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# Three-dimensional Markov Chain Model to Help Reduce the Spread of COVID-19 in IoT Environment

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*Abstract*: **The COVID-19 pandemic has spread with alarming speed and resulted in substantial loss of lives. In the COVID-19 pandemic response, the Internet of Things (IoT) technology provides an extensive integrated network of IoT devices to prevent the spread of contagion. Rapid access of the data transmitted by COVID-19 IoT devices simply means better decision-making and better response plans. Unfortunately, packets collision further delays the arrival of that data and thus the large-wide spreading of the pandemic. In this paper, we propose an approach that ensures fast and successful transmission of the data transmitted by COVID-19 IoT devices in a scenario comprising other types of IoT devices. We have proposed a three-dimensional Markov chain for modeling the proposed IoT network. Detailed mathematical expressions of the average throughput of transmitted packets, the average delay of transmitted packets, the average throughput of the backlogged packets, and the average delay of backlogged packets are given in order to compare the performance of each type of IoT devices. Numerical results show that our approach achieves a significant throughput increase and a low average delay for COVID-19 IoT devices due to the priority channel access given to that type of devices. Thereby mitigating the spread of the pandemic.**

*Keywords*: Internet of Things, Markov chain, Performance evaluation, COVID-19

## I. Introduction

The COVID-19 is an acronym for "Coronavirus Disease2019", it is a human infectious illness caused by infection with the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) virus strain [1, 2]. This novel coronavirus emerged out of Wuhan city of China in December 2019 [3]. It rapidly became a devastating pandemic, spreading throughout the world, resulting in multiple fatalities. As of April 2021, there are more than 140 million infected patients. And over 3 million deaths worldwide [4]. The COVID-19 virus is transmitted directly (from human to human) through inhalation of respiratory droplets from symptomatic people or from COVID-19 patients, also indirectly through polluted objects or surfaces [5, 6]. The virus causes mild respiratory infections in about 80% of people infected, though about half will have pneumonia. Another 15% of COVID-19 patients (older people, and those with underlying medical problems like malignancies, HIV, diabetes, cardiovascular diseases (CVD), chronic respiratory disease, hypertension, and cancer) develop severe illness, and 5% need critical care [7, 8].

As COVID-19 spreads more and more rapidly, symptom diagnosis, quarantine monitoring, contact tracing, social distancing, COVID-19 outbreak forecasting, and SARS-CoV-2 mutation tracking are overly critical for the containment and mitigation of the epidemic. In this regard, the Internet of Things (IoT) can help with all these things [9]. IoT technology [10], in particular, and along with other technologies like Blockchain technology[11] and Artificial intelligence (AI)[12], is one of the technological solutions that can help control the spread of the global COVID-19 pandemic using existing and new technology [13, 14]. The IoT refers to the interconnection of physical devices and the Internet. IoT devices are not only able to sense and record, but can also monitor and respond. The data collected by these devices are stored in the raw form and transmitted through the Internet to be analyzed for patterns or trends [15].

IoT technology plays a major role in beating infectious diseases. At the rise of the COVID-19 pandemic, the most urgent task is to trace and isolate people who possibly made contact with the infected. Tracking patients using smartphones is a new approach widely used during the COVID-19 pandemic to mitigate and control the spread of the virus [16]. The Bluetooth-based contact tracing app is a contact tracing tool that uses Bluetooth technology to find out if the app user has been in close contact with someone who tested positive for COVID-19 [17, 18]. Since there are more than 3.5 billion active smartphones in the world, these smartphone apps could be very efficient in beating COVID-19. There are already many smartphone apps in use in response to COVID-19 such as COVIDSafe [19], Corona-Warn-App [20], Tracetogether [21], Aarogya setu [22], Wiqaytna [23], Corona 100m [24], GH COVID-19 Tracker App [25], Tabaud App [26], Covid Watch App [27], and BeAware [28].

Any violation of self-quarantine can present a great risk and cost to public health [29]. Another critical task in pandemic response is quarantine monitoring of the infected people with the virus. IoT-enabled/linked devices/applications can be used to ensure infected people (or the potentially infected persons) compliance once they enter into quarantine where Public health personnel can monitor which patients remain quarantined, and which patients have breached the quarantine. IoT-based sensors can be used to ensure COVID-19 patient won't leave their homes while in quarantine. If a patient leaves the quarantine area, an alert will be sent to notify the authorities. These devices provide data to support compliance with local, state, and federal health guidelines. During a quarantine, there are also IoT devices such as IoT buttons that can provide adequate healthcare remotely to COVID-19 patients [30]. Smart robots also can be used in different ways, such as capturing respiratory signs and assisting patients with their treatments or food without the physical presence of the health workers [31].

Social distancing is one of the tasks identified by public health officials as critical to addressing the COVID-19 pandemic [32]. Social distancing also called "physical distancing," means increasing the space (6 feet) between people in order to decrease the chance of spreading illness. IoT devices such as drones devices can be used to monitor the density in closed places [33]. Wearable smart devices are also used to alert on social distancing.

Another task no less important than others presented above is early diagnosis. Diagnose COVID-19 early is the most effective way to curb its spread because there is no cure or vaccine available against it. IoT devices help in this phase to the proactive screening of contacts and early diagnosis in order to isolate patients. An IoT-based wearable device used for early detection of COVID-19 symptoms providing remote continuous monitoring of patients. Also, the smart thermometer can be used to help get ahead of the COVID-19 virus. This device can capture fever data in real-time to produce daily maps showing where the virus is moving [34].

