \\ f2 = -((a_4) * \big(y(\mathrm{t})\big) + ((a_5) * \big(x(\mathrm{t})\big) + ((a_6) * \big(f(\mathrm{t})\big) - ((a_7) * \big(x(tt)\big) * \big(p(\mathrm{t})\big)), \\ f3 = ((a_8) * \big(z(\mathrm{t})\big) + \Big((a_9) * \big(f(\mathrm{t})\big)\Big) + \Big((a_{10}) * \big(x(\mathrm{t})\big) * \big(m(\mathrm{t})\big)\Big) + \Big(\big(y(\mathrm{t})\big) * \big(v(\mathrm{t})\big)\Big) \\ f4 = -((a_{11}) * (w(\mathrm{t}))) + ((a_{12}) * (f(\mathrm{t}))) - ((a_4) * (p(\mathrm{t}))) - ((a_6) * (v(\mathrm{t})) * (m(\mathrm{t}))) \\ f5 = -\Big((a_{13}) * \big(v(\mathrm{t})\big)\Big) + \Big((a_{10}) * \big(u(\mathrm{t})\big)\Big) - ((a_6) * \big(y(\mathrm{t})\big) * \big(m(\mathrm{t})\big)) - ((a_{10}) * \big(z(\mathrm{t})\big) * \big(u(\mathrm{t})\big)) \\ f6 = -\Big((a_1) * \big(u(\mathrm{t})\big)\Big) - \Big((a_6) * \big(m(\mathrm{t})\big)\Big) + \big(f(\mathrm{t})\big) - \Big((a_{10}) * \big(z(\mathrm{t})\big) * \big(w(\mathrm{t})\big)\Big), \\ f7 = -\Big((a_6) * \big(m(\mathrm{t})\big)\Big) - \Big((a_6) * \big(v(\mathrm{t})\big)\Big) + \Big(\big(x(\mathrm{t})\big) * \big(y(\mathrm{t})\big)\Big) + \Big((a_{12}) * \big(x(\mathrm{t})\big) * \big(v(\mathrm{t})\big)\Big), \\ f8 = -\Big((a_{14}) * \big(f(\mathrm{t})\big)\Big) + \Big((a_1) * \big(y(\mathrm{t})\big)\Big) - \Big((a_2) * \big(w(\mathrm{t})\big)\Big) + \Big(\big(u(\mathrm{t})\big) * \big(m(\mathrm{t})\big)\Big) \\ f9 = -\Big((a_{15}) * \big(p(\mathrm{t})\big)\Big) + ((a_1) * (y(\mathrm{t}))) - ((a_7) * (f(\mathrm{t}))) - ((a_8) * (x(\mathrm{t})) * (f(\mathrm{t}))) \end{cases} \tag{10}
$$

The trajectory arena f diverges on $(\boldsymbol{\mathcal{R}})^{\mathbf{9}}$ and we take from the formula

$$(\nabla_* \cdot f_*) = \frac{\partial(f)_1}{\partial x} + \frac{\partial(f)_2}{\partial y} + \frac{\partial(f)_3}{\partial z} + \frac{\partial(f)_4}{\partial w} + \frac{\partial(f)_5}{\partial v} + \frac{\partial(f)_6}{\partial u} + \frac{\partial(f)_7}{\partial m} + \frac{\partial(f)_8}{\partial f} + \frac{\partial(f)_9}{\partial p} \tag{17}$$

We note that$(\nabla_* \cdot f_*)$ Under the flow, the rate of change of volumes is measured $((\Phi)^*)_t$ of $((\boldsymbol{f})_*)$.

take $(\mathcal{D}_*(\boldsymbol{t}))$ be the district in $(\boldsymbol{\mathcal{R}})^{\mathbf{9}}$ and the flat border and let $(((\mathcal{D})_*)(\boldsymbol{t})) = ((\Phi)^*)_{(t)})(t)$,the photo of $((\mathcal{D})_*)$ below $((\Phi)^*)_{(t)}$, Let $((\boldsymbol{V})_*)(\boldsymbol{t})$ be the volume of $((\mathcal{D})_*(\boldsymbol{t}))$the time (t) of the flowing of $((f)_*)$. . By Liouville's theorem [13], we get:

$$\frac{d\,((\mathcal{V})_*)}{d\,t} = \int_{((\mathcal{D})_*)\,(t))} ((\nabla)_* \cdot (\not{f})_*) * (d\,\mathcal{X}\;\; d\mathcal{Y}\;\; d\mathcal{Z}\;\; d\mathcal{W}\;\; d\mathcal{V}\;\; d\mathcal{U}\, dm\, df\, d\wp) \tag{18}$$

For the model (6), we find that

$$(\nabla_* \cdot f_*) = \frac{\partial(f)_1}{\partial x} + \frac{\partial(f)_2}{\partial y} + \frac{\partial(f)_3}{\partial z} + \frac{\partial(f)_4}{\partial w} + \frac{\partial(f)_5}{\partial v} + \frac{\partial(f)_6}{\partial u} + \frac{\partial(f)_7}{\partial m} + \frac{\partial(f)_8}{\partial f} + \frac{\partial(f)_9}{\partial p}$$

$$= -(a_1 + a_4 + a_8 + a_{11} + a_{13} + a_1 + a_6 + a_{14} + a_{15}) < 0 \tag{19}$$

Due $a_1, a_4, a_8, a_6, a_{11}, a_{13}, a_{14}$ *and* a_{15} are positive coefficient.

And by replacement (19) into (18) and to simplify we obtain

$$\frac{d(V)(t)}{dt} = -\begin{pmatrix} a_1 + a_4 + a_8 + a_{11} + a_{13} + a_1 + a_6 + a_{14} + \\ a_{15} \end{pmatrix} \int_{((D)(t))} dx\, dy\, dz\, dw\, dv\, du\, dm\, df\, dp \ldots$$

………. (20)

$$\frac{d(V)}{dt} = -(a_1 + a_4 + a_8 + a_{11} + a_{13} + a_1 + a_6 + a_{14} + a_{15})((V)(t) + \mathrm{c}_{\sigma}$$

To obtain the unique solution of equation (18), we solve the first Linear Differential Equation$((V(t)) = (k)e^{-(a_1+a_4+a_8+a_{11}+a_{13}+a_1+a_6+a_{14}+a_{15})t}$

$$= (k)\, \boldsymbol{e}^{\boldsymbol{-61.1t}}. \tag{21}$$

Any $((V_*)(t))$ -size should shrink exponentially to zero, as equation (21) shows, over time.

Therefore, the model is dis sipa tive, which is described in (7) for the dynamical model.

6. The Stability of the suggested new system with The Routh-Hurwitz criterion

Now, for $\boldsymbol{a_{14}} = \mathbf{0}$ There is just one equilibrium point in the modren model at $(f)_{(0).} = \{((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).)\}$ (the basis).

For $\boldsymbol{a_{14}} \neq \mathbf{0}$ and single of the succeeding a_1, a_2, a_3, a_8, a_9 *and* a_{15} are (= 0), there are 4 equilibrium points in the proposed messy model, one of which is the basis, a real numeral.

$f_{1.}(\mathrm{t})_{.}((\mathcal{X}(\mathrm{t}))_{1.}, (\mathcal{Y}(\mathrm{t}))_{1.}, (\mathcal{Z}(\mathrm{t}))_{1.}, (\mathcal{W}(\mathrm{t}))_{1.}, (\mathcal{V}(\mathrm{t}))_{1.}, (\mathcal{U}(\mathrm{t}))_{1.}, (m(\mathrm{t}))_{1.}, (\oint(\mathrm{t}))_{1.}, (\wp(\mathrm{t}))_{1.})$,

$f_{2.}(\mathrm{t})\ ((\mathcal{X}(\mathrm{t}))_{2.}, (\mathcal{Y}(\mathrm{t}))_{2.}, (\mathcal{Z}(\mathrm{t}))_{2.}, (\mathcal{W}(\mathrm{t}))_{2.}, (\mathcal{V}(\mathrm{t}))_{2.}, \mathcal{U}(\mathrm{t}))_{2.}, (m(\mathrm{t}))_{2.}, (\oint(\mathrm{t}))_{2.}, (\wp(\mathrm{t}))_{2.})$,

$f_{3.}(\mathrm{t})((\mathcal{X}(\mathrm{t}))_{3.}, (\mathcal{Y}(\mathrm{t}))_{3.}, (\mathcal{Z}(\mathrm{t}))_{3.}, (\mathcal{W}(\mathrm{t}))_{3.}, (\mathcal{V}(\mathrm{t}))_{3.}, (\mathcal{U}(\mathrm{t}))_{3.}, (m, (\oint(\mathrm{t}))_{3.}, (\wp(\mathrm{t}))_{3.})$,

$f_{4.}(\mathrm{t})((\mathcal{X}(\mathrm{t}))_{4.}, (\mathcal{Y}(\mathrm{t}))_{4.}, (\mathcal{Z}(\mathrm{t}))_{4.}, (\mathcal{W}(\mathrm{t}))_{4.}, (\mathcal{V}(\mathrm{t}))_{4.}, (\mathcal{U}(\mathrm{t}))_{4.}, (m(\mathrm{t}))_{4.}, (\oint(\mathrm{t}))_{4.}, (\wp(\mathrm{t}))_{4.})$,

When $a_1, a_2, a_3, a_8, a_9\ and\ a_{15}$ do ($\neq\ \mathbf{0}$), the basis is 1 of the 4 equilibrium positions in the new messy model. The equilibrium points are each Consist of real numbers. Meanwhile the coefficient $a_1, a_4, a_8, a_{11}, a_{13}, a_6, a_{14}, and\ a_{15}$ are also $(+)$ otherwise $(-)$ but are ($\neq\ \mathbf{0}$), When all its real parts are not equal to zero. The Jacobian matrix contains eigenvalues at the origin, this is in the example given. When the origin is linearly stable, if they are all negative eigenvalues, linear stability determines the signals of the real elements (If the model is hyperbolic and unreal, then it is locally stable). in the case the several of them positive then the basis is not stable with Since the model is every so often messy, we anticipate that the origin will be an unstable saddle point with both eigenvalues which positive and negative. We evaluate the local stability using linear stability because the equilibrium point is hyperbolic the development of global messy comportment is allowed because in most messy models the point of origin is unstable (saddle).

