

Decision-Support Framework for Type-2 Fuzzy Multi-Choice Transportation Models

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Abstract: The Multi-Choice Transportation Problem (MCTP) is an important extension of the classical transportation problem in which multiple transportation cost alternatives are available between each source and destination. In practical transportation systems, transportation costs, supplies and demands are often uncertain and cannot be represented accurately by crisp values. To address this issue, this study presents a Type-2 fuzzy framework for solving the MCTP under uncertain environments. The transportation costs, supplies and demands are represented as multi-choice interval Type-2 fuzzy numbers to capture the higher-order uncertainty. The Karnik-Mendel (KM) type-reduction algorithm is implemented to convert the Type-2 fuzzy parameters into the precise values using the defuzzification. The resulting crisp transportation problems are solved using Vogel's Approximation Method (VAM) to obtain an initial basic feasible solution and apply the Modified Distribution (MODI) method to determine the optimal transportation plan. Further, the obtained optimal solutions are verified using the LINGO software. Numerical computations involving four transportation cost choices to demonstrates the working of the presented framework. The computational results show that the proposed approach efficiently handles uncertainty while producing reliable and optimal transportation plans. Furthermore, the comparative analysis identifies the transportation choice corresponding to the minimum transportation cost, thereby providing an effective decision-support framework for transportation planning under uncertainty. ..

Keywords: Multi-Choice Transportation Problem, Type-2 Fuzzy Sets, Karnik–Mendel Algorithm, Defuzzification, Optimization, LINGO..

1. Introduction

Numerous fields, including logistics, supply chain management, production scheduling, distribution systems and the network optimization, use the different kind of transportation models. It is essential to decision-making processes concerning resource allocation, cost reduction and operational effectiveness. Many extensions of the classical transportation problem have been developed because of its practical significance. Hitchcock (1941) was the first to formulate the linear programming problem of the classical TP. Later, Zadeh (1965) proposed the idea of a fuzzy set, which offers a potent mathematical foundation for simulating the ambiguity and uncertainty present in many real-world issues. In the transport models such as transportation cost, time, supply, demand etc. may not be crisp in the day-to-day life. So, the researchers addressed uncertain problem by including the fuzzy parameters, stochastic factors and interval-valued data in response to the increasing complexity of real-world systems. This has improved the model's realism and ability to adapt to uncertain settings. The transportation problem is now much more applicable in contemporary decision-support systems thanks to these developments. As fuzzy set theory developed over time, Type-1 and Type-2 FS - each with varying degrees of uncertainty handling capability - were created.



Significant progress has been made in solving TPs under uncertain environments, several research gaps remain (Table 1). Most existing studies focus either on T1FS or T2FS transportation models independently, while only limited attention has been devoted to integrating **MCTP** with **interval Type-2 fuzzy representations**. Furthermore, many available approaches employ ranking functions or simplified defuzzification techniques that may not adequately capture the uncertainty associated with T2F parameters. In addition, comparative investigations involving multiple transportation cost choices under a unified T2F framework are relatively scarce. These limitations motivate the development of a systematic methodology capable of handling multiple transportation alternatives while preserving the uncertainty inherent in transportation costs, supplies, and demands.

This study suggests a T2FS framework for solving the MCTP in order to overcome these difficulties. The TP costs, supplies and demands are represented using IT2F numbers, and the KM type-reduction algorithm is employed to obtain crisp equivalents through centroid-based defuzzification. The resulting crisp transportation problems are solved using VAM to generate an initial basic feasible solution and the MODI method to determine the optimal transportation plan. The obtained optimal solutions are further verified using LINGO software.

This study's primary contributions are as follows:

A modified mathematical formulation of the MCTP under an IT2F environment is developed.

A systematic solution methodology integrating KM defuzzification, VAM, MODI and LINGO verification is presented for solving MCTP.

A comparative analysis of different transportation cost choices is performed to identify the minimum-cost transportation plan under uncertainty.

The remainder of this document is structured as follows: The fundamental ideas of T1FS and T2FS as well as the KM defuzzification technique are presented in Section 2. Section 3 develops the mathematical formulations of the classical, T1FS and T2FS MCTP. Section 4 describes the proposed solution methodology. Section 5 presents the numerical computations, while Section 6 discusses the computational results. The work is finally concluded in Section 7, which also suggests potential future research directions.

List of Abbreviations

Notations	Full Form
T1FS	Type-1 Fuzzy Set
T2FS	Type-2 Fuzzy Set
IT2FS	Interval Type-2 Fuzzy Set
MF	Membership Function
TP	Transportation Problem
MCTP	Multi-choice Transportation Problem
VAM	Vogel's Approximation Method
KM	Karnik–Mendel
MCMOTP	Multi-choice Multi-objective Transportation Problem

Table-1: Analysis of recent studies related to multi-choice TP

Authors	TP Model	Uncertainty environment	Type-2 Fuzzy	Multi-choice
Nasseri & Bavandi (2020)	Multi-commodity TP	Fuzzy random hybrid uncertainty	-	√
Gupta (2021)	Bilevel TP	Type-1 fuzzy programming	-	√

Mondal et al. (2023)	Multi-item Step Fixed-Charge STP	Intuitionistic fuzzy	-	√
El Sayed & Baky (2023)	Fractional Multi-objective TP	Stochastic	-	√
Jain & Dhodiya (2024)	Five-Dimensional TP	Type-1 fuzzy	-	√
Kumar et al. (2024)	Transshipment Problem	Intuitionistic Type-2 fuzzy	√	-
Sifaoui & Aïder (2024)	Sustainable STP	Stochastic	-	√
Bavandi et al. (2025)	Maritime Transportation System	Hybrid fuzzy stochastic	-	√
Biswas et al. (2026)	Stochastic TP	Probability distributions	-	√
Proposed	Multi-Choice TP	Interval Type-2 Fuzzy	√	√