The collision occurs due to simultaneous transmission by two or more IoT devices. This constraint greatly limits the applicability of IoT technology facing the COVID-19 pandemic. In this paper, we extend the approach introduced in [35]. Our IoT network contains $N$ types of IoT devices. The proposed approach gives the possibility of the packet transmitted by IoT devices used to stop the spread of COVID-19 (COVID-19 IoT devices) to be received even if it intersects in time with packets from other devices. Furthermore, the proposed IoT network scenario has been modeled using a multi-dimensional Markov chain to derive the analytical expression of the performance metrics of interest.

The rest of the paper is organized as follows. In Sect. II, we comprehensively describe the context of the problem addressed in this work. In Sect. III, we present the IoT scenario considered in this work. In Sect. IV, we propose a three-dimensional Markov chain for modelling the proposed IoT network. The performance metrics are obtained in Sect. V. Numerical results and conclusions are given in Sect. VI and Sect. VII, respectively.

## II. Problem Statement

The Internet of Things (IoT) concept is not just limited to smart grids or smart cities, but it can already help detect and control infectious disease outbreaks in real-time. IoT technology also can be used to stop the spread of the COVID-19 virus. IoT devices can provide early detection for the virus, isolating those who are positive with the virus, contact tracing, and isolating those who had contact with someone who later tested positive with the virus. Data from COVID-19 IoT devices are providing an inclusive view of the COVID-19 pandemic as well as can save lives. However, any delay in the arrival of these data could cause the large-wide spreading of the virus and therefore many otherwise preventable deaths. Packets collision is generally the responsible for all these delays. The collision occurs due to simultaneous transmission by two or more IoT devices.

There are several IoT devices in the market. An IoT device could be something as simple as a smart bicycle, or as important as medical sensors and smart fire alarm, etc. Some of these IoT devices are used to avoid disaster scenarios such as leaks and floods, and others are capable to stop the spread of infectious diseases such as COVID-19. And others are used to makes sure that the user does not lack important household items like soft drinks and grocery material. Thus, the data collected by IoT devices have not the same importance and have not the same effect. However, the collision of these data increases the delayed arrival of the important data. Therefore, the important data have to speedily decoded.

To better understand the issue addressed in this paper, we present the following example. Consider four IoT devices, the first one is IoT wearable bands that used to alert the authorities that a COVID-19 patient left the quarantine area. The second IoT device is a smart fire alarm that notifies the authorities if there is a fire. The third IoT device is a smart refrigerator that provides information and data for the user fridge. The last IoT device considered in this example is a fitness tracker that used for monitoring and tracking fitness-related metrics. Suppose all these four devices transmit their data at the same time, resulting in a collision. Therefore, all this data is considered damaged. Thus, the COVID-19 patient will infect others, and also the fire will spread with dramatic speed.

## III. System Model

We consider an IoT network consisting of an IoT gateway and $N$ types of IoT devices. We assume that the type 1

IoT devices are the devices used to stop the COVID-19 pandemic (COVID-19 IoT devices) and the other $N-1$ types of IoT devices are used in different IoT applications. In our network model, we assume that each IoT device has a finite buffer. We assume that all these devices use the same channel access mode during the transmission to the gateway. Furthermore, when two or more devices transmit their packets at the same time then they collide. Packets that are involved in the collision are assumed to be corrupted (backlogged) and are scheduled for retransmission after a random time. For collision avoidance, we assume that all the devices run the Slotted ALOHA [36, 37] as a random access protocol [38]. In the Slotted ALOHA protocol, time is divided into time-slots of a fixed length. Also, in this protocol, all devices are synchronized, in the sense that they can only transmit at the beginning of each time slot. And if a device misses transmitting in the current time slot, it waits for the following time slot. If the gateway successfully receives the packet, it re-sends an acknowledgment (ACK) to the device, informing it that it was a successful transmission. If a device receives any ACK, that means there has been a collision, and re-transmission is needed after a random time.
As we mentioned above, this work considers an IoT network consisting of $N$ types of devices (numbered $1, 2, \ldots, N$ ) such that if $i < j$ a type $i$ IoT device always has priority over type $j$ IoT device. That means, when a type $i$ IoT device and one or more type $j$ IoT devices transmit their packets simultaneously, the transmitted packet by the type $i$ IoT device can be successfully received and the other(s) packet cannot. And the same rule is verified by three or more types of IoT devices. Indeed, we assume that the $N$ types of IoT devices transmit their packets at $N$ different power levels such that if $i < j$ a type $i$ signal always dominates a type $j$ signal.
Let $M_i,\ i = 1, 2, ..., N$ be the number of type $i$ IoT devices. Each type $i$ IoT device generates a new packet with the same probability $p_{g_i}$. After the collision, the type $i$ IoT device retransmit their previous packet in each subsequent time slot with a re-transmission probability $p_{r_i}$.

