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Article

An Optimization Framework for Balanced Network Coverage Using Mean Absolute Deviation Domination in Graphs

M. Yasmin Nirosha[1], S. Firthous Fatima[2]

[1]Department of Mathematics, Sadakathullah Appa College (Autonomous), Affiliated to Manonmaniam Sundaranar University, Tamil Nadu, India.
Email: yasminnirosha@gmail.com
ORCID: 0009-0009-6494-4864
[2]Department of Mathematics, Sadakathullah Appa College (Autonomous), Manonmaniam Sundaranar University, Tamil Nadu, India.
Email: firthous2020maths@sadakath.ac.in
ORCID: 0009-0009-2388-6151
Corresponding Author: M. Yasmin Nirosha

Abstract: Domination in graphs is a fundamental concept in graph theory with important applica-tions in network design and resource allocation. Classical parameters such as the domination number and equitable domination number measure the size and fairness of dominating sets but do not quantify the imbalance in domination coverage. To address this limitation, we introduce a deviation-based measure called the mean absolute deviation of domination, de-noted by MAD(D), which evaluates the uniformity of domination across vertices. Two mixed integer linear programming formulations are proposed to compute minimum-MAD dominat-ing sets and equitable domination numbers. Computational experiments on several graph families demonstrate that graph topology significantly influences domination balance and computational performance..
Keywords: domination in graphs, mean absolute deviation, MAD domination number, equitable domination, mixed integer linear programming.

1. Introduction

Domination theory in graphs has become a fundamental area of research due to its wide range of applications in network design, resource allocation, facility location, communication systems, and biological and social networks. The classical domination number $\gamma(G)$ measures the mini-mum number of vertices required to dominate all others. Over the years, numerous extensions such as total domination, Roman domination, secure domination, and distance domination have been introduced to capture diverse structural and applied scenarios.

One important refinement that has gained increasing attention is equitable domination, which emphasizes fairness in the distribution of domination influence among vertices. This perspective is crucial in applications where balanced coverage is as significant as efficiency, such as wireless sensor networks and equitable resource management. Early contributions such as Swaminathan and Dharmalingam [1] studied degree equitable domination, while Polinar-Sandueta [2] analyzed equitable domination in several graph classes. Further variants include total near equitable domination [3], total equitable domination [4], and equitable and paired equitable domination in inflated graphs [5]. Other extensions include independent equitable

domination [6], equitable isolate domination [7], and global equitable domination [8]. These studies highlight the importance of equitable domination as a refinement of classical domination that helps evaluate fairness and balance in networks.

Another line of investigation in domination theory concerns *average domination parameters*, which provide insights from a distributional or statistical perspective. Blidia et al. [10] ex-amined relations between average lower independence and domination numbers. Later, Aslan and Kirlangic [11] formally introduced the average lower domination number, and Dogan [12] studied this parameter for middle graphs. Turaci et al. [13] investigated the average lower 2-domination number for several graph families including wheel, helm, gear, and sunflower graphs. Meenal, Ilaiyaraja, and Jose [9] further studied the average total domination number for trees. These contributions demonstrate that averaging techniques provide useful tools for analyzing irregularities in domination.

A complementary direction involves *optimization-based formulations* for domination param-eters. Such approaches enable the exact computation of domination measures in complex graph classes. Duraisamy and Esakkimuthu [14] proposed linear programming models for several domination parameters, while Pandiaraja et al. [15] developed formulations for generalized domination problems. Esakkimuthu et al. [16] studied strong restrained domination using al-gorithmic and LP-based techniques, and Sivakumar et al. [17] analyzed radial and antipodal domination numbers in trees. In addition, Ma et al. [18] proposed integer linear programming models for weighted total domination, and Cai et al. [19] developed ILP formulations for double Roman domination. These works illustrate the increasing role of mathematical programming in domination research.

Although mean absolute deviation has previously appeared in graph theory, particularly in the study of variability in degree sequences, its use in the context of domination parameters has not been explored. In contrast to earlier works that measure dispersion of vertex degrees, the present study considers the deviation of domination-degrees of vertices outside a dominating set. This perspective provides a way to evaluate how uniformly the domination responsibility is distributed across the graph and leads to the notion of deviation-minimal dominating sets.

Motivated by this observation, we introduce a deviation-based parameter called the *mean absolute deviation of domination*, denoted by MAD(D), which measures the irregularity in dom-ination relative to the average coverage level. This parameter complements existing equitable domination concepts and provides a refined perspective for applications in network balancing, load distribution, and monitoring systems.

The remainder of the paper is organized as follows. Section 2 presents the necessary pre-liminaries and notation. Section 3 introduces the deviation from mean domination parameter and establishes its fundamental properties. Section 4 develops computational models, discusses complexity aspects, and presents simulation results. Finally, Section 5 concludes the paper and outlines directions for future research.

Preliminaries

Let $G = (V, E)$ be a simple, finite, undirected graph with vertex set $V(G)$ and edge set $E(G)$. The order of G is $n = |V(G)|$ and its size is $m = |E(G)|$. For a vertex $v \in V(G)$ the open neighborhood of v is denoted by $N(v) = \{u \in V(G) : uv \in E(G)\}$ and the closed neighborhood by $N[v] = N(v) \cup \{v\}$. The degree of v in G is written as $\deg_G(v) = |N(v)|$.

A subset $D \subseteq V(G)$ is called a dominating set of G if every vertex in $V(G) \setminus D$ has at least

one neighbor in D. The minimum cardinality of a dominating set is the domination number $\gamma(G)$. For a given dominating set D and a vertex $v \in V(G) \setminus D$ we define the domination degree of v relative to D as

$d_D(v) = |N(v) \cap D|$,

that is, the number of vertices in Dadjacent to v.

The mean domination degree with respect to $D$is

$$\mu_D = \frac{1}{|V \setminus D|} \sum_{v \in V \setminus D} d_D(v).$$

The mean absolute deviation of domination degrees with respect to $D$is defined by

$$\mathrm{MAD}(D) = \frac{1}{|V \setminus D|} \sum_{v \in V \setminus D} |d_D(v) - \mu_D|.$$

Remark 2.1. If $D = V(G)$, then $V(G) \setminus D = \emptyset$, and the quantities μ_D and $\mathrm{MAD}(D)$ are not defined through the above formulas because the denominator $|V(G) \setminus D|$ is zero. In this trivial case, there are no non-dominated vertices, and we adopt the convention that

$$\mathrm{MAD}(D) = 0.$$

Throughout the theoretical results of this paper, we assume that

$$V(G) \setminus D \neq \emptyset,$$

unless stated otherwise.

A dominating set D is said to be a deviation-minimal dominating set if MAD(D) is minimum among all dominating sets of G. Thus, this notion emphasizes not only the covering ability of a dominating set but also the uniformity of domination it induces. Such sets are useful in situations where balance is as important as efficiency.

In what follows, we use the term *deviation-minimal dominating set* for theoretical discussions, while the equivalent term *minimum-MAD dominating set* is used in the computational context. The study of deviation-minimal dominating sets is best motivated by examining their be-havior in familiar classes of graphs. Establishing results for standard families such as complete graphs, stars, paths, cycles, bipartite graphs, wheels, and grids serves two purposes. First, these examples illustrate how the new parameter MAD(D) interacts with well-understood graph structures and confirm that deviation-minimal dominating sets naturally arise in many cases.

