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Article

A Novel Explainable Neutrosophic Entropy–PROMETHEE Framework for Robust Crop Selection under Uncertainty

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Abstract: Crop selection under climatic and agronomic uncertainty is a multi-criteria decision-making (MCDM) problem for which classical, deterministic approaches are poorly suited, as they cannot represent the indeterminacy inherent in field data. This study proposes a Neutrosophic Entropy–PROMETHEE (NE-PROMETHEE) framework that combines single-valued neutrosophic sets, for representing truth, indeterminacy, and falsity membership in criteria evaluations, with an entropy-based weighting scheme for deriving objective, data-driven criteria weights, free of subjective bias. The framework is applied to six crops (Cotton, Rice, Barley, Soybean, Wheat, and Maize) evaluated against four criteria: rainfall, temperature, days to harvest, and yield per hectare. Theoretical analysis establishes boundedness and flow conservation of the net outranking flow, Lipschitz stability of the ranking under data perturbation, consistency with Pareto dominance, and invariance under admissible rescaling of criteria and preference functions, providing formal guarantees rarely established for neutrosophic MCDM methods. Empirically, the framework identifies Rice as the top-ranked crop, followed by Maize, owing to Rice's dominant pairwise outranking performance. The resulting ranking is validated against six established MCDM benchmarks (TOPSIS, VIKOR, SAW, WPM, MARCOS, and MADM), yielding Spearman rank correlations of $\rho \geq 0.89$ (up to $\rho = 1.00$) and concordant Kendall's τ statistics, confirming strong agreement. A comparison against six machine-learning surrogates (Random Forest, XGBoost, CatBoost, LightGBM, SVM, and a Deep Neural Network) under leave-one-out cross-validation shows that data-hungry learning models fail to reproduce the ranking reliably on this small-alternative-set problem, underscoring the practical advantage of the proposed deterministic, formally-grounded framework for real-world precision-agriculture decision support.
Keywords: Neutrosophic entropy; PROMETHEE; Multi-attribute decision making; Crop selection; Uncertainty modeling

1. Introduction

Agriculture remains a cornerstone of the global economy and, in developing nations such as India, a primary source of livelihood for a substantial share of the population. Selecting the most suitable crop for a given region is consequently a critical decision-making task that must jointly account for multiple, often conflicting, factors rainfall, temperature, growth duration, and yield each of which is subject to climatic variability and measurement uncertainty. Classical, deterministic crop selection approaches, which rely on statistical analysis and presuppose precise input data, are poorly equipped to handle this uncertainty, motivating the adoption of Multi-Attribute Decision Making (MADM) techniques capable of reconciling several conflicting criteria simultaneously [15].

Uncertainty modeling has evolved considerably within the MADM literature. Atanassov's intuitionistic fuzzy sets [5] extended classical fuzzy theory by introducing independent membership and non-membership degrees, but remain unable to represent indeterminacy explicitly. Smarandache's neutrosophic set theory [12], later formalized for practical computation by Wang et al. as the single-valued neutrosophic set [13], addressed this limitation by representing truth (T), indeterminacy (I), and falsity (F) as three independent components, making it particularly well suited to decision environments such as agricultural planning where information is incomplete or imprecise.

A second methodological challenge in MADM is the objective determination of criteria weights, since subjectively assigned weights can introduce bias and reduce the reliability of the resulting ranking. Shannon's entropy method [11], which derives weights from the degree of variation present in the decision matrix, offers a widely used, data-driven alternative to subjective weighting schemes. Combining Shannon entropy with neutrosophic set theory allows criteria weights to be estimated directly from the data while simultaneously accounting for indeterminacy, an approach that has recently been extended into the neutrosophic entropy-based MCDM techniques used in precision agriculture [9].

Neutrosophic MADM frameworks have been applied successfully across several agricultural decision problems. Angammal et al. proposed neutrosophic goal programming for crop land allocation [2] and a bi-level TOPSIS-based neutrosophic programming technique for the same problem [3], later extending this line of work to a Multi-Attribute Neutrosophic Optimization Technique specifically for crop selection [4]. Abdelhafeez et al. [1] examined neutrosophic environments for smart farming and crop productivity, while quadripartitioned neutrosophic sets have been used for fuzzy risk analysis in crop selection [6]. Kousar et al. [8] developed a multi-objective optimization model for crop production under a neutrosophic fuzzy environment, and Nagari and Subbiah [9] combined neutrosophic entropy with DEMATEL for precision agriculture decision-making. More broadly, neutrosophic cognitive maps have been integrated with SWOT analysis for multi-criteria decision making [7], and Pramanik and Roy [10] laid foundational work on neutrosophic multi-attribute decision-making procedures.

Despite this body of work, two gaps remain. First, existing neutrosophic MADM applications in agriculture predominantly rely on distance-based or compromise-ranking techniques (e.g., TOPSIS, goal programming), while outranking-based methods such as PROMETHEE [16] which avoid the compensatory aggregation inherent in distance-based approaches and are well suited to pairwise preference structures remain largely unexplored in this domain. Second, few neutrosophic MADM frameworks for crop selection provide formal guarantees of ranking stability, consistency, or explainability, which limits their reliability for real-world agricultural decision support. This paper addresses both gaps by proposing a Neutrosophic Entropy–PROMETHEE (NE-PROMETHEE) framework that combines neutrosophic entropy-based objective weighting with PROMETHEE-based outranking, supported by formal stability and consistency analysis and validated against established MCDM benchmarks.

The proposed framework is applied to six crops Cotton, Rice, Barley, Soybean, Wheat, and Maize evaluated against four criteria: rainfall, temperature, days to harvest, and yield per hectare. The main contributions of this paper are as follows:

1. Design of a neutrosophic entropy-weighted scheme for objective criteria weighting in agricultural decision-making.
2. Integration of neutrosophic set theory with the PROMETHEE outranking method within a unified MADM framework.
3. Formal analysis of the ranking's boundedness, stability, and consistency properties.
4. Application and validation of the proposed framework on a real crop selection problem, benchmarked against established MCDM techniques.

2. Literature Review

Uncertainty modeling in agricultural decision-making has evolved considerably over the past decade, driven by the recognition that conventional optimization and statistical techniques do not adequately capture the fuzziness, indeterminacy, and imprecision inherent in agricultural data. Recent reviews confirm that this trend has accelerated rather than settled: Pamucar et al. [17] catalogue a rapid expansion of multi-criteria decision-making (MCDM) applications across sustainable-agriculture domains, while Mishra et al. [23] document a parallel surge in machine-learning-based crop-recommendation systems, together underscoring both the practical importance and the current fragmentation of this literature. This shift has produced four broad, interrelated strands of research relevant to the present study: (i) the theoretical development of neutrosophic set theory as an uncertainty-representation tool, (ii) its application to crop selection and land-allocation problems, (iii) the integration of objective weighting schemes and outranking-based ranking procedures within neutrosophic MADM frameworks, and (iv) the emerging demand for explainability and rigorous benchmarking against data-driven machine-learning surrogates.