## IV. Markov Model

Now for modeling the considered IoT network, we consider a sequence of random variables $\{Y_1, Y_2, \ldots, Y_N\}$, where $Y_i,\ i = 1, 2, ..., N$ describes the number of backlogged type $i$ IoT devices (or backlogged packets) at the beginning of a slot. We assume that the minimum value of $Y_i$ is 0, and the maximum is $M_i$ representing the number of type $i$ IoT devices. For any choice of values $p_{r_i} \in ]0; 1]$, the N-dimensional process $\{Y_1, Y_2, \ldots, Y_N\}$ is a discrete-time Markov chain. Indeed, it is easy to check that the past and future are conditionally independent, given the present state (Markov property). The Markov chain $\{Y_1, Y_2, \ldots, Y_N\}$ is irreducible with a state space of $\{0, 1, \ldots, M_1\} \times \{0, 1, \ldots, M_2\} \times \ldots \times \{0, 1, \ldots, M_N\}$.
Let $P_{r_i}(j, Y_i)$ be the probability that $j$ out of the $Y_i,\ i = 1, \ldots, N$ backlogged packets of type $i$ IoT devices are re-

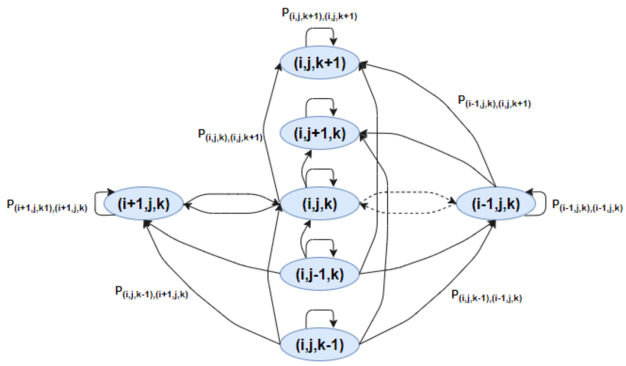


**Figure. 1**: The state transition diagram for the Markov chain model.

transmitted in a given slot. It is given by:

$$P_{r_i}(j, Y_i) = \binom{Y_i}{j} (1 - p_{r_i})^{Y_i - j} p_{r_i}^j, \tag{1}$$

Let $P_{g_i}(j, Y_i)$ be the probability that $j$ out of the $M_i - Y_i,\ i = 1, \ldots, N$ unbacklogged type $i$ IoT devices transmit their packets in a given slot. It is given by:

$$P_{g_i}(j, Y_i) = \binom{M_i - Y_i}{j} (1 - p_{g_i})^{M_i - Y_i - j} p_{g_i}^j, \tag{2}$$

And let $P_{g_i}(1, M_i) = 0$ and $P_{r_i}(1, 0) = 0$.
As a result of the proposed approach , the collision probability for type 1 IoT devices (COVID-19 IoT devices) can then be expressed as

$$\begin{aligned} P_{c_1} &= P_{g_1}(j, Y_1) + P_{r_1}(j, Y_1) \\ &\quad + P_{g_1}(1, Y_1)[1 - P_{r_1}(0, Y_1)] \\ &\quad + P_{r_1}(1, Y_1)[1 - P_{g_1}(0, Y_1)],\ 2 \le j \le M_1 \end{aligned} \tag{3}$$

and the collision probability for type 2 IoT devices is given by :

$$\begin{aligned} P_{c_2} &= P_{g_2}(j, Y_2) + P_{r_2}(j, Y_2) + P_{g_2}(1, Y_2) \\ &\quad \times [1 - P_{r_2}(0, Y_2)] + P_{r_2}(1, Y_2)[1 - P_{g_2}(0, Y_2)] \\ &\quad + [P_{g_2}(1, Y_2) + P_{r_2}(1, Y_2)][P_{g_1}(i, Y_1) \\ &\quad + P_{r_1}(i, Y_1)],\ 2 \le j \le M_2 \text{ and } 1 \le i \le M_1 \end{aligned} \tag{4}$$

and the collision probability for type $N$ IoT devices is given by :

$$\begin{aligned} P_{c_N} &= P_{g_N}(j, Y_N) + P_{r_N}(j, Y_N) + P_{g_N}(1, Y_N)[1 \\ &\quad - P_{r_N}(0, Y_N)] + P_{r_N}(1, Y_N)[1 - P_{g_N}(0, Y_N)] \\ &\quad + [P_{g_N}(1, Y_N) + P_{r_N}(1, Y_N)] \times [\sum_{n=1}^{N-1} P_{g_n}(i, Y_n) \\ &\quad + P_{r_K}(i, Y_n)],\ 2 \le j \le M_N \text{ and } 1 \le i \le M_n \end{aligned} \tag{5}$$

To simplify the analysis, we study the case where $N = 3$. The state transition diagram of the Markov chain model is shown in Fig. 1. The transition probability

$P_{(Y_1,Y_2,Y_3),(X_1,X_2,X_3)}$ is defined as the probability of the transition from state $(Y_1, Y_2, Y_3)$ to $(X_1, X_2, X_3)$, where $Y_1, X_1 \in \{0, 1, \ldots, M_1\}$, and $Y_2, X_2 \in \{0, 1, \ldots, M_2\}$, and $Y_3, X_3 \in \{0, 1, \ldots, M_3\}$. Then, the transition probabilities associated to the Markov chain are given in appendix A.

Since the state space is finite and all the states communicate among themselves, then the Marko chain is ergodic, and therefore the stationary distribution exists. Let $\pi_{ij}^{(k)}(q_{r_1}, q_{r_2}, q_{r_3}) = P(Y_1 = i, Y_2 = j, Y_3 = k)$, $i = 0, 1, \ldots, M_1$, $j = 0, 1, \ldots, M_2$ and $k = 0, 1, \ldots, M_3$ be the stationary distribution of the Markov chain. $\pi_{ij}^{(k)}$ represents the probability of $i$ backlogged type 1 devices, $j$ backlogged type 2 devices, and $k$ backlogged type 3 devices. Thus, the steady state distribution of the Markovian process can be obtained by solving the following system:

$$\begin{cases} \pi(p_r) = \pi(p_r)P(p_r) \\ \pi_{ij}^{(k)}(p_r) \geq 0,\ i = 0, \ldots, M_1,\ j = 0, \ldots, M_2, \\ \qquad \text{and } k = 0, \ldots, M_3 \\ \sum_{i=0}^{M_1}\sum_{j=0}^{M_2}\sum_{k=0}^{M_3} \pi_{ij}^{(k)}(p_r) = 1. \end{cases} \quad (6)$$

where $p_r = (p_{r_1}, q_{r_2}, q_{r_3})$ and $P(p_r)$ is the matrix of the transition probabilities $P_{(Y_1,Y_2,Y_3),(X_1,X_2,X_3)}$.