Second, they provide extremal or benchmark cases that guide the development of general theory in later sections. We therefore begin by presenting some basic results for classical graphs.

Proposition 2.1. If G is a complete graph K_n with $n \geq 2$, then every dominating set D with

$|D| = 1$ is a deviation-minimal dominating set and MAD(D) = 0.

Proof. Let $D = \{v\}$ for some $v \in V(K_n)$. Since K_n is complete, every vertex $u \in V \setminus D$ is adjacent to v, hence $d_D(u) = 1$ for all $u \in V \setminus D$. Therefore $\mu_D = 1$ and MAD(D) = 0. No dominating set can yield a smaller value, hence such sets are deviation-minimal. □

Proposition 2.2. If G is a star graph S_n with $n \geq 3$ and center c, then the dominating set {c}

is deviation-minimal with MAD({c}) = 0.

Proof. Let $D = \{c\}$. Each leaf $u \in V \setminus D$ is adjacent only to c, so $d_D(u) = 1$. Hence $\mu_D = 1$ and MAD(D) = 0. If a dominating set D contains more than one leaf and not the center, then some vertices in $V \setminus D$ have domination degree equal to 0 while others have degree at least 1, yielding MAD(D) > 0. Thus the set $\{c\}$ is deviation-minimal. □

Proposition 2.3. Let P_n be the path on $n \geq 3$ vertices. There exist dominating sets obtained by selecting every second vertex (an alternating selection) that satisfy MAD(D) = 0. These alternating sets have size [n/2| and therefore need not be minimum (since $\gamma(P_n)$ = [n/3|).

Proof. Label the vertices of P_n as $v_1, v_2, \ldots, v_n$ along the path. Let $D = \{v_2, v_4, v_6, \ldots\}$ (includ-ing v_n if n is even). Every vertex not in D has exactly one neighbour in D, hence $d_D(v) = 1$ for all $v \in V \setminus D$. Thus $\mu_D = 1$ and MAD(D) = 0.

Since $|D|$ = [$n/2$|, while $\gamma(P_n)$ = [$n/3$|, these sets are generally larger than minimum dominating sets. □

Proposition 2.4. *Let C_n be the cycle on $n \geq 3$ vertices. There exists a dominating set D with* $\mathrm{MAD}(D) = 0$ *(equivalently: every vertex of $V \setminus D$ has the same domination-degree) if and only if n is divisible by* 2 *or by* 3. *More precisely:*

If $2 \mid n$, the alternating set $D = \{v_1, v_3, \ldots, v_{n-1}\}$ is a dominating set and every vertex of $V \setminus D$ has domination-degree 2, *hence* $\mathrm{MAD}(D) = 0$.

If $3 \mid n$, the set $D = \{v_1, v_4, v_7, \ldots\}$ (every third vertex) is a dominating set and every vertex of $V \setminus D$ has domination-degree 1, *hence* $\mathrm{MAD}(D) = 0$.

If n is not divisible by 2 *or* 3, *then no dominating set D satisfies* $\mathrm{MAD}(D) = 0$.

Proof. Label the vertices of C_n cyclically as $v_1, v_2, \ldots, v_n$. For any dominating set D and any vertex $u \in V \setminus D$, the domination-degree $d_D(u)$ is either 1 or 2. It cannot be 0 since D dominates the graph, and it cannot exceed 2 in a cycle.

Thus if $\mathrm{MAD}(D) = 0$, all vertices of $V \setminus D$ must have the same domination-degree, which must be $r \in \{1, 2\}$. We consider the two possible cases.

Case $r = 2$**.** Every non-dominator has both neighbours in D. Hence no two vertices of $V \setminus D$ are adjacent, and D must consist of every other vertex around the cycle. This is possible exactly when n is even, taking $D = \{v_1, v_3, \ldots, v_{n-1}\}$. In this construction every $u \in V \setminus D$ has $d_D(u) = 2$, and therefore $\mathrm{MAD}(D) = 0$. If n is odd, such an alternating pattern is impossible, so no dominating set with $r = 2$ exists.

Case $r = 1$**.** Every non-dominator has exactly one neighbour in D. Consider the cyclic sequence of vertices and group them into consecutive blocks of non-dominators separated by dominators. Internal vertices of a block would have two non-dominator neighbours and hence domination-degree 0, which is impossible. Therefore block lengths cannot exceed 2.

A block of length 1 (a single non-dominator between two dominators) would have both neighbours in D and thus domination-degree 2, which contradicts $r = 1$. Hence each block of non-dominators must have length exactly 2. If there are k dominators, then there are k such blocks, so the total number of vertices is

$$n = k + 2k = 3k.$$

Thus $3 \mid n$.

Conversely, if $n = 3k$, then taking $D = \{v_1, v_4, v_7, \ldots, v_{3k-2}\}$ (every third vertex) yields exactly k blocks of two non-dominators each, and every non-dominator has exactly one neighbour in D. Hence $d_D(u) = 1$ for all $u \in V \setminus D$, and therefore $\mathrm{MAD}(D) = 0$.

Combining the two cases shows that a dominating set with $\mathrm{MAD}(D) = 0$ exists if and only if $2 \mid n$ or $3 \mid n$.

□

Remark 2.2. The structure described in the proof can be visualized along the cycle. If every non-dominator must have exactly one neighbour in the dominating set ($r = 1$), then a single non-dominator between two dominators would be adjacent to both of them and hence have domination-degree 2. On the other hand, three consecutive non-dominators would leave the middle vertex with domination-degree 0. Consequently, the only possible configuration is that dominators appear periodically every third vertex, producing blocks of exactly two consecutive non-dominators between them.

Proposition 2.5. *Let $G = K_{m,n}$ be the complete bipartite graph with partite sets X and Y, where $|X| = m$, $|Y| = n$, and $m, n \geq 1$. If a dominating set D contains all of X (respectively all of Y), then every vertex in $V \setminus D$ has the same domination degree, and hence* $\mathrm{MAD}(D) = 0$. *In particular, such sets are deviation-minimal.*

Proof. Assume $X \subseteq D$. Then every vertex $y \in Y = V \setminus D$ is adjacent to all m vertices of X, so $d_D(y) = m$ for every $y \in Y$. Thus $\bar{d}_D = m$, and each deviation $|d_D(y) - \bar{d}_D| = 0$, so $\mathrm{MAD}(D) = 0$. Since $\mathrm{MAD}(D) \geq 0$ always, achieving zero proves that D is deviation-minimal. The argument is symmetric if $Y \subseteq D$.

□

Theorem 2.1. *Let G be a k-regular bipartite graph with bipartition* (X, Y)*. If $D = X$, then every vertex $v \in V \setminus D$ satisfies $d_D(v) = k$, and hence* $\mathrm{MAD}(D) = 0$.

Proof. If $D = X$, then every vertex $y \in Y$ is adjacent to exactly k vertices in X. Thus $d_D(y) = k$ for all $y \in Y$. Hence the average domination degree is k, and therefore all deviations are zero. Consequently $\mathrm{MAD}(D) = 0$. $\square$

Remark 2.3. The conclusion of Theorem 2.1 does not generally extend to non-bipartite regular graphs. For instance, consider the cycle C_5, which is 2-regular and non-bipartite. Any domi-nating set D leaves vertices in $V \setminus D$ with different domination degrees (for example, 1 and 2), so $\mathrm{MAD}(D) > 0$. This phenomenon is consistent with the characterization for cycles established later in Proposition 2.4, which shows that a cycle admits a dominating set with $\mathrm{MAD}(D) = 0$ only when its order is divisible by 2 or 3.

