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Article

A Fuzzy M/M/1 Queueing Model for Single-Charger Electric Vehicle Charging Stations: An α-Cut Approach

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Abstract: The growing deployment of electric vehicle (EV) charging infrastructure has renewed interest in queueing-theoretic models of charging-station congestion, yet the arrival and service rates that drive these models are rarely known with certainty in practice. This paper develops a fuzzy M/M/1 queueing model for a single-charger EV station in which the arrival rate and the charging (service) rate are represented as triangular fuzzy numbers rather than fixed constants. The α-cut method is used as the principal computational tool: each classical M/M/1 performance measure is shown to be monotone in the arrival and service rates, so that its fuzzy counterpart's α-cut is obtained in closed form by evaluating the classical expression at the corresponding extreme combination of the arrival- and service-rate α-cuts. Using this approach, we derive the α-cut representations of the fuzzy traffic intensity, the fuzzy expected number of vehicles in the system and in the queue, and the fuzzy waiting times in the system and in the queue, and we prove a fuzzy stability condition and a fuzzy form of Little's Law that holds as an exact interval identity at every membership level, without additional independence assumptions. Each theorem is shown to reduce to the corresponding classical M/M/1 formula when the fuzzy parameters collapse to crisp numbers. A numerical illustration with $\lambda = (3, 4, 5)$ vehicles/hour and $\mu = (6, 8, 10)$ vehicles/hour is computed at eleven α-levels and cross-validated against the crisp limit, together with a sensitivity analysis across four illustrative demand and service scenarios. The results show that the width of the fuzzy performance measures narrows monotonically as $\alpha \to 1$ and widens sharply as the traffic intensity approaches saturation, indicating that a single-charger station is particularly exposed to worst-case parameter combinations under the assumed uncertainty. The model and its validated α-cut formulas provide station operators with an interval-based, rather than single-point, description of expected congestion, which is directly useful for capacity and reliability planning at single-charger EV facilities.

Keywords: Fuzzy queueing theory; M/M/1 queue; electric vehicle charging; α-cut method; triangular fuzzy number; waiting time; charging station; uncertainty modelling

1 Introduction

1.1 *Electric Vehicle Charging Infrastructure*

The transition from internal-combustion-engine vehicles to electric vehicles (EVs) has accel-erated over the past decade, driven by tightening emissions regulations, government incentive programmes, and falling battery costs. This growth in the EV fleet has been accompanied by rapid, but still incomplete, deployment of public and private charging infrastructure. Unlike conventional refuelling, which typically takes only a few minutes, EV charging can occupy a charging point for anywhere between several minutes, for direct-current (DC) fast charging, and several hours, for Level 1 residential charging, and the number of charging points at a given station is generally constrained by available space, electrical capacity, and installation cost.

A particularly common and economically important configuration is the single-charger sta-tion: a residential charging point, a single-bay workplace charger, a small retail or hospitality charging point, or a backup charger at a larger facility. In such configurations, any variation in arrival intensity or in charging duration is not smoothed by pooling across multiple servers, and the station can transition from lightly loaded to heavily congested over comparatively short de-mand fluctuations. Understanding, quantifying, and planning for the resulting congestion and waiting-time behaviour of single-charger stations is therefore of direct practical importance, both to station operators seeking to size their infrastructure appropriately and to EV users who must decide whether and when to use a given charging point.

1.2 Queueing Theory

Queueing theory is the branch of applied probability concerned with the mathematical analysis of waiting-line systems, characterized by an arrival process, a service mechanism, a queue dis-cipline, and (where relevant) a system capacity and calling population. In Kendall's notation, a queueing system is described as *A/B/c*, where *A* denotes the inter-arrival time distribution, *B* the service-time distribution, and *c* the number of parallel servers; the symbol *M* denotes a Markovian (exponential) distribution. The classical M/M/1 queue — Poisson arrivals at rate λ, a single server with exponentially distributed service times at rate μ, unlimited waiting capacity, and first-come-first-served (FCFS) discipline — is the simplest non-trivial queueing model and remains a standard building block for analyzing congestion in service systems, including EV charging stations. For $\rho = \lambda/\mu < 1$ (the stability condition), the classical M/M/1 model gives closed-form expressions for the expected number of customers in the system, $L = \rho/(1 - \rho)$, the expected number in the queue, $L_q = \rho^2/(1-\rho)$, the expected time in the system, $W = 1/(\mu-\lambda)$, and the expected time in the queue, $W_q = \rho/(\mu - \lambda)$, related to one another through Little's Law, $L = \lambda W$ and $L_q = \lambda W_q$.

1.3 Uncertainty in EV Charging Systems

The classical M/M/1 model assumes that λ and μ are known precisely and remain constant over the period of analysis. For an EV charging station, this assumption is difficult to justify. The arrival rate depends on local traffic conditions, time of day, weather, and the routing decisions of EV drivers, all of which vary in ways that are only partially captured by a single fitted Poisson rate. The service (charging) rate depends on battery state-of-charge on arrival, battery chemistry and health, ambient temperature, and the specific charger technology in use, and a station's operating history is frequently too short — particularly for newly deployed stations — to support reliable statistical estimation of a stationary service-rate distribution. In such settings, arrival and service rates are more naturally described as approximately known quantities, for example "around 4 vehicles per hour, plausibly between 3 and 5," than as precisely estimated statistical parameters. Crisp queueing models, by construction, cannot represent this kind of parameter-level imprecision; they can represent randomness in the arrival and service *processes* conditional on a rate, but not uncertainty about the rate itself.

1.4 Fuzzy Queueing Models

Fuzzy set theory, introduced by Zadeh [10], provides a mathematical framework for representing this kind of imprecision through membership functions rather than through classical binary set membership or a fully specified probability distribution. A fuzzy number represents a quantity as a range of possible values, each carrying a degree of membership in $[0, 1]$, and is well suited to situations in which only an approximate or expert-elicited range is available, rather than an estimated distribution.

The integration of fuzzy set theory with queueing theory was initiated by Li and Lee [7], who proposed a general approach to queueing systems in a fuzzy environment based on Zadeh's extension principle, the possibility concept, and fuzzy Markov chains, and who presented ana-lytical results for fuzzy M/F/1 and FM/FM/1 systems. Negi and Lee [8] extended this analy-sis, combining analytical α-cut methods with simulation to study fuzzy queues more broadly. Buckley [2] developed an alternative, possibility-theory-based formulation of elementary queue-ing theory. Buckley and Qu [3] formalized the use of α-cuts for evaluating functions of fuzzy numbers, showing that for a continuous function that is monotone in each argument, the α-cut of the induced fuzzy quantity can be obtained by evaluating the function at the extreme combinations of the argument α-cuts — the central computational tool used throughout the present paper. Foundational treatments of fuzzy numbers, α-cuts, and interval arithmetic more generally are given by Kaufmann [5], Dubois and Prade [4], Zimmermann [12], and Klir and Yuan [6].

Separately from the fuzzy queueing literature, classical (crisp) queueing models have been applied directly to EV charging infrastructure. Bae and Kwasinski [1] modelled the arrival of vehicles at a rapid-charging station near a highway exit using an M/M/s queue, combined with a fluid-dynamic traffic model to forecast the time-varying arrival rate. Zhu et al. [11] used an M/M/s queueing formulation, together with a Voronoi-diagram-based service-area partition and a genetic-algorithm optimization, to plan the location and charger capacity of EV charging stations subject to a user waiting-time patience constraint. Varshney et al. [9] developed a continuous-time-Markov-chain queueing framework for EV charging stations that incorporates customer impatience, balking, feedback, and state-dependent service policies within a finite-population, multi-server setting. These studies illustrate the maturity and continuing relevance of queueing-theoretic modelling for EV charging infrastructure, but — consistent with the great majority of the EV-charging queueing literature — they retain the assumption that the arrival and/or service rate parameters entering the queueing model are precisely specified (whether as fixed constants or as fitted, stationary distributional parameters), rather than representing genuine imprecision in those rates directly.

1.5 Research Gap

Two distinct bodies of literature therefore exist largely in parallel. On one side, fuzzy queueing theory ([7, 8, 2, 3]) provides a mature methodology for representing uncertain arrival and service rates using fuzzy numbers and deriving fuzzy performance measures, but has been developed and illustrated primarily for generic single- or multi-server queues rather than for EV charging specifically. On the other side, EV-charging queueing studies ([1, 11, 9]) address the specific operational and infrastructure-planning questions relevant to charging stations, but do so using classical, non-fuzzy queueing formulations, leaving the acknowledged uncertainty in arrival and charging-rate parameters outside the mathematical model itself. To the extent that fuzzy queueing methodology has been applied to EV charging specifically, existing treatments in the broader fuzzy-queueing literature have not, to the authors' knowledge, combined a full α-cut derivation of the fuzzy M/M/1 performance measures — including an explicit, monotonicity-based proof for each measure, a fuzzy stability condition, and a fuzzy form of Little's Law

— with a numerically validated, single-charger EV charging illustration and an accompanying sensitivity analysis. This gap, at the intersection of the two literatures identified above, is the specific target of the present paper.