The area of a modren model $(f)_{(0).} = \{((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).)\}$ is always superfluous in a state, therefore the Jacobian matrix of the new system((7) of $(f)_{(0).} = \{((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).)\}$ is obtained by linearization.

$$J_0 = \begin{bmatrix} -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_1 \\ a_5 & -a_4 & 0 & 0 & 0 & 0 & 0 & a_6 & 0 \\ 0 & 0 & -a_8 & 0 & 0 & 0 & 0 & a_9 & 0 \\ 0 & 0 & 0 & -a_{11} & 0 & 0 & 0 & a_{12} & -a_4 \\ 0 & 0 & 0 & 0 & -a_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_1 & -a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_6 & 0 & 0 \\ 0 & a_1 & 0 & -a_2 & 0 & 0 & 0 & -a_{14} & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 & 0 & -a_7 & -a_{15} \end{bmatrix} \qquad (16)$$

This its distinguishing eq.

$$\Delta(\lambda.) = ((\lambda.) + (a_{(8)})((\lambda.)^{(8)} + (\mathcal{A})_{(1.)}(\lambda.)^{(7)} + (\mathcal{A})_{(2.)}(\lambda.)^{(6)} + (\mathcal{A})_{(3.)}(\lambda.)^{(5)}$$
$$+(\mathcal{A})_{(4.)}(\lambda.)^{(4)} + (\mathcal{A})_{(5.)}(\lambda.)^{(3)} + (\mathcal{A})_{(6.)}\lambda^{(2)} + (\mathcal{A})_{(7.)}\lambda + (\mathcal{A})_{(8.)} = 0 \qquad (17)$$

$$(\mathcal{A})_1 = ((a)_{15} + (a)_{14}) + ((a)_6 + (a)_{13}) + ((a)_{11} + (a)_4) + (a)_1 + (a)_1.$$

$$(\mathcal{A}.)_{(2)} = ((a)_6 + (a)_{13})[((a)_{15} + (a)_{14}) + ((a)_{11} + (a)_4) + 2(a)_1] + 2(a)_1$$
$$[((a)_{15} + (a)_{14}) + ((a)_{11} + (a)_4)]\ ((a)_{11} + (a)_4) * ((a)_{15} + (a)_{14}) +$$
$$(a)_{12} * ((a)_2 - (a)_1) + (a)_{14} * (a)_{15} + (a)_{13} * (a)_6 + (a)_4 * (a)_{11} + {(a)_1}^2$$

$$(\mathcal{A})_3 = ((a)_6 + (a)_{13}) * \left((a)_{14} * (a)_{15} + (a)_4 * (a)_{11} + {(a)_1}^2\right) + ((a)_{15} + (a)_{14}) *$$
$$\left((a)_{13} * (a)_6 + * a)_4 * (a)_{11} + {(a)_1}^2\right) + ((a)_{11} + (a)_4) * ((a)_{14} * (a)_{15} + (a)_{13}$$
$$* (a)_6 + {(a)_1}^2) + ((a)_6 + (a)_{13}) * ((a)_{15} + (a)_{14}) * \left(((a)_{11} + (a)_4) + 2(a)_1\right)$$
$$+2(a)_1 * ((a)_{11} + (a)_4) * \left(((a)_{15} + (a)_{14}) + ((a)_6 + (a)_{13})\right) - (a)_{10} * {(a)_6}^2$$
$$+((a)_{15} + (a)_6) * ((a)_2 * (a)_{12} - (a)_1 * (a)_6) + (a)_2 * (a)_{12} * ((a)_{13} + (a)_4)$$
$$+2(a)_1 * ((a)_{14} * (a)_{15} + (a)_{13} * (a)_6 + (a)_4 * (a)_{11} + (a)_2(a)_{12}) + (a)_1 *$$
$$(a)_6 * \left((a)_{13} + (a)_{11}\right) - 2(a)_1).$$

$$(\mathcal{A})_4 = (a)_{14} * (a)_{15} * \left((a)_{13} * (a)_6 + (a)_4 * (a)_{11} + {(a)_1}^2\right) + ((a)_6 + (a)_{13}) * [(a)_{14} * (a)_{15} * ((a)_{11} + (a)_4) + (a)_4 * (a)_{11}[((a)_{15} + (a)_{14}) + 2(a)_1 * ((a)_{14} * (a)_{15} + (a)_4 * (a)_{11}) + {(a)_1}^2 * ((a)_{15} + (a)_{14})] + (a)_4 * (a)_{11} * (a)_{13} * (a)_6 + ((a)_{11} + (a)_4) * [((a)_{15} + (a)_{14}) + \left((a)_{13} * (a)_6 + {(a)_1}^2\right) + {(a)_1}^2 \left(((a)_6 + (a)_{13}) + 2(a)_1 * (a)_{14} * (a)_{15} + (a)_{13} * (a)_6\right)\Big] + ((a)_{13} * (a)_6 + (a)_4 * (a)_{11}) * \left[2(a)_1 * ((a)_{15} + (a)_{14}) + {(a)_1}^2\right] + 2(a)_1 * ((a)_{11} + (a)_4) * ((a)_6 + (a)_{13}) * ((a)_{15} + (a)_{14}) - (a)_{10} * {(a)_6}^2 * ((a)_{15} + ((a)_{14} + (a)_{11}) + ((a)_4 + (a)_1)) + (a)_2 * (a)_{12} * ((a)_6 * (a)_{15} + (a)_{13} * (a)_4 + 2(a)_1 * ((a)_{15} + (a)_6) + {(a)_1}^2) + ((a)_{13} + (a)_4) * [(a)_2(a)_{12} * ((a)_{15} + (a)_6) + 2(a)_1) + (a)_4 * (a)_{15} * (a)_7] + (a)_4 * (a)_{15} * (a)_7 * (1 + 2(a)_4) - (a)_1 * (a)_6 * [(a)_6 * (a)_{15} + ((a)_{13} + (a)_{11}) * ((a)_{15} + (a)_6) + (a)_{11} * (a)_{13} + 2(a)_1 * ((a)_{15} + (a)_6) + 2(a)_1 * ((a)_{13} + (a)_{11}) + {(a)_1}^2 + (a)_6(a)_4].$$

$$(\mathcal{A})_5 = ((a)_{11} + (a)_4)[(a)_{14} * (a)_{15} * \left((a)_{13} * (a)_{15} + 2(a)_1 * ((a)_6 + (a)_{13}) + {(a)_1}^2\right) + 2(a)_1 * (a)_{13} * (a)_6 * ((a)_{15} + (a)_{14}) + {(a)_1}^2 * ((a)_6 + (a)_{13}) * ((a)_{15} + (a)_{14}) + {(a)_1}^2 * (a)_{13}(a)_{15}] + ((a)_{15} + (a)_{14}) * [(a)_{13} * (a)_{15} * \left((a)_4 * (a)_{11} + {(a)_1}^2\right) {(a)_1}^2 * (a)_4 * (a)_{11}] + ((a)_6 + (a)_{13}) * [(a)_1 * (a)_4 * (2(a)_{11} * ((a)_{15} + (a)_{14}) - (a)_6 * (a)_2 + {(a)_1}^2 * ((a)_{14} * (a)_{15} + (a)_4 * (a)_{11})] + (a)_{14} * (a)_{15} + [(a)_4 * (a)_{11} + 2(a)_1 * ((a)_{13} * (a)_6 + (a)_4 * (a)_{11})] + 2(a)_1 * (a)_4 * (a)_{11} * (a)_{13} * (a)_6 + 2(a)_1 * (a)_2 * (a)_{12} * ((a)_6 * (a)_{15} + (a)_{13} * (a)_4) + ((a)_{15} + (a)_6) * [(a)_2 * (a)_{12} * \left((a)_{13} * (a)_4 + {(a)_1}^2\right) - (a)_1 * (a)_6 * \left((a)_{11} * (a)_{13} + {(a)_1}^2\right] - (a)_1 * (a)_6 ((a)_{13} + (a)_{11}) * \left[(a)_6 * (a)_{15} + 2(a)_1 * ((a)_{15} + (a)_6) + {(a)_1}^2\right] - (a)_{10} * {(a)_6}^2 * [(a)_{15} * ((a)_{14} + (a)_{11}) + (a)_{14} * (a)_{11} + ((a)_4 + (a)_1) * ((a)_{15} + ((a)_{14} + (a)_{11}) + (a)_1 * (a)_4] + (a)_4 * (a)_{15} * (a)_7 * \left(1 + (a)_4 * (a)_{13} + 2(a)_1 * (a)_6 + {(a)_1}^2\right) - (a)_{10}(a)_6 * \left((a)_{12} * (a)_2 - (a)_6 * {(a)_1}^2\right) - 2{(a)_1}^2 * (a)_6) * [(a)_6 * (a)_{15} + (a)_{11} * (a)_{13} * [-(a)_4(a)_{15}] + {(a)_1}^2 * (a)_5.$$

$$(\mathcal{A})_6 = (a)_{14} * (a)_{15} * \left[(a)_4 * (a)_{11} * (a)_{13} * (a)_6 + {(a)_1}^2 * ((a)_{13} * (a)_6 + (a)_4 * (a)_{11})\right] + (a)_1 * ((a)_{11} + (a)_4) * [(a)_{14} * (a)_{15} * (2(a)_{13} * (a)_6 + (a)_1 ((a)_6 + (a)_{13}) + (a)_1 * (a)_{13} * (a)_6 * ((a)_{15} + (a)_{14})] + (a)_1 * (a)_4 * (a)_{11} * (a)_{13} * 2(a)_6 * ((a)_{15} + (a)_{14}) * ((a)_1 * (a)_{15}) + (a)_1 * (a)_4 * ((a)_6 + (a)_{13}) [2(a)_{11} * (a)_{14} * (a)_{15} + (a)_1 * (a)_{11} * ((a)_{15} + (a)_{14}) - 2(a)_6 * (a)_1(a)_2] - {(a)_6}^2 * (a)_{10} * [(a)_6(a)_1 * ((a)_{11} + (a)_1) - (a)_{15} * ((a)_{14} * (a)_{11} + (a)_1(a)_4 + (a)_{12} * (a)_2 - (a)_6 * (a)_1) - (a)_4 * (a)_{15} * (a)_7] - (a)_{10} * {(a)_6}^2 * (a)_1(a)_4 * ((a)_{14} + (a)_{11}) - (a)_{10} * {(a)_6}^2 * ((a)_4 + (a)_1) * [(a)_{15} * ((a)_{14} + (a)_{11}) +$$

$(a)_{14} * (a)_{11} + (a)_{12} * (a)_2] + (a)_1 * ((a)_{13} + (a)_4) * [(a)_2 * (a)_{12} * (2(a)_{15} *$
$(a)_6 + (a)_1 * ((a)_{15} + (a)_6) + (a)_4 * (a)_{15} * (a)_7 * (2 + (a)_1)] + 2(a)_1 * (a)_{13}$
$* ((a)_{15} + (a)_6) * ((a)_2 * (a)_{12} * (a)_4 - (a)_1 * (a)_6 * (a)_{11}) + (a)_1 * ((a)_{13} +$
$(a)_4) * \; [(a)_2 * (a)_{12} * (2(a)_{15} * (a)_6 + (a)_1 * ((a)_{15} + (a)_6) + (a)_4 * (a)_{15} *$
$(a)_7 * (2(a)_6 + (a)_1)] + (a)_2(a)_{12} * [(a)_1{}^2 * (a)_6 * (a)_{15} + (a)_{13} * (a)_4 *$
$((a)_6 * (a)_{15} + (a)_1{}^2)] + (a)_4 * (a)_{15} * (a)_7 * [(a)_4 * (a)_{13} * ((a)_6 + 2(a)_1)$
$+(a)_1{}^2 * (a)_6] + (a)_1{}^2 * ((a)_{13} + (a)_{11})\big[(a)_6{}^2 * (a)_{15} + (a)_1 * (a)_6 * ((a)_{15} + \quad (a)_6) +$
$(a)_5\big] - (a)_1 * (a)_6{}^2 * (a)_{13} * ((a)_{11} * (a)_{15} + (a)_4 * (a)_{15}) - (a)^3 *$
$[(a)_{11} * (a)_{13} + (a)_6 * ((a)_6 * (a)_{15} + (a)_4 * (a)_{15})] + (a)_1{}^2 * ((a)_{12}$
$+(a)_{14} - (a)_7).$