Proposition 2.6. *For a wheel graph W_n $(n \geq 4)$, the dominating set consisting of the central vertex alone is deviation-minimal with* $\mathrm{MAD}(D) = 0$.

Proof. The wheel W_n is obtained from a cycle C_{n-1} by adding a central vertex c adjacent to all others. Let $D = \{c\}$. Then for each rim vertex $v \in V \setminus D$, we have $d_D(v) = 1$ since v is adjacent only to c from D. Hence $\mu_D = 1$ and $\mathrm{MAD}(D) = 0$. No dominating set can yield a smaller deviation, so $\{c\}$ is deviation-minimal. $\square$

Proposition 2.7. *Let $G = P_m \ P_n$ be the $m \times n$ grid with $m, n \geq 2$. The checkerboard partite class D is a dominating set, but in general the domination-degrees of vertices in $V \setminus D$ are not constant, so* $\mathrm{MAD}(D) > 0$ *except in trivial or special cases.*

Proof. Index vertices by coordinates (i, j). The grid is bipartite with parity $i + j$. Choose

$D = \{(i, j) : i + j \text{ even}\}$. This dominates all vertices.

However, vertices in $V \setminus D$ have varying degrees: interior vertices are adjacent to 4 dom-inators, boundary vertices to 2 or 3, etc. Thus the domination-degrees are not uniform, so

$\mathrm{MAD}(D) > 0$ in general. $\square$

Remark 2.4. If we take the toroidal grid $C_m \ C_n$ with both m, n even, then the graph is 4-regular and bipartite. Choosing one partite class as D makes every $v \in V \setminus D$ adjacent to 4 dominators, hence $\mathrm{MAD}(D) = 0$. This is the main regular exception.

Remark 2.5. The Cartesian toroidal grid $C_m \ C_n$ is 4-regular for every $m, n \geq 3$. Moreover, $C_m \ C_n$ is bipartite if and only if both m and n are even. When $m, n \geq 3$ are even, the graph is 4-regular and bipartite; choosing one entire partite class D then yields $d_D(v) = 4$ for every $v \in V \setminus D$, hence $\mathrm{MAD}(D) = 0$. If one of m, n is odd the product is not bipartite and the above checkerboard argument does not apply. Small or degenerate cases (e.g. very narrow grids or exceptional small values) should be checked separately.

Theorem 2.2. *The following families admit a dominating set D with* $\mathrm{MAD}(D) = 0$*:*

K_n, S_n, $K_{m,n}$, bipartite regular graphs, W_n,

even cycles C_{2k}, toroidal grids $C_m \ C_n$ with m, n even.

For paths P_n, alternating selections yield $\mathrm{MAD}(D) = 0$ *but these sets are not minimum. For odd cycles C_n and rectangular grids $P_m \ P_n$, no minimum dominating set achieves* $\mathrm{MAD}(D) = 0$.

It is important to highlight the distinction between deviation-minimal dominating sets and equitable dominating sets. An equitable dominating set requires that all domination degrees differ by at most one, a rigid structural condition that many graphs fail to satisfy. In contrast, a deviation-minimal dominating set minimizes the overall deviation from uniformity, even if perfect equitability is not attainable. For instance, in a cycle C_n or a grid $P_m \ P_n$, deviation-minimal dominating sets achieve $\mathrm{MAD}(D) = 0$, which coincides with equitable domination. However, in graphs such as stars S_n or wheels W_n, no equitable dominating set exists because the center and leaves (or rim) necessarily have domination degrees differing by more than one, yet a deviation-minimal dominating set still exists with $\mathrm{MAD}(D) = 0$, reflecting perfect balance among the dominated vertices. Thus, our notion extends the scope of equitable domination by capturing fairness in domination where equitable sets do not exist, providing a more flexible and widely applicable tool for analyzing balance in networks.

General Properties and Bounds

Throughout this section we assume that $V(G) \setminus D \neq \emptyset$, so that μ_D and $MAD(D)$ are well defined.

In this section we present some general observations, inequalities, and bounds for deviation-minimal dominating sets. The aim is to place this concept within the broader framework of domination theory and to illustrate how $\mathrm{MAD}(D)$ interacts with classical graph parameters such as the order, minimum degree, and maximum degree.

Lemma 3.1. *For any graph G, we have* $\mathrm{MAD}(D) = 0$ *if and only if $d_D(v)$ is constant for all $v \in V \setminus D$.*

Proof. If $d_D(v)$ is constant for all $v \in V \setminus D$, then μ_D equals this common value, and each term

$|d_D(v) - \mu_D|$ vanishes, so $\mathrm{MAD}(D) = 0$. Conversely, if $\mathrm{MAD}(D) = 0$, then each $|d_D(v) - \mu_D| = 0$, which implies $d_D(v) = \mu_D$ for all $v \in V \setminus D$. Hence $d_D(v)$ is constant. □

Theorem 3.1. *Not every regular graph admits a deviation-minimal dominating set D with* $\mathrm{MAD}(D) = 0$. *However, every bipartite k-regular graph admits such a set, namely either partite class.*

Proof. Let G be k-regular. In general, it is not guaranteed that we can choose a dominating set so that every $v \in V \setminus D$ has the same number of dominators. However, if G is bipartite and k-regular, then either partite set is a dominating set, and every vertex outside the set has domination degree exactly k. Thus $\mu_D = k$ and $\mathrm{MAD}(D) = 0$. This shows the claim for bipartite regular graphs and provides a counterexample to the more general statement.

Proposition 3.1. *For any graph G of order n and dominating set D, we have* □

$$0 \leq \mathrm{MAD}(D) \leq \frac{|D| - 1}{2}.$$

Proof. For each $v \in V \setminus D$, the domination degree $d_D(v)$ is an integer between 1 and $|D|$. Hence the spread of possible values of $d_D(v)$ is at most $|D| - 1$. The mean absolute deviation cannot exceed half of this spread, so $\mathrm{MAD}(D) \leq (|D| - 1)/2$. The lower bound is immediate. □

Theorem 3.2. *If G has a unique minimum dominating set D, then D is deviation-minimal if and only if for every other dominating set D',* $\mathrm{MAD}(D) \leq \mathrm{MAD}(D')$.

Proof. By definition, deviation-minimal dominating sets minimize $\mathrm{MAD}(D)$ among all dom-inating sets. If G has a unique minimum dominating set D, then the only candidates for deviation-minimality are D itself and dominating sets of larger size. If $\mathrm{MAD}(D) \leq \mathrm{MAD}(D')$ for all dominating sets D', then D is deviation-minimal. Conversely, if D is deviation-minimal,

then it must have $\mathrm{MAD}(D) \leq \mathrm{MAD}(D')$ for all D', otherwise there exists a dominating set with smaller deviation, contradicting minimality. □

Proposition 3.2. *For any graph G, there exists a deviation-minimal dominating set D satisfying*

$$\mathrm{MAD}(D) \leq \frac{n - 1}{2}.$$

Proof. For each $v \in V \setminus D$, the domination degree $d_D(v)$ is an integer between 1 and $|D|$. Thus the maximum possible difference between $d_D(v)$ and the mean μ_D is at most $(|D| - 1)/2$. Since

$|D| \leq n - 1$, the bound follows. □

Example 3.1. Consider the star graph S_n with center c and leaves $l_1, \ldots, l_{n-1}$. The dominating set $D = \{c\}$ yields $d_D(l_i) = 1$ for each i, so $\mu_D = 1$ and $\mathrm{MAD}(D) = 0$. Thus D is deviation-minimal. This agrees with Proposition 2.2 from Section 2 and also shows that stars belong to the family of graphs for which deviation-minimal dominating sets coincide with minimum dominating sets.