1.6 Research Contribution

This paper makes the following contributions:

1. Construction of a fuzzy M/M/1 queueing model for a single-charger EV charging station, with arrival and service rates represented as triangular fuzzy numbers.
2. Derivation, via the α-cut method, of closed-form lower and upper bounds for the fuzzy traffic intensity, fuzzy expected system size, fuzzy expected queue length, and fuzzy wait-ing times in the system and the queue (Theorems 5.1–5.5), each proved explicitly using interval arithmetic and monotonicity arguments.
3. A fuzzy stability condition (Theorem 5.2) stated in terms of the α-cut upper bound of the fuzzy traffic intensity.
4. A fuzzy form of Little's Law (Theorem 5.6), together with a corollary establishing that, for this model, the identity holds as an *exact* interval relation at every α-level, rather than only as an approximate or independence-based relation.
5. Mathematical validation that every derived fuzzy performance measure reduces exactly to its classical M/M/1 counterpart when $\alpha = 1$ (Remarks throughout Section 5), together with a full numerical study at eleven α-levels, an explicit machine-precision check of the fuzzy Little's Law identity, and a four-scenario sensitivity analysis with respect to the arrival and service rates.

2 Preliminaries

This section fixes the definitions and notation used throughout the paper.

Definition 2.1 (Fuzzy Set)**.** *A fuzzy set A on a universe of discourse X is characterized by a membership function* $\mu_A : X \to [0, 1]$*, where* $\mu_A(x)$ *denotes the degree to which x belongs to A. Unlike classical set membership, which is binary, fuzzy membership may take any value in* $[0, 1]$.

Definition 2.2 (Fuzzy Number)**.** *A fuzzy number is a normalized (*$\sup_x \mu_A(x) = 1$*), convex fuzzy set on* $\mathbb{R}$ *with a continuous, piecewise-smooth membership function and bounded support.*

Definition 2.3 (Triangular Fuzzy Number)**.** *A triangular fuzzy number (TFN) is denoted* $\tilde{A} = (a_1, a_2, a_3)$*, with* $a_1 \le a_2 \le a_3$*, and membership function*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The value a_2 *is the modal (most likely) value.*

Definition 2.4 (α-Cut)**.** *For $\alpha \in [0, 1]$, the α-cut of a fuzzy number $\tilde{A}$ is the crisp interval*

$$[\tilde{A}]_\alpha = \{x \in \mathbb{R} : \mu_{\tilde{A}}(x) \geq \alpha\}. \tag{2}$$

For a TFN $\tilde{A} = (a_1, a_2, a_3)$,

$$[\tilde{A}]_\alpha = [A_L(\alpha), A_U(\alpha)] = \quad a_1 + (a_2 - a_1)\alpha,\ a_3 - (a_3 - a_2)\alpha\ , \qquad 0 \leq \alpha \leq 1. \tag{3}$$

At $\alpha = 0$ the α-cut is the full support $[a_1, a_3]$; at $\alpha = 1$ it collapses to the single point $\{a_2\}$.

Definition 2.5 (Arithmetic of α-Cuts)**.** *For fuzzy numbers $\tilde{A}$, $\tilde{B}$ with α-cuts $[A_L(\alpha), A_U(\alpha)]$ and $[B_L(\alpha), B_U(\alpha)]$, and for α-cuts consisting of non-negative reals, interval addition, subtraction, multiplication, and division (division only where $0 \notin [B_L(\alpha), B_U(\alpha)]$) are defined by*

$$[\tilde{A}]_\alpha + [\tilde{B}]_\alpha = [A_L(\alpha) + B_L(\alpha), A_U(\alpha) + B_U(\alpha)], \tag{4}$$

$$[\tilde{A}]_\alpha - [\tilde{B}]_\alpha = [A_L(\alpha) - B_U(\alpha), A_U(\alpha) - B_L(\alpha)], \tag{5}$$

$$[\tilde{A}]_\alpha \cdot [\tilde{B}]_\alpha = [A_L(\alpha)B_L(\alpha), A_U(\alpha)B_U(\alpha)], \tag{6}$$

$$[\tilde{A}]_\alpha/[\tilde{B}]_\alpha = [A_L(\alpha)/B_U(\alpha), A_U(\alpha)/B_L(\alpha)], \qquad B_L(\alpha) > 0. \tag{7}$$

More generally, by the result of Buckley and Qu [3], if $f : \mathbb{R}^n \to \mathbb{R}$ is continuous and monotone in each argument, then the α-cut of $\tilde{Y} = f(\tilde{X}_1, \ldots, \tilde{X}_n)$ is obtained by evaluating f at the two extreme combinations of the argument α-cut endpoints — each argument set to its lower or upper bound according to whether f is increasing or decreasing in that argument — rather than by generic interval arithmetic applied term-by-term to an algebraically rearranged expression. This distinction matters whenever a variable appears more than once in an expression (as λ and μ each do, implicitly, in L, L_q, W, and W_q below), since naive term-by-term interval arithmetic on such expressions can produce intervals wider than the true α-cut. Section 4 verifies the required monotonicity explicitly for every performance measure used in this paper before applying this result.

Definition 2.6 (M/M/1 Queue)**.** *An M/M/1 queue has Poisson arrivals at rate $\lambda > 0$, a single server with exponentially distributed service times at rate $\mu > 0$, unlimited queue capacity, and FCFS discipline.*

Definition 2.7 (Traffic Intensity)**.** *The traffic intensity of an M/M/1 queue is $\rho = \lambda/\mu$. The queue is stable, in the sense of possessing a proper steady-state distribution, if and only if $\rho < 1$.*

Definition 2.8 (Fuzzy Arrival Rate)**.** *The fuzzy arrival rate is a triangular fuzzy number $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$, $\lambda_1 \leq \lambda_2 \leq \lambda_3$, representing the (imprecisely known) mean rate at which EVs arrive at the charging station.*

Definition 2.9 (Fuzzy Service Rate)**.** *The fuzzy service rate is a triangular fuzzy number $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$, $\mu_1 \leq \mu_2 \leq \mu_3$, representing the (imprecisely known) mean rate at which the charging point completes service.*

Definition 2.10 (Fuzzy Traffic Intensity)**.** *The fuzzy traffic intensity is $\tilde{\rho} = \tilde{\lambda}/\tilde{\mu}$, with α-cut, by Definition 2.5,*

$$[\tilde{\rho}]_\alpha = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)}\right], \qquad \mu_L(\alpha) > 0. \tag{8}$$

All definitions above are used consistently in the theorem statements and

3 Model Assumptions

The proposed model rests on the following assumptions.

1. There is a single charging server ($c = 1$).
2. In the classical (crisp) limit, EV arrivals follow a Poisson process with rate λ.
3. In the classical (crisp) limit, charging (service) times follow an exponential distribution with rate μ.
4. The charging station has a single charging point, shared by all arriving vehicles.
5. Vehicles are served on a first-come-first-served (FCFS) basis.
6. Queue capacity and the calling population are unlimited.
7. The arrival rate and the service rate are uncertain and are represented by triangular fuzzy numbers $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and $\mu = (\mu_1, \mu_2, \mu_3)$, respectively; no other source of uncertainty (e.g. in the queue discipline or in the arrival process type) is modelled.
8. Unless a different representation is separately justified for a given application, the fuzzy parameters are taken to be triangular fuzzy numbers, consistent with the definitions of Section 2.
9. The system is required to be stable at every α-level under consideration; this is verified ex-plicitly (Theorem 5.2) and checked numerically for the illustrative example of Section 5.1.
10. At every α-level, the α-cut interval endpoints used in any quotient are checked to ensure the denominator ($\mu_U(\alpha)$, $\mu_L(\alpha)$, or $\mu_U(\alpha) - \lambda_L(\alpha)$, $\mu_L(\alpha) - \lambda_U(\alpha)$, as relevant) is strictly positive; this is a restatement, in the notation of this model, of the stability requirement of Assumption 9.
    Queue capacity is taken to be unlimited (Assumption 6) because a single charging point with a physically bounded waiting area is nonetheless well approximated by an infinite-capacity queue whenever the probability of the (finite) waiting area being exceeded is negligible at the traffic intensities considered; this is verified to be the case for the illustrative parameters of Section 5.1, where $\rho_U(\alpha) < 1$ with a comfortable margin at every α-level reported.

