International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(15s) 2026 pp. 481-490
https://doi.org/10.70917/ijcisim-2026-4431
Article

Radio Analytic Mean Odd Graceful Number of Star Cycle Graphs

P.Malliga[1,2], V.Helan Sinthiya[1*], P.Poomalai[3] , P.Senthilkumar[4]

[1]Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Samayapuram, Trichy -621 112 ,India. Email Id: helanv.Set@dsuniversity.ac.in
[2]Department of Mathematics Dhanalakshmi Srinivasan college of Arts and science for women (Autonomous), Tamilnadu, India. Email Id:malligakavinesh@gmail.com :
[3]Department of Mathematics, NPR College of Engineering &Technology (Autonomous), Tamilnadu, India Email Id: poomalaip@nprcolleges.org
[4]Department of Mathematics, Vel Tech High Tech Dr.Rangrajan Dr.Sakunthala Engineering College, Tamilnadu ,India Email Id:psenthil9159115957@gmail.com
Corresponding author *:helanv.Set@dsuniversity.ac.in

Abstract: In this paper we investigate radio analytic mean odd graceful number in cycle related graphs. The star cycle graph obtained by joining a cycle attached some pendent Vertices on the boundary. The radio analytic mean number ramgn (f) is the maximum integer allocated to any node of G, and this node is referred to as the radio analytic mean graceful number.In this article, we investigate the ramgn of certain star and Bistar-related graphs, exploring how these graphs satisfy the radio analytic mean condition and examining their properties. Whered(u,v) is distance between any two nodes of G

Keywords: Analytic Mean , Odd graceful, Starcycle ,diameter

1. Introduction:

Consider the graph G(V,E) Where the vertex set and edge set of the Graph G respectively. The basic Terminology and notation are noted byHarary (1). In 1967 Rosa represent the concept of graph labeling (2).Offers a thoroughanalysis of graph labeling.The radio channel Assignment problems is one of the most significant applicant oriented of nodes labelling of graphs with positive integrs.The problem is to find a way to assign radio channels to transmitters, so neighbors channels have significant frequency difference ,where channels far from one another might have minor frequency Difference. The radio Channel assignment problem can be easily converted into graph labelling by consider radio network problem. We consider vertices correspond to transmitter locations, and two vertices are made adjacent if the locations of the radio stations corresponding to these vertices are close.Further Radio mean graceful was introduced by K.N.Meera.Gananajothi in 1991 introduced the concept odd Graceful graphs. They proved so many path union and ladder odd graceful graphs. In 2017 Sushantkumar introduced the new type odd graceful graphs.`In latter several authors was developed by Radio Geometric graceful, Radio Heronian graceful. Radio mean graceful graphs

Definition: 1.1

A Graph G=(V,E) with q edges is said to admit odd graceful labeling f:V→ $\{0,1,2 \ldots..2q-1\}$Such that the induced edges f:E→ $\{1,3,\ldots..2q-1\}$defined as f(ab)=$|f(a)-f(b)|$is bijective.

A graph which admits odd graceful labelling is called an odd graceful graph.

Definition: 1.2

A simple graph G in p vertices and q edges is graceful if there is a labeling of vertices with distinct integers from the set $\{0,1,2 \ldots.n\}$ so that induced edge labeling 'e 'defined by e(uv)=$|f(u)-f(v)|$is bijective. A graph which admits graceful labelling is called angraceful graph

Definition: 1.3

A star cycle graph consisting boundary line vertices. One or more boundary vertices having $v_i{}^j$ pendent vertices. This graph is called star cycle graph.

Definition:1.4

A Radio Analytic mean labeling is a bijective f:V(G)→ $\{1,2,3 \ldots\ldots\}$ satisfying the condition

d(u,v)+$\frac{|f(u^2)-f(v)^2|}{2} \geq$ 1+diam(G) for u,v∈ G.Any odd graceful graph satisfied this condition that graph is called RAMOG.