Example 3.2. Consider the cycle C_6 with vertex set $V(C_6) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ arranged cyclically.

First take the set $D = \{v_1, v_3, v_5\}$. Then the vertices outside D are $\{v_2, v_4, v_6\}$. Each of these vertices is adjacent to two vertices of D, so $d_D(v) = 2$ for all $v \in V \setminus D$. Hence the domination degrees are uniform and therefore $\mathrm{MAD}(D) = 0$. Thus C_6 has deviation-minimal dominating sets of size 3.

Now consider a minimum dominating set $D^r = \{v_1, v_3\}$ of size 2. The vertices outside D^r are

$\{v_2, v_4, v_5\}$. Their domination degrees are

$$dD'(v_2) = 2, \quad dD'(v_4) = 1, \quad dD'(v_5) = 0.$$

Hence the mean domination degree is

$$\mu D' = \frac{2 + 1 + 0}{3} = 4/3.$$

Therefore,

$$\mathrm{MAD}(D') = \frac{1}{3}\left(|2 - 4/3| + |1 - 4/3| + |0 - 4/3|\right) = 1/3.$$

Thus $\gamma(C_6) = 2$, but the minimum dominating sets yield $\mathrm{MAD}(D') = 1/3$. Therefore deviation-minimal dominating sets need not coincide with minimum dominating sets.

Example 3.3. For the path $P_5 = v_1v_2v_3v_4v_5$, let $D = \{v_2, v_4\}$. Then $dD(v_1) = 1$, $dD(v_3) = 2$, $dD(v_5) = 1$. The mean domination degree is

$$\mu D = \frac{1 + 2 + 1}{3} = 4/3,$$

and the mean absolute deviation is

$$\mathrm{MAD}(D) = \frac{1}{3}\left(|1 - 4/3| + |2 - 4/3| + |1 - 4/3|\right) = 2/9.$$

Thus $\mathrm{MAD}(D) > 0$, showing that not all natural dominating sets in paths achieve perfect uniformity. This contrasts with longer paths, where alternating selections produce $\mathrm{MAD}(D) = 0$.

Corollary 3.1. *If G is a tree in which every leaf is adjacent to a unique support vertex of degree at least two, then G admits a deviation-minimal dominating set D with* $\mathrm{MAD}(D) = 0$.

Proof. Let D be the set of all support vertices. Each leaf is dominated exactly once, so $d_D(v) = 1$ for every leaf v. Every non-leaf vertex not in D is adjacent to at least one support vertex and hence also receives equal domination. Therefore $\mu_D = 1$ and $\mathrm{MAD}(D) = 0$. $\square$

Corollary 3.2. *If a graph G admits an equitable dominating set D, then D is necessarily deviation-minimal.*

Proof. Suppose D is an equitable dominating set of G. By definition, the domination degrees of vertices in $V(G) \setminus D$ differ by at most one. Therefore there exists an integer k such that

$$d_D(v) \in \{k,\ k + 1\} \qquad \text{for all } v \in V(G) \setminus D.$$

Hence the mean domination degree μ_D satisfies

$k \le \mu_D \le k + 1.$

Consequently, the deviation of each domination degree from the mean is bounded:

$|d_D(v) - \mu_D| \le 1.$

Thus the domination degrees are distributed as uniformly as possible. Any dominating set that is not equitable would have domination degrees differing by more than one, resulting in a larger spread and therefore a larger mean absolute deviation. Hence an equitable dominating set minimizes $MAD(D)$ and is therefore deviation-minimal. □

Corollary 3.3. *If $\gamma(G) = 1$, then G admits a deviation-minimal dominating set D with*

$\mathrm{MAD}(D) = 0.$

Proof. If $\gamma(G) = 1$, then G has a universal vertex u. Taking $D = \{u\}$ yields $d_D(v) = 1$ for all $v \in V \setminus D$, so $\mathrm{MAD}(D) = 0.$ □

Proposition 3.3. *Let G be a graph of order n and maximum degree $\Delta(G)$, and let D be a dominating set such that $V(G) \setminus D \models \emptyset$. Then*

$1 \le \mu_D \le \Delta(G).$

Proof. Let $v \in V \setminus D$. Since D is a dominating set, every such vertex has at least one neighbour in D, so $d_D(v) \ge 1$.

Moreover, the domination degree of v cannot exceed the total degree of v, so

$d_D(v) \le \deg(v).$

Since $\deg(v) \le \Delta(G)$ for every vertex of G, it follows that

$d_D(v) \le \Delta(G).$

Thus every domination degree satisfies

$1 \le d_D(v) \le \Delta(G).$

Since μ_D is the average of the values $d_D(v)$ over all $v \in V \setminus D$, it follows that

$1 \le \mu_D \le \Delta(G).$

□

Corollary 3.4. *If G is δ-regular, then for any deviation-minimal dominating set D we have $\mu_D = \delta$ and* $\mathrm{MAD}(D) = 0.$

Further Properties and Bounds

We next establish inequalities relating deviation-minimal dominating sets to classical graph invariants.

Theorem 3.3. *Let $G = (V, E)$ be a finite graph and let $D \subseteq V$ be a dominating set. Assume $V \setminus D \models \emptyset$. Then*

$0 \le \mathrm{MAD(D)} \le (|\mathrm{D}| - 1) / 2$

Proof. The lower bound follows immediately since $MAD(D)$is an average of nonnegative quantities.

For the upper bound, note that for every $v \in V \setminus D$, we have

$$1 \le d_D(v) \le |D|.$$

Thus, the domination degrees of vertices in $V \setminus D$lie in the interval $[1, |D|]$, whose length is

$$|D| - 1.$$

Let $m = \mid V \setminus D \mid$ and write the domination degrees as $x_1, x_2, \ldots, x_m \in \{1, 2, \ldots, \mid D \mid\}$ with mean

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x_i .$$

For any collection of real numbers contained in an interval of length L, the mean absolute deviation from their mean is at most $L/2$. Since here $L = \mid D \mid -1$, it follows that

$$MAD(D) = \frac{1}{m}\sum_{i=1}^{m} \mid x_i - \mu \mid \leq \frac{\mid D \mid -1}{2}.$$

Therefore,

$$0 \leq MAD(D) \leq \frac{\mid D \mid -1}{2},$$

as claimed.

Theorem 3.4. Let G be a graph of order n and maximum degree $\Delta(G)$. Then every dominating set D of G satisfies

$$\mid D \mid \geq \frac{n}{\Delta(G) + 1}.$$

In particular, this bound applies to any deviation-minimal dominating set.

Proof. Each vertex $v \in D$, together with its closed neighborhood $N[v]$, can cover at most $\Delta(G) + 1$ distinct vertices of V. Since the closed neighborhoods of the vertices in a dominating set D cover the entire vertex set V, we have therefore

$$\mid \boldsymbol{D} \mid \cdot (\boldsymbol{\Delta}(\boldsymbol{G}) + \mathbf{1}) \geq \boldsymbol{n}.$$

and the inequality follows.