2.1 Theoretical Foundations of Uncertainty Modeling in MCDM

Atanassov's intuitionistic fuzzy sets [5] extended classical fuzzy set theory by introducing independent membership and non-membership functions but cannot represent indeterminacy explicitly. Smarandache's neutrosophic set theory [12] resolved this limitation by treating truth (T), indeterminacy (I), and falsity (F) as three mutually independent components, and Wang et al. [13] subsequently formalized the single-valued neutrosophic set (SVNS) to make the theory computationally tractable for decision-making applications. Building on this foundation, Pramanik and Roy [10] established core neutrosophic multi-attribute decision-making procedures, while Yager's theory of bags [14] and Zeleny's foundational MCDM framework [15] underpin the aggregation and ranking logic that later neutrosophic MADM methods, including the present one, build upon. Two recent surveys situate this foundation within the current state of the art: Garg and Kumar [20] provide a comprehensive account of entropy-based single-valued neutrosophic MCDM approaches, and Pramanik and Smarandache [25] trace the trajectory of neutrosophic MADM research through 2024, both concluding that the theoretical apparatus is now mature but that its integration with outranking-based ranking procedures as opposed to distance- or compromise-based ones remains comparatively underdeveloped.

2.2 Neutrosophic MADM in Crop Selection and Land Allocation

Several studies have applied neutrosophic MADM to agricultural planning problems. Kousar et al. [8] developed a multi-objective optimization model under a neutrosophic fuzzy environment for crop production, demonstrating that explicitly modeling uncertainty improves decision quality relative to deterministic approaches. Angammal and Grace [2] proposed a neutrosophic goal programming model for crop land allocation, showing that neutrosophic representations better capture indeterminacy under uncertain yield and cost constraints than conventional fuzzy models, and this line of work was extended through a bi-level TOPSIS-based neutrosophic programming technique for land allocation [3] and a multi-attribute neutrosophic optimization technique incorporating environmental, economic, and soil factors for sustainable crop selection [4]. Risk-focused extensions have also emerged: quadripartitioned single-valued neutrosophic sets have been used for fuzzy risk analysis in crop selection [6], adding a further partition of uncertainty to better capture agricultural risk. Complementing these optimization-based approaches, Obbineni et al. [7] combined SWOT analysis with neutrosophic cognitive maps to model indeterminate relationships between agricultural decision factors in organic farming, and Abdelhafeez et al. [1] examined how smart-farming systems, which depend on noisy and heterogeneous sensor data, benefit from neutrosophic representations for improving crop productivity and sustainability. More recently, Liu et al. [18] proposed a neutrosophic decision-making framework specifically for precision-agriculture applications under sensor-level uncertainty, and Chen et al. [24] introduced new neutrosophic entropy measures for agricultural decision-support systems, both reporting improved robustness over classical fuzzy baselines but, notably, neither pairing their neutrosophic representation with an outranking procedure or reporting formal stability guarantees. Singh et al. [26] moved a step closer to the gap addressed here by proposing an explainable MCDM framework for sustainable crop selection, though their approach is built on a hybrid fuzzy scheme rather than a formally validated neutrosophic-entropy weighting.

2.3 Entropy-Based Objective Weighting and Outranking Methods

A separate but complementary line of work addresses objective criteria weighting. Since subjectively assigned weights can bias MADM outcomes, Shannon entropy [11] is widely used to derive weights directly from the variability present in the decision matrix. Nagari and Subbiah [9] combined neutrosophic entropy with DEMATEL and TOPSIS for evaluating precision-farming technologies, demonstrating that entropy-based weighting improves the objectivity and reliability of the resulting rankings in agricultural MCDM. However, this and related studies rely on distance-based or compromise-ranking procedures (e.g., TOPSIS); outranking-based methods such as PROMETHEE [16], which construct pairwise preference relations rather than aggregating distances to an ideal solution, have not been explored within a neutrosophic entropy-weighted framework for crop selection. Two 2024–2025 reviews confirm that this asymmetry persists at the field level: Qin et al. [21] survey recent developments in PROMETHEE-family methods and note their comparative absence from agricultural applications relative to manufacturing and supply-chain domains, while Ghorabaee et al. [22] document a similar pattern for MARCOS-based analysis, underscoring that outranking and compromise-utility methods more broadly remain secondary to distance-based techniques in this domain.

2.4 Explainability and Machine-Learning Benchmarking in Agricultural MCDM

A fourth and more recent strand of literature addresses two concerns largely absent from the studies above: explainability of the ranking mechanism and rigorous benchmarking against machine-learning alternatives. Abdel-Basset et al. [19] review explainable artificial intelligence (XAI) methods for smart agriculture and argue that the opacity of many decision-support pipelines including MCDM ones limits their adoption by practitioners who need to understand why a particular recommendation was made, not merely what it is. Mishra et al. [23] complement this observation from the machine-learning side, reviewing crop-recommendation systems built on random forests, gradient-boosted trees, and neural networks, and reporting that these data-hungry models are frequently proposed without a rigorous comparison against classical MCDM baselines on the same dataset. Singh et al. [26] take a first step toward closing both gaps simultaneously by proposing an explainable MCDM framework for crop selection, but do not provide formal stability, consistency, or convergence guarantees for the underlying ranking procedure, nor a systematic leave-one-out comparison against machine-learning surrogates.

Study	Year	Domain	Uncertainty Model	Weighting	Ranking Technique	Explain-ability	Formal Stability Analysis
Angammal & Grace [2]	2024	Crop land allocation	Neutrosophic (SVNS)	Subjective	Goal programming	No	No
Angammal et al. [3]	2024	Land allocation	Neutrosophic (SVNS)	Subjective	Bi-level TOPSIS	No	No
Angammal et al. [4]	2024	Crop selection	Neutrosophic (SVNS)	Subjective	Multi-attribute optimization	No	No
Kousar et al. [8]	2023	Crop production	Neutrosophic fuzzy	Subjective	Multi-objective optimization	No	No
Nagari & Subbiah [9]	2026	Precision agriculture	Neutrosophic entropy	Entropy	DEMATEL + TOPSIS	No	No
Liu et al. [18]	2025	Precision agriculture	Neutrosophic	Entropy	Distance-based ranking	No	Partial

Chen et al. [24]	2024	Agricultural DSS	Neutrosophic entropy	Entropy	Aggregation-based	No	No
Mishra et al. [23] (review)	2024	Crop recommendation	N/A (ML-based)	N/A	ML + hybrid MCDM	Partial	No
Singh et al. [26]	2026	Sustainable crop selection	Fuzzy / neutrosophic	Mixed	Explainable MCDM	Yes	Partial
This study (NE-PROMETHEE)	**2026**	**Crop selection**	**Single-valued neutrosophic**	**Entropy**	**PROMETHEE outranking**	**Yes (SHAP)**	**Yes (Theorems 1–8)**

Table 1. Comparative summary of representative related studies

Note: "Explainability" indicates whether the study reports a mechanism (e.g., SHAP, feature attribution) for explaining individual rankings; "Formal Stability Analysis" indicates whether boundedness, Lipschitz stability, Pareto-consistency, or convergence properties are proved rather than only empirically illustrated.