Definition1.5: If d(v_i, v_j) means the distance between any pair of two vertices.

2. MAIN RESULTS

We consider the odd graceful graphs star like circlegraph, star tree graph, star fan graphs satisfied the RAMC.The highest integer of odd graceful graph is called the RAMOGN.

We are already proved by the following results.

Preposition 2.1 :The cycle (c_n) graph is admitted ramgn

Preposition 2.2: The Bistar$(B_{n,n})$,graph is admitted ramgn.

Preposition 2.3:The friendship graph $(fl_n)$is admitted ramgn.

Preposition 2.4: The Bipartite $(B_{x,y})$ graph is admitted ramgn**.**

3. Algorithm:

Step1: Consider $a_1, a_2, a_3 \ldots \ldots \ldots a_n$ be boundary vertices of cycle graph SC_8.and $a_i{}^j$ be the pendent vertices of cycle $S\ C_8$

Step2: We assign to labels boundary vertices and pendent vertices

Step3:we check the Given the graph satisfied odd graceful condition .ie) f:V→ $\{0,1,2 \ldots\ldots 2q-1\}$Such that the induced function f:E→ $\{1,3, \ldots\ldots 2q-1\}$ defined as f(ab)=$|f(a)-f(b)|$is bijective

Step4: If found the diam (G)in given the graph.

Step5: If Ensure given the graph is odd graceful graph ,then we apply ramc**.**

Step6 : If satisfied $\frac{[|\theta(u^2)-\theta(v^2)|]}{2} \geq 1 + diam(G) - d(u,v)$.We get RAMOGN

Step 7: If one or two arbitrary nearby vertices not satisfied ramc , we calculate the d(v_i, v_j)anti Clockwise direction.

Step8 : Then we get RAMOGN

Theorem 3.1 : The Radio Analytic mean odd graceful number of Sc_n is 2q-1 ,n≥ 8

Proof:

Let $v_1, v_2, v_3 \ldots \ldots \ldots v_n$ be the vertices of cycle graph, $v_i{}^j$ are switch of one vertex boundary pendent vertices of star cycle graph .we construct the vertex labeling f: V(G) → $\{0,1,2, \ldots .2q-1\}$ where q is the total no edges . The diameter of $sc_8$is 6 .

$f(v_i) = i - 1, i = 1,3,5,7$

$f(v_i) = 2q - (3i - 1)$,i=2,4,6

$f(v_7)$=$\frac{q}{2} - 2$,

$f(v_8) = 1$

$f(v_i{}^j) = 2q - (2i - 1), i, j = 1,2$,

$$f(v_i{}^j) = 2q - (2i + 1), i, j = 3,4$$

$$f(v_i{}^j) = 2q - (2i + 3), i, j = 5,6$$

$$f(v_i{}^j) = 2q - (3i - 2), i, j = 7$$

$f(v_i{}^j)$=3 ,i,j=8 . we construct the function defined induces edge labels are distinct.

f: E(G)→ $\{1,3,5 \dots \dots .2q - 1\}$.Therefore f is an odd graceful graph. Then we will Checkradio analytic mean condition on odd graceful graph.

Table 1: we will check the Radio Analytic Mean Condition

$Distance\ between\ any\ two\ vertices d(v_i, v_j)$	$Analytic\ \ mean\ \ condition$ $\frac{[\lvert\theta(u^2) - \theta(v^2)\rvert]}{2}$	$1 + diam\ (G)$- $d(v_i, v_j)$
$d(v_i, v_j)$ =1	480, 364, 362, 218, 212,……..	6
$d(v_i, v_j)$ =2	481,364, 60, 56, 52, 38,18……..	5
$d(v_i, v_j)$ =3	478, 462, 418, 402, 312,…….	4
$d(v_i, v_j)$ =4	260,368,216,200,220……..	3
$d(v_i, v_j)$ =5	478, 412,294,83………	2
$d(v_i, v_j)$ =6	336,150,180,260……..	1

Hereodd graceful graphin all the pairs of $d(v_i, v_j)$ satisfied Radio analytic mean condition. Therefore highest node is called Radio Analyticmean odd graceful number. Therefore RAMOGN is 2q-1.