## V. Performance Evaluation

In this section, we evaluate the performance of each type of IoT devices considered in this paper.

### *A. Performance Metrics of COVID-19 IoT Devices*

As we mentioned before, the type 1 IoT devices are the devices used to stop COVID-19 (COVID-19 IoT devices). And they have constantly higher priority to access the channel over other types of IoT devices.

Let $Th_{covid}(p_r)$ be the normalized throughput of COVID-19 IoT devices, defined as the average number of packets transmitted by COVID-19 IoT devices that are successfully received by the IoT gateway in a given time slot. It is given by:

$$Th_{covid}(p_r) = \sum_{i=0}^{M_1}\sum_{j=0}^{M_2}\sum_{k=0}^{M_3} P^i_{succ_1} \pi_{ij}^{(k)}(p_r), \quad (7)$$

where $P^i_{succ_1}$ represents the successful transmission probability. It is defined as the average number of the packets transmitted by the COVID-19 IoT devices that are successfully received at state $i$ (for $i = 0, 1, 2, ..., M_1$) and it is given by:

$$P^i_{succ_1} = P_{g_1}(1, i)P_{r_1}(0, i) + P_{r_1}(1, i)P_{g_1}(0, i). \quad (8)$$

According to the normalization condition: $\sum_{i=0}^{M_1}\sum_{j=0}^{M_2}\sum_{k=0}^{M_3} \pi_{ij}^{(k)}(p_r) = 1$, the normalized throughput of the COVID-19 IoT devices can be re-written as follows:

$$Th_{covid}(p_r) = p_{g_1}[M_1 - N_{Bcovid}(p_r)], \quad (9)$$

where $N_{Bcovid}$ represents the average number of backlogged COVID-19 IoT devices (backlogged COVID-19 packets). It is given by:

$$N_{Bcovid}(p_r) = \sum_{i=0}^{M_1} i \sum_{j=0}^{M_2}\sum_{k=0}^{M_3} \pi_{ij}^{(k)}(p_r). \quad (10)$$

The average delay for successfully transmitted packets is defined as the average time interval from a packet generation to its successful transmission. Applying Little's result [39], the average packet transmission delay of the COVID-19 IoT devices $De_{covid}$, is given by:

$$\begin{aligned} De_{covid}(p_r) &= \frac{Th_{covid}(p_r) + N_{Bcovid}(p_r)}{Th_{covid}(p_r)} \\ &= 1 + \frac{N_{Bcovid}(p_r)}{Th_{covid}(p_r)}. \end{aligned} \quad (11)$$

Another way to evaluate IoT devices performance, is to analyze the performance metrics of backlogged packets. The throughput of the backlogged packets related to COVID-19 IoT devices is given by:

$$Th_{Bcovid}(p_r) = Th_{covid}(p_r) - Th^{covid}_{succ}, \quad (12)$$

where $Th^{covid}_{succ}(p_r)$ is the throughput of packets related to the COVID-19 IoT devices that are successfully transmitted in the first attempt. And it is calculated by:

$$Th^{covid}_{succ}(p_r) = \sum_{i=0}^{M_1} i \sum_{j=0}^{M_2}\sum_{k=0}^{M_3} P_{g_1}(1, i)P_{r_1}(0, i)\pi_{ij}^{(k)}(p_r). \quad (13)$$

The average delay of backlogged packets is defined as the average time, in slots, that a backlogged packet takes to go from the device to the IoT gateway. Applying Little's result, the average delay of backlogged packets related to COVID-19 IoT devices $De_{Bcovid}$, is given by:

$$\begin{aligned} De_{Bcovid}(p_r) &= \frac{Th_{Bcovid}(p_r) + N_{Bcovid}(p_r)}{Th_{Bcovid}(p_r} \\ &= 1 + \frac{N_{Bcovid}(p_r)}{Th_{Bcovid}(p_r)}. \end{aligned} \quad (14)$$

### *B. Performance Metrics of Type 2 IoT Devices*

Similarly, the normalized throughput of the type 2 IoT devices is given by:

$$Th_{\text{type2}}(p_r) = \sum_{j=0}^{M_2} j \sum_{i=0}^{M_1}\sum_{k=0}^{M_3} P^j_{succ_2} \pi_{ij}^{(k)}(p_r), \quad (15)$$

where $P^j_{succ_2}$ represents the successful transmission probability. It is defined as the average number of the packets transmitted by the type 2 IoT devices that are successfully received at state $j$ (for $j = 0, 1, 2, ..., M_2$). It is given by:

$$P_{succ_2}^{j} = P_{g_2}(1,j)P_{r_2}(0,j)P_{g_1}(0,i)P_{r_1}(0,i) + P_{r_2}(1,j)P_{g_2}(0,j)P_{g_1}(0,i)P_{r_1}(0,i). \quad (16)$$