4 Proposed Fuzzy M/M/1 EV Charging Model

Let $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ denote the fuzzy EV arrival rate and $\mu = (\mu_1, \mu_2, \mu_3)$ the fuzzy charging (service) rate at a single-charger EV station, with α-cuts

$$[\tilde{\lambda}]_\alpha = [\lambda_L(\alpha), \lambda_U(\alpha)], \qquad [\tilde{\mu}]_\alpha = [\mu_L(\alpha), \mu_U(\alpha)], \qquad 0 \leq \alpha \leq 1, \tag{9}$$

where $\lambda_L(\alpha) = \lambda_1 + (\lambda_2 - \lambda_1)\alpha$, $\lambda_U(\alpha) = \lambda_3 - (\lambda_3 - \lambda_2)\alpha$, and similarly for $\mu_L(\alpha)$, $\mu_U(\alpha)$.

Physically, $\tilde{\lambda}$ represents the imprecisely known mean arrival intensity of EVs at the station

(for example, “about 4 vehicles per hour, plausibly between 3 and 5”), and μ represents the imprecisely known mean rate at which the single charging point completes a charging session.

The fuzzy traffic intensity $\tilde{\rho} = \tilde{\lambda}/\tilde{\mu}$ has α-cut, from Definition 2.10

$$[\tilde{\rho}]_\alpha = [\rho_L(\alpha), \rho_U(\alpha)] = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)}\right]. \tag{10}$$

The correctness of using $\lambda_L(\alpha)$ paired with $\mu_U(\alpha)$ for the lower bound (and $\lambda_U(\alpha)$ paired with $\mu_L(\alpha)$ for the upper bound), rather than some other pairing of endpoints, follows because $g(\lambda, \mu) = \lambda/\mu$ is strictly increasing in λ and strictly decreasing in μ on $\lambda, \mu > 0$: $\partial g/\partial\lambda = 1/\mu > 0$ and $\partial g/\partial\mu = -\lambda/\mu^2 < 0$. By the extension-principle result of Buckley and Qu [3] (Definition 2.5), the minimum of g over the box $[\lambda_L(\alpha), \lambda_U(\alpha)] \times [\mu_L(\alpha), \mu_U(\alpha)]$ is therefore attained at $(\lambda_L(\alpha), \mu_U(\alpha))$ and the maximum at $(\lambda_U(\alpha), \mu_L(\alpha))$, which is precisely (10). This monotonicity-based justification, rather than blind substitution of lower and upper endpoints, is used throughout Section 5 for every derived performance measure.

5 Main Results

Definition 5.1 (Fuzzy Traffic Intensity)**.** *The fuzzy traffic intensity of the proposed model is* $\tilde{\rho} = \tilde{\lambda}/\tilde{\mu}$, *with α-cut as given in* (10).

Theorem 5.1 (α-Cut Representation of Fuzzy Traffic Intensity)**.** *Let* $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ *and* $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$ *be triangular fuzzy numbers with* $\lambda_1, \mu_1 > 0$. *Then the α-cut of* $\tilde{\rho} = \tilde{\lambda}/\tilde{\mu}$ *is*

$$[\tilde{\rho}]_\alpha = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)}\right]. \tag{11}$$

Proof. Write $g(\lambda, \mu) = \lambda/\mu$. On the domain $\lambda > 0$, $\mu > 0$, g is continuous, and

$$\frac{\partial g}{\partial \lambda} = \frac{1}{\mu} > 0, \qquad \frac{\partial g}{\partial \mu} = -\frac{\lambda}{\mu^2} < 0,$$

so g is strictly increasing in λ and strictly decreasing in μ. For fixed α, g is therefore minimized over the rectangle $[\lambda_L(\alpha), \lambda_U(\alpha)] \times [\mu_L(\alpha), \mu_U(\alpha)]$ at the corner $(\lambda_L(\alpha), \mu_U(\alpha))$ and maximized at $(\lambda_U(\alpha), \mu_L(\alpha))$; no interior or other boundary point can give a smaller (respectively larger) value, since g is monotone in each coordinate separately and the rectangle is a product of intervals. By Zadeh's extension principle applied to a continuous function of fuzzy arguments (as formalized for monotone functions by Buckley and Qu [3]),

$$[\tilde{\rho}]_\alpha = [\tilde{\lambda}/\tilde{\mu}]_\alpha = \left[\min_{(\lambda,\mu)\in[\lambda_L,\lambda_U]\times[\mu_L,\mu_U]} g(\lambda, \mu), \max_{(\lambda,\mu)\in[\lambda_L,\lambda_U]\times[\mu_L,\mu_U]} g(\lambda, \mu)\right] = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)}\right].$$

Remark 5.1. At $\alpha = 1$, $\lambda_L(1) = \lambda_U(1) = \lambda_2$ and $\mu_L(1) = \mu_U(1) = \mu_2$, so $[\tilde{\rho}]_1 = \{\lambda_2/\mu_2\}$, the classical crisp traffic intensity at the modal rates. This crisp-limit reduction holds for every performance measure derived below and is not repeated at each theorem.

Theorem 5.2 (Fuzzy Stability Condition)**.** *The fuzzy M/M/1 queueing system of Section 4 possesses finite α-cut performance measures at level α if and only if*

$$\rho_U(\alpha) = \frac{\lambda_U(\alpha)}{\mu_L(\alpha)} < 1. \tag{12}$$

A sufficient condition for the system to be stable at every $\alpha \in [0, 1]$ *is* $\rho_U(0) = \lambda_3/\mu_1 < 1$.

Proof. Each of the classical performance measures derived below (Theorems 5.3–5.5) is finite at a crisp parameter pair (λ, μ) if and only if $\rho = \lambda/\mu < 1$, since each measure contains a factor of $1/(1-\rho)$ or $1/(\mu-\lambda)$ that diverges as $\rho \to 1^-$ and is undefined for $\rho \geq 1$. By Theorem 5.1, the α-cut of $\tilde{\rho}$ at level α is $[\rho_L(\alpha), \rho_U(\alpha)]$, and every performance measure's α-cut upper bound is evaluated (Theorems 5.3–5.5) at the pair $(\lambda_U(\alpha), \mu_L(\alpha))$, i.e. at traffic intensity $\rho_U(\alpha)$. This upper bound is finite if and only if $\rho_U(\alpha) < 1$; since $\rho_L(\alpha) \leq \rho_U(\alpha)$ by construction, $\rho_U(\alpha) < 1$ also guarantees $\rho_L(\alpha) < 1$ and hence finiteness of the lower bound. Because $\rho_U(\alpha)$ is, by the monotonicity argument of Theorem 5.1, non-increasing in α for a triangular fuzzy number's standard α-cut construction (the endpoints $\lambda_U(\alpha)$ and $\mu_L(\alpha)$ move toward the modal values as α increases, and $\rho_U(\alpha) = \lambda_U(\alpha)/\mu_L(\alpha)$ is thus bounded above by $\rho_U(0) = \lambda_3/\mu_1$), the condition $\rho_U(0) < 1$ is sufficient for stability at every $\alpha \in [0, 1]$.

Remark 5.2. For an EV charging station, Theorem 5.2 means that stability must hold even under the least favourable but still plausible combination of parameters — the highest plausible

□

arrival rate together with the lowest plausible service rate — rather than merely at the modal (most likely) rates. A station that is stable in the classical crisp sense at its modal parameters may nonetheless be fuzzy-unstable (i.e. have $\rho_U(0) \geq 1$) if its plausible parameter range is wide enough: this is a genuinely more conservative and, for infrastructure planning, more informative requirement than the crisp stability check alone.

Theorem 5.3 (Fuzzy Expected Number of EVs in the System). *The fuzzy expected number of EVs in the system, $\tilde{L} = \tilde{\rho}/(1 - \tilde{\rho})$, has α-cut*

$$[\tilde{L}]_\alpha = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha) - \lambda_L(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha) - \lambda_U(\alpha)}\right], \qquad \mu_L(\alpha) > \lambda_U(\alpha). \tag{13}$$

Proof. Write $h(\rho) = \rho/(1 - \rho)$ for $\rho \in [0, 1)$. Then $h'(\rho) = 1/(1 - \rho)^2 > 0$, so h is strictly increasing in ρ. Since h is a strictly increasing function of the single variable ρ, and ρ has α-cut $[\rho_L(\alpha), \rho_U(\alpha)]$ by Theorem 5.1, the extension-principle result applied to the composition $h \circ g$ (with $g(\lambda, \mu) = \lambda/\mu$ as in Theorem 5.1) gives

$$[\tilde{L}]_\alpha = \left[h(\rho_L(\alpha)),\ h(\rho_U(\alpha))\right] = \left[\frac{\rho_L(\alpha)}{1 - \rho_L(\alpha)}, \frac{\rho_U(\alpha)}{1 - \rho_U(\alpha)}\right]$$