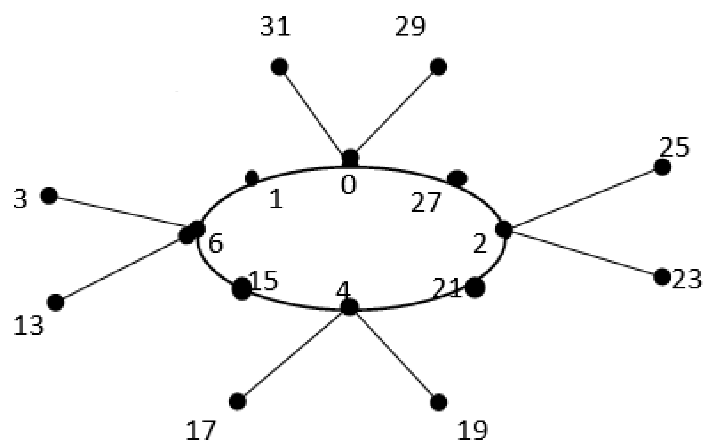

Figure 1. RAMOGN =2q-1

Theorem3.2:

The Switch of three boundary vertices in odd graceful number of $sc_n$is Ramogn is $2q - 1$,n≥ 12

Proof:

Let $v_1, v_2, v_3 \dots \dots \dots v_n$be the vertices of cycle graph, $v_i{}^j$ are switch ofone vertex boundary pendent vertices of star cycle graph .we construct the vertex labeling f: V(G) → $\{0,1,2, \dots .2q - 1\}$ where q is the total no edges . The diameter of sc_{12} is 7.

$$f(v_i) = i - 1, i = 1,3,5,7,9,11$$

$$f(v_i) = 2q - (2n + i) + 1 , i = 2,4$$

$$f(v_i) = (q + n) - (i - 3), i = 6,8$$

$$f(v_{10})=15, f(v_{12}) = 1$$

we construct the pendent edges as follows:

$$f({v_i}^j) = \begin{cases} 2q - 2j + 1 , i = 1 \ and \ 1 \le j \le n \\ 2q - 2n - 2j - 3, i = 5 \ and \ 1 \le j \le n \\ 2q - 4n - 2j - 7 , i = 9 \ and \ 1 \le j \le n \end{cases}$$

$$f(v_9) = 11 \ , i = 9, j = n$$

Here sc_{12} all edges having distinct odd labels.ie) $f(E(G) \rightarrow \{1,3,5,7 \dots .2q - 1\}$,the star cycle graph admitted odd graceful condition. Next we apply step 4 , we apply radio analytic mean condition in odd graceful graph. If satisfied step 5. We have valid RAMOGN 2q-1.

Table 3.2: The Radio Analytic Mean Condition is Verified

$Distance\ between\ any\ two\ vertices d(v_i, v_j)$	$Analytic\ mean\ condition$ $\frac{[\lvert\theta(u^2) - \theta(v^2)\rvert]}{2}$	$1 + diam\ (G) - d(v_i, v_j)$
$d(v_i, v_j)$ =1	925, 922,333,839………..	7
$d(v_i, v_j)$ =2	104, 204, 188, 84,14………..	6
$d(v_i, v_j)$=3	1402, 1153,1010,498,32……..	5
$d(v_i, v_j)$=4	564,460,10,116,36……..	4
$d(v_i, v_j)$=5	728,652,580,512……..	3
$d(v_i, v_j)$=6	700,504,392,280…….	2
$d(v_i, v_j)$=7	1386,1282, 1182,1086………	1