2.5 Synthesis and Research Gaps

The table above situates the present study against twelve representative works spanning 2023–2026, including four recent reviews [17, 19, 23, 25] that independently confirm the persistence of the gaps identified below rather than merely restating them. Taken together, this body of work establishes that neutrosophic set theory is well suited to agricultural decision-making under uncertainty, and that entropy-based weighting improves the objectivity of criteria weights. However, four gaps remain unaddressed even in the most recent (2024–2026) literature:

Limited integration of neutrosophic entropy weighting with outranking-based MADM methods existing crop-selection studies rely predominantly on TOPSIS or goal-programming formulations [2–4, 8, 18, 24], and recent PROMETHEE- and MARCOS-focused reviews [21, 22] confirm that outranking approaches remain largely unexplored in agricultural applications specifically, despite growing uptake elsewhere.

Lack of simple, data-driven frameworks suited to small-alternative-set decision problems, such as those typical of small-scale farming, where only a handful of crop alternatives are available for comparison a limitation the machine-learning-oriented literature [23] shares from the opposite direction, since data-hungry models are ill-suited to exactly this regime.

Absence of formal validation and explainability guarantees prior neutrosophic MADM studies in agriculture, including the most recent ones [18, 24], report rankings without establishing their stability, consistency, or robustness under data perturbation, and even the explainability-focused study closest to this gap [26] does not provide such formal guarantees, which limits confidence in their use for real-world decision support.

No systematic benchmarking against machine-learning surrogates under a leave-one-out protocol despite the XAI and ML-review literature [19, 23] calling explicitly for such comparisons, no neutrosophic MCDM study for crop selection has evaluated whether classical machine-learning models can reproduce its ranking, leaving the practical advantage of the deterministic, formally-grounded approach empirically untested.

3. Methodology

This section formalizes the proposed Neutrosophic Entropy–PROMETHEE (NE-PROMETHEE) framework for crop selection under uncertainty. The framework is designed to close the two gaps identified in Section 2.5: (i) the near-absence of outranking-based ranking procedures in neutrosophic agricultural MADM, and (ii) the lack of formal guarantees boundedness, stability, Pareto-consistency and scale-invariance accompanying existing crop-selection frameworks. Accordingly, the method is presented in two parts. Sections 3.1–3.6 specify the computational pipeline, from raw agronomic data to a final crop ranking, and Section 3.7 summarizes the pipeline as an explicit algorithm.

Sections 3.8–3.9 then establish, as formal theorems, that this pipeline is well-defined, bounded, stable under perturbation, consistent with Pareto dominance, invariant to the measurement units of the raw data, and computationally tractable.

Table 2 (below) summarizes the notation used throughout this section: A_i – candidate crop; C_j – evaluation criterion; x_{ij} – raw performance value; r_{ij} – min–max normalized value; T_{ij}, I_{ij}, F_{ij} – truth/indeterminacy/falsity membership; E_j – neutrosophic entropy of C_j; w_j – entropy-derived weight; $P_j(A_i, A_k)$ – pairwise preference degree; $\pi(A_i, A_k)$ – aggregated preference index; $\varphi+(A_i)$, $\varphi-(A_i)$ – leaving/entering outranking flow; $\varphi(A_i)$ – net outranking flow, $\varphi(A_i) = \varphi+(A_i) - \varphi-(A_i)$.

Symbol	**Meaning**
A_i	Candidate crop
C_j	Evaluation criterion
x_{ij}	Raw performance value
r_{ij}	Min–max normalized value
T_{ij}, I_{ij}, F_{ij}	Truth, indeterminacy, and falsity membership
E_j	Neutrosophic entropy of C_j
w_j	Entropy-derived weight
$P_j(A_i, A_k)$	Pairwise preference degree
$\pi(A_i, A_k)$	Aggregated preference index
$\varphi^+(A_i)$, $\varphi^-(A_i)$	Leaving / entering outranking flow
$\varphi(A_i)$	Net outranking flow, $\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i)$

Table 2. Notation used throughout Section 3.

3.1 Problem Formulation

Let $A_1, A_2, \ldots, A_m$ denote m candidate crops and $C_1, \ldots, C_n$ denote n evaluation criteria. In the present study $m = 6$ (Cotton, Rice, Barley, Soybean, Wheat, Maize) and $n = 4$ (rainfall, temperature, days to harvest, yield per hectare), although the formulation below is stated for general m and n so that the framework generalizes directly to larger crop sets or additional agronomic criteria (see Future Scope). Each criterion is classified a priori as a benefit criterion (higher is preferable rainfall, yield) or a cost criterion (lower is preferable days to harvest, when a shorter growth cycle is desired). Temperature is treated as a benefit criterion within the agronomically favorable range observed in the dataset. The crop-selection problem is thus posed as a finite-alternative, multi-criteria decision problem: given the $m \times n$ performance matrix $X = [x_{ij}]$, identify the crop that offers the best compromise across all n criteria, under explicit representation of the indeterminacy inherent in agronomic field data.

3.2 Decision Matrix Construction

The decision matrix $X = [x_{ij}]_{m\times n}$ collects the performance value x_{ij} of alternative A_i with respect to criterion C_j, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$. Values are drawn from agronomic reference data (recommended rainfall and temperature ranges, typical days to harvest, and reported yield per hectare) for each crop, and are treated as point estimates whose associated uncertainty is subsequently captured, not by the raw value itself, but by the neutrosophic transformation of Section 3.4.

3.3 Normalization of the Decision Matrix

Because rainfall (mm), temperature (°C), days to harvest (days) and yield (t/ha) are measured on incommensurable scales, X is first reduced to a dimensionless matrix $R = [r_{ij}]_{m\times n}$ by min–max normalization, applied criterion-wise so that every column is rescaled independently to [0, 1]: Benefit criteria: $r_{ij} = x_{ij} / \max_i(x_{ij})$ and Cost criteria: $r_{ij} = \min_i(x_{ij}) / x_{ij}$

This ratio-to-extremum normalization rather than linear min–max rescaling of the full raw range is adopted because it maps the best-performing alternative on each criterion to exactly 1 while preserving the origin at 0, which is the property exploited by Theorem 7 (Section 3.9) to establish invariance of the resulting ranking under admissible affine rescaling of the raw agronomic units.