Based on the normalization condition, the normalized throughput of the type 2 IoT devices can be re-written as follows:

$$Th_{\text{type2}}(q_r) = p_{g_2}[M_2 - N_{Btype2}(q_r)], \quad (17)$$

where $N_{Btype2}$ represents the average number of backlogged type 2 IoT devices. It is given by:

$$N_{Btype2}(q_r) = \sum_{j=0}^{M_2} j \sum_{i=0}^{M_1} \sum_{k=0}^{M_3} \pi_{ij}^{(k)}(p_r). \quad (18)$$

By little's formula, the average packet transmission delay of the type 2 devices $De_{type2}$, is given by:

$$De_{type2}(p_r) = \frac{Th_{type2}(p_r) + N_{Btype2}(p_r)}{Th_{type2}(p_r)} = 1 + \frac{N_{Btype2}(p_r)}{Th_{type2}(p_r)}. \quad (19)$$

The throughput of the backlogged packets related to the IoT devices of type 2 is given by:

$$Th_{Btype2}(q_r) = Th_{type2}(q_r) - Th_{succ}^{type2}(q_r), \quad (20)$$

where $Th_{succ}^{type2}(q_r)$ is the throughput of packets related to the IoT devices of type 2 that are successfully transmitted in the first attempt is calculated by:

$$Th_{succ}^{type2}(q_r) = \sum_{j=0}^{M_2} j \sum_{i=0}^{M_1} \sum_{k=0}^{M_3} P_{g_2}(1,j)P_{r_2}(0,j) \times P_{g_1}(0,i)P_{r_1}(0,i)\pi_{ij}^{(k)}(p_r). \quad (21)$$

Thus, the average delay of backlogged packets related to the IoT devices of type 2 $De_{Btype2}$, is given by:

$$De_{Btype2}(q_r) = \frac{Th_{Btype2}(q_r) + N_{Btype2}(q_r)}{Th_{Btype2}(q_r)} = 1 + \frac{N_{Btype2}(q_r)}{Th_{Btype2}(q_r)}. \quad (22)$$

### C. Performance Metrics of Type 3 IoT Devices

The normalized throughput of the type 3 IoT devices is given by:

$$Th_{\text{type3}}(p_r) = \sum_{k=0}^{M_3} k \sum_{i=0}^{M_1} \sum_{j=0}^{M_2} P_{succ_3}^{k} \pi_{ij}^{(k)}(p_r), \quad (23)$$

where $P_{succ_3}^{k}$ represents the successful transmission probability. It is defined as the average number of the packets transmitted by the type 3 IoT devices that are successfully received at state $k$ (for $k = 0, 1, 2, ..., M_3$). It is given by:

$$P_{succ_3}^{k} = P_{g_3}(1,k)P_{r_3}(0,k)P_{g_2}(0,k)P_{r_2}(0,j) \times P_{g_1}(0,i)P_{r_1}(0,i) + P_{r_3}(1,k)P_{g_3}(0,k) \times P_{g_2}(0,k)P_{r_2}(0,j)P_{g_1}(0,i)P_{r_1}(0,i). \quad (24)$$

Based on the normalization condition, the normalized throughput of the type 3 IoT devices can be re-written as follows:

$$Th_{\text{type3}}(q_r) = p_{g_3}[M_3 - N_{Btype3}(q_r)], \quad (25)$$

where $N_{Btype3}$ represents the average number of backlogged type 3 IoT devices. It is given by:

$$N_{Btype3}(q_r) = \sum_{k=0}^{M_3} k \sum_{i=0}^{M_1} \sum_{j=0}^{M_2} \pi_{ij}^{(k)}(p_r). \quad (26)$$

By little's formula, the average packet transmission delay of the type 3 devices $De_{type2}$ as follows:

$$De_{type3}(p_r) = \frac{Th_{type3}(p_r) + N_{Btype3}(p_r)}{Th_{type3}(p_r)} = 1 + \frac{N_{Btype3}(p_r)}{Th_{type3}(p_r)}. \quad (27)$$

The throughput of the backlogged packets related to the IoT devices of type 3 is given by:

$$Th_{Btype3}(q_r) = Th_{type3}(q_r) - Th_{succ}^{type3}(q_r) \quad (28)$$

where $Th_{succ}^{type3}(q_r)$ is the throughput of packets related to the IoT devices of type 3 that are successfully transmitted in the first attempt, it is calculated by:

$$Th_{succ}^{type3}(q_r) = \sum_{k=0}^{M_3} k \sum_{i=0}^{M_1} \sum_{j=0}^{M_2} P_{g_3}(1,k)P_{r_3}(0,k) \times P_{g_2}(0,k)P_{r_2}(0,j)P_{g_1}(0,i) \times P_{r_1}(0,i)\pi_{ij}^{(k)}(p_r). \quad (29)$$

Thus, the average delay of backlogged packets related to the IoT devices of type 3 $De_{Btype3}$, is given by:

$$De_{Btype3}(q_r) = \frac{Th_{Btype3}(q_r) + N_{Btype3}(q_r)}{Th_{Btype3}(q_r)} = 1 + \frac{N_{Btype3}(q_r)}{Th_{Btype3}(q_r)}. \quad (30)$$

### D. Performance Metrics of Overall Network

The average number of backlogged packets in the proposed network is given by:

$$N_{\text{Backlogged}}(q_r) = N_{Bcovid}(q_r) + N_{Btype2}(q_r) + N_{Btype3}(q_r). \quad (31)$$