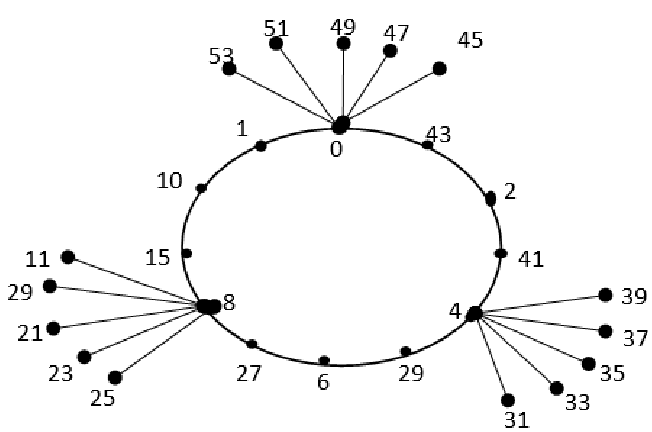

Figure 2: ramogn in sc_n

Theorem 3.3 :

The Radio Analytic mean odd graceful number of SC_n is 2q-1 ,n≥12

Proof:

$Let\ a_1, a_2 \dots \dots \dots \dots . a_{12}$ be theboundary vertices of cycle graph.$a_i{}^j$be the pendent vertices of SC_n.we construct the vertex labeling f: V(G) → $\{0,1,2, \dots .2q-1\}$ where q is the total no edges . The diameter of sc_{12} is 8.

$$f(a_i) = i-1, i = 1,3,5,7,9,11$$

$$f(a_1{}^j)= 2q-i, i = 1,3,5,7,9 \dots.$$

$$f(a_3{}^j)=2q-(n+i), i = 1,3,5,7,9$$

$$f(a_5{}^j)=2q-2n-i, i = 1,3,5,7,9$$

$$f(a_7{}^j)=2q-3n-i, i = 1,5,7,9,11$$

$$f(a_9{}^j)=2n+(2+i), i= 1,3,5,7,9$$

$$f(a_{11}{}^j)= n+i, i = 1,2,5,9,11$$

$$f(a_i)= 2q-(nj-1), j = 1,2,3,4, i=2,4,6,8,10$$

$$f(a_{12})=1 \quad .$$

$Distance\ between\ any\ two\ verticesd(v_i, v_j)$	$Analytic\ \ mean\ \ condition$ $\frac{[\lvert\theta(u^2)-\theta(v^2)\rvert]}{2}$	$1 + diam\ (G) - d(v_i, v_j)$
$d(v_i, v_j)$ =1	2665,3120,2964,3445,3280…..	8
$d(v_i, v_j)$ =2	6,16,14,18,50…….	7
$d(v_i, v_j)$=3	2657, 676, 2513,1299……	6
$d(v_i, v_j)$=4	836,780,720,656……	5
$d(v_i, v_j)$=5	683,758,838,922…..	4
$d(v_i, v_j)$=6	18, 42,48,1980…….	3
$d(v_i, v_j)$=7	256,212,136,105…..	2
$d(v_i, v_j)$=8	2340, 2356, 2280,2128……	1