$(\mathcal{A})_7 = 2a_1a_{13}a_6[a_4a_{15}(a_{11}a_{14} + a_2a_{12} + a_4a_7) - a_1a_6(a_{11} + a_4a_{15})] \; + a_1{}^2a_6a_{15}$
$(a_{13} + a_4)(a_{13}a_{14} + a_2a_{12} + a_4a_7) + a_1{}^2a_4(a_6 + a_{13})(a_{11}a_{14}a_{15} - a_6a_1a_2)$
$+a_1{}^2a_{13}(a_4a_{11}a_6(a_{15} + a_{14}) + a_4{}^2a_{15}a_7) - a_{10}a_6{}^2a_1a_4(a_{14}a_{11} + a_{15}(a_{14} + a_{11})$
$+a_{12}a_2) + a_1{}^2a_{13}(a_{15} + a_6)(a_2a_{12}a_4 - a_1a_{11}) - a_{10}a_6{}^2a_2(a_4 + a_1)$
$(a_{12}a_{15} + a_4a_7) - a_1{}^2(a_{13} + a_{11})(a_1a_6{}^2a_{15} + a_5a_7 - a_5a_{14}) \; + a_6{}^3a_{10}a_1$
$(a_1a_{11} + a_{15}(a_{11} + a_1) + a_4{}^2) + a_1{}^2(\, a_{11}a_{13} + a_{10}a_6{}^2(a_7 - 1)) + a_{12}a_2(a_2 + a_{13}).$

$(\mathcal{A})_8 = a_1{}^2a_4a_{15}a_{13}a_6(a_{11}a_{14} + a_4a_7) + a_1{}^2(a_7(a_{10}a_6{}^2 - a_{13}) + a_{13}a_{14}) + a_1{}^2$
$a_2a_{12}a_{13}(a_4a_6a_{15} + a_1a_5) + a_1{}^2a_6{}^2a_{11}a_{15}(a_6a_{10} - a_1a_{13}) - a_{10}a_6{}^2[a_{14}$
$a_{11}a_{15}(a_4 + a_1) + a_1a_2a_4(a_{12}a_{15} + a_7a_4) + a_1{}^2a_5(a_{14} + a_{12}a_2)] + a_1{}^2a_6{}^2$
$a_4(a_6a_{10}a_2 - a_1a_{13}).$

Now let's put $\alpha = a_{15} + a_{14}$, $\beta = a_6 + a_{13}$, $\delta = a_{11} + a_4$, $\Psi = a_2 - a_1$, $\phi = a_{15} + a_6$, $\eta = a_{13} + a_4$, $\mu = a_{13} + a_{11}$, $\Omega = a_{14} + a_{11}$, $\rho = a_4 + a_1$, $\upsilon = a_{11} + a_1$, $\sigma = a_{13} + a_{11}$, $\theta = a_2 + a_{13}$, $\varphi = a_1 + a_6$, $\gamma = a_1 + a_{13}$,

$\omega = a_8 + a_{13}$, $\xi = a_8 + a_{11}$, $\vartheta = a_1 + a_{13}$, $\psi = a_{11} + a_6$, $\varepsilon = a_{14} - a_7$.

Implies that

$(\boldsymbol{\mathcal{A}})_{(\mathbf{1}).} = (\alpha) + (\beta) + (\delta) + 2(a)_1.$

$(\boldsymbol{\mathcal{A}})_{\mathbf{2}} = (\beta) * [(\alpha) + (\delta) + 2(a)_1] + 2(a)_1[(\alpha) + (\delta)] + (\delta) * (\alpha) + (l) * (\Psi) + (a)_{14}$
$* (a)_{15} + (a)_{13} * (a)_6 + (a)_4 * (a)_{11} + (a)_1{}^2.$

$(\boldsymbol{\mathcal{A}})_{\mathbf{3}} = (\beta)\big((a)_{14} * ((a)_{15} + (a)_4 * (a)_{11} + (a)_1{}^2\big) + (\alpha) * ((a)_{13} * (a)_6 + (a)_4 * (a)_{11}$
$+(a)_1{}^2) + (\delta) * \big((a)_{14} * (a)_{15} + (a)_{13} * (a)_6 + (a)_1{}^2\big) + (\beta) * (\alpha) * ((\delta) + 2(a)_1)$
$+2(a)_1 * (\delta) * \; \big((\alpha) + (\beta)\big) - (a)_{10} * (a)_6{}^2 + (\phi) * ((a)_2 * (a)_{12} - (a)_1 * (a)_6) +$
$(a)_2 * (a)_{12} * (\eta) + 2(a)_1((a)_{14} * (a)_{15} + (a)_{13} * (a)_6 + (a)_4 * (a)_{11} + (a)_2 *$
$(a)_{12}) + \; (a)_1 * (a)_6 * ((\sigma) - 2(a)_{14}).$

$$(\boldsymbol{\mathcal{A}})_{\mathbf{4}} = (a)_{14} * (a)_{15} * \left((a)_{13} * (a)_6 + (a)_4 * (a)_{11} + {(a)_1}^2\right) + (\beta) * [(a)_{14}(a)_{15} * (\delta)$$

$$+(a)_4(a)_{11} * ((\alpha) + 2\left((a)_1 * ((a)_{14} * (a)_{15} + (a)_4(a)_{11}) + {(a)_1}^2 (\alpha)\right] +$$

$$(a)_4 * (a)_{11} * (a)_{13} * (a)_6 + (\delta) * [(\alpha) + \left((a)_{13} * (a)_6 + {(a)_1}^2\right) + {(a)_1}^2 *$$

$$\left((\beta) + 2(a)_1 * ((a)_{14} * (a)_{15} + (a)_{13} * (a)_6)\right] + ((a)_{13} * (a)_6 + (a)_4 * (a)_{11})$$

$$\left[2(a)_1 * ((\alpha) + {(a)_1}^2\right] + 2(a)_1 * (\delta) * (\beta) * (\alpha) - \ (a)_{10} * {(a)_6}^2 *$$

$$((a)_{15} + (\Omega) + (\rho)) + (a)_2 * (a)_{12} * ((a)_6 * (a)_{15} + (a)_{13} * (a)_4 + 2(a)_1 * (\phi)$$

$$+{(a)_1}^2) + (\eta) * [(a)_2 * (a)_{12} * (\phi) + 2(a)_1) + (a)_4 * (a)_{15} * (g)] + (a)_4 * (a)_{15}$$

$$(a)_5 * (1 + 2(a)_4) - (a)_1 * (a)_6 * [(a)_6 * (a)_{15} + (\sigma) * (\phi) + (a)_{11} * (a)_{13} + 2(a)_1$$

$$+2(a)_1 * (\mu) + {(a)_1}^2 + (a)_6 * (a)_4].$$

$$(\boldsymbol{\mathcal{A}})_{\mathbf{5}} = (\delta) * [(a)_{14} * (a)_{15} * \left((a)_{13} * (a)_{15} + 2(a)_1 * (\beta) + {(a)_1}^2\right) + 2(a)_1 * (a)_{13}$$

$$* (a)_6 * (\alpha) + {(a)_1}^2 * (\beta) * (\alpha) + {(a)_1}^2 * (a)_{13} * (a)_{15}] + (\alpha) * [(a)_{13} * (a)_{15}$$

$$\left((a)_4 * (a)_{11} + {(a)_1}^2 \right) + (a)_{14} * (a)_{15} * [(a)_4 * (a)_{11} + 2(a)_1 *$$

$$((a)_{13} * (a)_6 + (a)_4(a)_{11})] + 2(a)_1 * (a)_4 * (a)_{11} * (a)_{13} * (a)_6 + 2(a)_1 *$$

$$(a)_2 * (a)_{12} * ((a)_6 * (a)_{15} + (a)_{13} * (a)_4) + (\phi) * [(a)_2 * (a)_{12} *$$

$$\left((a)_{13} * (a)_4 + {(a)_1}^2\right) - a_1 a_6 (a_{11} a_{13} + {a_1}^2) - \ (a)_1 * (a)_6 * (\mu) * [(a)_6 *$$

$$(a)_{15} + 2(a)_1 * (\phi) + {(a)_1}^2] - (a)_{10} * {(a)_6}^2 * [(a)_{15} * (\Omega) + (a)_{14} * (a)_{11}$$

$$+(\rho) * ((a)_{15} + (\Omega) + (a)_1 * (a)_4] + \ (a)_4 * (a)_{15} * (a)_7 * (1 + (a)_4 * (a)_{13}$$

$$+2(a)_1 * (a)_6 + {(a)_1}^2) - (a)_{10} * (a)_6 * \left((a)_{12} * (a)_2 - (a)_6 * {(a)_1}^2\right) - 2{(a)_1}^2$$

$$* (a)_6 * [(a)_6 * (a)_{15} + (a)_{11} * (a)_{13} - (a)_4 * (a)_{15}] + {(a)_1}^2 * (a)_5.$$

$$(\boldsymbol{\mathcal{A}})_{\mathbf{6}} = (a)_{14} * (a)_{15} * \left[(a)_4 * (k) * (a)_{13} * (a)_6 + {(a)_1}^2 * ((a)_{13} * (a)_6 + (a)_4(a)_{11})\right]$$

$$+(a)_1 * (\delta) * [(a)_{14} * (a)_{15} * (2(a)_{13}(a)_6 + (a)_1 * (\beta) + (a)_1 * (a)_{13} * (a)_6 * (\alpha)]$$

$$+(a)_1 * (a)_4 * (a)_{11} * (a)_{13} * (2(a)_6 * (\alpha) + (a) * (a)_{15}) + (a)_1 * (a)_4 * (\beta) * [2$$

$$(a)_{11} * (a)_{14} * (a)_{15} + (a)_1 * (a)_{11} * (\alpha) - 2(a)_6 * (a)_1 * (a)_5] - {(a0_6}^2 * (a)_{10}$$

$$[(a)_6 * (a)_1 * (\nu) - (a)_{15} * ((a)_{14} * (a)_{11} + (a)_1 * (a)_4 + (a)_{12} * (a)_2 - (a)_6 * \qquad (a)_1) - (a)_4 *$$

$$(a)_{15} * (a)_7] - (a)_{10} * {(a)_6}^2 * (a)_1 * (a)_4 * (\Omega) - (a)_{10} * {(a)_6}^2$$

$$(a)_{15} * \ [(a)_{15} * (\Omega) + (a)_{14} * (a)_{11} + (a)_{12} * (a)_2] + (a)_1 * (\eta)[2(a)_2 * (a)_{12} *$$

$$(2(a)_{15} * (a)_6 + (a)_1 * (\phi) + (a)_4 * (a)_{15} * (a)_7(2 + (a)_1)] + ((a)_2 * (a)_{12} * (a)_4$$

$$-(a)_1 * (a)_6(a)_{11}) + \ (a)_1 * (\eta) * [(a)_2(a)_{12} * (2(a)_{15} * (a)_6 + (a)_1 * (\phi)$$

$$+(a)_4 * (a)_{15} * (a)_7 * \ (2a_6 + a_1)] + (a)_2 * (a)_{12} \ [{(a)_1}^2 * (a)_6 * (a)_{15} + (a)_{13} *$$

$$(a)_4 * ((a)_6 * (a)_{15} + {(a)_1}^2)] + (a)_4 * (a)_{15} * (a)_7[(a)_4 * (a)_{13} * ((a)_6 + 2(a)_1)$$

$$+{(a)_1}^2 * (a)_6] + {(a)_1}^2 * (\mu) \left[{(a)_6}^2 * ((a)_{15} + (a)_1 * (a)_6 * (\phi) + (a)_5\right] - (a)_1$$

$$* {(a)_6}^2 * (a)_{13} * ((a)_{11} * (a)_{15} + (a)_4 * (a)_{15}) - {(a)_1}^3 * [(a)_{11} * (a)_{13} + (a)_6$$

$$((a)_6 * (a)_{15} * (a)_4 * (a)_5)] + {(a)_1}^2 * ((a)_{12} * ((a)_2 + (\varepsilon)).$$