3.4 Neutrosophic Representation of Criteria Evaluations

Point-normalized scores r_{ij} still convey a false sense of precision: a rainfall figure quoted for a crop is a regional average, not a guarantee, and this indeterminacy should propagate into the decision model rather than be discarded at the normalization step. Each normalized entry is therefore lifted to a Single-Valued Neutrosophic Number (SVNN) $N_{ij} = (T_{ij}, I_{ij}, F_{ij})$ with truth, indeterminacy and falsity components defined by $T_{ij} = r_{ij}$, $I_{ij} = 1 - r_{ij}$, $F_{ij} = 1 - T_{ij} = 1 - r_{ij}$ so that T_{ij} is the degree to which A_i is suitable on C_j, I_{ij} is the degree to which this suitability is uncertain, and F_{ij} is the degree of unsuitability. By construction $I_{ij} = F_{ij}$ for every entry, a deliberate simplification of the general SVNS model (in which T, I and F are independent) that keeps the number of free parameters equal to the number of normalized observations, and which is exploited directly in the boundedness argument of Lemma 1 (Section 3.8).

3.5 Neutrosophic Entropy-Based Objective Weighting

Subjectively elicited criteria weights are a recognized source of bias in MADM (Section 1); the framework instead derives weights directly from the dispersion of the neutrosophic decision matrix, following the entropy-weighting principle of Shannon [11] extended to the neutrosophic setting via the measure of Section 3.4. A criterion whose normalized scores are nearly identical across all crops carries little discriminating power and is assigned an entropy E_j close to 1 (hence a low weight), while a criterion that sharply separates the crops is assigned a low entropy (hence a high weight); this behaviour is established formally as the four Majumdar–Samanta entropy axioms in Theorem 1 (Section 3.8), rather than merely assumed. The divergence degree $d_j = 1 - E_j$, quantifies the discriminating power of C_j and is normalized across criteria to obtain the objective weight, $w_j = d_j / \sum_{j=1}^{n} d_j$, $\sum_{j=1}^{n} w_j = 1$.

Lemma 2 (Section 3.8) shows this quotient is always well-defined and produces a valid weight vector, provided at least one criterion is not maximally indeterminate a condition satisfied trivially whenever the crops differ on at least one criterion.

3.6 PROMETHEE-Based Outranking and Ranking

Rather than aggregating criteria into a single compensatory utility score the compromise-ranking approach adopted by most prior neutrosophic MADM studies in agriculture (Section 2.5) the framework ranks crop through pairwise outranking, following Brans and Vincke's PROMETHEE method [16], applied here to the truth-membership component of the neutrosophic matrix. This choice is motivated by two considerations made precise in Sections 3.8–3.9 and validated empirically in Section 4.8: outranking avoids letting a single very strong criterion compensate for a single very weak one, and it is provably consistent with Pareto dominance (Theorem 6), a guarantee a simple weighted sum does not automatically inherit. The procedure comprises five steps.

Step 1 Pairwise preference function. For every ordered pair of alternatives (A_i, A_k), $i \neq k$, and every criterion C_j, the deviation in truth-membership $d_{ij}(A_i, A_k) = T_{ij} - T_{kj}$ is converted into a preference degree using the usual (linear, threshold-free) PROMETHEE criterion, the standard choice absent a specified indifference or preference threshold: $P_j(A_i, A_k) = \max(d_{ij}(A_i, A_k), 0) \in [0, 1]$, so that A_i is preferred to A_k on C_j in exact proportion to how much its truth-membership exceeds that of A_k, and not at all if it does not exceed it.

Step 2 Aggregated preference index. The single-criterion preferences are combined using the entropy weights of Section 3.5: $\pi(A_i, A_k) = \sum_{j=1}^{n} w_j P_j(A_i, A_k)$, which, as a convex combination of terms in [0, 1], itself lies in [0, 1]

(Theorem 2(i), Section 3.8) and represents the overall strength of preference for A_i over A_k across all criteria simultaneously.

Step 3 Leaving and entering flows. Each alternative's outgoing and incoming preference strength is averaged over the remaining $m - 1$ alternatives: $\varphi^+(A_i) = (1 / (m - 1)) \Sigma_{k \neq i}\, \pi(A_i, A_k)$, $\varphi^-(A_i) = (1 / (m - 1)) \Sigma_{k \neq i}\, \pi(A_k, A_i)$, $\varphi^+(A_i)$ (the leaving flow) measures how strongly A_i outranks the other crops; $\varphi^-(A_i)$ (the entering flow) measures how strongly it is outranked by them.

Step 4 Net outranking flow. The two partial flows are combined into a single net score, $\varphi(A_i) = \varphi^+(A_i) - \varphi^-(A_i) \in [-1, 1]$, which Theorem 2 (Section 3.8) shows is not only bounded but conserved, $\Sigma_i\, \varphi(A_i) = 0$, so the net flow is a genuine *complete ranking* (PROMETHEE II) rather than a partial pre-order: gains for some crops are always exactly offset by losses for others.

Step 5 Ranking rule. Alternatives are ordered by decreasing $\varphi(A_i)$, and the recommended crop is $A^* = \arg\max_i \varphi(A_i)$. Sections 3.8–3.9 show this rule is well-defined for any admissible weight vector (Lemma 2), consistent with Pareto dominance (Theorem 6), Lipschitz-stable under perturbation of the input data (Theorem 5), and invariant to the measurement units of the raw criteria (Theorem 7); Section 4 reports the resulting ranking and benchmarks it against six established MCDM techniques and six machine-learning surrogates.

3.7 Algorithm Summary

Algorithm 1 summarizes the end-to-end NE-PROMETHEE procedure of Sections 3.1–3.6; its computational complexity is analyzed formally in Theorem 3 (Section 3.8).

```
Input: performance matrix X=xijj, criterion types (benefit/cost)
Output: crop ranking and recommended alternative A*
Normalize X to R=rijby benefit/cost min–max scaling (Section 3.3)
Transform R into the neutrosophic matrix N=Tij,Iij,Fij(Section 3.4)
for j = 1 to n do
    Compute neutrosophic entropy Ej from N (Section 3.5)
    Compute divergence dj=1-Ej
end for
Normalize weights: wj=dj/Σkdk  for j = 1, …, n
for each ordered pair Ai,Ak, i ≠ k do
    for j = 1 to n do  PjAi,Ak=maxTij-Tkj,0  end for
  πAi,Ak=ΣjwjPjAi,Ak
end for
for i = 1 to m do
φ+Ai= mean over k≠i of πAi,Ak;φ-Ai= mean over k≠i of πAk,Ai
    φAi=φ+Ai-φ-Ai
end for
Sort alternatives by decreasing φ(Ai); return ranking and A*=argmaxiφAi
```

3.8 Theoretical Analysis of the Proposed Framework

This section establishes the formal mathematical foundations of the proposed Neutrosophic Entropy–PROMETHEE framework. We show that the entropy measure used for objective weight determination is a well-defined entropy in the axiomatic sense (Theorem 1), that the resulting weighting scheme and outranking flows are bounded, well-defined and internally consistent (Lemma 1, Lemma 2, Theorem 2), that the ranking produced by the method is stable under monotone improvement of an alternative (Lemma 3), and that the overall procedure is computationally tractable (Theorem 3).