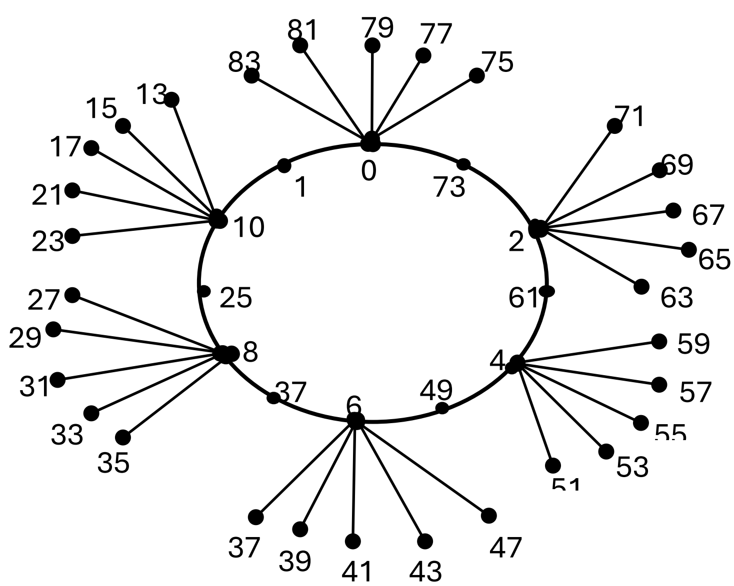

Figure 3 :ramogn in sc_{n}

Theorem 3.4 :The Radio Analytic mean odd graceful number of SC_n is 2q-1 ,n≥8

Proof: Let $a_1, a_2 \ldots \ldots \ldots \ldots . a_8$ be the vertices of the star cycle graph.$a_i{}^j$be the pendent vertices of star cycle are attached any arbitrary vertexa_i.we construct the vertex labeling f: V(G) → $\{0,1,2, \ldots .2q-1\}$ where q is the total no edges . The diameter of sc_8 is 6.The edge labels we define by

f: E(G)→ $\{1,3,5 \ldots \ldots .2q-1\}$& edge labels are distinct. So in this graph admitted odd graceful graph. Then we apply Radio analytic mean condition &we get valid RAMOGN.

$$f(a_i) = i-1, i = 1,3,5,7$$

$$f(a_i{}^j) = 2q-(2i-1)\ , \ \ i = 1, j = 1,2,3,4,5,6$$

$$f(a_{2i}) = \tfrac{q}{2}\text{+n- (2i-1) ,i=1,2,3}$$

$$f(a_i{}^j) = q-(2i-1)\text{,i=5,j=1,2,3,4,5,6}$$

$$f(a_7)\text{=1}$$

Table 3.4 : In all the cases $d(v_i, v_j)$ Satisfied Ramc

$Distance\ between\ any\ two\ vertices d(v_i, v_j)$	$Analytic\ \ mean\ \ condition$ $\frac{[|\theta(u^2)-\theta(v^2)|]}{2}$	$1 + diam\ (G) - d(v_i, v_j)$
$d(v_i, v_j)$ =1	544,612,684,760…..	6
$d(v_i, v_j)$ =2	10,52,346,364,	5
$d(v_i, v_j)$=3	23,59,83,143…….	4
$d(v_i, v_j)$=4	232,300,372,448……	3
$d(v_i, v_j)$=5	25,61,85,113…….	2
$d(v_i, v_j)$=6	336,500,624,736…..	1

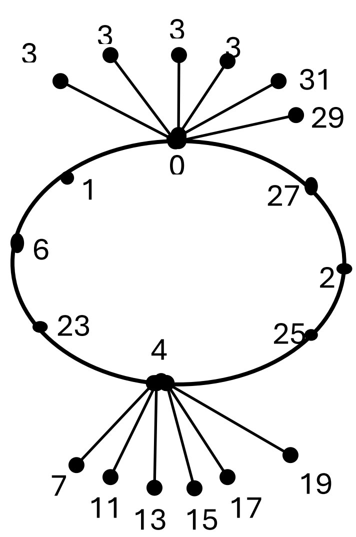

Figure 4: ramogn in sc_{n}

Theorem 3.5:The Radio Analytic mean odd graceful number of SC_n is 2q-1 ,n≥12

Proof:

Let $a_1, a_2 \ldots \ldots \ldots \ldots . a_{12}$ be the vertices of the star cycle graph.$a_i{}^j$be the pendent vertices of star cycle are attached any arbitrary vertex a_i.we construct the vertex labeling f: V(G) → $\{0,1,2, \ldots .2q-1\}$ where q is the total no edges . The diameter of sc_{12} is 7.The edge labels we define by f: E(G)→ $\{1,3,5 \ldots \ldots .2q-1\}$& edge labels are distinct. So in this graph admitted odd graceful graph. Then we apply Radio analytic mean condition on odd graceful graph&we get valid RAMOGN