$(\boldsymbol{\mathcal{A}})_{\mathbf{7}} = 2a_1a_{13}a_6[a_4a_{15}(a_{11}a_{14} + a_2a_{12} + a_4a_7) - a_1a_6(a_{11} + a_4a_{15})] + a_1{}^2a_6a_{15}\eta(a_{13}a_{14}$
$+a_2a_{12} + a_4a_7) + a_1{}^2a_4\beta(a_{11}a_{14}a_{15} - a_6a_1a_2) + a_1{}^2a_{13}(a_4a_{11}a_6\alpha + a_4{}^2a_{15}a_7) -$
$a_{10}a_6{}^2a_1a_4(a_{14}a_{11} + a_{15}\Omega + a_{12}a_2) + a_1{}^2a_{13}\phi(a_2a_{12}a_4 - a_1a_{11}) - a_{10}a_6{}^2a_2\rho$
$(a_{12}a_{15} + a_4a_7) - a_1{}^2\sigma\,(a_1a_6{}^2a_{15} + a_5a_7 - a_5a_{14}) + a_6{}^3a_{10}a_1(a_1a_{11} + a_{15}\nu + a_4{}^2)$
$+a_1{}^2(a_{11}a_{13} + a_{10}a_6{}^2(a_7 - 1)) + a_{12}a_2\theta.$

$(\boldsymbol{\mathcal{A}})_{\mathbf{8}} = a_1{}^2a_4a_{15}a_{13}a_6(a_{11}a_{14} + a_4a_7) + a_1{}^2(a_7(a_{10}a_6{}^2 - a_{13}) + a_{13}a_{14}) + a_1{}^2a_2a_{12}a_{13}$
$(a_4a_6a_{15} + a_5) + a_1{}^2a_6{}^2a_{11}a_{15}(a_6a_{10} - aa_{13}) - a_{10}a_6{}^2[a_{14}a_{11}a_{15}\rho + a_1ba_4$
$(la_{15} + a_7a_4) + a_1{}^2a_5(a_{14} + a_{12}a_2)] + a_1{}^2a_6{}^2a_4(a_6a_{10}a_2 - a_1a_{13}).$

And this is what we get $(\lambda) = -a_8$ with

$$\Delta_0(\lambda) = \lambda^{(8)} + (\mathcal{A})_{(1).}\lambda^{(7)} + (\mathcal{A})_{(2).}\lambda^{(6)} + (\mathcal{A})_{(3).}\lambda^{(5)} + (\mathcal{A})_{(4).}\lambda^{(4)} + (\mathcal{A})_{(5).}\lambda^{(3)} + (\mathcal{A})_{(6).}\lambda^{(2)} + (\mathcal{A})_{(7).}\lambda + (\mathcal{A})_{(8).} = 0 \qquad (18)$$

The Routh-Hurwitz criterion cases that every root and a real part in the form $\Delta_0(\lambda)$ is negative if and only if $(\mathcal{A})_1 > 0$, $(\mathcal{A})_8 > 0$ Put $(\mathcal{D})_1 = \left|(\mathcal{A})_1\right| > 0$, $(\mathcal{D})_2 = \begin{vmatrix} (\mathcal{A})_1 & 1 \\ (\mathcal{A})_3 & (\mathcal{A})_2 \end{vmatrix} > 0$,

$$(\mathcal{D})_3 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 \\ (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 \end{vmatrix} > 0,\ (\mathcal{D})_4 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 \\ (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 \\ (\mathcal{A})_7 & (\mathcal{A})_6 & (\mathcal{A})_5 & (\mathcal{A})_4 \end{vmatrix} > 0,$$

$$(\mathcal{D})_5 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 & 0 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 \\ (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 \\ (\mathcal{A})_7 & (\mathcal{A})_6 & \mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 \\ 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & \mathcal{A})_6 & \mathcal{A})_5 \end{vmatrix} > 0\,,$$

$$(\mathcal{D})_6 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 & 0 & 0 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 & 0 \\ (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 \\ (\mathcal{A})_7 & (\mathcal{A})_6 & (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 \\ 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & \mathcal{A})_6 & (\mathcal{A})_5 & (\mathcal{A})_4 \\ 0 & 0 & 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & (\mathcal{A})_6 \end{vmatrix} > 0\ ,$$

$$(\mathcal{D})_7 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 & 0 & 0 \\ (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 \\ (\mathcal{A})_7 & (\mathcal{A})_6 & (\mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 \\ 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & (\mathcal{A})_6 & \mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 \\ 0 & 0 & 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & (\mathcal{A})_6 & (\mathcal{A})_5 \\ 0 & 0 & 0 & 0 & 0 & (\mathcal{A})_8 & (\mathcal{A})_7 \end{vmatrix} >\ 0\,,$$

$$(\mathcal{D})_8 = \begin{vmatrix} (\mathcal{A})_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 & 0 & 0 & 0 \\ \mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 & 0 & 0 \\ (\mathcal{A})_7 & (\mathcal{A})_6 & \mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 & (\mathcal{A})_1 & 1 \\ 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & \mathcal{A})_6 & \mathcal{A})_5 & (\mathcal{A})_4 & (\mathcal{A})_3 & (\mathcal{A})_2 \\ 0 & 0 & 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & \mathcal{A})_6 & \mathcal{A})_5 & (\mathcal{A})_4 \\ 0 & 0 & 0 & 0 & 0 & (\mathcal{A})_8 & (\mathcal{A})_7 & \mathcal{A})_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (\mathcal{A})_8 \end{vmatrix} > 0\,,\ \ \boldsymbol{i.e}$$

$(\mathcal{D})_{(1).} = (\mathcal{A})_{(1).} > 0$

$(\mathcal{D})_{(2).} = (\mathcal{A})_{(1).} * (\mathcal{A})_{(2).} - (\mathcal{A})_{(3).} > 0\,,$

$(\mathcal{D})_{(3).} = -(\mathcal{A})_{(3).}{}^2 - (\mathcal{A})_{(1).}{}^2 * (\mathcal{A})_{(4).} + (\mathcal{A})_{(1).} * \big((\mathcal{A})_{(2).} * (\mathcal{A})_{(3).} + (\mathcal{A})_{(5).}\big) > 0\,,$

$$\begin{aligned}(\mathcal{D})_{(4).} = {} & (\mathcal{A})_{(1).} * (\mathcal{A})_{(2).} * (\mathcal{A})_{(3).} * (\mathcal{A})_{(4).} - (\mathcal{A})_{(1).} * (\mathcal{A})_{(2).}{}^2 * (\mathcal{A})_{(5).} - (\mathcal{A})_{(1).}{}^2 \\ & * (\mathcal{A})_{(4)}{}^2 + (\mathcal{A})_{(1).}{}^2 * (\mathcal{A})_{(2).} * (\mathcal{A})_{(6).} + 2(\mathcal{A})_{(1).} * (\mathcal{A})_{(4).} * (\mathcal{A})_{(5).} - \\ & (\mathcal{A})_{(1).} * (\mathcal{A})_{(3).} * (\mathcal{A})_{(6).} - (\mathcal{A})_{(3).}{}^2 * (\mathcal{A})_{(4).} + (\mathcal{A})_{(2).} * (\mathcal{A})_{(3).} * (\mathcal{A})_{(5).} - \\ & (\mathcal{A})_{(1).} * (\mathcal{A})_{(2).} * (\mathcal{A})_{(7).} - * (\mathcal{A})_{(5).}{}^2 + (\mathcal{A})_{(3).} * (\mathcal{A})_{(7)} > 0\,,\end{aligned}$$

and so on to find the rest of the values of the determinants $(D)_{(5)}$, $(D)_{(6).}$, $(D)_{(7).}$, $(D)_{(8).}$ and $(\boldsymbol{\mathcal{A}})_{(\mathbf{8}).} > 0$ from these inequalities one gets

$(\alpha) + (\beta) + (\delta) + 2(a)_1 > 0\,,$

$$\left(\begin{array}{c} \begin{pmatrix}(\alpha)+(\beta)+\\(\delta)+2(a)_1\end{pmatrix} * (\beta) * \begin{bmatrix}(\alpha)+(\delta)\\+2(a)_1\end{bmatrix} + \\ 2(a)_1[(\alpha)+(\delta)] + (\delta)*(\alpha) + (l)*(\Psi) + (a)_{14}*(a)_{15} + (a)_{13}(a)_6 + (a)_4(a)_{11} + (a)_1{}^2) \\ -\begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+a_1{}^2) + \alpha(a_{13}a_6+a_4a_{11}+a_1{}^2) + \delta(a_{14}a_{15}+a_{13}a_6+a_1{}^2) \\ +\beta\alpha(\delta+2a_1) + 2a_1\delta\,(\alpha+\beta) - a_{10}a_6{}^2 + \phi(a_2a_{12}-a_1a_6) + a_2a_{12}\eta + 2a_1 \\ (a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12}) + \ a_1a_6(\sigma-2a_{14})\end{pmatrix}\end{array}\right) > 0\,,$$

$$\left(\begin{array}{c} -\begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+a_1{}^2) + \alpha(a_{13}a_6+a_4a_{11}+a_1{}^2) + \delta(a_{14}a_{15}+a_{13}a_6+a_1{}^2) \\ +\beta\alpha(\delta+2a_1) + 2a_1\delta\,(\alpha+\beta) - a_{10}a_6{}^2 + \phi(a_2a_{12}-a_1a_6) + a_2a_{12}\eta + 2a_1 \\ (a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12}) + \ a_1a_6(\sigma-2a_{14})\end{pmatrix}^2 \\ -\begin{pmatrix}\alpha+\beta+\\\delta+2a_1\end{pmatrix}^2 * (T)_0 + \begin{pmatrix}\alpha+\beta+\\\delta+2a_1\end{pmatrix} * \begin{pmatrix}\beta[\alpha+\delta+2a_1] + 2a_1[\alpha+\ \delta] + \ \delta\alpha \\ +l\Psi + a_{14}a_{15} + a_{13}a_6 + a_4a_{11} + a_1{}^2\end{pmatrix} * \\ \begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+a_1{}^2) + \alpha(a_{13}a_6+a_4a_{11}+a_1{}^2) + \delta(a_{14}a_{15}+a_{13}a_6+a_1{}^2) \\ +\beta\alpha(\delta+2a_1) + 2a_1\delta\,(\alpha+\beta) - a_{10}a_6{}^2 + \phi(a_2a_{12}-a_1a_6) + a_2a_{12}\eta + 2a_1 \\ (a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12}) + \ a_1a_6(\sigma-2a_{14})\end{pmatrix} \\ (T)_1)\end{array}\right) > 0$$