□

Proposition 3.4. *Let G be a graph of order n and let $D \subseteq V(G)$ be a dominating set with $V \setminus D \neq \emptyset$. Then the mean domination degree satisfies*

$1 \leq \mu_D \leq \Delta(G)$,

where $\Delta(G)$ denotes the maximum degree of G.

Proof. For each $v \in V \setminus D$ we have $d_D(v) = |N(v) \cap D| \geq 1$ because D dominates v, and also $d_D(v) \leq \deg_G(v) \leq \Delta(G)$. Averaging these inequalities over all $v \in V \setminus D$ yields

$1 \leq \mu_D \leq \Delta(G)$.

The lower bound is strict only when $V \setminus D \neq \emptyset$; if $V \setminus D = \emptyset$ (so $D = V$) then μ_D is an average over an empty set and should be treated according to the convention adopted in the paper.

Corollary 3.5. *If G is a δ-regular bipartite graph with bipartition (X, Y), then taking $D = X$ or $D = Y$ yields a deviation-minimal dominating set with $\mu_D = \delta$ and* $\mathrm{MAD}(D) = 0$.

Proof. Suppose G is a δ-regular bipartite graph with bipartition (X, Y). If $D = X$, then every vertex $y \in Y = V \setminus D$ has exactly δ neighbors in X. Hence $d_D(y) = \delta$ for all $y \in Y$, and therefore $\mu_D = \delta$ and $\mathrm{MAD}(D) = 0$. The same argument holds symmetrically if $D = Y$. Since $\mathrm{MAD}(D) \geq 0$ for any dominating set, such sets D are deviation-minimal.

Computational Formulation and Simulation □

The use of linear and integer linear programming has become an important methodology in the study of domination parameters in graphs. Such formulations allow classical domination numbers and their generalizations to be computed exactly, while also providing a framework for designing algorithms and deriving bounds. Recent research demonstrates the effectiveness of this approach for various domination-related parameters. For example, Duraisamy and Esakkimuthu

[14] developed linear programming models for different domination parameters, while Pandi-araja, Shanmugam, and Sivakumar [15] provided formulations for generalized domination num-bers. Algorithmic approaches have also been used to handle special domination types such as strong restrained domination on trees and product graphs [16], and studies continue to extend this framework to newer parameters, including radial and antipodal domination [17]. From a broader perspective, integer linear programming has also been applied to weighted domination problems [18] and specialized variants like double Roman domination [19]. These works illustrate that linear programming is not only a theoretical tool but also a practical method for computing domination parameters, motivating its use in our formulation of deviation-minimal dominating sets. The problem of identifying a deviation-minimal dominating set can be approached through a two-stage mixed integer linear programming model. In the first stage, we determine the dom-ination number of the graph. In the second stage, we search among all dominating sets of minimum cardinality for the one that minimizes the deviation MAD(D).

This formulation highlights the dual advantage of the mixed integer linear programming ap-proach: it not only ensures the exact computation of the classical domination number but also provides a systematic way to incorporate additional balance-oriented criteria such as deviation. By integrating these objectives, the method advances beyond traditional domination studies and opens up new directions for analyzing fairness in graph-theoretic coverings. Moreover, the adaptability of MILP techniques makes this approach extendable to other generalized domina-tion parameters, thereby offering a unifying framework that is both theoretically rigorous and computationally practical.

The computational complexity of finding a minimum-MAD dominating set is closely related to the classical domination problem. It is well known that determining a minimum dominating set in a general graph is NP-hard. Since the computation of deviation-minimal dominating sets requires identifying dominating sets and optimizing an additional objective based on domination degrees, the minimum-MAD domination problem is also NP-hard in general. Similarly, the equitable domination problem is known to be computationally difficult. Consequently, exact methods such as mixed integer linear programming (MILP) provide a practical framework for solving these problems on graphs of moderate size.

Stage 1: Domination Number

We formulate the computation of $\gamma(G)$ as the following binary integer programming model:

(P1)

$$\min \sum_{v\in V} x_v$$

$$\text{s.t.} \quad \sum_{u\in N[v]} x_u \geq 1, \qquad \forall v \in V, \qquad (1)$$

$x_v \in \{0,1\}$, $\forall v \in V$. where $x_v = 1$ if $v \in D$ and 0 otherwise. The objective minimizes the size of the dominating set, and the constraints ensure that every vertex is dominated. The optimal objective value of (P1) is the domination number $\gamma(G)$.

Stage 2: Minimization of Deviation MAD(D)

Given $\gamma(G)$ from Stage 1, we formulate Stage 2 as follows. For each $v \in V$, let

$y_v = 1 - x_v$, so that $y_v = 1$ if $v \in V \setminus D$,

d_v = domination degree of v, defined as $d_v = \sum_{u \in N(v)} x_u$,

z_v = auxiliary variable representing $|d_v - \mu|$ when $y_v = 1$, and 0 otherwise,

$w_{v,u}$ = linearization variable for $y_v \cdot x_u$, ensuring feasibility of mean computation.

$$\text{(P2)} \qquad \min \sum_{v \in V} z_v \qquad (2)$$

$$\text{s.t.} \quad \sum_{u \in N[v]} x_u \geq 1, \qquad \forall v \in V, \qquad (3)$$

$$\sum_{v \in V} x_v = \gamma(G), \qquad (4)$$

$$x_v + y_v = 1, \qquad \forall v \in V, \qquad (5)$$

$$d_v = \sum_{u \in N(v)} x_u, \qquad \forall v \in V, \qquad (6)$$

$$w_{v,u} \leq x_u, \; w_{v,u} \leq y_v, \; w_{v,u} \geq x_u + y_v - 1, \qquad \forall v \in V, \; u \in N(v), \; (7)$$

$$(n - \gamma(G))\mu = \sum_{v \in V} \sum_{u \in N(v)} w_{v,u}, \qquad \text{if } n - \gamma(G) > 0, (8)$$

$z_v \geq d_v - \mu - M(1 - y_v)$,	$\forall v \in V$,	(9)
$z_v \geq \mu - d_v - M(1 - y_v)$,	$\forall v \in V$,	(10)
$x_v, y_v, w_{v,u} \in \{0, 1\}$,		(11)
$d_v \in \{0, \ldots, \gamma(G)\}$,		(12)
$z_v, \mu \geq 0$.		(13)

The objective minimizes the sum of deviations, which is proportional to MAD(D) since $|V \setminus D| = n - \gamma(G)$ is constant. The constraints guarantee that D is a dominating set of size $\gamma(G)$ and ensure a correct linearization of the mean domination degree. This yields a pure MILP formulation capable of exactly computing deviation-minimal dominating sets.