The overall throughput of the proposed network is denoted as the total throughput of all subnetworks. Therefore, the overall network throughput can be calculated as follows:

$$Th(p_r) = Th_{\text{covid}}(p_r) + Th_{\text{type2}}(p_r) + Th_{\text{type3}}(p_r). \quad (32)$$

The expected delay of transmitted packets $De$, is defined as the average time, in slots, that a packet (whatever its source) takes from its source to the IoT gateway. It is given by:

$$\begin{aligned} De(q_r) &= \frac{Th(q_r) + N_{\text{Backlogged}}(q_r)}{Th(q_r)} \\ &= 1 + \frac{N_{\text{Backlogged}}(q_r)}{Th(q_r)}. \end{aligned} \tag{33}$$

The throughput of the backlogged packets of the proposed network is given by:

$$\begin{aligned} Th_{\text{Backlogged}}(q_r) = Th_{Bcovid}(q_r) + Th_{Btype2}(q_r) \\ + Th_{Btype3}(q_r) \end{aligned} \tag{34}$$

The expected delay of backlogged packets of the proposed network is given by:

$$\begin{aligned} De_{\text{Backlogged}}(q_r) = &= \frac{Th_{\text{Backlogged}}(q_r) + N_{\text{Backlogged}}(q_r)}{Th_{\text{Backlogged}}(q_r)} \\ &= 1 + \frac{N_{\text{Backlogged}}(q_r)}{Th_{\text{Backlogged}}(q_r)}. \end{aligned} \tag{35}$$

## VI. Numerical Results

In this section, we evaluate the performance of our IoT network model in terms of the normalized throughput of transmitted packets, the delay of transmitted packets, the throughput of the backlogged packets, and the delay of backlogged packets. Our proposed network contains three types of IoT devices: type 1 IoT devices (COVID-19 IoT devices), consumer type 2 IoT devices, and consumer type 3 IoT devices. COVID-19 IoT devices have a higher priority over other types of IoT devices, and also the consumer type 2 IoT devices have priority over type 3 IoT devices. The devices are arranged in a star fashion in the network. All the devices transmit with the same generation probability ($p_{g_1} = p_{g_2} = p_{g_3}$) and re-transmit with the same probability ($p_{r_1} = p_{r_2} = p_{r_3}$). The following figures plot the performance evaluation of the metrics studied with variation in the number of type 3 IoT devices $M_3$ (from 0 to 10) and with a fixed number of both COVID-19 IoT devices and type 2 IoT devices ($M_1 = M_2 = 5$). We vary the number of consumer IoT devices to investigate the following two cases:

- **First case:** COVID-19 IoT devices are more than consumer IoT devices.
- **Second case:** COVID-19 IoT devices are fewer than consumer IoT devices.

In Fig. 2, we plot the normalized throughput of transmitted packets $Th_{covid}$, $Th_{type2}$ and $Th_{type3}$ versus $M_3$. This figure demonstrates that of the three types of IoT devices under consideration, the COVID-19 IoT devices have the best throughput performance. Around the start point, about $1 < M_3 < 4$, the COVID-19 IoT devices outperform the type 2 IoT devices by a factor of 14.15 and the type 3 IoT devices by a factor of 16.00. This is due to the fact that COVID-19 IoT devices have a priority over other types of IoT devices. At small traffic, i.e., $0 < M_3 < 5$, type 2 IoT devices have almost better throughput than type 3 IoT devices. We also report a slight decrease for $Th_{type2}$ when $3 < M_3 < 8$. And when $M_3 > 8$, type 2 IoT devices have nearly the same throughput as type 3 IoT devices, due to the increased number of type 3 IoT devices.

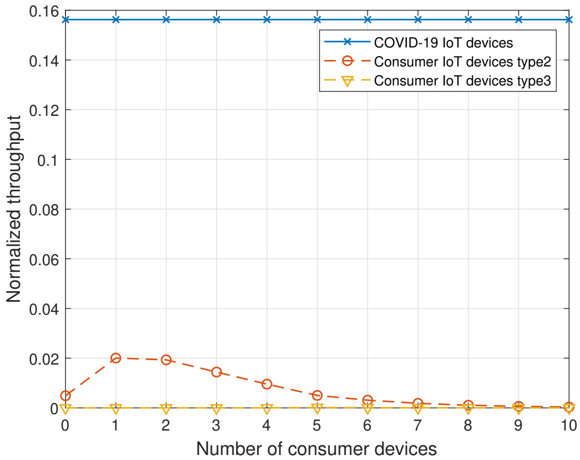


**Figure. 2**: Normalized throughput of transmitted packets versus network size.

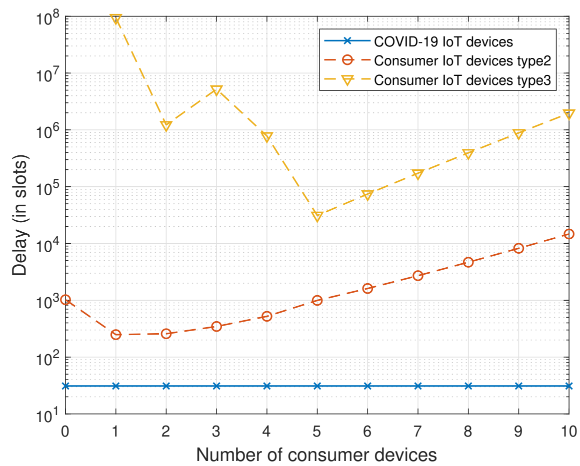


**Figure. 3**: Average delay of transmitted packets versus network size.

Fig. 3 depicts the delay of transmitted packets, $De_{covid}$ $De_{type2}$, and $De_{type3}$, as a function of the number of type 3 IoT devices $M_3$. This figure gives information about the data transmission delay, which is crucial for IoT devices used to contain COVID-19. With the increase of $M_3$, both of $De_{type2}$ and $De_{type3}$ increase, while $De_{covid}$ remains constant. The obtained results are similar to those expected, that is, COVID-19 IoT devices with higher priority have a lower delay in accessing the channel than both the type 2 and type 3 IoT devices with lower priority. That can be explained as follows, the channel access will be faster for COVID-19 IoT devices during a transmission compared with the other types of IoT devices.