f(a_i)=i-1,i=1,3,5……11

f($a_i{}^j$)=2j-1, i=1,j=2,4,6,7,8,f(a_{i+1})=2q-i,i=1,3,5,7,9

Table 3.5 : we will Check the RAMC

$Distance\ between\ any\ two\ vertices d(v_i, v_j)$	$Analytic\ \ mean\ \ condition$ $\frac{[\lvert\theta(u^2)-\theta(v^2)\rvert]}{2}$	$1 + diam\ (G) - d(v_i, v_j)$
$d(v_i, v_j)$ =1	5,13, 25,61,85…….	7
$d(v_i, v_j)$ =2	540,532,520,484…..	6
$d(v_i, v_j)$=3	480,462,356,494……	5
$d(v_i, v_j)$=4	16, 116, 108, 32…..	4
$d(v_i, v_j)$=5	18,526,362,305…..	3
$d(v_i, v_j)$=6	42,336,408,416…..	2
$d(v_i, v_j)$=7	6, 7,14, 43…….	1

Iiiustration

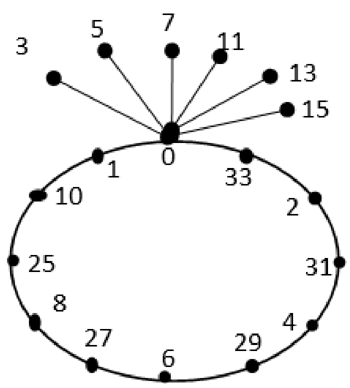

Figure 5: ramogn in sc_n

Theorem3.6:The Radio Analytic mean odd graceful number Star Fan graph

Proof:

Let G be a graph obtained by attaching the star fan graph.Let number of vertices and number of edges are denoted by V (G) andE (G) respectively. Let v_1 ,v_2,………v_n denote the pendent vertices of SF_n. Consider

w_1,w_2,………$w_n$be the path vertices of star fan graph. The graph G has p=4r vertices and q=4m-1 edges. The odd graceful condition is edge labels distinct. Here diameter of the star fan graph is 3.

Edge odd graceful graph satisfied radio analytic mean condition. & we get valid RAMOGN

We construct the vertex label follow the equality condition

f(w_i)=2(i-1), 1≤ $i \le r$

$f(v_i)$=2r+2(i-1) , 1$\leq i \leq m+1$

$f(u_i)$=2q-2m-(i+1),for i=2k, 1$\leq k \leq n$

$f(s_i)$=2q-(2r+1)-2(i-1), 1$\leq i \leq m+1$

f(t)=2q-1

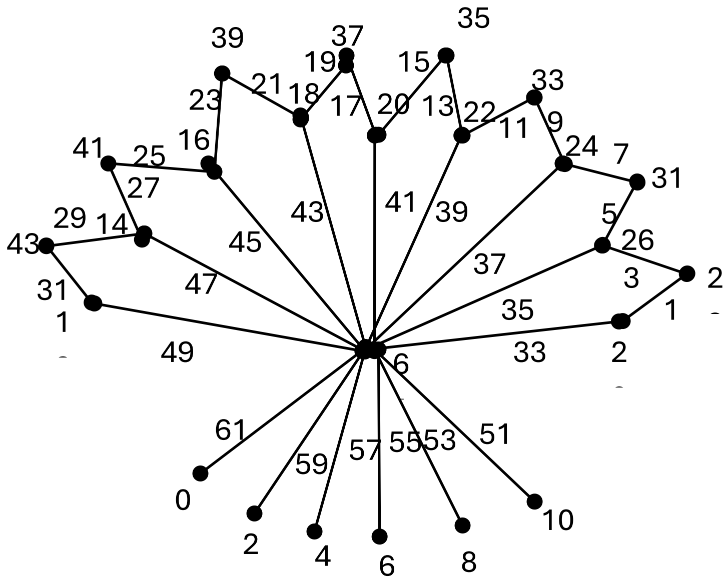

Figure 6: ramogn in sc_n

Table 3.6: we will check in all the cases satisfied RAMC

$Distance\ between\ any\ two\ vertices d(v_i, v_j)$	$Analytic\ \ mean\ \ condition$ $\frac{[|\theta(u^2) - \theta(v^2)|]}{2}$	$1 + diam\ (G) - d(v_i, v_j)$
$d(v_i, v_j)$ =1	853,742,632…..	3
$d(v_i, v_j)$ =2	26,30,34,38…..	2
$d(v_i, v_j)$=3	698,612,544……..	1