2. Review of literature

Hitchcock (1941) presented the mathematical formulation of the classical TP using linear programming to establish the foundation for transportation optimization. Zadeh (1965) introduced the concept of FS which provides a mathematical framework for modelling uncertainty and imprecision in optimization problems. Zadeh (1973) proposed a new framework for analysing complex systems and decision-making processes under uncertainty through fuzzy logic. Zadeh (1975) introduced the concept of linguistic variables and approximate reasoning, significantly extending the applications of fuzzy logic in intelligent decision-making. Karnik and Mendel (1998) introduced T2F logic systems to model higher levels of uncertainty that cannot be effectively represented using conventional T1FS. Mendel and Liu (2007) developed the KM iterative algorithm for efficiently computing the centroid of IT2FS. Castillo et al. (2007) presented the theoretical foundations and practical applications of T2F logic systems in intelligent control and optimization. Castillo (2010) demonstrated the effectiveness of interval T2F logic in control applications by improving system robustness under uncertain environments.

Maity and Roy (2014) proposed a utility function approach for solving MCMOTP under uncertain environments. Chakraborty et al. (2015) developed a solution methodology for MOMC multi-item TP using Atanassov's intuitionistic FS and chance operators. Maity and Roy (2016) investigated a MOTP with nonlinear transportation costs and MC demand under a fuzzy environment. Roy et al. (2017) proposed a conic scalarization approach for solving MCMOTP with interval goals. Gupta et al. (2018) investigated MCMO capacitated TP under uncertain supply and demand through a practical case study. Ranarahu et al. (2018) developed a computational framework for solving MCMO fuzzy probabilistic TP. Nasser and Bavandi (2020) proposed a MC linear programming model under fuzzy-random hybrid uncertainty and demonstrated its application to multi-commodity TP. Nasser and Bavandi (2020) further presented a fuzzy programming approach for solving MOMC stochastic TP. Gupta (2021) developed a fuzzy programming methodology for solving MC bilevel TP. Mondal et al. (2023) proposed an intuitionistic fuzzy approach for sustainable MO multi-item MC step fixed-charge solid TP. El Sayed and Baky (2023) investigated fractional stochastic multi-objective TP within a multi-choice optimization framework. Jain and Dhodiya (2024) proposed variants of genetic algorithms for solving fuzzy MCMO multi-item five-dimensional TP. Kumar et al. (2024) introduced a dynamic ranking function for optimizing transshipment problems under intuitionistic T2F and T1F fuzzy environments. Sifaoui and Aïder (2024) investigated sustainable MC stochastic MO multi-item solid TP under uncertain environments. Bavandi et al. (2025) developed a hybrid fuzzy-stochastic MC framework for maritime transportation by simultaneously considering travel time and reliability. Biswas et al. (2026) proposed a soft computing-based optimization framework for MC stochastic TP involving exponential and logistic probability distributions.

Basic Concept

2.1 Type-1 Fuzzy Sets

Let X be the universal set under consideration. A FS $\tilde{A}$ on X is characterized by a MF $\mu_{\tilde{A}}: X \rightarrow [0,1]$ which associates each member $\mu_{\tilde{A}}(z)$ with a membership grade ranging from 0 to 1. Consequently, the FS is represented as a collection of order pair, where each pair contains an element and its corresponding MF value, describing the grade to which the element belongs to $z \in \tilde{A}$.

2.2 Interval-valued Type-1 fuzzy set

In a universal set X , an interval T1FS, symbolized as $\tilde{\tilde{A}}$, an ordered pair collection defines $\tilde{\tilde{A}}$. An element z from X makes up each pair and its matching interval-valued MF in $\tilde{\tilde{A}}$, stated by $\mu_{\tilde{\tilde{A}}}(z) = [\underline{\mu}_{\tilde{\tilde{A}}}(z), \bar{\mu}_{\tilde{\tilde{A}}}(z)]$, where $0 \leq \underline{\mu}_{\tilde{\tilde{A}}}(z) \leq \bar{\mu}_{\tilde{\tilde{A}}}(z) \leq 1$. Thus, the MF is defined as

$$\mu_{\tilde{\tilde{A}}}: X \rightarrow \mathcal{I}([0,1]),$$

where $\mathcal{I}([0,1])$ symbolizes the set of all closed subintervals of $[0,1]$. The interval-valued MF represents the range of possible MF degrees of each element $z \in X$, thereby capturing the uncertainty associated with assigning a precise membership value.

Equivalently, the interval T1FS can be represented as

$$\tilde{\tilde{A}} = \left\{ \left(z, [\underline{\mu}_{\tilde{\tilde{A}}}(z), \bar{\mu}_{\tilde{\tilde{A}}}(z)] \right) \mid z \in X \right\}$$

2.3. Type-2 fuzzy set

In a universal set X , a T2FS, symbolized as $\tilde{\tilde{\tilde{A}}}$, is defined by a set of ordered pairs in which each element z from X is associated with a fuzzy MF grade. Unlike a T1FS, where the MF degree is a crisp value in $[0,1]$, the MF degree of a T2FS is itself a fuzzy set in $[0,1]$. It is expressed by the T2FS MF $\mu_{\tilde{\tilde{\tilde{A}}}}(z, u)$, where $z \in X$ and $u \in J_z \subseteq [0,1]$. Thus, the MF is defined as

$$\mu_{\tilde{\tilde{\tilde{A}}}}: X \times [0,1] \rightarrow [0,1]$$

Here, u represents the primary MF of z , and $\mu_{\tilde{\tilde{\tilde{A}}}}(z, u)$ represents the secondary membership grade corresponding to u . Therefore, T2FS is represented as

$$\tilde{\tilde{\tilde{A}}} = \left\{ \left((z, u), \mu_{\tilde{\tilde{\tilde{A}}}}(z, u) \right) \mid z \in X, u \in J_z \subseteq [0,1] \right\}$$

Equivalently, it may be written as:

$$\tilde{\tilde{\tilde{A}}} = \int_{z \in X} \int_{u \in J_z} \frac{\mu_{\tilde{\tilde{\tilde{A}}}}(z, u)}{(z, u)}$$

where J_z denotes the primary MF domain of z , and the secondary MF $\mu_{\tilde{\tilde{\tilde{A}}}}(z, u)$ describes the uncertainty associated with the primary membership value.