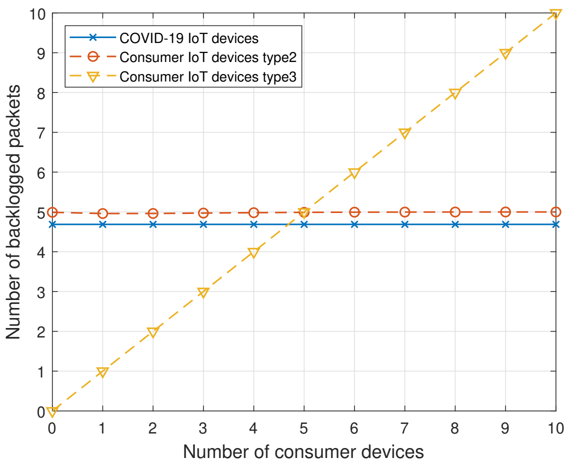


**Figure. 4**: Average number of backlogged packets versus network size.

Fig. 4 plots the average number of backlogged packets $N_{Bcovid}$, $N_{Btype2}$ and $N_{Btype3}$ versus $M_3$. We report that the number of backlogged type 3 devices increases linearly, with the increase of $M_3$ which is due to the reason that this type of devices has lower priority (all type 3 devices will be backlogged). Also, we can see that, the number of backlogged COVID-19 IoT devices much less than both the number of backlogged type 2 and type 3 IoT devices. That can be explained as follows, a COVID-19 packet gets to collide only with a packet of the same type. While type 3 packets gets to collide with packets of other types.

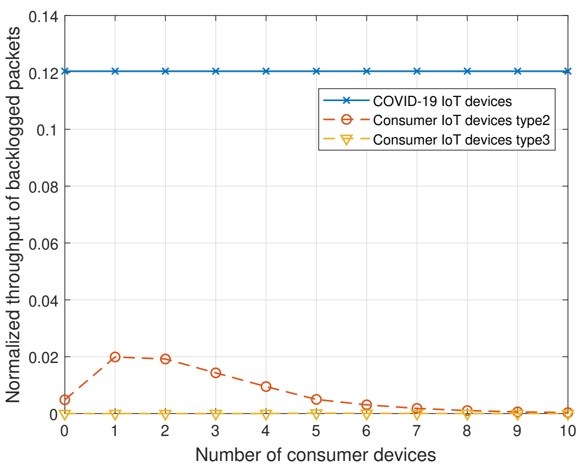


**Figure. 5**: Average throughput of backlogged packets versus network size.

Next we plot in Fig. 5 the throughput of backlogged packets, $Th_{Bcovid}$ $Th_{Btype2}$, and $Th_{Btype3}$, as a function of the number of type 3 IoT devices $M_3$. We observe that COVID-19 IoT devices perform better than other types of IoT devices in terms of the throughput of backlogged packets. This is due to the fact that backlogged COVID-19 devices have a priority over other types during the retransmission process. Also, we can observe that when $0 < M_3 < 5$ the throughput of the backlogged type 2 packets is significantly better than that of backlogged type 3 packets. But when load is very high: $M_3 > 8$, almost the same throughput for both type 2 and type 3 of IoT devices are obtained.

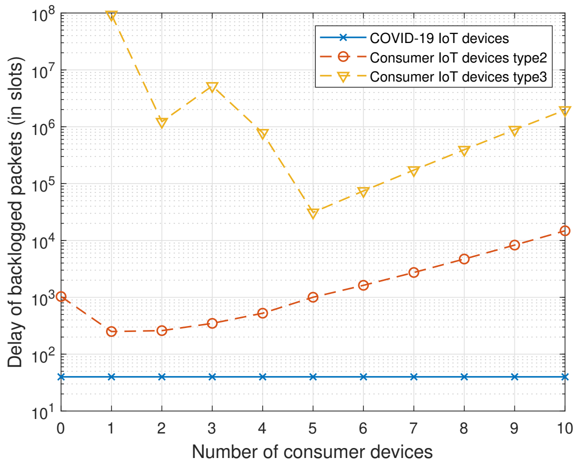


**Figure. 6**: Average delay of backlogged packets versus network size.

Lastly, Fig. 6 plots the delay of backlogged packets $De_{Bcovid}$, $De_{Btype2}$ and $De_{Btype3}$ versus $M_3$. As the COVID-19 packet can get to collide with another packet of the same type, the delay of backlogged packets is a really important factor to analyze the applicability of our approach. We remark the delay of backlogged packets of both type 2 and type 3 of IoT devices increases as the number of consumer IoT devices increases but it remains constant for COVID-19 IoT devices ($10^2$ slots). As can also be seen, we have a lower delay for COVID-19 IoT devices compared with other types of IoT devices. One of the major factors in higher delay for type 3 IoT devices is that the collision probability is high for that type of devices.