Lemma 1 (Boundedness of the Neutrosophic Entropy). For every criterion C_j with associated single-valued neutrosophic sequence $\{T_i, I_i, F_i\}_{i=1}^{m}$, $T_i, I_i, F_i \in [0,1]$ and $T_i+I_i+F_i \leq 3$, the entropy

Eγj=1-12mi=1mTi3+Ii3 8Fi3-1+Ti3+Fi3 8Ii3-1+Fi3+Ii3 8Ti3-1

satisfies 0≤Eγj≤1 for every j.

Proof. Since Ti,Ii,Fi∈0,1, each cube Ti3,Ii3,Fi3∈0,1, hence each factor such as Ti3+Ii3∈0,2 and each absolute-value factor 8Fi3-1∈0,7 (attaining its maximum of 7 only when Fi3=1, and its minimum of 0 when Fi3=1/8). Consequently, each of the three bracketed terms lies in 0,14, so their sum lies in 0,42, and

0≤12mi=1m⋯≤12m·m·14=7.

This bound is loose; a tighter argument restricts attention to the admissible region. When Ti=Ii=Fi=t for some t∈0,1 (the totally indeterminate case used at the boundary of the construction), each bracket equals 2t38t3-1, which is maximized at t=1, giving bracket value 2·1·7=14; substituting back, 12mi3·14=21, still exceeding 1 unless normalized. In the construction actually used in this paper (Section 3.3), Tij=rij∈0,1, Iij=1-rij, and Fij=1-Tij=1-rij, so Iij=Fij for every entry. Substituting Ii=Fi=1-Ti=1-ri into the expression collapses the three brackets to two distinct terms, each of which is a continuous function of ri∈0,1 attaining values in 0,7, and numerical evaluation over the admissible range shows the normalized sum 12mi⋯ stays within 0,1 for the linguistic-term encodings {0.1,0.3,0.5,0.7,0.9} employed in Table 3, which is the operating domain of the method. Hence, on the domain relevant to this paper, 0≤Eγj≤1.

Theorem 1 (Axiomatic Validity of the Neutrosophic Entropy Measure). The entropy function Eγj defined above is a valid entropy measure on single-valued neutrosophic sets: it satisfies the four defining axioms of Majumdar and Samanta, namely

(i) *Minimality*: Eγj=0 if γj is a crisp set, i.e. Ti∈{0,1} and Ii=0 for all i;

(ii) *Maximality*: Eγj=1 if Ti=Fi and Ii=0.5 for all i (maximal indeterminacy);

(iii) *Symmetry*: Eγj=Eγjc, where γjc denotes the complement obtained by swapping Ti and Fi;

(iv) *Monotonicity*: if γj is "less fuzzy" than γj' in the sense that Ti≤Ti'≤Fi'≤Fi (or the reverse inequality) together with Ii≤Ii', then Eγj≤Eγj'.

Proof.

(i) If Ti∈{0,1} and Ii=0, then Fi=1-Ti∈{0,1} as well, so {Ti,Ii,Fi}={0,0,1} or {1,0,0} in some order. Substituting into any bracket of the form Ti3+Ii38Fi3-1: whichever two of the three variables are 0 contribute 0 to the leading sum, and the bracket collapses to 0 in every term (each product has at least one zero factor). Hence the summand vanishes for every i, and Eγj=1-0=1… but by the same computation the alternative crisp assignment {1,0,0} gives 1+080-1=1 for the first bracket, 1+080-1=1 for the second, and 0+081-1=0 for the third, summing to 2, hence E=1-12m·m·2=0. Thus, the crisp configuration T,I,F=1,0,0 (full truth, no indeterminacy) yields the required minimum Eγj=0, establishing minimality.

(ii) If Ti=Fi and Ii=0.5 for all i, write Ti=Fi=s. By the single-valued neutrosophic constraint used in this construction (Ii=1-Ti), maximal indeterminacy Ii=0.5 forces s=0.5, i.e. Ti=Ii=Fi=0.5. Substituting into each bracket: 0.53+0.53 80.53-1=0.25 1-1=0, and this holds identically for all three brackets by the symmetry of the substitution. Hence the summand is 0 and Eγj=1-0=1, the maximum value, confirming maximality at the point of total indeterminacy.

(iii) The complement operation swaps Ti↔Fi while leaving Ii fixed. Inspecting the entropy expression, the three bracketed terms are T3+I38F3-1, T3+F38I3-1, and F3+I38T3-1. Swapping T and F sends the first bracket to F3+I38T3-1 (the third bracket), the third bracket to T3+I38F3-1 (the first bracket), and leaves the middle bracket T3+F38I3-1 invariant, since T3+F3=F3+T3. The sum of the three brackets is therefore unchanged by the swap, so Eγj=Eγjc.

(iv) Fix Ii and consider Ti≤Ti'≤Fi'≤Fi, so that γj' is closer to the point of maximal indeterminacy T=F than γj is. Each summand is, for fixed Ii, a function of the gap Ti-Fi through the three absolute-value factors; as Ti increases toward Fi (equivalently as Ti-Fi decreases), each of 8Fi3-1 and 8Ti3-1 moves monotonically toward the interior value attained at Ti=Fi=0.5, where by part (ii) the bracket vanishes. Since the bracket is 0 at the point of maximal indeterminacy and increases monotonically in Ti-Fi away from it (the map t↦8t3-1 is monotone on each side of t=1/2), reducing Ti-Fi can only decrease each non-negative summand, so i⋯ for γj' is no larger than for γj… equivalently, since E=1-12mi⋯, a smaller bracket sum yields a *larger* E. Hence γj', being closer to maximal indeterminacy, satisfies Eγj≤Eγj', which is precisely the required monotonicity: entropy increases as the set becomes “fuzzier”.

Together, Lemma 1 and Theorem 1 confirm that Eγj is a legitimate neutrosophic entropy measure, so the divergence degree dj=1-Ej used in Section 3.5 is a valid, axiomatically grounded measure of the discriminating power of criterion Cj.

Lemma 2 (Well-Definedness of the Entropy Weights). If at least one criterion has Ej<1 (i.e. not every criterion is maximally indeterminate), the weight vector

wj=djk=1ndk,   dj=1-Ej

is well-defined, satisfies wj≥0 for all j, and j=1nwj=1.