Substituting $\rho_L(\alpha) = \lambda_L(\alpha)/\mu_U(\alpha)$ and $\rho_U(\alpha) = \lambda_U(\alpha)/\mu_L(\alpha)$ and simplifying each bound as a single fraction,

$$\frac{\rho_L(\alpha)}{1 - \rho_L(\alpha)} = \frac{\lambda_L(\alpha)/\mu_U(\alpha)}{1 - \lambda_L(\alpha)/\mu_U(\alpha)} = \frac{\lambda_L(\alpha)}{\mu_U(\alpha) - \lambda_L(\alpha)}$$

and likewise for the upper bound, gives the stated result. Equivalently, and consistently, $L(\lambda, \mu) = \lambda/(\mu - \lambda)$ is directly verified to be increasing in λ ($\partial L/\partial \lambda = \mu/(\mu - \lambda)^2 > 0$) and decreasing in μ ($\partial L/\partial \mu = -\lambda/(\mu - \lambda)^2 < 0$), so the same corner pairing (λ_L, μ_U) for the lower bound and (λ_U, μ_L) for the upper bound applies by the argument of Theorem 5.1. □

Theorem 5.4 (Fuzzy Expected Number of EVs in Queue). *The fuzzy expected queue length,*

$$[\tilde{L}_q]_\alpha = \left[\frac{\lambda_L(\alpha)^2}{\mu_U(\alpha)\left(\mu_U(\alpha) - \lambda_L(\alpha)\right)}, \frac{\lambda_U(\alpha)^2}{\mu_L(\alpha)\left(\mu_L(\alpha) - \lambda_U(\alpha)\right)}\right]. \tag{14}$$

Proof. Let $k(\rho) = \rho^2/(1 - \rho)$ for $\rho \in [0, 1)$. Then

$$k'(\rho) = \frac{2\rho(1 - \rho) + \rho^2}{(1 - \rho)^2} = \frac{\rho(2 - \rho)}{(1 - \rho)^2} > 0 \quad \text{for } \rho \in (0, 1),$$

so k is strictly increasing on $\rho \in [0, 1)$. As in the proof of Theorem 5.3, monotonicity of k together with Theorem 5.1 gives $[L_q]_\alpha = [k(\rho_L(\alpha)), k(\rho_U(\alpha))]$, and substituting $\rho_L(\alpha) = \lambda_L(\alpha)/\mu_U(\alpha)$, $\rho_U(\alpha) = \lambda_U(\alpha)/\mu_L(\alpha)$ and simplifying gives the stated bounds. Equivalently, writing $L_q(\lambda, \mu) = \lambda^2/(\mu(\mu - \lambda))$, direct differentiation gives $\partial L_q/\partial \lambda = \lambda(2\mu - \lambda)/(\mu(\mu - \lambda)^2) > 0$ for $0 < \lambda < \mu$ (since then $2\mu - \lambda > \mu > 0$), and $\partial L_q/\partial \mu = -\lambda^2(2\mu - \lambda)/(\mu^2(\mu - \lambda)^2) < 0$, confirming that L_q is increasing in λ and decreasing in μ, so the corner pairing $(\lambda_L, \mu_U) \mapsto$ lower bound, $(\lambda_U, \mu_L) \mapsto$ upper bound applies as claimed.

□

Theorem 5.5 (Fuzzy Waiting Time in the System and in the Queue)**.** *The fuzzy time in the system,* $\tilde{W} = 1/(\tilde{\mu} - \tilde{\lambda})$*, and the fuzzy time in the queue,* $\tilde{W}_q = \tilde{\rho}/(\tilde{\mu} - \tilde{\lambda})$*, have α-cuts*

$$[\widetilde{W}]_\alpha = \left[\frac{1}{\mu_U(\alpha) - \lambda_L(\alpha)}, \frac{1}{\mu_L(\alpha) - \lambda_U(\alpha)}\right], \tag{15}$$

$$[\widetilde{W}_q]_\alpha = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)\ \mu_U(\alpha) - \lambda_L(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)\ \mu_L(\alpha) - \lambda_U(\alpha)}\right]. \tag{16}$$

Consequently $W_q = W - 1/\mu$ *at every α-level, and* $W \neq W_q$ *in general:* W *measures total time in the system (waiting plus charging), while* W_q *measures waiting time in the queue only, prior to charging.*

Proof. Write $W(\lambda, \mu) = 1/(\mu - \lambda)$. Then $\partial W/\partial\lambda = 1/(\mu - \lambda)^2 > 0$ and $\partial W/\partial\mu = -1/(\mu - \lambda)^2 < 0$, so W is increasing in λ and decreasing in μ; by the corner argument of Theorem 5.1, $[W]_\alpha = [W(\lambda_L(\alpha), \mu_U(\alpha)), W(\lambda_U(\alpha), \mu_L(\alpha))]$, which is the stated result.

Write $\tilde{W}_q(\lambda, \mu) = \lambda/(\mu(\mu - \lambda))$. Differentiating,

$$\frac{\partial W_q}{\partial\lambda} = \frac{1}{(\mu - \lambda)^2} > 0,$$

so W_q is increasing in λ. For the derivative with respect to μ, write $W_q = \lambda/(\mu^2 - \lambda\mu)$; then

$$\frac{\partial W_q}{\partial\mu} = \frac{-\lambda(2\mu - \lambda)}{(\mu^2 - \lambda\mu)^2} = \frac{-\lambda(2\mu - \lambda)}{\mu^2(\mu - \lambda)^2} < 0 \quad \text{for } 0 < \lambda < \mu,$$

since $2\mu - \lambda > \mu > 0$. Hence W_q is increasing in λ and decreasing in μ, and the corner argument of Theorem 5.1 gives the stated α-cut.

Finally, for any crisp pair (λ, μ) with $\lambda < \mu$, $W(\lambda, \mu) - W_q(\lambda, \mu) = \frac{1}{\mu-\lambda} - \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\mu-\lambda}{\mu(\mu-\lambda)} = \frac{1}{\mu}$, so $W = W_q + 1/\mu$ identically. Applying this identity at the corner $(\lambda_L(\alpha), \mu_U(\alpha))$ gives $W_L(\alpha) = W_{q,L}(\alpha) + 1/\mu_U(\alpha)$, and at $(\lambda_U(\alpha), \mu_L(\alpha))$ gives $W_U(\alpha) = W_{q,U}(\alpha) + 1/\mu_L(\alpha)$; since $1/\mu_U(\alpha)$ and $1/\mu_L(\alpha)$ are respectively the lower and upper bounds of $[1/\tilde{\mu}]_\alpha$ (as $1/\mu$ is decreasing in μ), this is exactly the interval identity $[\widetilde{W}]_\alpha = [\widetilde{W}_q]_\alpha + [1/\tilde{\mu}]_\alpha$, i.e. $\widetilde{W} = \widetilde{W}_q + 1/\tilde{\mu}$ at every α-level. □

Theorem 5.6 (Fuzzy Little's Law)**.** *At every* $\alpha \in [0, 1]$*, the fuzzy performance measures of Theorems 5.3–5.5 satisfy the exact interval identities*

$$L_L(\alpha) = \lambda_L(\alpha)\, W_L(\alpha),\ L_U(\alpha) = \lambda_U(\alpha)\, W_U(\alpha),\ L_{q,L}(\alpha) = \lambda_L(\alpha)\, W_{q,L}(\alpha),\ L_{q,U}(\alpha) = \lambda_U(\alpha)\, W_{q,U}(\alpha). \tag{17}$$

That is, the fuzzy Little's Law $\tilde{L} = \tilde{\lambda}\tilde{W}$ *and* $\tilde{L}_q = \tilde{\lambda}\tilde{W}_q$ *holds as an exact relation between the α-cut endpoints of* L *(resp.* L_q*) and the corresponding α-cut endpoints of* λW *(resp.* λW_q*), evaluated by ordinary (independent) interval multiplication as in Definition 2.5.*

Proof. By Theorem 5.3, $L_L(\alpha) = \lambda_L(\alpha)/(\mu_U(\alpha) - \lambda_L(\alpha))$, and by Theorem 5.5, $W_L(\alpha) = 1/(\mu_U(\alpha) - \lambda_L(\alpha))$; hence

$$\lambda_L(\alpha)\, W_L(\alpha) = \lambda_L(\alpha) \cdot \frac{1}{\mu_U(\alpha) - \lambda_L(\alpha)} = \frac{\lambda_L(\alpha)}{\mu_U(\alpha) - \lambda_L(\alpha)} = L_L(\alpha).$$

The identity $L_U(\alpha) = \lambda_U(\alpha)W_U(\alpha)$ follows identically, using $\lambda_U(\alpha)$, $\mu_L(\alpha)$ in place of $\lambda_L(\alpha)$, $\mu_U(\alpha)$. The identities for L_q and W_q follow the same substitution using Theorems 5.4 and 5.5.