2.4 Interval-valued Type-2 fuzzy set

In a universal set X , an interval-valued T2FS, symbolized as $\tilde{\tilde{\tilde{\tilde{A}}}}$, an ordered pair collection defines $\tilde{\tilde{\tilde{\tilde{A}}}}$, in which each element z from X is associated with an interval-valued secondary MF. Unlike a general T2FS, where the secondary membership grades may take any value in $[0,1]$, the secondary MF grades of an interval-valued T2FS are all equal to unity. Consequently, the uncertainty is represented solely by the interval of primary membership values. The T2FS MF is expressed as

$$\mu_{\tilde{\tilde{\tilde{\tilde{A}}}}}(z, u) = 1, \forall u \in J_z \subseteq [0,1],$$

where $J_z = [\underline{\mu}_{\tilde{A}}(z), \bar{\mu}_{\tilde{A}}(z)]$ denotes the interval of primary membership values corresponding to the element z . Thus, the MF is defined as

$$\mu_{\tilde{A}}: X \times J_z \rightarrow \{1\}$$

Accordingly, an interval-valued T2FS is represented as

$$\tilde{A} = \{(z, u), 1 \mid z \in X, u \in J_z\},$$

where $J_z = [\underline{\mu}_{\tilde{A}}(z), \bar{\mu}_{\tilde{A}}(z)]$ is referred to as the primary membership interval, and the collection of all such intervals forms the Footprint of Uncertainty (FOU), given by

$$\text{FOU}(\tilde{A}) = \bigcup_{z \in X} J_z = \bigcup_{z \in X} [\underline{\mu}_{\tilde{A}}(z), \bar{\mu}_{\tilde{A}}(z)]$$

2.5 Karnik–Mendel (KM) Defuzzification Method

According to Mendel and Liu, defuzzification of an interval T2FS is performed through type-reduction first. The centroid of an IT2FS is an interval: $C_{\tilde{A}} = [c_l, c_r]$.

here c_l, c_r are obtained using the Karnik–Mendel iterative algorithm. For a discrete interval T2FS, the left endpoint is:

$$c_l = \frac{\sum_{i=1}^k x_i \bar{\mu}(x_i) + \sum_{i=k+1}^N x_i \underline{\mu}(x_i)}{\sum_{i=1}^k \bar{\mu}(x_i) + \sum_{i=k+1}^N \underline{\mu}(x_i)}$$

and the right endpoint is:

$$c_r = \frac{\sum_{i=1}^k x_i \underline{\mu}(x_i) + \sum_{i=k+1}^N x_i \bar{\mu}(x_i)}{\sum_{i=1}^k \underline{\mu}(x_i) + \sum_{i=k+1}^N \bar{\mu}(x_i)}$$

After obtaining c_l and c_r , the final crisp value is: $C = \frac{c_l + c_r}{2}$

Figure 1 depicts the working of KM method.