By the similar way then the following above we obtain: $(\mathcal{D})_{(4).} > 0$ where

$$\begin{aligned}(\boldsymbol{T})_{(\mathbf{0}).} = {} & (a)_{14} * (a)_{15} * \big((a)_{13} * (a)_6 + (a)_4 * (a)_{11} + (a)_1{}^2\big) + (\beta) * [(a)_{14} * (a)_{15} * (\delta) \\ & +(a)_4 * (a)_{11} * (\ 9\alpha) \ + 2(a)_1 * ((a)_{14} * (a)_{15} + (a)_4 * (a)_{11}) + \big(a)_1{}^2 * (\ \alpha)\big] + \\ & (a)_4 * (a)_{11} * (a)_{13} * (a)_6 + (\delta) * [\ (\alpha) + \big((a)_{13} * (a)_6 + (a)_1{}^2\big) + (a)_1{}^2 * \\ & ((\beta) + 2(a)_1 * ((a)_{14} * (a)_{15} + (a)_{13} * (a)_6)] + ((a)_{13} * (a)_6 + (a)_4 * (a)_{11}) \\ & \big[2(a)_1 * (\ \alpha) + (a)_1{}^2\big] + 2(a)_1 * (\delta) * (\beta) * (\alpha) - \ (a)_{10} * (a)_6{}^2 * ((a)_{15} + (\Omega)\end{aligned}$$

$+(\rho)) + (a)_2 * (a)_{12}\big((a)_6 * (a)_{15} + (a)_{13} * (a)_4 + 2(a)_1 * (\phi) + {(a)_1}^2\big) + (\eta) *$
$[(a)_2 * (a)_{12} * (\phi) + 2(a)_1) + (a)_4 * (a)_{15} * (g)] + (a)_4 * (a)_{15} * (a)_5(1 + 2(a)_4)$
$-(a)_1 * (a)_6 * [(a)_6 * (a)_{15} + (\sigma) * (\phi) + (a)_{11} * (a)_{13} + 2(a)_1 + 2(a)_1 * (\mu) +$
${(a)_1}^2 + (a)_6 * (a)_4].$

$(\boldsymbol{T})_{\mathbf{1}.} = (\delta) * [(a)_{14} * (a)_{15} * \big((a)_{13} * (a)_{15} + 2(a)_1 * (\beta) + {(a)_1}^2\big) + 2(a)_1 * (a)_{13} * (a)_6$
$* (\alpha) + {(a)_1}^2 * (\beta) * (\alpha) + {(a)_1}^2 * (a)_{13} * (a)_{15}] + (\alpha) * [(a)_{13} * (a)_{15} * ((a)_4 * (a)_{11}$
$+{(a)_1}^2) + (a)_{14} * (a)_{15} * [(a)_4 * (a)_{11} + 2(a)_1 * ((a)_{13} * (a)_6 + (a)_4 * (a)_{11})] +$
$2(a)_1 * (a)_4 * (a)_{11} * (a)_{13} * (a)_6 + 2(a)_1 * (a)_2 * (a)_{12} * ((a)_6 * (a)_{15} + (a)_{13} *$
$(a)_4) + (\phi) * [(a)_2 * (a)_{12} * \big((a)_{13} * (a)_4 + {(a)_1}^2\big) - (a)_1 * (a)_6 * ((a)_{11} * (a)_{13} +$

${(a)_1}^2)] - (a)_1 * (a)_6 * (\mu) * [(a)_6 * (a)_{15} + 2(a)_1 * (\phi) + {(a)_1}^2] - (a)_{10} * {(a)_6}^2$
$* [(a)_{15} * (\Omega) + (a)_{14} * (a)_{11} + (\rho) * ((a)_{15} + (\Omega) + (a)_1 * (a)_4] + (a)_4 * (a)_{15}$
$* (a)_7 * \big(1 + (a)_4 * (a)_{13} + 2(a)_1 * (a)_6 + {(a)_1}^2\big) - (a)_{10} * (a)_6 * ((a)_{12} * (a)_2$
$-(a)_6 * ({a)_1}^2) - 2{(a)_1}^2 * (a)_6[(a)_6 * (a)_{15} + (a)_{11} * (a)_{13} - (a)_4 * (a)_{15}]$
$+{(a)_1}^2(a)_5.$

$(\boldsymbol{T})_{\mathbf{2}.} = (a)_{14} * (a)_{15} * \big[(a)_4 * (k) * (a)_{13} * (a)_6 + {(a)_1}^2 * ((a)_{13} * (a)_6 + (a)_4 * (a)_{11})\big] +$
$(a)_1 * (\delta) * [(a)_{14} * (a)_{15} * (2(a)_{13} * (a)_6 + (a)_1 * (\beta) + (a)_1 * (a)_{13} * (a)_6 * (\alpha)]$
$+(a)_1 * (a)_4 * (a)_{11} * (a)_{13} * (2(a)_6 * (\alpha) + (a) * (a)_{15}) + (a)_1 * (a)_4 * (\beta) * [2 *$
$(a)_{11} * (a)_{14} * (a)_{15} + (a)_1 * (a)_{11} * (\alpha) - 2(a)_6 * (a)_1 * (a)_5] - {(a)_6}^2 * (a)_{10}$
$[(a)_6 * (a)_1 * (\nu) - (a)_{15} * ((a)_{14} * (a)_{11} + (a)_1 * (a)_4 + (a)_{12} * (a)_2 - (a)_6 * \quad (a)_1) - (a)_4 *$
$(a)_{15} * (a)_7] - (a)_{10} * {(a)_6}^2 * (a)_1 * (a)_4 * (\Omega) - (a)_{10} * {(a)_6}^2 *$
$(a)_{15} * [(a)_{15} * (\Omega) + (a)_{14} * (a)_{11} + (a)_{12} * (a)_2] + (a)_1 * (\eta) * [(a)_2 * (a)_{12} *$
$(2(a)_{15} * (a)_6 + (a)_1 * (\phi) + (a)_4 * (a)_{15} * (a)_7 * (2 + (a)_1)] + ((a)_2 * (a)_{12} *$
$(a)_4 - (a)_1 * (a)_6 * (a)_{11}) + (a)_1 * (\eta) * [(a)_2 * (a)_{12} * (2(a)_{15} * (a)_6 + (a)_1 *$
$(\phi) + (a)_4 * (a)_{15} * (a)_7 * (2(a)_6 + (a)_1)] + (a)_2 * (a)_{12} * [{(a)_1}^2 * ((a)_6 * (a)_{15}$
$+(a)_{13} * (a)_4 * ((a)_6 * (a)_{15} + {(a)_1}^2)] + (a)_4 * (a)_{15} * (a)_7 * [(a)_4 * (a)_{13} *$
$((a)_6 + 2(a)_1) + {(a)_1}^2 * (a)_6] + {(a)_1}^2 * (\mu) * [{(a)_6}^2 * (a)_{15} + (a)_1 * (a)_6 * (\phi)$
$+(a)_5] - (a)_1 * {(a)_6}^2 * (a)_{13} * ((a)_{11} * (a)_{15} + (a)_4 * (a)_{15}) - {(a)_1}^3 * [(a)_{11} *$
$(a)_{13} + (a)_6 * ((a)_6 * (a)_{15} * (a)_4 * (a)_5)] + {(a)_1}^2 * ((a)_{12} * (a)_2 + (\varepsilon)).$

From the discussion above the following characteristic is obtained

Theorem 1: If $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}$ *and* a_{15} are $(\neq \mathbf{0})$, the proposed model takes a hyper bolic evenness

$(f)_{(0).} = \{((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).), ((0).)\}$ (the basis)., with if $(\alpha) + (\beta) + (\delta) + 2(a)_1 > 0$, Approximately the hyperbolic equilibrium of $(f(t))$ is steady

$$\left(\begin{array}{c}\begin{pmatrix}(\alpha)+(\beta)+\\(\delta)+2(a)_1\end{pmatrix}\left((\beta)*\begin{bmatrix}(\alpha)+(\delta)\\+2(a)_1\end{bmatrix}+2a_1[(\alpha)+(\delta)]+\ \delta\alpha+l\Psi+a_{14}a_{15}+a_{13}a_6+a_4a_{11}+{a_1}^2\right)\\ -\begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+{a_1}^2)+\alpha(a_{13}a_6+a_4a_{11}+{a_1}^2)+\delta(a_{14}a_{15}+a_{13}a_6+{a_1}^2)\\+\beta\alpha(\delta+2a_1)+2a_1\delta\,(\alpha+\beta)-a_{10}{a_6}^2+\phi(a_2a_{12}-a_1a_6)+a_2a_{12}\eta+2a_1\\(a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12})+\ a_1a_6(\sigma-2a_{14})\end{pmatrix}\end{array}\right)>0\,,$$

$$\left(\begin{array}{c}-\begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+{a_1}^2)+\alpha(a_{13}a_6+a_4a_{11}+{a_1}^2)+\delta(a_{14}a_{15}+a_{13}a_6+{a_1}^2)\\+\beta\alpha(\delta+2a_1)+2a_1\delta\,(\alpha+\beta)-a_{10}{a_6}^2+\phi(a_2a_{12}-a_1a_6)+a_2a_{12}\eta+2a_1\\(a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12})+\ a_1a_6(\sigma-2a_{14})\end{pmatrix}^2\\ -\begin{pmatrix}\alpha+\beta+\\\delta+2a_1\end{pmatrix}^2*(T)_0+\begin{pmatrix}\alpha+\beta+\\\delta+2a_1\end{pmatrix}*\begin{pmatrix}\beta[\alpha+\delta+2a_1]+2a_1[\alpha+\ \delta]+\ \delta\alpha\\+l\Psi+a_{14}a_{15}+a_{13}a_6+a_4a_{11}+{a_1}^2\end{pmatrix}*\\ \begin{pmatrix}\beta(a_{14}a_{15}+a_4a_{11}+{a_1}^2)+\alpha(a_{13}a_6+a_4a_{11}+{a_1}^2)+\delta(a_{14}a_{15}+a_{13}a_6+{a_1}^2)\\+\beta\alpha(\delta+2a_1)+2a_1\delta\,(\alpha+\beta)-a_{10}{a_6}^2+\phi(a_2a_{12}-a_1a_6)+a_2a_{12}\eta+2a_1\\(a_{14}a_{15}+a_{13}a_6+a_4a_{11}+a_2a_{12})+\ a_1a_6(\sigma-2a_{14})\end{pmatrix}\\(T)_1)\end{array}\right)>0$$

Where $(T)_0$, $(T)_1$, $(T)_2$ up top Except that in deterministic $(f(\mathrm{t}))$ there is an instability, so a statement $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}$ *and* a_{15} are ($\neq$ **0**),, the proposed new model and there is 4 equilibrium point are real numbers.