It is important to note that this two-stage approach provides both structural and balance-oriented insights into dominating sets. While Stage 1 guarantees the minimality of the dom-inating set size, Stage 2 ensures fairness in terms of domination distribution among the non-dominated vertices. This balance is captured through the deviation measure MAD(D), which quantifies how evenly the domination responsibilities are spread. By integrating both objectives into a sequential optimization framework, the method not only identifies an optimal dominat-ing set but also refines it to achieve improved uniformity, making the model suitable for both theoretical analysis and practical applications where balanced coverage is essential

Algorithm 1 Two-stage computation of deviation-minimal dominating set

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1: **Input:** Graph $G = (V, E)$ with adjacency list adj

Output: Domination number $\gamma(G)$, deviation-minimal dominating set $D_{\min}$, deviation $\mathrm{MAD}(D_{\min})$

Stage 1: Compute $\gamma(G)$

Define binary variables $x_v \in \{0,1\}$ for all $v \in V$

Objective: minimize $\sum_{v\in V} x_v$

$\sum_{u\in N[v]} x_u \geq 1$ for all $v \in$

Solve MILP; let $\gamma(G)$ = optimal value

Stage 2: Minimize deviation $\mathrm{MAD}(D)$

Define variables $x_v, y_v, d_v, z_v, \mu, w_{v,u}$

Objective: minimize $\sum_{v\in V} z_v$

Constraints:

(a) Domination: $\sum_{u\in N[v]} x_u \geq 1$ for all v

(b) $\sum_{v\in V} x_v =$

(c) Partition: $x_v + y_v = 1$ for all v

(d) Domination degrees: $d_v = \sum_{u\in N(v)} x_u$ for all v

(e) Linearization: $w_{v,u} \leq x_u$, $w_{v,u} \leq y_v$, $w_{v,u} \geq x_u + y_v - 1$

(f) Mean: $(n-\gamma(G))\mu = \sum_{v\in V} \sum_{u\in N(v)} w_{v,u}$

(g) Deviation: $z_v \geq d_v - \mu - M(1-y_v)$ and $z_v \geq \mu - d_v - M(1-y_v)$

Solve MILP; output $D_{\min} = \{v \in V : x_v = 1\}$ and $\mathrm{MAD}(D_{\min}) = \frac{1}{n-\gamma(G)} \sum_{v\in V} z_v$.

Equitable Domination: MILP Formulation

We now present an integer linear programming model for computing the equitable domination number $\gamma_{eq}(G)$.

(P3)	$\min \sum_{v\in V} x_v$		(14)
s.t.	$\sum_{u\in N[v]} x_u \geq 1,$	$\forall v \in V,$	(15)
	$x_v + y_v = 1,$	$\forall v \in V,$	(16)
	$d_v = \sum_{u\in N(v)} x_u,$	$\forall v \in V,$	(17)
	$d_v \geq d_{\min} - M(1-y_v),$	$\forall v \in V,$	(18)
	$d_v \leq d_{\max} + M(1-y_v),$	$\forall v \in V,$	(19)
	$d_{\max} - d_{\min} \leq 1,$		(20)

$x_v, y_v \in \{0, 1\}$,	$\forall v \in V$,	(21)
$d_v \in \{0, 1, \ldots, \|V\|\}$,	$\forall v \in V$,	(22)
$d_{\min}, d_{\max} \in \{0, 1, \ldots, \|V\|\}$.		(23)

The objective minimizes the size of the dominating set. The first constraint ensures that D is a dominating set. The variables y_v identify non-dominating vertices, while d_v represents the domination degree of vertex v. The variables $d_{\min}$ and $d_{\max}$ capture the minimum and maximum domination degrees among vertices in $V \setminus D$. The constraint $d_{\max} - d_{\min} \leq 1$ enforces the equitable condition.

The constant M denotes a sufficiently large upper bound used in the big-M constraints that activate the bounds on d_v only when $y_v = 1$ (that is, when $v \in V \setminus D$). Since the domination degree of any vertex cannot exceed its total degree and is therefore bounded by $|V|$, a safe choice is $M = |V|$. In the computational implementation we adopt this value.

Remark 4.1. If $\gamma(G) = n$, then the dominating set is $D = V$ and there are no vertices outside

D. In this trivial case the mean domination degree μ is undefined (division by zero). By convention we may set $\mathrm{MAD}(D) = 0$, and Stage 2 should be skipped. In implementation, the mean constraint

$$(n-\gamma(G))\mu = \sum_{v\in V} \sum_{u\in N(v)} w_{v,u}$$

is only enforced when $n-\gamma(G) > 0$. Note that the variable μ is declared as a continuous variable in the MILP formulation, since it represents the mean domination degree of vertices in $V \setminus D$, which may in general be non-integer.

Algorithm 2 MILP for Equitable Domination

1: **Input:** Graph $G = (V, E)$ with adjacency list adj

2: **Output:** Equitable domination number $\gamma_{\mathrm{eq}}(G)$, equitable dominating set D_{eq}

3: Define binary variables $x_v \in \{0, 1\}$ for all $v \in V$ (1 if $v \in D$)

4: Define binary variables $y_v \in \{0, 1\}$ for all $v \in V$ (1 if $v \notin D$) 5: Define integer variables $d_v \geq 0$ for all $v \in V$(domination degree) 6: Define integer variables $d_{\min}, d_{\max}$ (minimum and maximum domination degrees among

non-dominators)

7: Objective: minimize $\sum_{v\in V} x_v$ (size of dominating set)

8: Constraints:

9: (a) Domination: $\sum_{u\in N[v]} x_u \geq 1$ for all $v \in V$

10: (b) Partition: $x_v + y_v = 1$ for all $v \in V$

11: (c) Domination degrees: $d_v = \sum_{u\in N(v)} x_u$ for all $v \in V$

12: (d) Activation of min/max:

13: $d_v \geq d_{\min} - M(1-y_v)$, $d_v \leq d_{\max} + M(1-y_v)$ for all $v \in V$

14: (e) Equitability: $d_{\max} - d_{\min} \leq 1$

15: Solve MILP; output $D_{eq} = \{v \in V : x_v = 1\}$ and $\gamma_{\mathrm{eq}}(G) = |D_{eq}|$

The complexity of exact algorithms for domination-related parameters grows rapidly with the size of the graph, making theoretical analysis alone insufficient to understand their behavior on larger instances. To complement the theoretical framework, we conducted a comprehensive simulation study. The primary objective was to investigate how the proposed deviation measure MAD behaves across different graph families and to compare it with the equitable domination number γ_{eq}. Since closed-form analysis is often infeasible for large graphs, simulation provides an effective way to observe patterns, trends, and computational feasibility. Furthermore, the exper-iments allow us to study how the two-stage MILP approach for minimizing MAD and the direct MILP model for computing γ_{eq} perform in terms of both solution quality and computational runtime.

For the simulation, we considered multiple families of graphs commonly used in structural and applied graph theory: Erdős–Rényi random graphs (ER), random geometric graphs (RGG), grid graphs, and random spanning trees. These graph models capture a wide spectrum of struc-tural diversity ranging from uniform randomness (ER) to spatially embedded networks (RGG) and regular topologies (grids). For each graph type, experiments were carried out for different orders of graphs, with the number of vertices ranging from n = 10 to n = 100. To accountfor variability, we repeated each experiment for multiple independent trials, typically with ten repetitions for each parameter setting. The values reported in the figures correspond to the mean over these trials, and the error bars represent one standard deviation of the measured quantities across the trials. In each trial, both the minimum MAD-dominating set (computed using a two-stage MILP formulation) and the equitable dominating set (computed via an alter-native MILP model) were obtained, subject to predefined time limits (30 seconds for Stage 1, 60 seconds for Stage 2, and 120 seconds for γ_{eq}). These settings ensured that the experiments remained computationally tractable while providing meaningful statistical comparisons across different graph types and sizes.