## VII. Conclusions

In this paper, we have proposed an approach to enhance the performance of the COVID-19 IoT devices in a scenario comprising other types of IoT devices. In such a scenario where devices compete for channel access, prioritizing COVID-19 IoT devices over other types of IoT devices can lead to significant performance improvement. We have also presented a novel analytical model based on Markov chain for modeling the proposed IoT network. Then we have obtained the analytical expressions of the normalized throughput of transmitted packets, the average delay of transmitted packets, the average throughput of the backlogged packets, and the average delay of backlogged packets for each type of IoT devices. Our numerical results showed that the performance of COVID-19 IoT devices outperforms the performance of the other types of IoT devices in terms of all performance metrics of interest. We believe that our contribution can help mitigate the spread of the COVID-19 pandemic.

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**Appendix A: Transition Probabilities** $P_{(Y_1,Y_2,Y_3),(Y_1+i,Y_2+j,Y_3+n)} =$

$$
\left\{
\begin{array}{l}
\left.
\begin{array}{l}
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)P_{g_1}(i,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(j,Y_2)P_{g_1}(i,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(j,Y_2)P_{g_1}(i,Y_1), & n=0
\end{array}\right\} j\geq 2\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(1,Y_2)P_{g_1}(i,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(1,Y_2)P_{g_1}(i,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(1,Y_2)P_{g_1}(i,Y_1), & n=0
\end{array}\right\} j=1\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(0,Y_2)P_{g_1}(i,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(0,Y_2)P_{g_1}(i,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(0,Y_2)P_{g_1}(i,Y_1), & n=0
\end{array}\right\} j=0
\end{array}
\right\} i\geq 2\\
\\
\left.
\begin{array}{l}
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(j,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(j,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=0
\end{array}\right\} j\geq 2\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(1,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(1,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(1,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=0
\end{array}\right\} j=1\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(0,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(0,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(0,Y_2)P_{g_1}(1,Y_1)[1-P_{r_1}(0,Y_1)], & n=0
\end{array}\right\} j=0
\end{array}
\right\} i=1\\
\\
\left.
\begin{array}{l}
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(1,Y_1))], & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(j,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(1,Y_1))], & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(j,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(1,Y_1))], & n=0
\end{array}\right\} j\geq 2\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(1,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(n,Y_3)P_{g_2}(1,Y_2)(1-P_{r_2}(0,Y_2))P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(1,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(1,Y_3)P_{g_2}(1,Y_2)(1-P_{r_2}(0,Y_2))P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(1,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(0,Y_3)P_{g_2}(1,Y_2)(1-P_{r_2}(0,Y_2))P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=0
\end{array}\right\} j=1\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(0,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(n,Y_3)[P_{g_2}(1,Y_2)P_{r_2}(0,Y_2)+P_{g_2}(0,Y_2)(1-P_{r_2}(1,Y_2))]P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(0,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(1,Y_3)[P_{g_2}(1,Y_2)P_{r_2}(0,Y_2)+P_{g_2}(0,Y_2)(1-P_{r_2}(0,Y_2)-P_{r_2}(1,Y_2))] & \\
\times P_{g_1}(0,Y_1)P_{r_1}(0,Y_1)+P_{g_3}(1,Y_3)(1-P_{r_3}(0,Y_3))P_{g_2}(0,Y_2)P_{r_2}(0,Y_2) & \\
\times P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(0,Y_2)[P_{g_1}(1,Y_1)P_{r_1}(0,Y_1)+P_{g_1}(0,Y_1)(1-P_{r_1}(0,Y_1)-P_{r_1}(1,Y_1))] & \\
+P_{g_3}(0,Y_3)[P_{g_2}(1,Y_2)P_{r_2}(0,Y_2)+P_{g_2}(0,Y_2)(1-P_{r_2}(0,Y_2)-P_{r_2}(1,Y_2))] & \\
\times P_{g_1}(0,Y_1)P_{r_1}(0,Y_1)+[P_{g_3}(1,Y_3)P_{r_3}(0,Y_3)+P_{g_3}(0,Y_3)(1-P_{r_3}(1,Y_3))] & \\
\times P_{g_2}(0,Y_2)P_{r_2}(0,Y_2)P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=0\\
P_{g_3}(0,Y_3)P_{r_3}(1,Y_3)P_{g_2}(0,Y_2)P_{r_2}(0,Y_2)P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=-1
\end{array}\right\} j=0\\
\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(0,Y_2)P_{r_2}(1,Y_1)P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(0,Y_2)P_{r_2}(1,Y_1)P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(0,Y_2)P_{r_2}(1,Y_1)P_{g_1}(0,Y_1)P_{r_1}(0,Y_1), & n=0
\end{array}\right\} j=-1
\end{array}
\right\} i=0\\
\\
\left.
\begin{array}{l}
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n\geq 2\\
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=1\\
P_{g_3}(n,Y_3)P_{g_2}(j,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=0
\end{array}\right\} j\geq 2\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(1,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(1,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(1,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=0
\end{array}\right\} j=1\\
\left.\begin{array}{ll}
P_{g_3}(n,Y_3)P_{g_2}(0,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n\geq 2\\
P_{g_3}(1,Y_3)P_{g_2}(0,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=1\\
P_{g_3}(0,Y_3)P_{g_2}(0,Y_2)P_{g_1}(0,Y_1)P_{r_1}(1,Y_1), & n=0
\end{array}\right\} j=0
\end{array}
\right\} i=-1
\end{array}
\right.
$$