$f_{1.}(\mathrm{t})_{.}((x(\mathrm{t}))_{1.}, (y(\mathrm{t}))_{1.}, (z(\mathrm{t}))_{1.}, (w(\mathrm{t}))_{1.}, (v(\mathrm{t}))_{1.}, (u(\mathrm{t}))_{1.}, (m(\mathrm{t}))_{1.}, (f(\mathrm{t}))_{1.}, (p(\mathrm{t}))_{1.})$,

$f_{2.}(\mathrm{t})\ ((x(\mathrm{t}))_{2.}, (y(\mathrm{t}))_{2.}, (z(\mathrm{t}))_{2.}, (w(\mathrm{t}))_{2.}, (v(\mathrm{t}))_{2.}, (u(\mathrm{t}))_{2.}, (m(\mathrm{t}))_{2.}, (f(\mathrm{t}))_{2.}, (p(\mathrm{t}))_{2.})$,

$f_{3.}(\mathrm{t})((x(\mathrm{t}))_{3.}, (y(\mathrm{t}))_{3.}, (z(\mathrm{t}))_{3.}, (w(\mathrm{t}))_{3.}, (v(\mathrm{t}))_{3.}, (u(\mathrm{t}))_{3.}, (m(\mathrm{t}))_{3.}, (f(\mathrm{t}))_{3.}, (p(\mathrm{t}))_{3.})$,

$f_{4.}(\mathrm{t})((x(\mathrm{t}))_{4.}, (y(\mathrm{t}))_{4.}, (z(\mathrm{t}))_{4.}, (w(\mathrm{t}))_{4.}, (v(\mathrm{t}))_{4.}, (u(\mathrm{t}))_{4.}, (m(\mathrm{t}))_{4.}, (f(\mathrm{t}))_{4.}, (p(\mathrm{t}))_{4.})$,

Similarity, in the stability, thus we just debate the steadiness of $(f(t))_1$

$J_1 =$

$$\begin{bmatrix}-(a)_1 & 0 & (a)_2v(\mathrm{t}) & 0 & (a)_2z(\mathrm{t})+(a)_3u(\mathrm{t}) & (a)_3v(\mathrm{t}) & 0 & 0 & -(a)_1\\ (a)_5-(a)_7p(\mathrm{t}) & -(a)_4 & 0 & 0 & 0 & 0 & 0 & (a)_6 & -(a)_7x(\mathrm{t})\\ (a)_{10}x(\mathrm{t}) & v(\mathrm{t}) & -(a)_8 & 0 & y(\mathrm{t}) & 0 & (a)_{10}x(\mathrm{t}) & (a)_9 & 0\\ 0 & 0 & 0 & -(a)_{11} & -(a)_6m(\mathrm{t}) & 0 & -(a)_6v(\mathrm{t}) & (a)_{11} & -(a)_4\\ 0 & -(a)_6m(\mathrm{tt}) & -(a)_{10}u(\mathrm{t}) & 0 & -(a)_{13} & (a)_{10}-(a)_{10}z(\mathrm{t}) & -(a)_6y(\mathrm{t}) & 0 & 0\\ 0 & 0 & -(a)_{10}w(\mathrm{t}) & -(a)_{10}z(\mathrm{t}) & 0 & -(a)_1 & -(a)_6 & 0 & 0\\ y(\mathrm{t})+(a)_{12}v(\mathrm{t}) & x(\mathrm{t}) & 0 & 0 & -(a)_6+(a)_{12}x(\mathrm{t}) & 0 & -(a)_6 & 0 & 0\\ 0 & (a)_1 & 0 & -(a)_2 & 0 & m(\mathrm{t}) & u(\mathrm{t}) & -(a)_{14} & 0\\ -(a)_8f(\mathrm{t}) & (a)_1 & 0 & 0 & 0 & 0 & 0 & -(a)_7-(a)_8x(\mathrm{t}) & -(a)_{15}\end{bmatrix}$$

…… …… (19)

Consequently, the linearization model possesses the succeeding distinguishing equation at equilibrium $(f(t))_1$.

$$\Gamma(\lambda) = (\lambda)^{(9)} + (\mathcal{B})_{(1).} * (\lambda)^{(8)} + (\mathcal{B})_{(2).} * (\lambda)^{(7)} + (\mathcal{B})_{(3).} * (\lambda)^{(6)} + (\mathcal{B})_{(4).} * (\lambda)^{(5)} +$$

$$(\mathcal{B})_{(5).} * (\lambda)^{(4)} + (\mathcal{B})_{(6).} * (\lambda)^{(3)} + (\mathcal{B})_{(7).} * (\lambda)^{(2)} + (\mathcal{B})_{(8).} * (\lambda) + (\mathcal{B})_{(9).} \qquad \textbf{(20)}$$

$(\mathcal{B})_{1.} = (\vartheta) + (\delta) + (\varphi) + (\alpha) + (a)_8$,

$(\mathcal{B})_{2.} = a_1\varphi + a_8\delta + \beta\alpha + a_8((\beta+\alpha) + 2a_1((\delta+\alpha+a_{13}) + \delta(\beta+\alpha) + a_4a_{11} + a_{14}a_{15}$
$+f(t)a_1a_8 + x(t)a_1a_7 - f(t)a_2a_{10} - p(t)a_3a_{10} + z(t)a_{10} + a_2a_9 + y(t){a_6}^2 -$
$p(t)a_6\, a_{12}$

Now, we make reference to all of a $(\mathcal{B})_{3.}, (\mathcal{B})_{4.}, (\mathcal{B})_{5.}, (\mathcal{B})_{6.}, (\mathcal{B})_{7.}, (\mathcal{B})_{8.}$ *and* $(\mathcal{B})_{9.}$, This gives

$$\Gamma_{(0).}(\lambda) = (\lambda)^{(9)} + (\mathcal{B})_{(1).} * (\lambda)^{(8)} + (\mathcal{B})_{(2).} * (\lambda)^{(7)} + (\mathcal{B})_{(3).} * (\lambda)^{(6)} + (\mathcal{B})_{(4).} * (\lambda)^{(5)} + (\mathcal{B})_{(5).} * (\lambda)^{(4)} + (\mathcal{B})_{(6).} * (\lambda)^{(3)} + (\mathcal{B})_{(7).} * (\lambda)^{(2)} + (\mathcal{B})_{(8).} * (\lambda) + (\mathcal{B})_{(9).} = 0 \quad (21)$$

The Routh-Hurwitz criterion states that the rea portions of each the origins (roots) λ in $\Gamma_{0.}(\lambda)$ are non-positive or nonzero if and only if

$$(\Delta)_1 = \mathcal{B}_{(1).} > 0 , \mathcal{B}_{(9).} > 0$$

$$(\Delta)_{\mathbf{2}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 \\ \mathcal{B}_{2.} & \mathcal{B}_{2.} \end{vmatrix} > 0 , (\Delta)_{\mathbf{3}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} \end{vmatrix} > 0 ,$$

$$(\Delta)_{\mathbf{4}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} \\ \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} \end{vmatrix} > 0 , (\Delta)_{\mathbf{5}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 & 0 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} \\ \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} \\ \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} \end{vmatrix} > 0 ,$$

$$(\Delta)_{\mathbf{6}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 & 0 & 0 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 & 0 \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 \\ \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} \\ \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} \\ 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} \end{vmatrix} > 0 ,$$

$$(\Delta)_{\mathbf{7}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 & 0 & 0 \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 \\ \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} \\ \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} \\ 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} \\ 0 & 0 & 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} \end{vmatrix} > 0 ,$$

$$(\Delta)_{\mathbf{8}} = \begin{vmatrix} \mathcal{B}_{1.} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 & 0 & 0 & 0 \\ \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 & 0 & 0 \\ \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} & \mathcal{B}_{1.} & 1 \\ \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} & \mathcal{B}_{3.} & \mathcal{B}_{2.} \\ 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} & \mathcal{B}_{5.} & \mathcal{B}_{4.} \\ 0 & 0 & 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} & \mathcal{B}_{7.} & \mathcal{B}_{6.} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{B}_{9.} & \mathcal{B}_{8.} \end{vmatrix} > 0 , \quad (22)$$

When condition (22) is satisfied, the characteristic polynomial (21) as a outcome has each roots with the real negative part

Theorem 2:

The suggested model has 4 hyperbolic equilibrium points $(f(t))_{1.}$, $(f(t))_{2.}$, $(f(t))_{3.}$, $(f(t))_{4.}$, $, (f(t))_6, (f(t))_7, (f(t))_8$ and $(f(t))_9$, for $a_1, a_2, a_4, a_6, a_7, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}$ and a_{15} that are $(\neq \mathbf{0})$,. If conditions (22) are met, the four equilibrium points become $(E)_0, (E)_1, (E)_2$ *and* $(E)_3$, which is independent of the value of the coefficient; otherwise, the four hyperbolic equilibrium points are unstable.

7. Phase portraits

The Figures (1-4) show the phase portraits. A very intriguing, intricate, and chaotic dynamical behavior seems to be displayed by the novel hyper chaotic attractor. Additionally, the numerical simulation has been finished using the “ ***MATHEMATICA*** “application. The complex and plentiful chaotic dynamics characteristics of this nonlinear system are demonstrated by the weird attractors in (3-D) Figure (3,4) and the strange attractors in (2-D) Figure **(**1,2). show that the topology resembles the shape of the odd attractor, which is how the term "Butterfly Effect" was created.