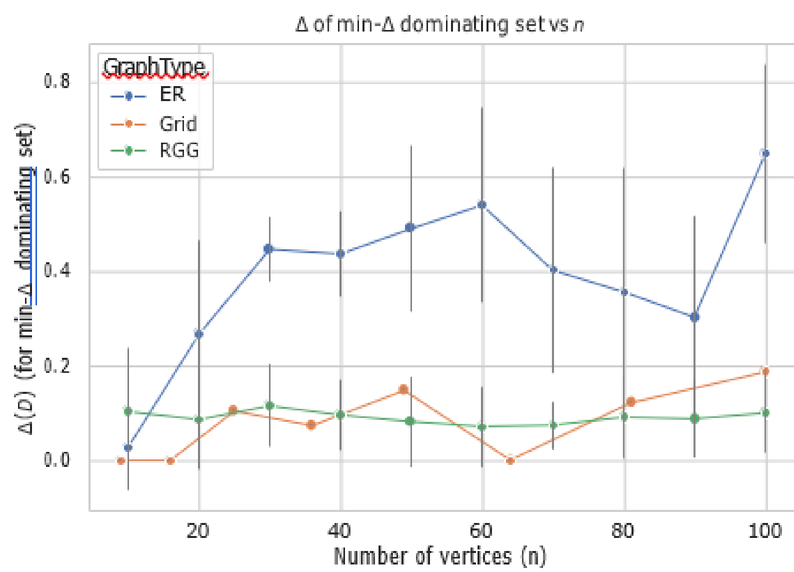

Figure 1: Variation of MAD(D) for minimum-MAD dominating sets across different graph families as the number of vertices increases. The values represent the mean over 10 independent trials, and the error bars indicate one standard deviation.

Figure 1 presents the variation of MAD(D) for minimum-MAD dominating sets as the number of vertices increases across three graph families: Erdős–Rényi (ER), grid graphs, and random geometric graphs (RGG). A clear distinction is observed between ER graphs and the other two families. For ER graphs, MAD(D) grows steadily with n, reaching values above 0.6 at n = 100, and exhibits considerable variability as indicated by the error bars. This suggests that the non-dominators in ER graphs experience significant imbalance in coverage by dominators, which intensifies as the graph size increases. By contrast, both grid and RGG graphs maintain com-paratively low values of MAD(D), generally below 0.2, with relatively small variability across trials. This indicates that spatially structured or regular networks provide more uniform domi-nation, whereas purely random connections in ER graphs yield larger deviations in domination balance.

Figure 2 compares the domination number γ obtained from the minimum-MAD sets with the equitable domination number γ_{eq} across different graph sizes. For small instances, both parameters remain close, typically in the range of 3 to 6, indicating that equitable solutions can be obtained without significant increase in set size. However, as the graph size grows, disparities emerge, with γ_{eq} occasionally requiring larger sets than γ, particularly in graphs with $n = 70$ and $n = 80$, where γ_{eq} exceeds 30 in some trials while γ remains closer to 20. The large error bars in these cases emphasize the variability of γ_{eq} across trials, suggesting sensitivity of equitable domination to graph structure. This highlights that enforcing equitable coverage often comes at the cost of larger dominating sets, especially in irregular graph families.

Figure 3 illustrates the computational runtime for solving the equitable domination MILP model as a function of graph size, plotted on a logarithmic scale. The results reveal a sharp increase in runtime for ER graphs, which quickly rise from below 0.1 seconds at $n = 20$ to over

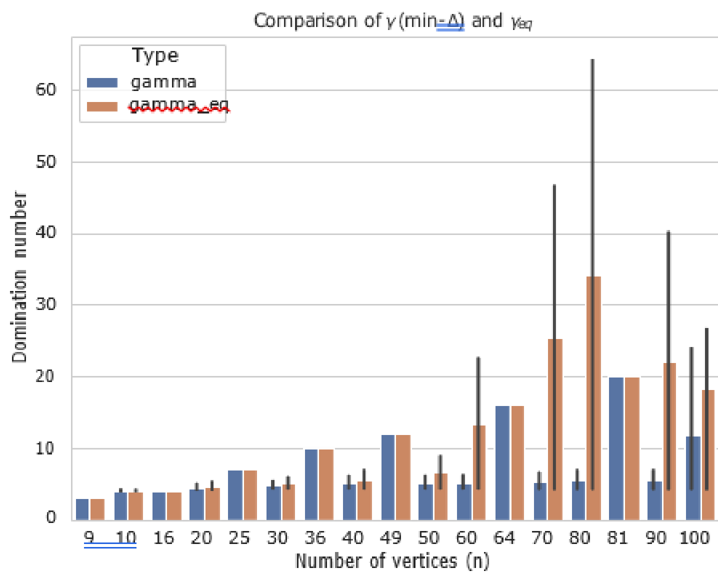

Figure 2: Comparison of domination number γ and equitable domination number γ_{eq} for varying graph sizes.

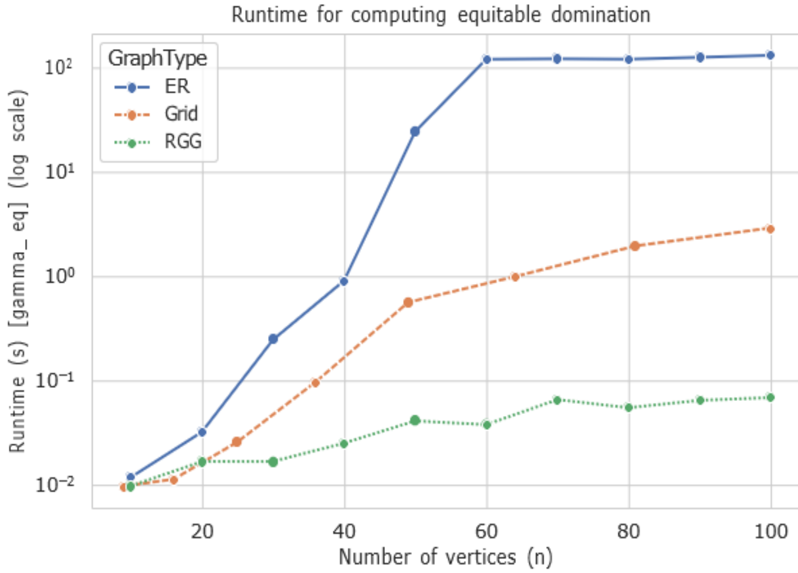

Figure 3: Runtime analysis of equitable domination MILP model as graph size increases (log scale).