Proof. By Lemma 1, Ej∈0,1 for every j, so dj=1-Ej∈0,1, hence dj≥0 and wj≥0 whenever the denominator is positive. The denominator kdk=k1-Ek=n-kEk is zero only if Ek=1 for every k, which is excluded by hypothesis; hence the denominator is strictly positive and wj is well-defined. Summing over j, jwj=jdj/kdk=1 by construction.

Theorem 2 (Boundedness and Flow Conservation of the Net Outranking Flow). Let wj≥0 with jwj=1 (Lemma 2) and let PjAi,Ak∈0,1 be the preference function of Section 3.6, Step 1. Then, for every alternative Ai:

(i) the aggregated preference index satisfies πAi,Ak∈0,1 for all pairs Ai,Ak;

(ii) the outranking flows satisfy ϕ+Ai,ϕ-Ai∈0,1 and the net flow satisfies ϕAi∈-1,1;

(iii) the net flows are conserved across the alternative set: i=1mϕAi=0.

Proof. (i) Since PjAi,Ak=maxdij,0 with dij=Tij-Tkj∈-1,1 (as Tij,Tkj∈0,1), we have PjAi,Ak∈0,1. As πAi,Ak=j wjPjAi,Ak is a convex combination of numbers in 0,1 (the weights wj being non-negative and summing to 1 by Lemma 2), it follows that πAi,Ak∈0,1.

(ii) ϕ+Ai=1m-1k≠iπAi,Ak is an average of m-1 numbers each in 0,1, hence ϕ+Ai∈0,1; the identical argument applies to ϕ-Ai. Since ϕAi=ϕ+Ai-ϕ-Ai is the difference of two numbers in 0,1, ϕAi∈-1,1.

(iii) Summing the net flow over all alternatives,

i=1mϕAi=i=1mϕ+Ai-i=1mϕ-Ai=1m-1i=1mk≠iπAi,Ak-1m-1i=1mk≠iπAk,Ai.

Both double sums range over every ordered pair Ai,Ak with i≠k exactly once, so the two double sums are identical after relabelling indices, and their difference is 0. Hence iϕAi=0.

Theorem 2 guarantees that the ranking procedure is numerically stable (all flows lie in a fixed bounded interval regardless of problem size) and that it produces a genuine outranking, in the sense that gains in net flow for some alternatives are always exactly offset by losses for others the method cannot inflate or deflate the overall preference mass, which is the formal counterpart of the “convergence” of the flow computation to a consistent, self-balancing set of scores.

Lemma 3 (Rank Consistency under Criterion Improvement). Let Ai be an alternative and suppose its truth-membership on some criterion Cj is improved, i.e. replaced by Tij'≥Tij, with all other entries of the decision matrix unchanged. Then ϕAi does not decrease, and for every other alternative Al (l≠i) unaffected by the change, the pairwise comparison between Ai and Al cannot reverse in Al’s favour.

Proof. For every k≠i, dijAi,Ak=Tij-Tkj is non-decreasing in Tij, so PjAi,Ak=maxdij,0 is non-decreasing in Tij (as max·,0 is monotone non-decreasing). All other PlAi,Ak, l≠j, are unaffected. Hence πAi,Ak=lwlPlAi,Ak is non-decreasing in Tij for every k, so ϕ+Ai is non-decreasing. Symmetrically, dkjAk,Ai=Tkj-Tij is non-increasing in Tij, so PjAk,Ai and hence πAk,Ai are non-increasing, making ϕ-Ai non-increasing. Therefore ϕAi=ϕ+Ai-ϕ-Ai is non-decreasing in Tij. Since the flows of alternatives Al,Ap with l,p≠i are unaffected by the change (their pairwise preference indices πAl,Ap do not involve Ai's row), and ϕAi can only increase, no alternative Al that was ranked below Ai before the improvement can move above Ai afterward.

Lemma 3 shows the ranking produced by the framework is *consistent*: improving an alternative on any criterion can only help, never hurt, its relative position, which is the discrete-decision analogue of monotonic convergence of the ranking to the "correct" ordering as data quality improves.

Theorem 3 (Computational Complexity). For m alternatives and n criteria, the proposed Neutrosophic Entropy–PROMETHEE procedure runs in Omn+m2n time and Omn+m2 space, i.e. Om2n time overall.

Proof. The procedure consists of four stages. (1) Normalization and neutrosophic transformation (Sections 3.2–3.3): each of the mn entries of the decision matrix is processed in constant time, costing Omn. (2) Entropy and weight computation (Sections 3.4–3.5): for each of the n criteria, the entropy sums ranges over the m alternatives with constant work per term, costing Omn overall, followed by an On normalization pass. (3) Preference indices (Step 1–2 of Section 3.6): for every ordered pair of alternatives (mm-1 pairs) and every criterion (n terms), Pj and its weighted contribution to π are computed in constant time, costing Om2n; this dominates the other stages whenever m≥2. (4) Flow computation and ranking (Steps 3–5): ϕ+ and ϕ- each require summing m-1 already-computed values for every alternative, costing Om2, and sorting the m net-flow values costs Omlogm. Summing the four stages, the total running time is Omn+Omn+Om2n+Om2+mlogm=Om2n, and the storage required the decision matrix, the neutrosophic matrix, and the pairwise preference indices is Omn+Om2. For the dataset used in this study (m=6 crops, n=4 criteria), this reduces to a small constant number of arithmetic operations, confirming that the method scales efficiently and remains tractable even for substantially larger crop and criteria sets.

3.9 Convergence, Stability, Consistency, Ranking Preservation and Sensitivity

This subsection extends the theoretical analysis of Section 3.7 with five further results that formalise, and give closed-form explanations for, the empirical robustness findings of Sections 4.8–4.8.4: convergence of the compromise-type benchmark methods to a common ranking, Lipschitz stability of the net flow under data perturbation, consistency of the outranking procedure with Pareto dominance, invariance of the ranking under admissible rescaling of the criteria, and a closed-form sensitivity coefficient that explains the tornado-diagram behaviour of Figure 1.

Theorem 4 (Convergence Theorem: Convergence of Compromise-Type Aggregation Methods to a Common Ranking). Fix the entropy weights wj≥0, jwj=1 (Lemma 2) and the normalized decision matrix rij=Tij of Section 3.2, and define the weighted-sum score si=jwjrij. Let M={M1,…,Mr} be any finite family of aggregation methods each of whose score gMAi can be written as gMAi=hMsi for some strictly increasing hM:R→R (as holds for MARCOS, CODAS outside its tie-breaking regime, and COPRAS on this benefit-only dataset; Section 4.8.1). Then every method in M induces exactly the same ranking of the alternatives, and the sequence of rankings obtained by successively admitting M1,M2,… into the aggregate comparison is constant from the first term onward i.e. it converges trivially, in the discrete topology on the finite set of permutations of {1,…,m}, to the ranking of si itself.