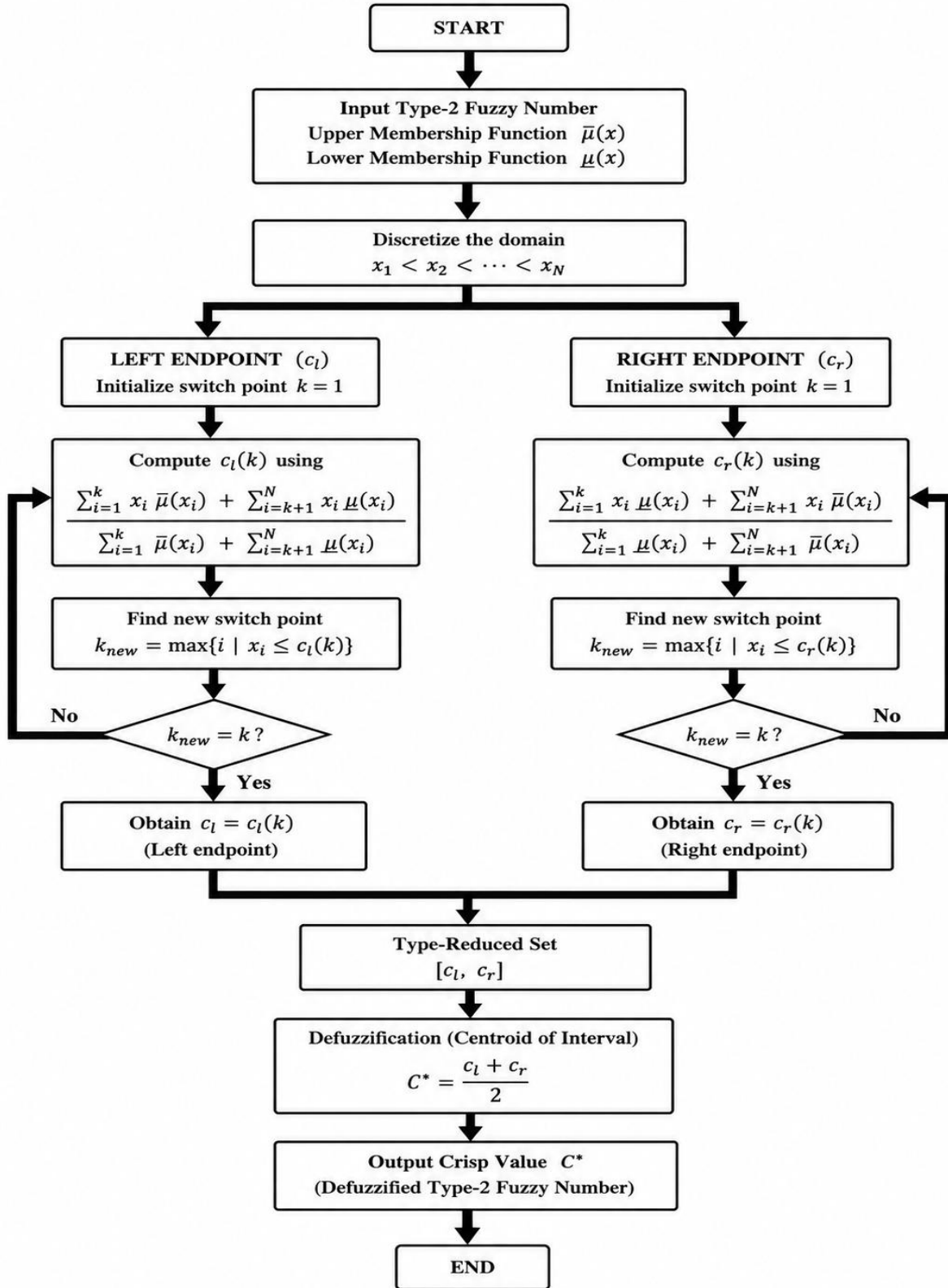


Figure 1: Flowchart of the KM-Defuzzification Method

3. Mathematical Formulations

3.1 Classical Multichoice Transportation Problem

The classical MCTP can be characterized through the following mathematical model:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n (C_{ij}^1 x_{ij} \text{ or } C_{ij}^2 x_{ij} \text{ or } \dots \text{ or } C_{ij}^p x_{ij}) \quad (1)$$

s.t.

$$\sum_{j=1}^n x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \dots \text{ or } a_i^r) \quad \text{for } i = 1, 2, \dots, m \quad (2)$$

$$\sum_{j=1}^n x_{ij} \geq (b_i^1 \text{ or } b_i^2 \text{ or } \dots \text{ or } b_i^s) \quad \text{for } i = 1, 2, \dots, m \quad (3)$$

$$x_{ij} \geq 0 \quad \forall i \& j$$

Here, $(c_{ij}^1, c_{ij}^2, \dots, c_{ij}^p)$ represent the possible crisp TP costs associated with the feasible routes from origin i to destination j ($i \leftrightarrow j$). Similarly, $(a_i^1, a_i^2, \dots, a_i^r)$ denote the alternative crisp supply capacities available at origin i , whereas $(b_j^1, b_j^2, \dots, b_j^s)$ represent the possible crisp demand values corresponding to destination j . The decision variable x_{ij} denotes the total crisp quantity of things transported from $(i \leftrightarrow j)$.

3.2 Type-1 Fuzzy Multichoice Transportation Problem

The Type-1 Fuzzy MCTP can be represented through the following mathematical model:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n (C_{ij}^1 x_{ij} \text{ or } C_{ij}^2 x_{ij} \text{ or } \dots \text{ or } C_{ij}^p x_{ij}) \quad (4)$$

Subject to constraints

$$\sum_{j=1}^n x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \dots \text{ or } a_i^r) \quad \text{for } i = 1, 2, \dots, m \quad (5)$$

$$\sum_{j=1}^n x_{ij} \geq (b_i^1 \text{ or } b_i^2 \text{ or } \dots \text{ or } b_i^s) \quad \text{for } i = 1, 2, \dots, m \quad (6)$$

$$x_{ij} \geq 0 \quad \forall i \& j$$

Here, $(c_{ij}^1, c_{ij}^2, \dots, c_{ij}^p)$ represent the possible T1FS TP costs associated with the feasible routes from $(i \leftrightarrow j)$. Similarly, $(a_i^1, a_i^2, \dots, a_i^r)$ denote the alternative T1FS supply capacities available at origin i , whereas $(b_j^1, b_j^2, \dots, b_j^s)$ represent the possible T1FS demand values corresponding to destination j . The decision variable x_{ij} denotes the total crisp quantity of goods transported from $(i \leftrightarrow j)$.

3.3 Type-2 Fuzzy Multichoice Transportation Problem

The Type-2 Fuzzy MCTP can be represented through the following mathematical model:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n (C_{ij}^1 x_{ij} \text{ or } C_{ij}^2 x_{ij} \text{ or } \dots \text{ or } C_{ij}^p x_{ij}) \quad (7)$$

Subject to constraints

$$\sum_{j=1}^n x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \dots \text{ or } a_i^r) \quad \text{for } i = 1, 2, \dots, m \quad (8)$$

$$\sum_{j=1}^n x_{ij} \geq (b_i^1 \text{ or } b_i^2 \text{ or } \dots \text{ or } b_i^s) \quad \text{for } i = 1, 2, \dots, m \quad (9)$$

$$x_{ij} \geq 0 \quad \forall i \& j$$

Here, $(c_{ij}^1, c_{ij}^2, \dots, c_{ij}^p)$ represent the possible T2FS TP costs associated with the feasible routes from $(i \leftrightarrow j)$. Similarly, $(a_i^1, a_i^2, \dots, a_i^r)$ denote the alternative T2FS supply capacities available at origin i , whereas $(b_j^1, b_j^2, \dots, b_j^s)$ represent the possible T2FS demand values corresponding to destination j . The decision variable x_{ij} denotes the total crisp quantity of goods transported from $(i \leftrightarrow j)$.