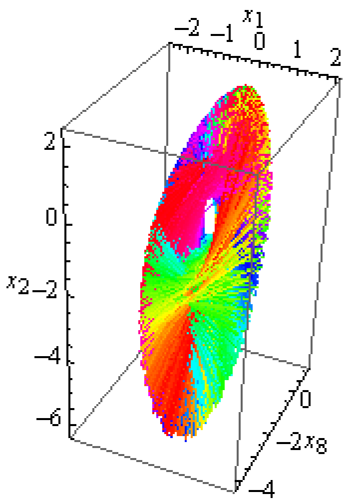

Figure 1: Chaotic attractors

(3-D) view $(f - x - y)$

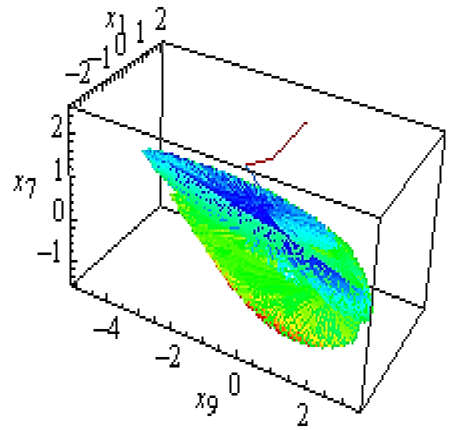

Figure 2: Chaotic attractors

(3-D) view $(p - x - m)$

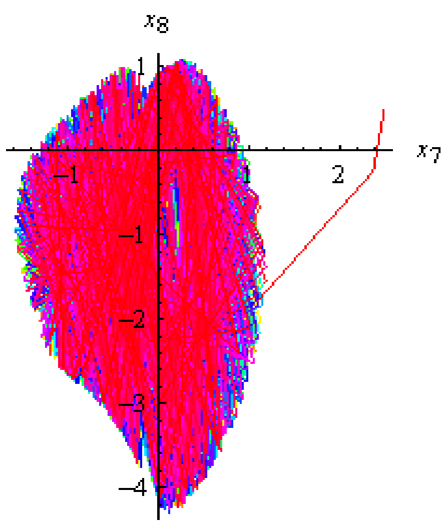

Figure 3: Chaotic attractors

$(m - f\ )$ phase plane

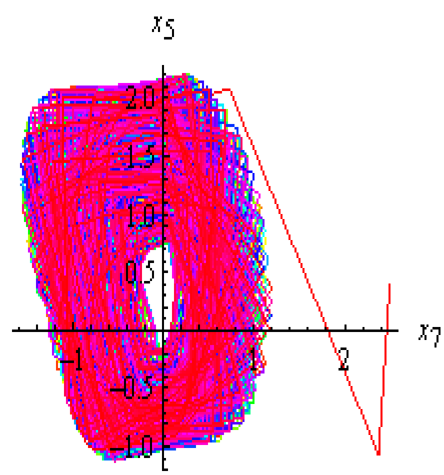

Figure 4: Chaotic attractors

$(m - v\ )$ phase plane

8. The new chaotic system’s waveform analysis

It is well known that the waveform of a chaotic system should be aperiodic. to demonstrate that the proposed system is chaotic. The plot of time versus states generated by the MATHEMATICA simulation is shown in Figure 5. In the time domain, Figure 5 displays the waveforms of $\{x(t), y(t), z(t), w(t), v(t), u(t),\ m(t), f(t), and\ p(t)\}$. "aperiodic" describes the waveforms of $\{\mathcal{X}(t), \mathcal{Y}(t), \mathcal{Z}(t), \mathcal{W}(t), \mathcal{V}(t), \mathcal{U}(t),\ m(t), f(t), and\ p(t)\}$.To differentiate between multiple periodic motion and chaotic motion, both of which can display complex behavior. Additionally, the time domain waveform's non-cyclical features are evident.

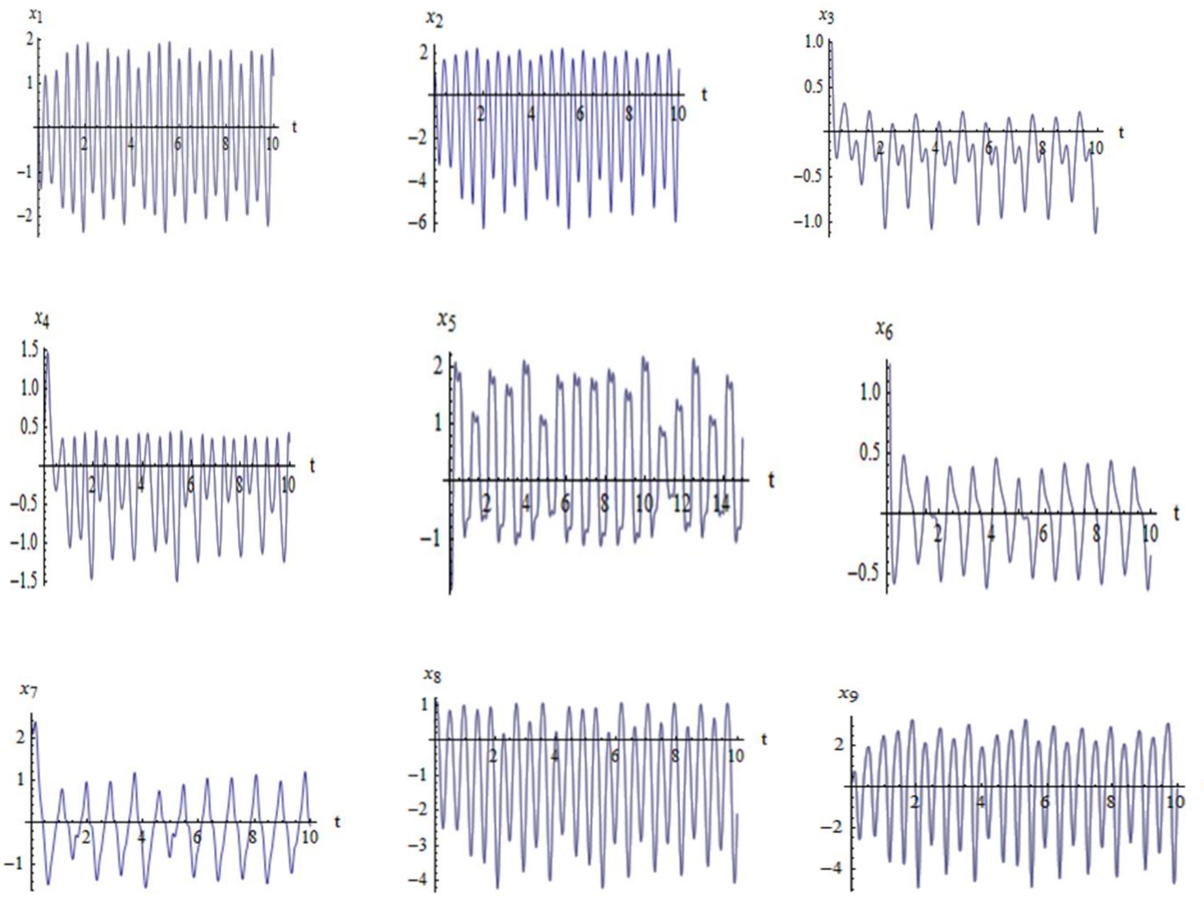

Figure 5: Time Compared to

$\{\mathcal{X}(\text{t}), \mathcal{Y}(\text{t}), \mathcal{Z}(\text{t}), \mathcal{W}(\text{t}), \mathcal{V}(\text{t}), \mathcal{U}(\text{t}), \mathcal{m}(\text{t}), \mathcal{f}(\text{t}), and\ \mathcal{p}(\text{t})\}$. of the emerging messy model

9. Sensitivity to initial condition

A chaotic system's long-term unpredictability, which result the solution' delicate reliance on initial conditions, is its most characteristic feature. Even if two initial conditions are identical at first they will quickly diverge significantly. Therefore, it will quickly become difficult to reliably predict the state of the system given any initial scenario and a finite number of accuracy digits. The creation of the chaos trajectory is significantly influenced by initial circumstances, as understood in Figure 6. The model's early valu es it assigned to:

$\{x\,(0.) = (0.6)\,, y(0.) = (0.5)\,, z(0.) = (3).\,, w\,(0.) = (0.6)\,, v\,(0.) = (0.4)\,, u(0.)$

$= (8.).\,, m(0.) = (2\ .5), f(0.) = (\ 0.5)$ and $p(0.) = (0\ .\ 2).\}$

for the rock-hard stroke with

$\{\mathcal{X}(0.) = 0.6\,, \mathcal{Y}(0.) = 0.5\,, \mathcal{Z}(0.) = 3.\mathbf{000\ 0000\ 000\ 0001}\,, \mathcal{W}(0.) = 0.6\ , \mathcal{V}(0.) = 0.4\,,$

$\mathcal{U}\,(0.) = 8.\,, \mathcal{m}\,(0.) = 2.5, \mathcal{f}(0.) = 0.\ 5\ and\ \mathcal{p}(0.) = 0.2.\}$ for the dashed stroke.

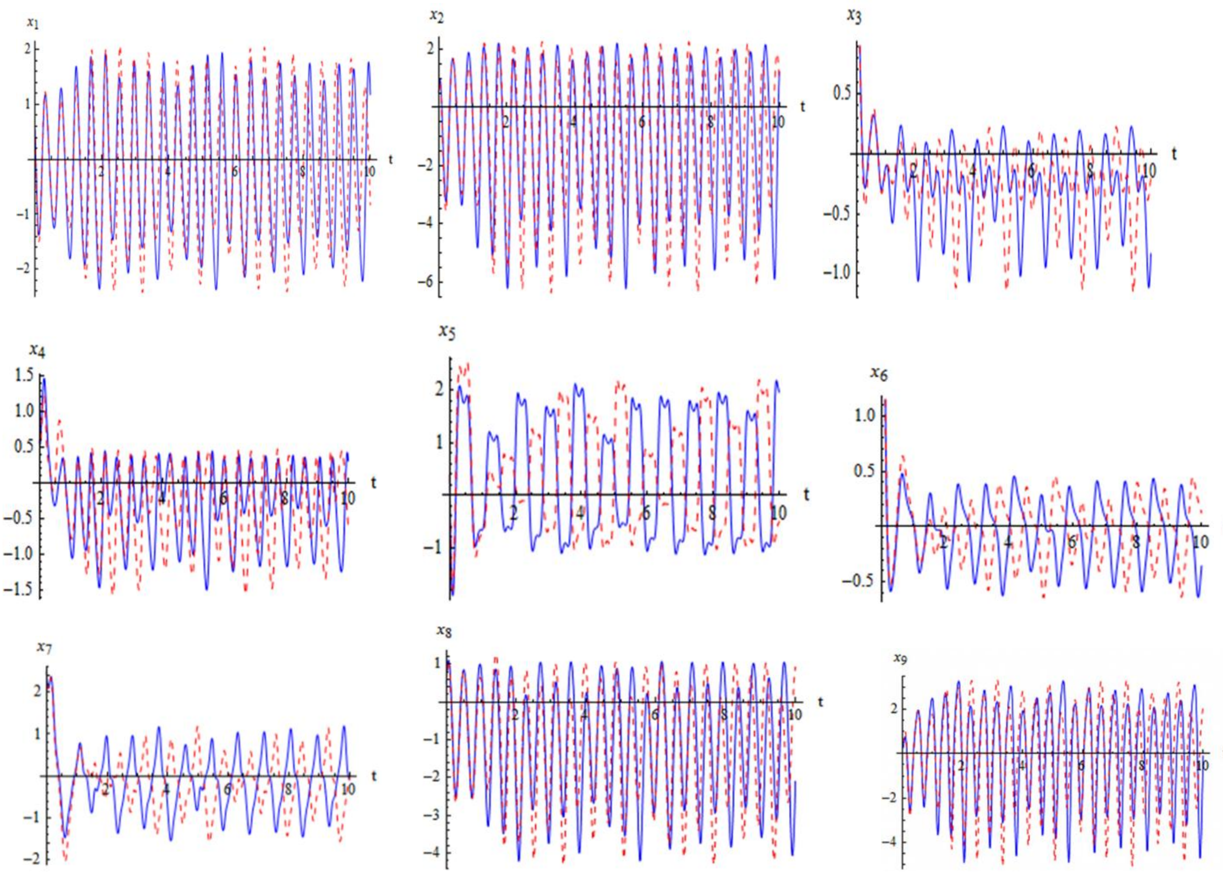

Figure 6 : Sensitivity tests of the new system

$\{x\ (\text{t}\ ), y\ (\text{t}), z(\ \text{t}\ ), w(\ \text{t}\ ), v\ (\ \text{t}\ ), u(\ \ \text{t}\ ), m\ (\ \text{t}\ ), f(\ \text{t}\ ), p(\text{t}\ \ )\}$