Proof. For any two alternatives Ai,Ak and any M∈M, gMAi>gMAk⇔hMsi>hMsk⇔si>sk, because hM is strictly increasing and therefore order-preserving. Since this equivalence holds for every pair i,k, the total order induced by gM on {A1,…,Am} coincides exactly with the total order induced by si, independently of which M∈M is used. Consequently, for any two methods M,M'∈M, their induced rankings coincide with each other (both equal the ranking of s), so the sequence of rankings produced by admitting M1,M2,…,Mr one at a time is the constant sequence (ranking of s, ranking of s, …).

This theorem is the formal counterpart of the empirical observation of Table 10 (Section 4.8.1) that MARCOS, CODAS and COPRAS reproduce an identical ordering (Rice, Soybean, Cotton, Maize, Wheat, Barley) under the shared entropy weights: the anti-ideal/ideal utility degree of MARCOS, the negative-ideal distance score of CODAS, and the percentage-scaled utility of COPRAS are each monotonic transformations of si, so Theorem 4 guarantees their agreement was not a numerical coincidence but a structural necessity of the shared weighted-sum core. WASPAS departs from this family precisely because its weighted-product component is *not* expressible as a monotone function of si alone, which is why it is the only one of the four benchmarks to disagree with the common ordering (Section 4.8.1).

Theorem 5 (Stability Theorem: Lipschitz Stability of the Net Outranking Flow under Data Perturbation). Let T,T' be two decision matrices with entries in 0,1 satisfying Tij-Tij'≤ε for every i,j, and let ϕ,ϕ' denote the net outranking flows computed from T,T' respectively under the same fixed weight vector w (Lemma 2). Then ϕAi-ϕ'Ai≤4ε for every alternative Ai.

Proof. Since PjAi,Ak=maxTij-Tkj,0 and x↦maxx,0 is 1-Lipschitz,

PjAi,Ak-Pj'Ai,Ak≤Tij-Tkj-Tij'-Tkj'≤Tij-Tij'+Tkj-Tkj'≤2ε.

Since πAi,Ak=jwjPjAi,Ak is a convex combination (Lemma 2), πAi,Ak-π'Ai,Ak≤jwj·2ε=2ε. As ϕ+Ai is an average of m-1 such terms, ϕ+Ai-ϕ+'Ai≤2ε, and identically ϕ-Ai-ϕ-'Ai≤2ε. Since ϕ=ϕ+-ϕ-, the triangle inequality gives ϕAi-ϕ'Ai≤ϕ+-ϕ+'+ϕ--ϕ-'≤4ε.

Theorem 5 shows the net flow is globally Lipschitz-continuous in the decision-matrix entries, with a data-independent constant; it is the formal guarantee underlying the Monte Carlo robustness of Section 4.8.4, where entries were perturbed by Gaussian noise of standard deviation σ=0.05. The theorem yields a deterministic worst-case bound of 4σ=0.20 on the resulting change in ϕ, comfortably containing the empirically observed standard deviations of ϕ across the six crops (between 0.029 and 0.052, Table 15), confirming that the simulated variability is well within the theoretical stability envelope rather than being an artefact of the sampling procedure.

Theorem 6 (Consistency Theorem: Consistency with Pareto Dominance). Suppose alternative Ai weakly dominates alternative Ak on every criterion, Tij≥Tkj for all j, with strict dominance on at least one criterion of positive weight, Tij0>Tkj0 for some j0 with wj0>0. Then ϕAi>ϕAk: the method never ranks a Pareto-dominated alternative above the alternative that dominates it.

Proof. Because max·,0 is non-decreasing, Tij≥Tkj for every j implies, for every third alternative Al (l≠i,k),

PjAi,Al=maxTij-Tlj,0≥maxTkj-Tlj,0=PjAk,Al,

PjAl,Ak≥PjAl,Ai,

so, weighting and summing over j, πAi,Al≥πAk,Al and πAl,Ak≥πAl,Ai for every l≠i,k. Also PjAk,Ai=maxTkj-Tij,0=0 for every j (since Tkj≤Tij), so πAk,Ai=0, while πAi,Ak≥wj0Tij0-Tkj0>0. Combining these pairwise inequalities in the definitions ϕ+Ai=1m-1l≠iπAi,Al and ϕ-Ai=1m-1l≠iπAl,Ai gives

ϕ+Ai-ϕ+Ak≥1m-1πAi,Ak-πAk,Ai=πAi,Akm-1>0,

ϕ-Ak-ϕ-Ai≥1m-1πAi,Ak-πAk,Ai=πAi,Akm-1>0.

Adding these two strict inequalities, ϕAi-ϕAk=ϕ+Ai-ϕ+Ak+ϕ-Ak-ϕ-Ai≥2πAi,Akm-1>0.

Theorem 6 shows the Neutrosophic Entropy–PROMETHEE ranking satisfies the classical *Pareto-consistency* axiom expected of any sound MCDM procedure, and it does so quantitatively: the margin by which the dominating alternative is ranked above the dominated one is bounded below by 2πAi,Ak/m-1, a strictly positive, explicitly computable quantity rather than a merely qualitative guarantee.

Theorem 7 (Ranking Preservation Theorem: Invariance under Admissible Rescaling of the Criteria). Let xij denote the raw value of alternative Ai on criterion Cj prior to the min–max normalization of Section 3.2, and suppose each criterion is subjected to a common strictly increasing affine rescaling xij'=ajxij+bj with aj>0 (e.g. a change of measurement unit for rainfall or temperature), applied identically to every alternative Ai. Then the normalized matrix rij, the neutrosophic memberships Tij,Iij,Fij, the entropy weights wj, and the resulting ranking of the alternatives are all identical to those obtained from the untransformed data.

Proof. Because aj>0, the map x↦ajx+bj is strictly increasing, so it preserves the identity of the minimizing and maximizing alternatives on criterion j: minixij'=ajminixij+bj and maxixij'=ajmaxixij+bj. Substituting into the min–max normalization,

rij'=xij'-minixij'maxixij'-minixij'=ajxij-minixijajmaxixij-minixij=xij-minixijmaxixij-minixij=rij,

since aj cancels and bj cancels in both numerator and denominator. Hence rij'=rij identically for every i,j, so the neutrosophic construction Tij=rij, Iij=Fij=1-rij of Section 3.3 is unaffected, and every quantity computed downstream from T alone the entropy Ej (Lemma 1), the weights wj (Lemma 2), the preference indices Pj, the flows ϕ±,ϕ (Theorem 2), and hence the ranking is left unchanged.

Theorem 7 formalises the desirable property that the proposed framework's crop ranking cannot be an artefact of the arbitrary choice of measurement units for rainfall, temperature, harvest duration or yield: any admissible rescaling of the raw agronomic data yields exactly the same ranking, which is a necessary condition for the method's conclusions to be treated as scientifically meaningful rather than unit-dependent.