Remark 5.3. The exactness in Theorem 5.6 relies on a structural feature of this specific model, and does not hold automatically for arbitrary fuzzy queueing quantities combined by ordinary (independent) interval multiplication: L and W are here derived from the *same* underlying crisp function of (λ, μ), evaluated at the *same* extreme corner of the (λ, μ) box for each bound — $\lambda_L(\alpha)$ paired with $\mu_U(\alpha)$ for the lower bound of both L and W, and $\lambda_U(\alpha)$ paired with $\mu_L(\alpha)$ for the upper bound of both. Because L and W (respectively L_q and W_q) are monotone in the same direction with respect to both λ and μ, this shared-corner structure is preserved, and the crisp identity $L = \lambda W$ carries over exactly to the interval endpoints. If two fuzzy quantities entering a product were instead derived from α-cuts of variables that are monotone in *opposite* directions relative to one another, or were otherwise not comonotonic in this sense, ordinary interval multiplication of their respective α-cuts would not in general coincide with the α-cut of their true joint extension, and could overstate the resulting interval's width.

Corollary 5.1. *Because Theorem 5.6 holds without any additional independence or comono-tonicity assumption on λ and W beyond the shared derivation described in the preceding remark, the fuzzy performance measures of Theorems 5.3–5.5 are mutually consistent under Little's Law at every α-level, and in particular at $\alpha = 1$, where the identity reduces to the classical crisp relations $L = \lambda W$ and $L_q = \lambda W_q$ at the modal parameters (λ_2, μ_2).*

5.1 Numerical Illustration

To illustrate the model of Sections 4–5, we consider a single-charger EV station with fuzzy arrival rate

$$\tilde{\lambda} = (3, 4, 5) \text{ vehicles/hour}$$

and fuzzy service rate

$$\tilde{\mu} = (6, 8, 10) \text{ vehicles/hour.}$$

These values are *illustrative and synthetic*, chosen to represent a plausible low-throughput single-charger station (residential, workplace, or backup charging) and to satisfy the fuzzy stability condition of Theorem 5.2 with a clear margin; they are not derived from field measurements at any specific station. Stability is confirmed at $\alpha = 0$: $\rho_U(0) = \lambda_3/\mu_1 = 5/6 = 0.8333 < 1$.

All computations were carried out independently in Python (using standard double-precision floating-point arithmetic) at eleven equally spaced α-levels, $\alpha = 0, 0.1, 0.2, \ldots, 1.0$. For each α-level we compute: (1) the α-cut of λ; (2) the α-cut of μ; (3) the α-cut of ρ (Theorem 5.1); (4) the α-cut of L (Theorem 5.3); (5) the α-cut of L_q (Theorem 5.4); (6) the α-cut of W; and (7) the α-cut of W_q (both from Theorem 5.5). As a further internal check, the exact interval identity of Theorem 5.6 was verified numerically at every α-level and at both bounds; the maximum absolute discrepancy between $L(\alpha)$ and $\lambda(\alpha)W(\alpha)$ (and between $L_q(\alpha)$ and $\lambda(\alpha)W_q(\alpha)$) across all 22 checks (11 α-levels × 2 bounds) was 1.8×10^{-15}, consistent with floating-point rounding error and confirming the exact identity of Theorem 5.6 to machine precision.

5.2 Tables

Table 1: Model Parameters

Parameter	Symbol	Description	Unit
Arrival rate	λ	Mean rate of EV arrivals at the station	vehicles/hour
Service rate	μ	Mean rate of charging-session completion	vehicles/hour
Traffic intensity	$\rho = \lambda/\mu$	Server utilization factor	dimensionless
System size	L	Expected number of EVs in the station	vehicles
Queue length	L_q	Expected number of EVs waiting	vehicles
System time	W	Expected total time in the station	hours (also reported in minutes)
Queue time	W_q	Expected waiting time before charging	hours (also reported in minutes)
Membership level	α	Confidence/possibility level of the α-cut	dimensionless, [0, 1]

Table 2: Fuzzy Parameters (Illustrative)

Parameter	Triangular Fuzzy Number	Interpretation
Fuzzy arrival rate $\tilde{\lambda}$	$(3, 4, 5)$ vehicles/hour	Plausibly 3 to 5, modal value 4
Fuzzy service rate $\tilde{\mu}$	$(6, 8, 10)$ vehicles/hour	Plausibly 6 to 10, modal value 8

Table 3: α-Cut Results: Traffic Intensity, System Size, and Queue Length

α	$[\lambda]_\alpha$	$[\mu]_\alpha$	$[\rho]_\alpha$	$[L]_\alpha$ (vehicles)	$[L_q]_\alpha$ (vehicles)
0.0	[3.000, 5.000]	[6.000, 10.000]	[0.3000, 0.8333]	[0.4286, 5.0000]	[0.1286, 4.1667]
0.1	[3.100, 4.900]	[6.200, 9.800]	[0.3163, 0.7903]	[0.4627, 3.7692]	[0.1464, 2.9789]
0.2	[3.200, 4.800]	[6.400, 9.600]	[0.3333, 0.7500]	[0.5000, 3.0000]	[0.1667, 2.2500]
0.3	[3.300, 4.700]	[6.600, 9.400]	[0.3511, 0.7121]	[0.5410, 2.4737]	[0.1899, 1.7616]
0.4	[3.400, 4.600]	[6.800, 9.200]	[0.3696, 0.6765]	[0.5862, 2.0909]	[0.2166, 1.4144]
0.5	[3.500, 4.500]	[7.000, 9.000]	[0.3889, 0.6429]	[0.6364, 1.8000]	[0.2475, 1.1571]
0.6	[3.600, 4.400]	[7.200, 8.800]	[0.4091, 0.6111]	[0.6923, 1.5714]	[0.2832, 0.9603]
0.7	[3.700, 4.300]	[7.400, 8.600]	[0.4302, 0.5811]	[0.7551, 1.3871]	[0.3249, 0.8060]
0.8	[3.800, 4.200]	[7.600, 8.400]	[0.4524, 0.5526]	[0.8261, 1.2353]	[0.3737, 0.6827]
0.9	[3.900, 4.100]	[7.800, 8.200]	[0.4756, 0.5256]	[0.9070, 1.1081]	[0.4314, 0.5825]
1.0	[4.000, 4.000]	[8.000, 8.000]	[0.5000, 0.5000]	[1.0000, 1.0000]	[0.5000, 0.5000]

Table 4: Waiting-Time Results (minutes)

α	$[W]_\alpha$ (minutes)	$[W_q]_\alpha$ (minutes)
0.0	[8.571, 60.000]	[2.571, 50.000]
0.1	[8.955, 46.154]	[2.833, 36.476]
0.2	[9.375, 37.500]	[3.125, 28.125]
0.3	[9.836, 31.579]	[3.453, 22.488]
0.4	[10.345, 27.273]	[3.823, 18.449]
0.5	[10.909, 24.000]	[4.242, 15.429]
0.6	[11.538, 21.429]	[4.720, 13.095]
0.7	[12.245, 19.355]	[5.268, 11.247]
0.8	[13.043, 17.647]	[5.901, 9.752]
0.9	[13.953, 16.216]	[6.636, 8.524]
1.0	[15.000, 15.000]	[7.500, 7.500]

Table 5: Sensitivity Analysis: Crisp Performance Measures under Four Illustrative Scenarios

Scenario	λ	μ	ρ	L (veh.)	L_q (veh.)	W (min)	W_q (min)
A (Low arrival)	3.0	8.0	0.3750	0.6000	0.2250	12.000	4.500
B (Moderate arrival)	4.0	8.0	0.5000	1.0000	0.5000	15.000	7.500
C (High arrival)	5.0	8.0	0.6250	1.6667	1.0417	20.000	12.500
D (Improved service)	4.0	10.0	0.4000	0.6667	0.2667	10.000	4.000

5.3 *Figures*

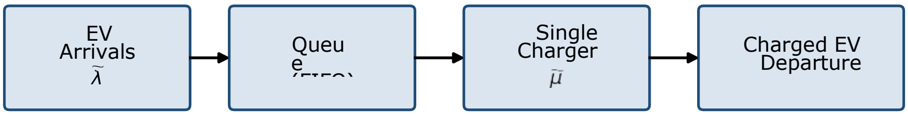

Figure 1: Schematic representation of a single-charger EV charging station: EVs arrive with fuzzy rate $\tilde{\lambda}$, join a FIFO queue, are served by a single charger at fuzzy rate $\tilde{\mu}$, and depart charged.