Theorem 3.7: The Radio Analytic mean odd graceful number of star tree graph

Proof:

Let G be the graph obtained by attaching star tree graph .Letw_1, w_2, ….$w_n$be the path vertices of G. The graph has v=2(r+1)+m(r+2)vertices and E=r(m+2)+(3m+1)edges.

we define the vertex label for the apex of SSG.Here we consider m=7,r=5,n=15.

f(v)=2q-1,

$f(u_i)$=2(i-1), i=1$\leq i \leq$ r

$f(v_i)$=2r+(i-1) for i=2k+1and 1 ≤k≤(n+1)/2

$f(w_i)$=2r+2n-(i+1) for i=2kand 1 ≤k≤(n+1)/2

This theorem proof is followed by previous case.

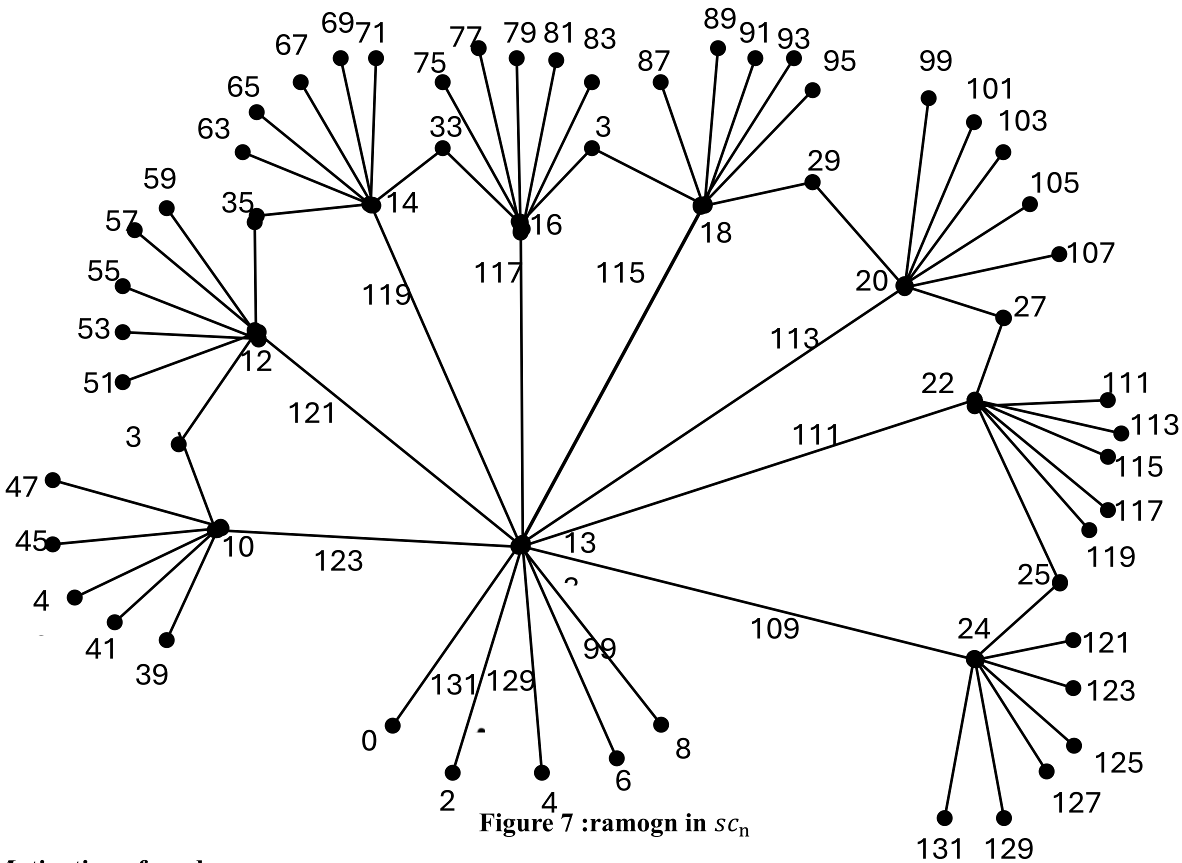

Figure 7 :ramogn in sc_n

Motivation of readers:

Here we proved only odd graceful satisfied radio analytic mean condition. In other types odd even graceful and edge odd graceful graphs, edge even graceful graphs fulfil the radio analytic mean conditiontry to various graphs.

Open problems:

1. Ramogn is w-tree n ≥ 2k-2

2.Ramognis disjoint union of w-tree w T(n,k)

3. Ramogn in Ladder graph L_n =p_n xp_n , n ≥ 4

4. Conclusion:

As in thisarticle we proved that star cycle ,star fan ,star tree graph are the odd graceful graphs,

then the odd graceful graphs involves in ramc.Hereafter the research can be developed on this result on other variations in edge even graceful labelling and edge delta graceful labelling.

References:

1. F.Harary,GraphTheory, Addison-Wesley, Reading, Massachusetts, 1972
2. Rosa .A ,'' On certain valuations of the vertices of a graph '' in proc.Int.Symp.Theory,Rome, Italy,Jul1967 pp 349-355
3. .Chartrand .G, Erwin.F, Radio labelling of graphs, 'Bull. Inst.Combinat, Appl,.Vol 33,pp 77-85, sep2001