Methodology

The following steps comprise the suggested solution methodology:

Step 1: Formulate the TP with m origins and n destinations. Represents the TP costs, supplies and demands as multi-choice T2FS numbers. Let $\tilde{c}_{ij}^{(k)}$ denote the k^{th} transportation cost alternative between source S_i and destination D_j , where $k = 1, 2, \dots, r$. Similarly, represent the supplies and demands as T2F numbers.

Step 2: Select one of the available transportation cost choices from the multi-choice environment for further analysis. Repeat the entire procedure for every available choice to determine the best transportation plan.

Step 3: Represent each selected transportation cost, supply and demand as T2F numbers. Assign appropriate secondary membership grades based on expert opinion, historical observations or uncertainty levels associated with the transportation network.

Step 4: Apply the KM iterative algorithm to every T2F transportation cost, supply and demand to obtain the left and right centroid endpoints, c_l and c_r . The T2FS number is therefore converted into the interval $[c_l, c_r]$.

Step 5: Obtain the crisp value of every TP cost, supply and demand using $C^* = \frac{c_l + c_r}{2}$. Construct the corresponding crisp transportation table.

Step 6: Solve the crisp TP by first obtaining an IBFS using VAM. Subsequently, apply the MODI method to determine the optimal transportation plan. Finally, verify the obtained optimal solution using LINGO software.

Step 7: Repeat Steps 2-6 for each available transportation cost choice in the multi-choice environment. Compare the optimal transportation costs obtained for all choices and select the transportation plan corresponding to the lowest total TP cost as the optimal solution.

Figure 2 depicts the working of presented methodology.

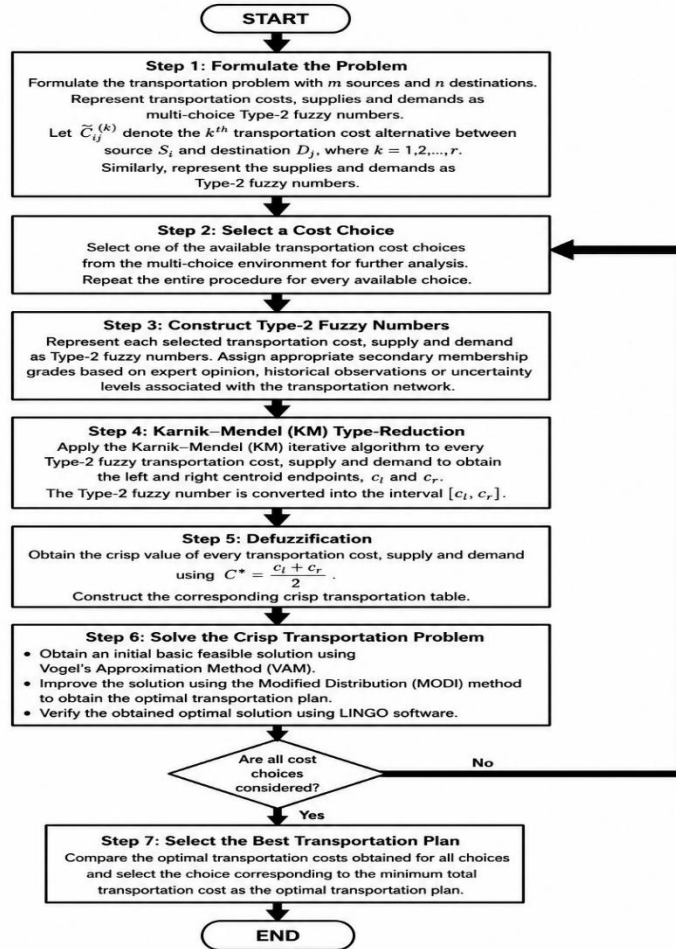


Figure 2: Flowchart of the methodology

Numerical Computations

Consider a transportation system with two sources (S_1, S_2) and the three destinations (D_1, D_2, D_3). The transportation costs are represented by multi-choice IT2F numbers (Table 2).

Table 2: MCTP (Choice 1)

	D_1	D_2	D_3
S_1	([2,4], [3,5])	([3,6], [4,7])	([1,3], [2,4])
S_2	([4,8], [5,9])	([2,5], [3,6])	([3,7], [4,8])

Type-2 Fuzzy Supplies: $\tilde{a}_1 = \{[30,40], [35,45]\}$, $\tilde{a}_2 = \{[20,30], [25,35]\}$

Type-2 Fuzzy Demands: $\tilde{b}_1 = \{[15,25], [20,30]\}$, $\tilde{b}_2 = \{[20,30], [25,35]\}$, $\tilde{b}_3 = \{[15,25], [20,30]\}$.