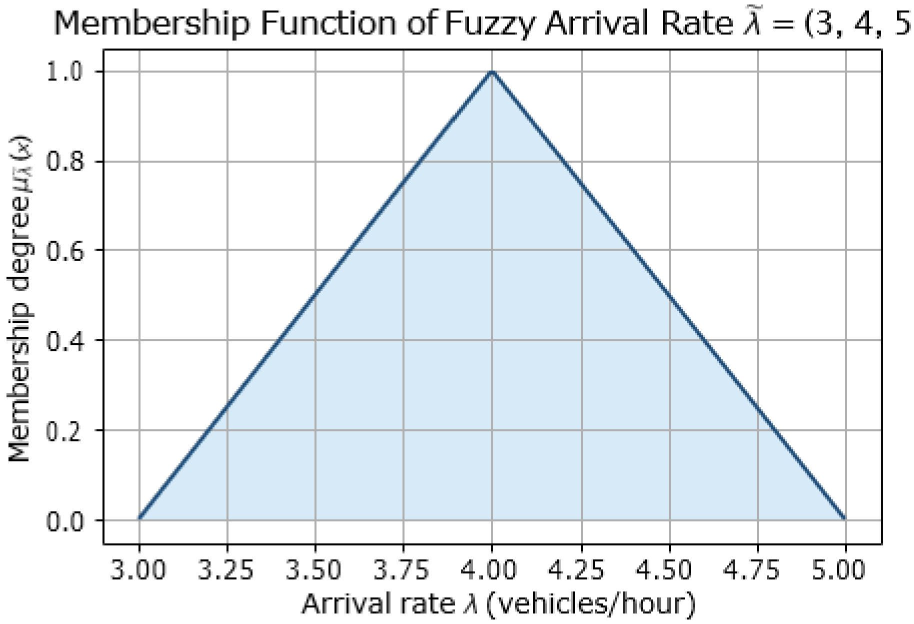

Figure 2: Membership function of the fuzzy arrival rate $\tilde{\lambda}$ = (3, 4, 5).

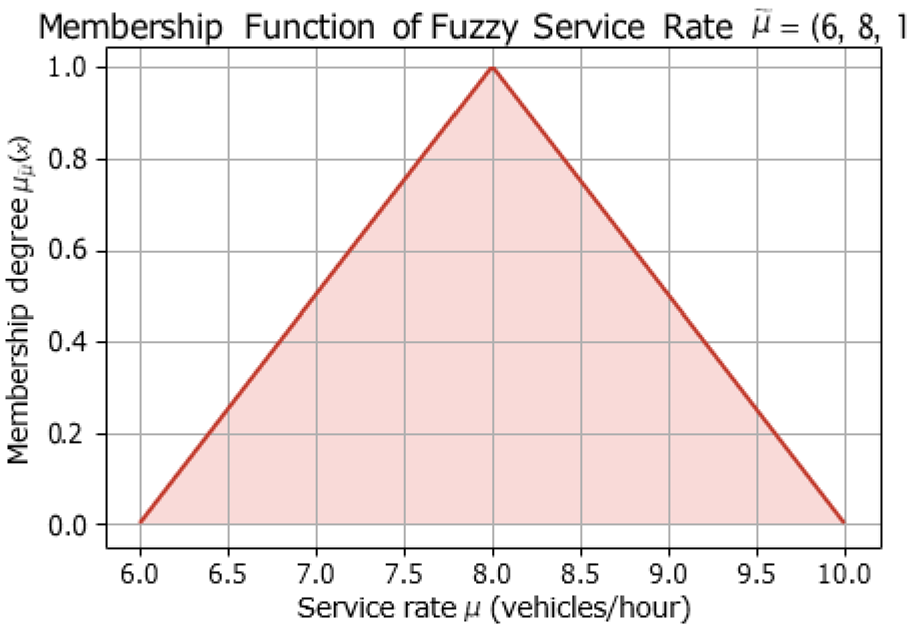

Figure 3: Membership function of the fuzzy service rate $\tilde{\mu}$ = (6, 8, 10).

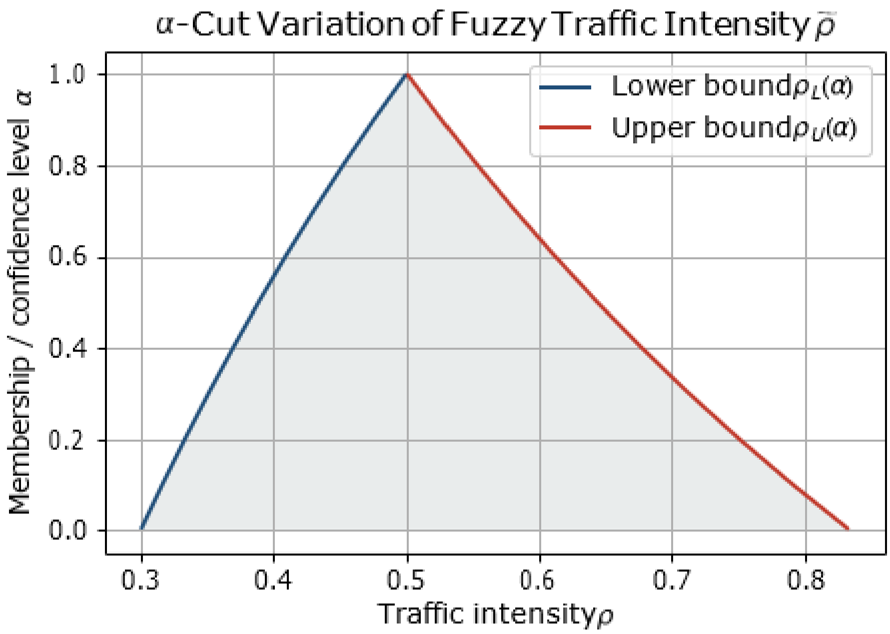

Figure 4: α-cut variation of the fuzzy traffic intensity $\tilde{\rho}$ (Theorem 5.1).

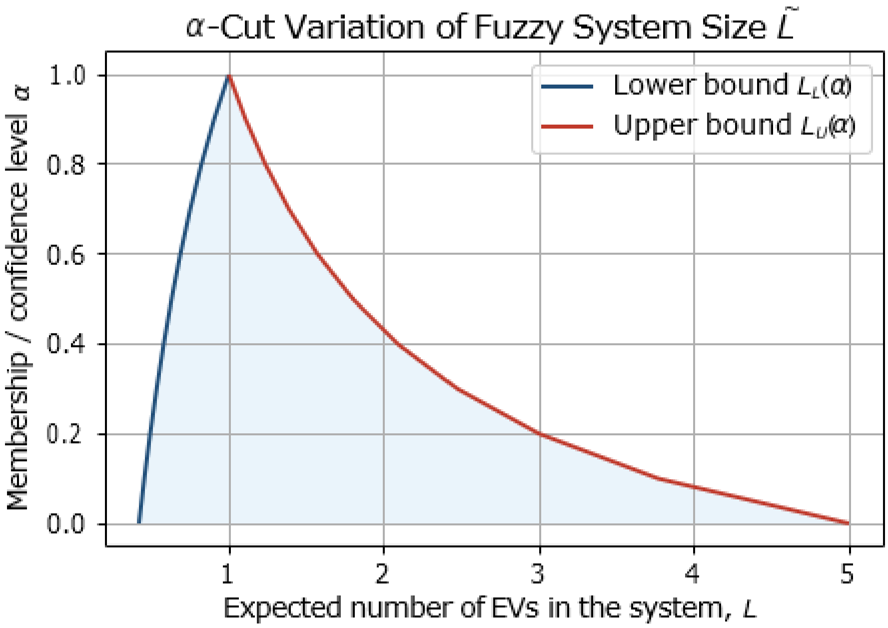

Figure 5: α-cut variation of the fuzzy expected number of EVs in the system, $\tilde{L}$ (Theorem 5.3).

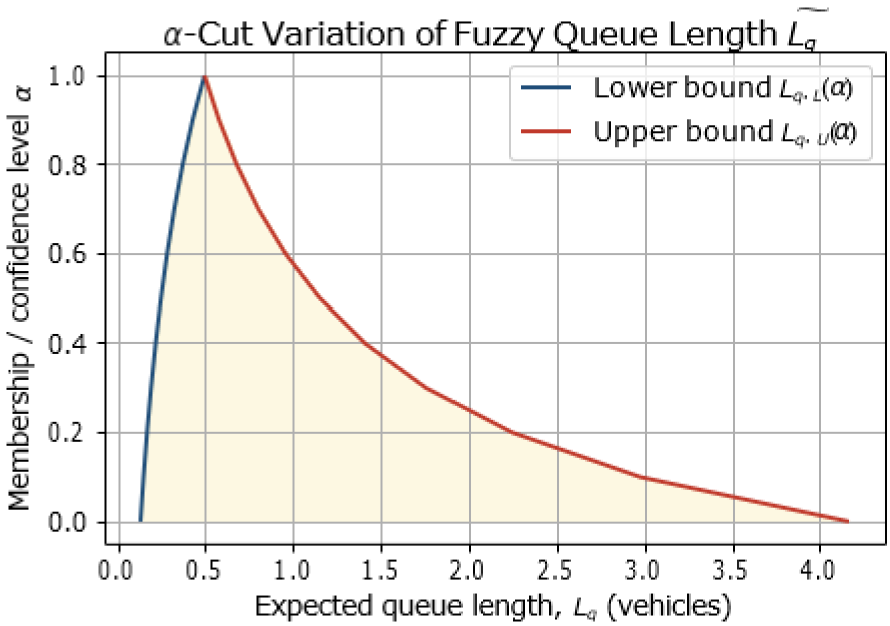

Figure 6: α-cut variation of the fuzzy expected queue length, $\tilde{L}q$ (Theorem 5.4).