100 seconds at n = 70 and beyond, where the solver frequently hits the imposed time limit. Grid graphs exhibit moderate growth in runtime, reaching only a few seconds at n = 100, while RGG graphs remain computationally the most tractable, with runtimes consistently under one second even for the largest tested sizes. This indicates that the structure of the graph plays a crucial role in solver performance: dense and irregular adjacency patterns (ER) lead to computational bottlenecks, whereas spatial and regular graphs (RGG, grids) allow the solver to scale efficiently. Figure 4 reports the runtime for computing minimum-MAD dominating sets using the two-stage MILP approach. Similar to the equitable domination case, ER graphs demand significantly more computational effort than structured graphs, with runtimes approaching 100 seconds for n = 100. Grid graphs also show a steep increase, but remain slightly below ER graphs at larger

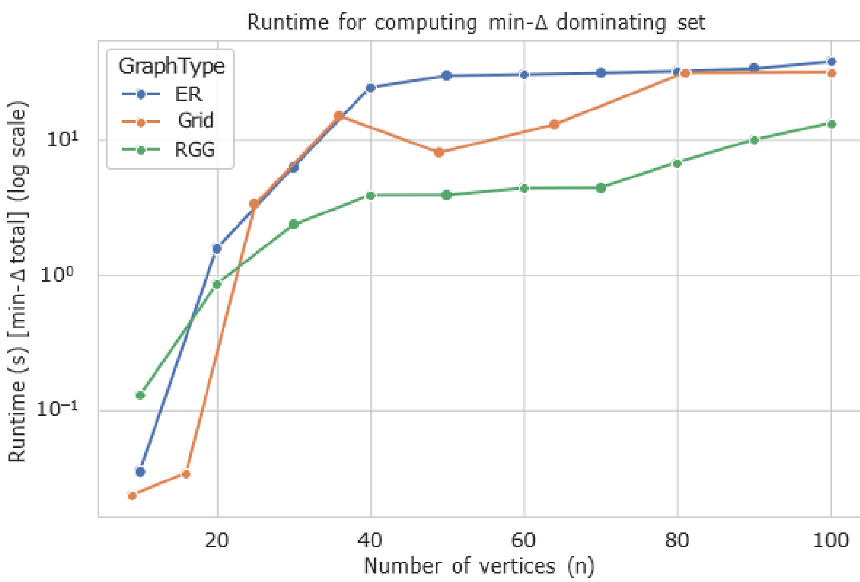

Figure 4: Runtime analysis of two-stage MILP for computing minimum-MAD dominating sets (log scale).

sizes, while RGG graphs demonstrate the best scalability, with runtimes around 10 seconds at $n = 100$. Interestingly, the curves for grid and ER graphs converge beyond $n = 60$, suggesting that the solver faces comparable challenges in these two families despite their different structures. Overall, the results emphasize that although the two-stage MILP approach is more stable than the equitable MILP, it still scales poorly for random and dense graphs, requiring careful time-limit management.

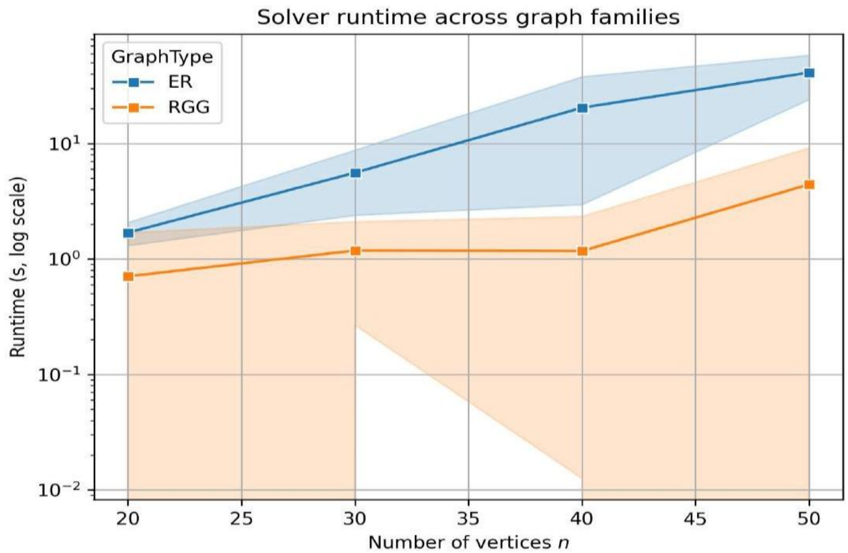

Figure 5: Consolidated runtime comparison of ER and RGG graphs for medium-sized instances (n = 20 to 50).

Figure 5 consolidates solver runtimes across ER and RGG graphs for medium-sized instances ($n = 20$ to 50). The log-scale representation highlights the pronounced difference in solver dif-ficulty between the two families. ER graphs exhibit exponential-like growth in runtime, with median values rising from approximately 2 seconds at $n = 20$ to nearly 30 seconds at $n = 50$, ac-companied by wide variability across trials. In contrast, RGG graphs remain significantly faster to solve, with runtimes rarely exceeding 5 seconds even at $n = 50$, and with much narrower vari-ability. This comparative analysis confirms that spatial constraints in RGG graphs lead to more predictable domination structures, whereas the unpredictability of ER connectivity amplifies solver complexity. Such differences underline the importance of graph topology in determining not only the structural outcomes of domination but also the feasibility of computation.

Taken together, Figures 1–5 provide a comprehensive picture of the behavior of domination parameters under different graph families and computational settings. The results consistently demonstrate that graph structure strongly influences both the balance of domination (as mea-sured by MAD) and the efficiency of MILP-based computation. Erdős–Rényi graphs, with their irregular edge distribution, tend to produce higher deviations MAD, larger equitable domination numbers γ_{eq}, and substantially longer runtimes, often approaching solver time limits. In con-trast, random geometric graphs display the most stable and tractable patterns, with low MAD, nearly coincident values of γ and γ_{eq}, and runtimes well within acceptable bounds. Grid graphs occupy an intermediate position: while they offer relatively balanced domination, their regular structure still poses computational challenges as graph size increases. Overall, the simulations reveal that while equitable domination can lead to significantly larger dominating sets in certain graph classes, especially ER graphs, spatial or structured families enable both better balance and faster solvability. These findings highlight the practical importance of graph topology in assessing the feasibility and performance of domination-based optimization models.

2. Conclusion

In this work, we introduced a deviation-based framework for analyzing domination in graphs, complementing the classical domination number and equitable domination number. By em-ploying mixed integer linear programming (MILP) models, we developed methods to identify dominating sets that minimize deviation and to compute equitable domination numbers subject to fairness constraints. Through extensive simulations on Erdős–Rényi, random geometric, grid, and tree graphs, we demonstrated that graph structure plays a decisive role in both the balance of domination and the computational tractability of the models. Erdős–Rényi graphs showed high deviation and frequent solver timeouts, while random geometric graphs consistently yielded balanced domination and efficient computation. Grid and tree graphs exhibited intermediate characteristics, highlighting structural diversity.

The concept of Mean Absolute Deviation (MAD) domination introduced in this work has potential applications in several real-world network scenarios. In communication and computer networks, MAD dominating sets can be used to identify optimal nodes for monitoring, control, or information dissemination while maintaining balanced structural properties of the network. The approach may also be useful in transportation and social network analysis, where selecting influential or controlling vertices with minimal deviation can improve efficiency and stability. Furthermore, the study of MAD domination may contribute to the design of fault-tolerant systems and resource allocation strategies in large-scale interconnected systems.

The study underscores that enforcing equitable domination often comes with increased dom-inating set size, especially in irregular graphs, while the deviation measure MAD provides a finer tool for quantifying fairness in domination. From a computational perspective, the results show that MILP formulations are viable for moderate graph sizes and structured families, but require time-limit strategies for dense and irregular graphs. Future research directions include developing approximation algorithms for MAD-domination, exploring parameterized complexity, and extending the deviation framework to weighted and directed graphs. Overall, this work lays the foundation for a deeper understanding of balanced domination and opens new avenues for applications where fairness in resource coverage is essential.

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