The transportation model is

$$\begin{aligned} \min \tilde{Z} &= \sum_{i=1}^2 \sum_{j=1}^3 \tilde{c}_{ij} x_{ij} \\ \text{s.t.} \quad \sum_{j=1}^3 x_{1j} &= \tilde{a}_1, \quad \sum_{j=1}^3 x_{2j} = \tilde{a}_2 \\ \sum_{i=1}^2 x_{i1} &= \tilde{b}_1, \quad \sum_{i=1}^2 x_{i2} = \tilde{b}_2, \quad \sum_{i=1}^2 x_{i3} = \tilde{b}_3 \\ x_{ij} &\geq 0, \quad \forall i = 1, 2, j = 1, 2, 3. \end{aligned}$$

$$x_{ij} \geq 0$$

Construct Interval Type-2 Membership Grades

Assume the secondary membership grades are

Interval	Secondary Grade
Lower interval	0.6
Upper interval	1.0

Example, $C_{11} = \{([2,4], 0.6), ([3,5], 1.0)\}$

Karnik–Mendel Type Reduction

For C_{11} , values: $x_1 = 3$ (midpoint of [2,4]), $x_2 = 4$ (midpoint of [3,5])

Membership grades: $\mu = 0.6$, $\bar{\mu} = 1.0$

Left Endpoint: $c_l = \frac{3(1.0)+4(0.6)}{1.0+0.6} = \frac{3+2.4}{1.6} = 3.375$

Right Endpoint: $c_r = \frac{3(0.6)+4(1.0)}{0.6+1.0} = \frac{1.8+4}{1.6} = 3.625$

Therefore: $C_{11}^{KM} = \frac{c_l + c_r}{2} = \frac{3.375 + 3.625}{2} = 3.50$

By applying the same KM type-reduction and defuzzification procedure to all T2F transportation costs, supplies and demands, the original T2F TP was converted into an equivalent crisp transportation problem. KM-Defuzzified Crisp Transportation Problem for Choice 1 is presented in Table 3.

Table 3: Defuzzified MCTP (Choice 1)

	D_1	D_2	D_3	Supply
S_1	3.50	5.00	2.50	37.50
S_2	6.50	4.00	5.50	27.50
Demand	22.50	27.50	15.00	65.00

$VAM=226.25$, $MODI=226.25$, $Lingo: 226.25$

Second Choice: MCTP (Table 4)

Table 4: Defuzzified MCTP (Choice 2)

	D_1	D_2	D_3
S_1	{[3,5], [4,6]}	{[4,7], [5,8]}	{[2,4], [3,5]}
S_2	{[5,9], [6,10]}	{[3,6], [4,7]}	{[4,8], [5,9]}

Type-2 Fuzzy Supplies: $\tilde{a}_1 = \{[32,42], [37,47]\}$, $\tilde{a}_2 = \{[18,28], [23,33]\}$

Type-2 Fuzzy Demands: $\tilde{b}_1 = \{[17,27], [22,32]\}$, $\tilde{b}_2 = \{[18,28], [23,33]\}$, $\tilde{b}_3 = \{[15,25], [20,30]\}$
KM-Defuzzified Crisp Transportation Problem for Choice 2 is presented in Table 5.

Table 5: Defuzzified MCTP (Choice 2)

	D_1	D_2	D_3	Supply
S_1	4.50	6.00	3.50	39.50
S_2	7.50	5.00	6.50	25.50
Dummy	0	0	0	7.50
Demand	24.50	25.50	22.50	72.50

$VAM=301.50$, $MODI=282.75$, $Lingo: 282.75$

Third Choice: MCTP (Table 6)

Table 6: Defuzzified MCTP (Choice 2)

	D_1	D_2	D_3
S_1	{[4,6], [5,7]}	{[5,8], [6,9]}	{[2,5], [3,6]}
S_2	{[6,10], [7,11]}	{[4,7], [5,8]}	{[5,9], [6,10]}

Type-2 Fuzzy Supplies: $\tilde{a}_1 = \{[34,44], [39,49]\}$, $\tilde{a}_2 = \{[21,31], [26,36]\}$

Type-2 Fuzzy Demands: $\tilde{b}_1 = \{[18,28], [23,33]\}$, $\tilde{b}_2 = \{[21,31], [26,36]\}$, $\tilde{b}_3 = \{[16,26], [21,31]\}$

KM-Defuzzified Crisp Transportation Problem for Choice 3 is presented in Table 7.

Table 7: Defuzzified MCTP (Choice 3)

	D_1	D_2	D_3	Supply
S_1	5.50	7.00	4.00	41.50
S_2	8.50	6.00	7.50	28.50
Dummy	0	0	0	7.50
Demand	25.50	28.50	23.50	77.50

$VAM=382.75$, $MODI=364$, *Lingo*: 364

Fourth Choice: MCTP (Table 8)

Table 8: Defuzzified MCTP (Choice 4)

	D_1	D_2	D_3
S_1	{[5,7], [6,8]}	{[6,9], [7,10]}	{[3,6], [4,7]}
S_2	{[7,11], [8,12]}	{[5,8], [6,9]}	{[6,10], [7,11]}

Type-2 Fuzzy Supplies: $\tilde{a}_1 = \{[36,46], [41,51]\}$, $\tilde{a}_2 = \{[22,32], [27,37]\}$

Type-2 Fuzzy Demands: $\tilde{b}_1 = \{[20,30], [25,35]\}$, $\tilde{b}_2 = \{[22,32], [27,37]\}$, $\tilde{b}_3 = \{[16,26], [21,31]\}$

KM-Defuzzified Crisp Transportation Problem for Choice 4 is presented in Table 9.

Table 9: Defuzzified MCTP (Choice 4)

	D_1	D_2	D_3	Supply
S_1	6.50	8.00	5.00	43.50
S_2	9.50	7.00	8.50	29.50
Dummy	0.00	0.00	0.00	7.50
Demand	27.50	29.50	23.50	80.50

$VAM=476.50$, $MODI=454$, *Lingo*: 454

4. Results and Discussion

To demonstrate the efficiency of the proposed methodology, four transportation cost choices of the multi-choice Type-2 fuzzy TP were considered. Each choice was represented using IT2F costs, supplies and demands. The KM type-reduction algorithm was first employed to convert the Type-2 fuzzy parameters into crisp values. The resulting crisp TP were subsequently solved using VAM to obtain an IBFS, followed by the MODI method to determine the optimal transportation plan. Further, the optimal solutions obtained through MODI are verified using the LINGO software. The TP costs corresponding to all four choices are summarized in Table 10, while the comparative presentation is illustrated in Figure 3.