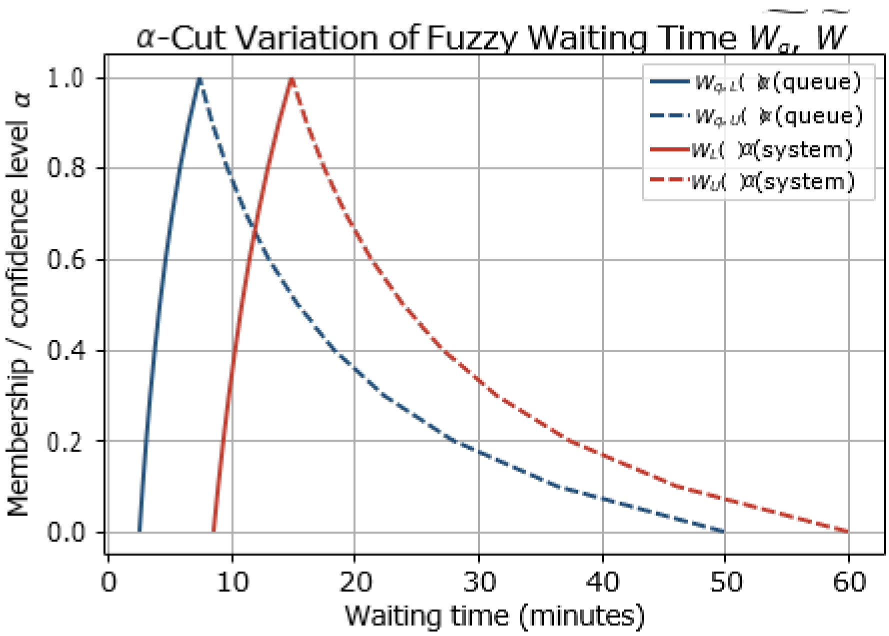

Figure 7: α-cut variation of the fuzzy waiting time in the queue, W_q, and in the system, W (Theorem 5.5).

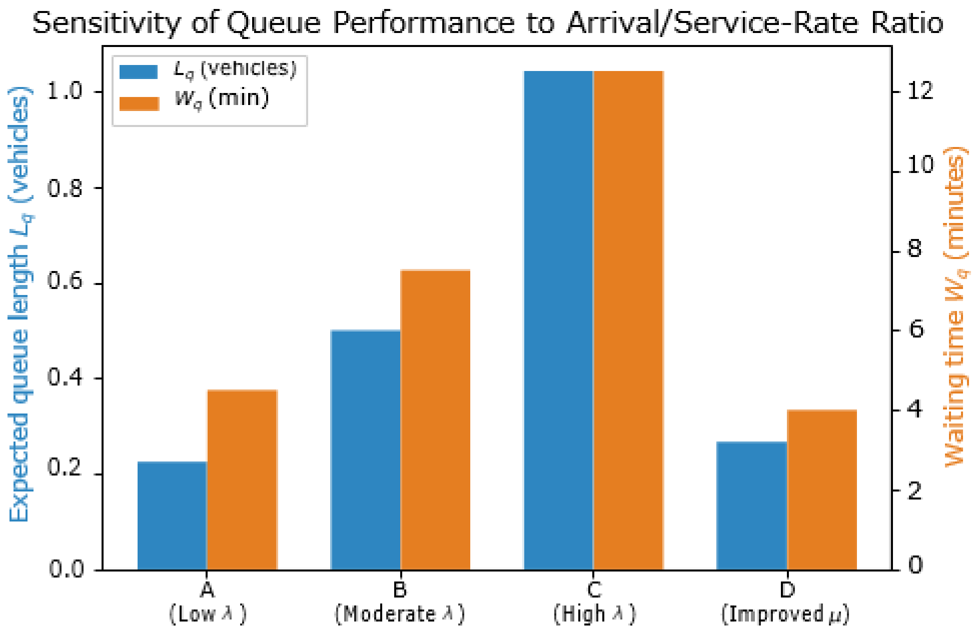

Figure 8: Sensitivity of the crisp expected queue length L_q and queue waiting time W_q across the four scenarios of Table 5 (Section 7).

6 Discussion of Results

Effect of arrival-rate uncertainty on congestion. Table 3 and Figure 4 show that the α-cut of the fuzzy traffic intensity widens from a single point $\{0.5\}$ at $\alpha = 1$ to the interval $[0.300, 0.833]$ at $\alpha = 0$. Because every downstream performance measure is an increasing function of ρ (Theorems 5.3–5.5), this widening propagates directly into L, L_q, W, and W_q: at $\alpha = 0$, the expected queue length ranges from 0.129 to 4.167 vehicles (Table 3), a spread of more than 32-fold between the lower and upper bound, compared with the single modal value of 0.5 vehicles at $\alpha = 1$. Uncertainty in the arrival rate alone — holding the service-rate uncertainty fixed

— therefore has a materially amplified effect on predicted congestion, because the M/M/1 performance measures are convex increasing functions of ρ near $\rho = 1$.

Effect of service-rate uncertainty on queue length. The same mechanism operates through μ: because L_q is decreasing in μ (Theorem 5.4), a plausible reduction in the charging rate — for example, from a battery-health or charger-degradation effect not captured by the modal service rate — pushes ρ and hence L_q upward. Table 5 quantifies this directly: moving from Scenario B (modal $\mu = 8$) to a service-rate shortfall is not separately tabulated, but the comparison between Scenario B and Scenario D (service rate raised from 8 to 10 vehicles/hour) shows that this size of service-rate change alone reduces L_q from 0.500 to 0.267 vehicles and W_q from 7.5 to 4.0 minutes, a proportionally larger change than the corresponding arrival-rate-only comparison between Scenarios A and B (L_q from 0.225 to 0.500 vehicles).

Role of the α-level. Figures 4–7 show a consistent qualitative pattern: every fuzzy perfor-mance measure's α-cut is a nested family of intervals, narrowing monotonically as α increases from 0 to 1 and collapsing to the classical crisp M/M/1 value at $\alpha = 1$. The rate at which this interval narrows is not uniform, however: because $\rho \mapsto L_q(\rho)$ is convex on $[0, 1)$ (Theorem 5.4 proof), the interval for L_q narrows comparatively slowly near $\alpha = 0$ (where ρ is close to its upper limit) and more rapidly as $\alpha \to 1$, visible in the pronounced curvature of the upper bound in Figure 6 relative to the lower bound.

Higher traffic intensity and increased waiting. The classical mechanism whereby waiting time increases sharply as $\rho \to 1^-$ is preserved, and indeed amplified, in the fuzzy model: at $\alpha = 0$ the upper bound of the fuzzy traffic intensity, $\rho_U(0) = 0.833$, is already within a comparatively narrow margin of the stability boundary $\rho = 1$, and the corresponding upper bound of W_q is 50.0 minutes — seven times the modal value of 7.5 minutes at $\alpha = 1$. This confirms, in the fuzzy setting, the well-known crisp result that $W_q = \rho/(\mu - \lambda)$ diverges as

$\tilde{\rho} \to 1$, and shows that this divergence is directly inherited by the $\alpha = 0$ upper bound whenever the fuzzy parameters admit a sufficiently high traffic intensity within their support.

Implications for charging-station operators. The fuzzy model reveals an operating envelope, rather than a single expected value, for waiting time and queue length. An operator relying solely on the modal (crisp) estimate — W_q = 7.5 minutes in the illustrative example — would not observe, from that estimate alone, that the same station could plausibly experience waiting times up to 50 minutes under a combination of parameters still consistent with the assumed uncertainty. This distinction is directly actionable: it supports setting service-level targets (e.g. a maximum acceptable $\alpha = 0$ upper-bound waiting time) rather than targets based on the modal estimate alone, and it quantifies the value of investment in charger reliability (rais-ing μ_1, the lower bound of the fuzzy service rate) relative to investment in demand-smoothing measures (lowering λ_3, the upper bound of the fuzzy arrival rate).