4. J.A.Gallian,Adynamicsurveyofgraphlabeling,TheElectronicjournalofcombinatory, 5(2002),
5. .Linlin Cui, Feng Li , Radio number of Cartesian Products of Complete Bipartite Graphs odd cycles, Utilitas Mathematical vol 121 , pp 137-150 ,2021
6. . Tharmaraj,P.B.Sarasija,AnalyticMeanGraphs.Int.Journal ofMath.Analysis,Vol.8, 2014,no.12, 595 - 609
7. S.K.Vaidya , N.H. Shah ,'' Graceful and odd graceful labelling of some graphs ,International journal of Mathematics and soft computing Vol.3,No1 (2013)

8. K.N.Meera,Radio Geometric Mean graceful Graphs IOP conference series in 2019.
9. Davidraj.C ,Mary Shalini .T, Radio Geometric Mean graceful labelling on Degree splitting of cycle related graphs, Mathematical Statistician and Engineering Applications ,Vol 70 No2, -2021
10. V.Brindhamary,C.Davidraj,''Even Radio Mean Graceful Labelling on Degree Splitting of Snake related graphs, South East Asian Journal of Mathematics and mathematical Science.Vol,18-2022
11. .P. Poomalai, R. Vikramaprasad , '' Radio Analytic Mean labelling on Degree splitting of some graphs ,International journal of Advanced Science and Technology Vol 29,No 7, -2020
12. .P. Poomalai, R. Vikramaprasad, "Radio analytic mean number of some subdivision graphs, " Journal of Advanced Research in Dynamical and control systems "vol 12, p. 577-583-2020
13. V.Helan Sinthiya, P.Malliga, P.Poomalai,''Radio Analytic Mean Graceful Number of Starand Bistar Related Graphs, 'South East European journal of Public Health, XXVI,S2,2025.
14. .Sushant Kumar Rout, DebdasMishra, 'Odd Graceful Labelling of some New type of graphs' 'International Journal of Mathematics Trends and Technology'' vol 41 number1-2017.