10. Bifurcation Drawing

The new model (7) is unraveled mathematically done the simulation of Mathematica package and we putting the values from the state $y(t)$ with take altering the values of the coefficient a_{10} we see witness the exciting area of $y(t)$ which the model vicissitudes its comportment and the values of y commence to Bifurcation, specifically among $a_{10} =\ 0.5$and $a_{10} = (0.6)$ with likewise in this "values" among $a_{10} = (0\ .3)$ with $a_{10} = (0\ .4)$, and the branching properties is one of the most significant features of messy models and it is accomplished in the newfangled model by way of exposed Figure 7:

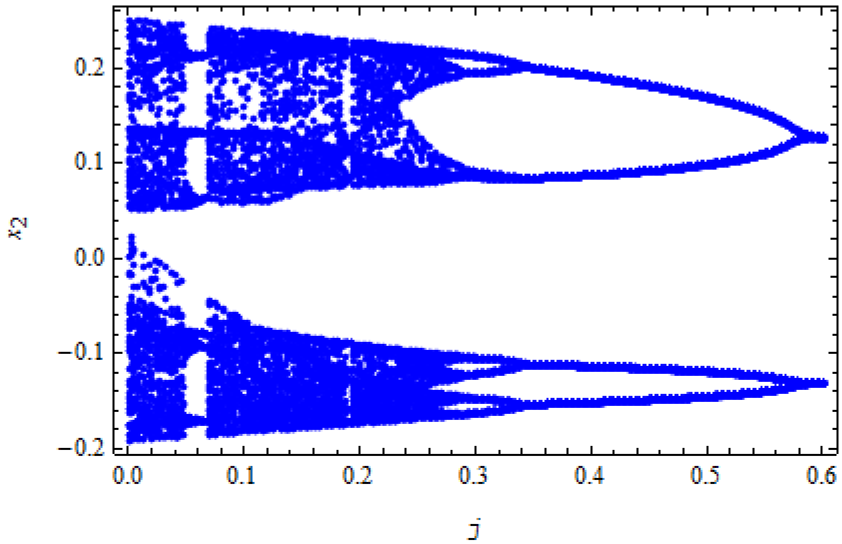

Figure7: Bifurcation diagram of the variable $y(t)$ for increases a_{10}

putting the values of the state $f(t)$ with when altering of the values from a positive constant n It is interesting that it can be noted area from $f(t)$ whatever model altering its comportment with of the values from a $f(t)$ begin to branching, specifically among $a_{14} = (9\,.6)$ and $a_{14} = (9\,.7)$ and likewise with of the values from an among $a_{14} = (9\,.\,8)$ and$a_{14} = (9\,.9)$, and the bifurcation characteristics is one of the greatest significant features of messy models and it is accomplished in the newfangled model as exposed Figure 8.

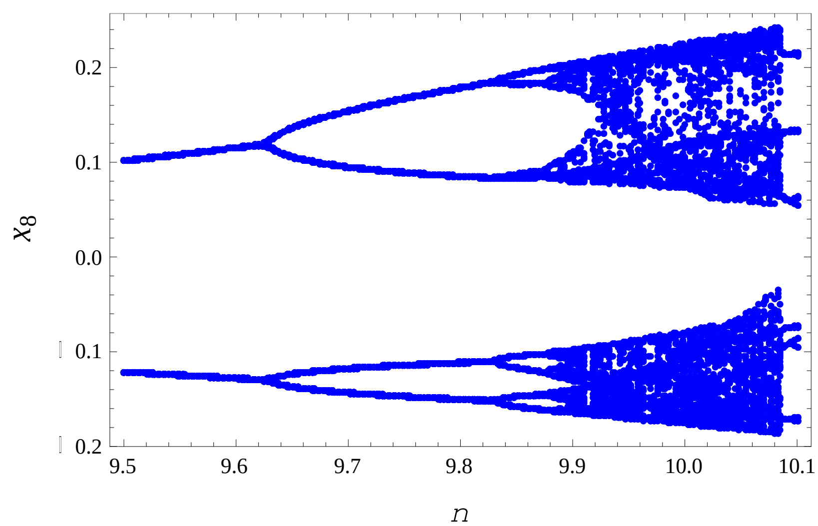

Fig**. 8**: Bifurcation drawing of the state $f(t)$ for increasing (n)

putting the values of the state $v(t)$with altering the values of the positive constant a_{15} we see witness the interesting area of $v(t)$ which the model altering its comportment and the values of $f(t)$ start to bifurcation, specifically among $a_{15} = (8\,.6)$ and $a_{15} = (8\,.8)$ its with of the values from a among $a_{15} = (9\,.0$)and $a_{15} = (\,9\,.2)$, and the bifurcation characteristics is a "greatest" important properties of messy models with it is attained in the newfangled model as exposed "Fig. 9".

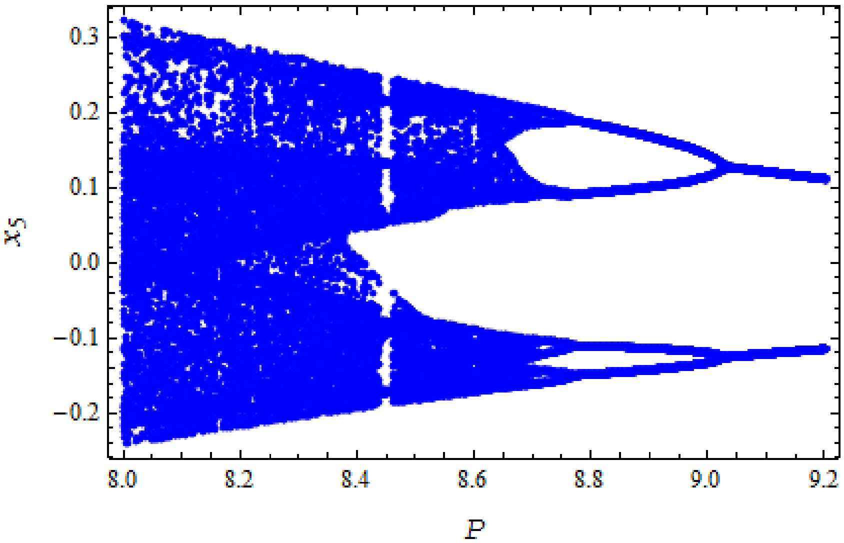

Figure 9: Bifurcation drawing of the state $v(t)$ for increasing a_{15}

11. Zero- One Test

In determinative dynamical models, the zero-one examination is secondhand to distinguish among regular and messy dynamic powers. This examination's information is a $(1-D)$ trajectory $\mathfrak{H}(\mathfrak{n})$

Where $\{\mathfrak{n}. = (1.)., (2.)., (3.). \ldots..\}$ the information $\mathfrak{H}(\mathfrak{n})$ is secondhand to limited the $(2-D)$ model

$$\mathfrak{N}(\mathfrak{n}+1). = \mathfrak{N}(\mathfrak{n}) + \mathfrak{H}(\mathfrak{n}) cos\ c\mathfrak{n}$$

$$\dot{M}(\mathfrak{n}+1). = \dot{M}(\mathfrak{n}) + \mathfrak{H}(\mathfrak{n})\ sin\ c\mathfrak{n}$$

Where $0 < (c) < 2\pi$. is coefficent. Which is the squared error means the average offset (MSD) i.e. determine through:

$$MSD(\mathfrak{n}) = \lim_{\mathfrak{n}\to\infty} \frac{1}{\mathfrak{N}} \sum_{j=1}^{\mathfrak{n}} [\mathfrak{N}(J+\mathfrak{n}) - \mathfrak{N}(J)]^2 + [\dot{M}(J+\mathfrak{n}) - \dot{M}(J)]^2 \qquad (23)$$

The Approximate Growing rate is:

$$\mathcal{K}_c = \frac{\log MSD(\mathfrak{n})}{\log \mathfrak{n}}$$

$$\mathcal{K} = median\ (\mathcal{K}_c)$$

$$\text{Decision} = \begin{cases} \mathcal{K} \cong 0 & the\ system\ is\ regular \\ \mathcal{K} \cong 1 & the\ system\ is\ chaotic \end{cases}$$

The $\mathcal{K}$ values of the enhanced model states $\{\mathcal{X}(t), \mathcal{Y}(t), \mathcal{Z}(t), \mathcal{W}(t), \mathcal{V}(t), \mathcal{U}(t), m(t), f(t)\ and\ p(t)\}$ were accomplished:

$$\mathcal{K}_{x(t)} = 0.9901, \mathcal{K}_{y(t)} = 0.9797, \mathcal{K}_{z(t)} = 0.9811\ , \mathcal{K}_{w(t)} = 0.9614\ , \mathcal{K}_{v(t)} = 0.9497$$

$$\mathcal{K}_{u(t)} = 0.9597\ , \mathcal{K}_{m(t)} = 0.9712\ , \mathcal{K}_{f(t)} = 0.9712\ and\ \mathcal{K}_{p(t)} = 0.9712$$

These outcomes expression that the enhancement model is messy because each of the computable values of $\mathcal{K}$ are neighboring to one.

12. Conclusion

This work presents a thirteen-term cross-product, fifteen positive parameters, nine-dimensional nonlinear hyperchaotic autonomous system. The three positive Lyapunov exponents of the suggested system make it highly chaotic. Furthermore, waveform analysis, numerical simulation, SDIC (Sensitivity Dynamic starting condition), and the complex chaotic attractor have all shown the chaotic behavior of the suggested system. The waveform analysis reveals noncyclical patterns in the time-domain waveform, further proving that the suggested system is aperiodic. Three Lyapunov positive exponents make the new chaotic system hyperchaotic. Bifurcations, a crucial aspect of chaos, appear in the suggested system when parameter values change. utilized the Zero-One test as well, and since all of the computed values of K are close to one, the planned system is chaotic. It was also examined for stable behavior using the Routh-Hurwitz criterion. Every fixed point in the system is unstable.

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