Single-charger configuration as a potential bottleneck. Because $c = 1$, none of the pooling benefits available to multi-server configurations are present in this model: any adverse combination of (λ, μ) within the assumed fuzzy support is directly reflected in the α-cut bounds, with no averaging across parallel servers to dampen the effect. This is visible in the comparatively wide $\alpha = 0$ intervals reported in Tables 3–4, and underscores that single-charger configurations, while economical, are structurally more exposed to worst-case parameter combinations than multi-server configurations would be under comparable relative uncertainty. **Importance of maintaining adequate service capacity.** Theorem 5.2 shows that fuzzy stability requires $\rho_U(0) < 1$, a strictly stronger requirement than crisp stability at the modal rates alone. For the illustrative parameters, $\rho_U(0) = 0.833$ leaves a comfortable but not unlim-ited margin; a station operator considering a modest increase in the plausible upper bound of demand (e.g. λ_3 rising from 5 to 5.5, all else equal) would push $\rho_U(0)$ to $5.5/6 = 0.917$, still stable but with the $\alpha = 0$ upper bound of W_q rising sharply, consistent with the convexity noted above. This illustrates concretely why service-capacity planning for single-charger sta-tions should be evaluated against the upper bound of plausible demand, not only the modal

estimate.

7 Sensitivity Analysis

Table 5 and Figure 8 report four illustrative crisp scenarios, evaluated at the classical M/M/1 formulas (i.e. at $\alpha = 1$ for the corresponding degenerate fuzzy parameters), to isolate the separate effects of arrival-rate and service-rate changes on queue performance.

Scenario A (Low arrival rate). With $\lambda = 3.0$, $\mu = 8.0$ (the lower bound of λ's support paired with the modal service rate), $\rho = 0.375$, $L_q = 0.225$ vehicles, and $W_q = 4.5$ minutes.

Scenario B (Moderate arrival rate). With $\lambda = 4.0$ (the modal value), $\mu = 8.0$, $\rho = 0.500$, $L_q = 0.500$ vehicles, and $W_q = 7.5$ minutes — the crisp baseline used as the modal ($\alpha = 1$) case throughout Sections 5.1–6.

Scenario C (High arrival rate). With $\lambda = 5.0$ (the upper bound of λ's support), $\mu = 8.0$,

$\rho = 0.625$, $L_q = 1.042$ vehicles, and $W_q = 12.5$ minutes. Moving from Scenario A to Scenario C

— a change in λ from 3.0 to 5.0, i.e. +66.7% — increases L_q by a factor of 4.63 and W_q by a factor of 2.78, illustrating the convex growth of both measures in λ established in the proofs of Theorems 5.4 and 5.5.

Scenario D (Improved service rate). With $\lambda = 4.0$ (modal), $\mu = 10.0$ (the upper bound of μ's support), $\rho = 0.400$, $L_q = 0.267$ vehicles, and $W_q = 4.0$ minutes. Relative to Scenario B, a +25% increase in μ (from 8.0 to 10.0) reduces L_q by 46.6% and W_q by 46.7%. The comparable equal−magnitude change in the arrival rate, a −25% reduction from $\lambda = 4.0$ to $\lambda =$

3.0 (Scenario A relative to Scenario B), reduces L_q by 55.0% and W_q by 40.0%. The two measures

therefore respond to equal-magnitude, opposite-direction changes in λ and μ asymmetrically, but not uniformly in one direction: queue length L_q is somewhat more responsive to the arrival-rate reduction (55.0% vs. 46.6%), while queue waiting time W_q is more responsive to the service-rate increase (46.7% vs. 40.0%). This is consistent with L_q and W_q being different (though related, via Theorem 5.5, $W_q = L_q/\lambda$) convex functions of ρ, and cautions against assuming a single, uniform asymmetry rule across all performance measures; the sign and magnitude of the effect should be checked separately for each measure of interest, using the monotonicity results of Section 5, rather than inferred from queue length alone.

Overall, the sensitivity analysis is consistent with, and numerically instantiates, the mono-tonicity results proved in Section 5: all four performance measures increase with λ and decrease with μ, and the magnitude of these effects grows as the system approaches saturation.

8 Limitations

The model and results presented here are subject to several limitations. First, the model as-sumes a single server ($c = 1$); it does not represent multi-charger stations, for which pooling effects change both the functional form of the performance measures and their sensitivity to parameter uncertainty. Second, the classical-limit arrival and service processes are assumed Poisson and exponential respectively; charging-time distributions in practice are influenced by battery chemistry, charger protocol, and state-of-charge in ways that may be better represented by general (non-exponential) service-time distributions in future work. Third, the fuzzy param-eters are represented as triangular fuzzy numbers; while this is a standard and computationally convenient choice, other membership-function shapes (e.g. trapezoidal, or empirically fitted shapes) may be more appropriate if sufficient elicitation or historical data become available. Fourth, the numerical illustration of Section 5.1 uses illustrative, synthetic parameter values rather than field-measured arrival and service-rate data from an operating charging station; the qualitative conclusions of Sections 6–7 (monotonicity, convexity, and the resulting amplification of uncertainty near saturation) are general properties of the model and do not depend on these specific values, but the numerical magnitudes reported do. Fifth, the model does not represent

possible dependence between the arrival and service processes (for example, correlated demand surges and charger strain during periods of high ambient temperature), nor does it represent time-varying (non-stationary) arrival rates, both of which are plausible features of real EV charging demand. Finally, customer behaviours such as balking (declining to join a long queue) and reneging (abandoning the queue after joining) are not modelled; these are known to be rele-vant to EV charging user behaviour and would modify the effective arrival process at high traffic intensities. The model presented here should accordingly be understood as a mathematically rigorous but stylized representation of a single-charger station, appropriate for illustrating the fuzzy α-cut methodology and for qualitative and order-of-magnitude planning insight, rather than as a validated predictive tool for any specific charging station without further calibration.

9 Conclusion

This paper has developed a fuzzy M/M/1 queueing model for single-charger electric vehicle charging stations, in which the arrival rate and the charging (service) rate are represented as triangular fuzzy numbers rather than fixed constants, and in which the α-cut method is used, together with an explicit monotonicity argument for each performance measure, to derive closed-form lower and upper bounds for the fuzzy traffic intensity, fuzzy expected system size, fuzzy expected queue length, and fuzzy waiting times in the system and the queue (Theorems 5.1, 5.3, 5.4, and 5.5). A fuzzy stability condition was derived (Theorem 5.2), requiring that the $\alpha = 0$ upper bound of the traffic intensity remain below unity — a strictly stronger requirement than crisp stability at the modal parameters alone. A fuzzy form of Little's Law was proved (Theorem 5.6) to hold as an exact interval identity at every membership level, a consequence of the shared extreme-corner structure underlying the derivation of L, L_q, W , and W_q, rather than an assumption imposed separately.

Each theorem was verified to reduce exactly to the corresponding classical M/M/1 formula at $\alpha = 1$, and the model was illustrated numerically for a single-charger station with fuzzy arrival rate $\lambda = (3, 4, 5)$ vehicles/hour and fuzzy service rate $\mu = (6, 8, 10)$ vehicles/hour, com-puted at eleven α-levels and cross-checked against the fuzzy Little's Law identity to machine precision. The numerical results show that uncertainty in the underlying parameters is ampli-fied, rather than merely propagated linearly, into the derived performance measures as traffic intensity approaches the stability boundary, and a four-scenario sensitivity analysis quantified the differential effect of arrival-rate and service-rate changes on queue length and waiting time. From a practical standpoint, the model offers EV charging-station operators an interval-based description of expected congestion — a full plausible operating envelope rather than a single point estimate — which is directly relevant to setting service-level targets, evaluating charger-reliability investment against demand-management measures, and assessing whether a given single-charger configuration provides an adequate stability margin under realistic param-

eter uncertainty, rather than only at the most likely (modal) operating point.

Several directions for future research follow naturally from the limitations noted in Section 8. These include extending the α-cut methodology to fuzzy M/M/c models for multi-charger sta-tions; developing fuzzy finite-capacity queueing models appropriate for stations with physically bounded waiting areas; incorporating time-dependent (non-stationary) fuzzy arrival rates to capture diurnal and seasonal demand patterns; extending the model to renewable-energy-powered charging stations, where the fuzzy service rate may itself depend on a further source of uncertainty (energy availability); developing hybrid stochastic-fuzzy models that combine probabilistic arrival/service processes with fuzzy parameter uncertainty within a single frame-work; calibrating and validating the model against real EV charging-station datasets as they become available; and using the derived fuzzy performance measures as the basis for formal optimization of charging-station capacity under uncertainty.

2020 Mathematics Subject Classification: 60K25; 90B22; 03E72; 94D05

Rationale. 60K25 (Queueing theory, aspects of probability theory) is relevant because the underlying classical model is a continuous-time M/M/1 queue. 90B22 (Queues and service systems, in operations research) is relevant because the paper's motivation and application are the operational planning of a service facility. 03E72 (Theory of fuzzy sets, fuzzy relations, and fuzzy set operations) is relevant because the arrival and service rates are represented and manipulated as fuzzy numbers via α-cuts. 94D05 (Fuzzy sets and logic, in connection with information, communication, or circuit theory) is relevant because the queueing system studied is, in essence, an information/service-flow system analyzed under fuzzy uncertainty.

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