From the Table 10, it is observed that for “Choice 1”, the transportation cost obtained by VAM, MODI and LINGO is 226.25. Since the VAM solution already satisfies the optimality conditions, the MODI method does not produce any further improvement and the identical result obtained by LINGO confirms the correctness of the solution.

For “Choice 2”, the initial transportation cost obtained using VAM is 301.50, whereas the MODI method reduces the transportation cost to 282.75, representing an improvement of 18.75 units (6.22%). The same optimal value is obtained using LINGO to validate the MODI solution.

For “Choice 3”, the initial VAM solution yields a transportation cost of 382.75, which is reduced to 364.00 after applying the MODI method. This corresponds to a reduction of 18.75 units (4.90%). Once again, the optimal solution obtained through MODI exactly matches the LINGO solution.

For “Choice 4”, the transportation cost decreases from 476.50 obtained by VAM to 454.00 after MODI optimization, resulting in an improvement of 22.50 units (4.72%). The identical value reported by LINGO further confirms the optimality of the proposed optimization procedure.

The computational results validate that the KM-based Type-2 fuzzy framework transforms the uncertain MCTP into an equivalent crisp transportation model without losing the uncertainty information. The VAM algorithm provides a good initial feasible solution and the MODI method consistently improves the transportation plan whenever the initial solution is non-optimal. Further, the MODI and LINGO solutions for all four transportation cost choices confirms the correctness and reliability of the proposed solution methodology.

Table 10: Comparative Transportation Costs corresponding to different choices

	VAM	MODI	LINGO
Choice 1	226.25	226.25	226.25
Choice 2	301.50	282.75	282.75
Choice 3	382.75	364.00	364.00
Choice 4	476.50	454.00	454.00

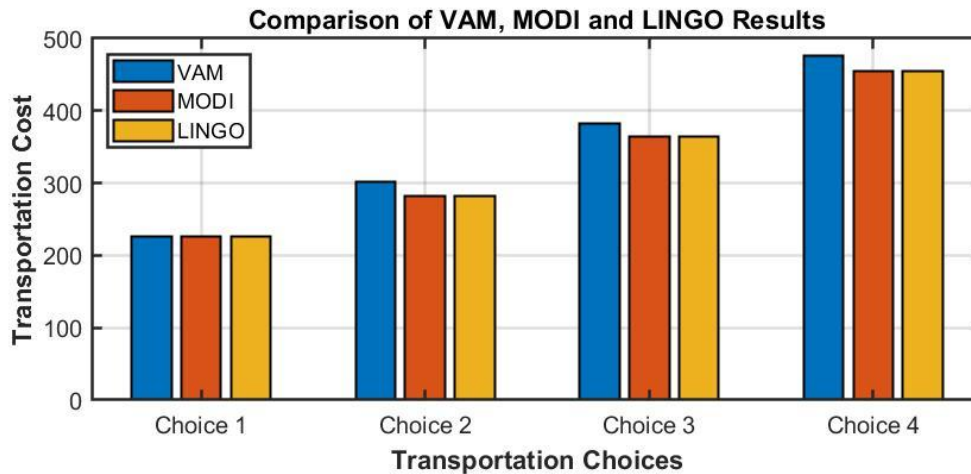


Figure 3: Comparison of the cost corresponding to different choices of TP

5. Conclusion

A framework to optimize the multi-choice TP in a T2FS environment proposed in this study. The presented method represents transportation costs, supplies and demands as multi-choice IT2FS numbers. The KM type-reduction algorithm is employed to convert the T2FS parameters into the crisp values. The defuzzified TP model then solved using the VAM to obtain an IBFS and the MODI method to determine the optimal transportation plan. The optimal solutions obtained through MODI were further verified using LINGO software.

Numerical computations involving four transportation cost choices are considered to validate the efficiency of the proposed methodology. The computational results showed that the KM defuzzification procedure efficiently transformed the uncertain transportation model into the crisp TP model while preserving the uncertainty information embedded in the T2FS parameters. The comparative analysis showed that the MODI method consistently produced

transportation costs that were either equal to or lower than those obtained using VAM. Also, the identical optimal solutions obtained by MODI and LINGO confirmed the correctness and reliability of the proposed solution procedure.

Future research may extend the presented framework to Pythagorean T2FS multi-choice TP, Type-3 Fuzzy multi choice TP. Furthermore, integrating the proposed T2FS framework with advanced metaheuristic optimization algorithms and intelligent decision-making techniques may further improve solution quality and computational efficiency for large-scale real-world transportation systems.

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Author's Contribution:

Kapil Kumar: Literature review, Original Draft, Methodology, Computations, critical revision of the manuscript.

M.K. Sharma: Conceptualization, supervision, Direction, interpretation, data analysis, manuscript review, editing and critical review of the final manuscript.

Alok Kapil: Software analysis, Visualisation, Data Curation, Methodology, review and editing of the manuscript.

References:

1. Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of mathematics and physics*, 20(1-4), 224-230. <https://doi.org/10.1002/sapm1941201224>
2. Zadeh L.A. Fuzzy sets. (1965). *Information and Control*, 8(3):338–53. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X).
3. Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on systems, Man, and Cybernetics*, (1), 28-44. DOI: 10.1109/TSMC.1973.5408575
4. Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning - I. *Information sciences*, 8(3), 199-249. [https://doi.org/10.1016/0020-0255\(75\)90036-5](https://doi.org/10.1016/0020-0255(75)90036-5)
5. Karnik, N. N., & Mendel, J. M. (1998, May). Introduction to type-2 fuzzy logic systems. In 1998 IEEE international conference on fuzzy systems proceedings. IEEE world congress on computational intelligence (Cat. No. 98CH36228) (Vol. 2, pp. 915-920). IEEE. DOI: 10.1109/FUZZY.1998.686240
6. Castillo, O., Melin, P., Kacprzyk, J., & Pedrycz, W. (2007, November). Type-2 fuzzy logic: theory and applications. In 2007 IEEE international conference on granular computing (GRC 2007) (pp. 145-145). IEEE. DOI: 10.1109/GrC.2007.118
7. Castillo, O. (2010, August). Interval type-2 fuzzy logic for control applications. In 2010 IEEE International Conference on Granular Computing (pp. 79-84). IEEE. DOI: 10.1109/GrC.2010.40
8. Mendel, J. M., & Liu, F. (2007). Super-exponential convergence of the Karnik–Mendel algorithms for computing the centroid of an interval type-2 fuzzy set. *IEEE Transactions on Fuzzy Systems*, 15(2), 309-320. DOI: 10.1109/TFUZZ.2006.882463
9. Maity, G., & Roy, S. K. (2014). Solving multi-choice multi-objective transportation problem: a utility function approach. *Journal of uncertainty Analysis and Applications*, 2(1), 11. <https://doi.org/10.1186/2195-5468-2-11>
10. Chakraborty, D., Jana, D. K., & Roy, T. K. (2015). A new approach to solve multi-objective multi-choice multi-item Atanassov's intuitionistic fuzzy transportation problem using chance operator. *Journal of intelligent & fuzzy systems*, 28(2), 843-865. <https://doi.org/10.3233/IFS-141366>
11. Maity, G., & Kumar Roy, S. (2016). Solving a multi-objective transportation problem with nonlinear cost and multi-choice demand. *International Journal of Management Science and Engineering Management*, 11(1), 62-70. <https://doi.org/10.1080/17509653.2014.988768>
12. Roy, S. K., Maity, G., Weber, G. W., & Gök, S. Z. A. (2017). Conic scalarization approach to solve multi-choice multi-objective transportation problem with interval goal. *Annals of Operations Research*, 253(1), 599-620. <https://doi.org/10.1007/s10479-016-2283-4>
13. Ranarahu, N., Dash, J. K., & Acharya, S. (2018). Computation of multi-choice multi-objective fuzzy probabilistic transportation problem. In *Operations research in development sector* (pp. 81-95). Singapore: Springer Singapore. https://doi.org/10.1007/978-981-13-1954-9_6
14. Nasser, S. H., & Bavandi, S. (2020). Multi-choice linear programming in fuzzy random hybrid uncertainty environment and their application in multi-commodity transportation problem. *Fuzzy Information and Engineering*, 12(1), 109-122. DOI: 10.1109/CFIS49607.2020.9238695

15. Nasseri, S. H., & Bavandi, S. (2020, September). Solving multi-objective multi-choice stochastic transportation problem with fuzzy programming approach. In 2020 8th Iranian joint congress on fuzzy and intelligent systems (CFIS) (pp. 207-210). IEEE. DOI: 10.1109/CFIS49607.2020.9238695
16. Gupta, S., Ali, I., & Ahmed, A. (2018). Multi-choice multi-objective capacitated transportation problem—a case study of uncertain demand and supply. *Journal of Statistics and Management Systems*, 21(3), 467-491. <https://doi.org/10.1080/09720510.2018.1437943>
17. Gupta, K. (2021). Fuzzy programming for multi-choice bilevel transportation problem. *Operations Research and Decisions*.
18. Mondal, A., Roy, S. K., & Midya, S. (2023). Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem. *Journal of ambient intelligence and humanized computing*, 14(6), 6975-6999. <https://doi.org/10.1007/s12652-021-03554-6>
19. El Sayed, M. A., & Baky, I. A. (2023). Multi-choice fractional stochastic multi-objective transportation problem: MA El Sayed, IA Baky. *Soft Computing*, 27(16), 11551-11567. <https://doi.org/10.1007/s00500-023-08101-3>
20. Jain, E., & Dhodiya, J. M. (2024, November). Fuzzy Multi-choice Multi-objective Multi-item Five-Dimensional Transportation Problem and Its Solution by Variants of Genetic Algorithm. In *International Conference on Advanced Engineering Optimization Through Intelligent Techniques* (pp. 431-444). Singapore: Springer Nature Singapore. https://doi.org/10.1007/978-981-96-8070-2_34
21. Kumar, T., Chaudhary, S., Kumar, K., Dhanuk, K., & Sharma, M. K. (2024). Dynamic ranking function to optimize transshipment costs in intuitionistic Type-2 and Type-1 fuzzy environments. *Systems and Soft Computing*, 6, 200153. <https://doi.org/10.1016/j.sasc.2024.200153>
22. Sifaoui, T., & Aïder, M. (2024, October). Solving Sustainable Multi-choice Stochastic Multi-objective Multi-Item Solid Transportation Problem. In *International Colloquium on Methods and Tools for Decision Support* (pp. 127-146). Singapore: Springer Nature Singapore. https://doi.org/10.1007/978-981-96-9808-0_8
23. Bavandi, S., Nasseri, S. H., Postorino, M. N., Paganelli, F., & Mantecchini, L. (2025). Navigating a transportation system challenge with hybrid fuzzy stochastic multi-choice travel time and reliability: an application to maritime transportation. *Transportmetrica B: Transport Dynamics*, 13(1), 2452204. <https://doi.org/10.1080/21680566.2025.2452204>
24. Biswas, S., Mishra, K., & Bera, U. K. (2026). Soft Computing-Enabled Optimization of Multi-Choice Stochastic Transportation Problem Involving Exponential and Logistic Distributions. *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, 1-23. <https://doi.org/10.1007/s40010-025-00978-z>