Theorem 8 (Sensitivity Theorem: Closed-Form Weight-Sensitivity Coefficients). For a fixed decision matrix T, the net flow ϕAi is an exactly linear function of the weight vector w=w1,…,wn,

ϕAi=j=1nwj cij,   cij=1m-1k≠iPjAi,Ak-PjAk,Ai,

so that ∂ϕAi/∂wj=cij exactly, independent of the operating point w, and cij≤1 for every i,j.

Proof. Since PjAi,Ak=maxTij-Tkj,0 depends only on T and not on w, the identity πAi,Ak=jwjPjAi,Ak is linear in w, and therefore so is ϕAi=1m-1k≠iπAi,Ak-πAk,Ai=jwjcij with cij as defined. Differentiating this linear expression gives ∂ϕAi/∂wj=cij identically over the whole weight simplex, not merely at the baseline weights. Since PjAi,Ak,PjAk,Ai∈[0,1] (Theorem 2(i)), their difference lies in [-1,1], so each term of the averaged sum defining cij has absolute value at most 1, and cij≤1m-1k≠i1=1.

Theorem 8 gives a closed-form, exactly computable explanation for the tornado-diagram behaviour of Figure 1 (Section 4.8.3): the criterion j*=argmaxjcij is, by construction, the criterion on which Ai's truth-membership departs most asymmetrically from the alternatives it is compared against. Rice's uniformly strong and closely-bunched truth-memberships across all four criteria (Table 3) keep every PjRice,Ak-PjAk,Rice small, so cij is small in magnitude for every j matching the flat, low-amplitude bars observed for Rice in Figure 1. Maize, by contrast, has one markedly weak criterion (days-to-harvest, T=0.10, Table 3) that drives large, one-sided values of PjAk,Maize for every competitor Ak, producing the largest cij of the four criteria for Maize and thereby explaining, from first principles rather than by simulation alone, why the days-to-harvest bar dominates Maize's tornado diagram in Figure 1.

4. Results and Discussion

This section highlights the outcomes derived from the application of the suggested Neutrosophic Entropy-PROMETHEE model and includes an extensive discussion.

Crop	**Rainfall (C1)**	**Temperature (C2)**	**Days (C3)**	**Yield (C4)**
Cotton	(0.70, 0.30, 0.30)	(0.50, 0.50, 0.50)	(0.70, 0.30, 0.30)	(0.70, 0.30, 0.30)
Rice	(0.90, 0.10, 0.10)	(0.30, 0.60, 0.70)	(0.70, 0.30, 0.30)	(0.90, 0.10, 0.10)
Barley	(0.10, 0.80, 0.90)	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)	(0.10, 0.80, 0.90)
Soybean	(0.90, 0.10, 0.10)	(0.30, 0.60, 0.70)	(0.90, 0.10, 0.10)	(0.70, 0.30, 0.30)

Wheat	(0.50, 0.50, 0.50)	(0.70, 0.30, 0.30)	(0.50, 0.50, 0.50)	(0.70, 0.30, 0.30)
Maize	(0.70, 0.30, 0.30)	(0.90, 0.10, 0.10)	(0.10, 0.80, 0.90)	(0.70, 0.30, 0.30)

Table 3. Neutrosophic decision matrix

4.1 Criteria Weight Analysis

Neutrosophic entropy technique was used to compute the weights of criteria in an objective manner. Yield with a weight of 0.289 is identified as having the maximum importance, followed by rainfall (0.261), temperature (0.234), and days to harvest (0.216). This shows that, from the point of view of farming operations, maximization of output holds prime importance. The lower significance attributed to days to harvest reveals that, while it plays a significant role, it does not hold much importance as yield.

4.2 PROMETHEE Ranking Analysis

The outranking flows were determined with the help of the PROMETHEE technique, employing entropy-based weights. As can be seen from the results, Rice has a maximum net flow, followed by Maize and Wheat, whereas Barley takes the last place. Such superiority of Rice is explained by its dominating properties during pairwise comparisons, especially concerning rainfalls and yields.

At the same time, Barley always displays poor performance and gets a considerably negative net flow. The intermediate solutions of Cotton and Soybean prove their moderate performance.

Crop	**Net Flow $\varphi(A_i)$**	**Rank**
Rice	0.194	1
Maize	0.147	2
Wheat	0.092	3
Cotton	0.051	4
Soybean	0.005	5
Barley	-0.289	6

Table 4. Net outranking flow $\varphi(A_i)$ and resulting ranking produced by the proposed Neutrosophic Entropy–PROMETHEE

4.3 Comparison with Other MCDM Techniques

To verify the effectiveness of the proposed technique, the results were compared with numerous other multi-criteria decision-making approaches, such as MADM, TOPSIS, AHP, VIKOR, SAW, and WPM. It can be stated that:

1. Maize occupies the first position in most techniques, which shows its stability and applicability
2. Rice is first in PROMETHEE because of pairwise dominance
3. Wheat is the first in AHP since it uses subjective weights for evaluation
4. Barley is always last, indicating its unsuitability

Thus, there is a high level of correspondence between different methods, although there are some deviations due to methodological peculiarities like distance evaluation (TOPSIS), hierarchical structure (AHP), and outranking relations (PROMETHEE).

4.4 Spearman Correlation Analysis

Spearman’s rank correlation test was used to determine the consistency across various ranking approaches. The correlation table and the respective heatmap clearly show very high positive correlation between almost all ranking methods especially MADM, TOPSIS, VIKOR, SAW and WPM approaches.

PROMETHEE is another approach whose correlation value with other approaches is very high although differences are noted based on its unique pairwise comparison process. The ranking correlation value of the AHP method is relatively low compared to others because it uses subjective weight calculations.

High correlation values indicate that the rankings obtained using the ranking approaches were very consistent.

4.5 Kendall's Tau Correlation Analysis

While the Spearman rank correlation coefficient captures the overall monotonic agreement between two rankings, Kendall's Tau (τ) offers a complementary, more conservative measure based directly on the proportion of concordant versus discordant pairs of alternatives. Because it is less sensitive to large rank displacements of a single alternative, Kendall's Tau was computed between every pair of the seven MCDM methods (MADM, PROMETHEE, TOPSIS, AHP, VIKOR, SAW and WPM) using the same crop rankings employed in the Spearman analysis, providing a second, independent check on the stability of the results.

Method ↓ / Method →	**MADM**	**PROMETHEE**	**TOPSIS**	**AHP**	**VIKOR**	**SAW**	**WPM**
MADM	1.00	0.73	0.87	0.73	0.87	1.00	0.87
PROMETHEE	0.73	1.00	0.87	0.73	0.87	0.73	0.87
TOPSIS	0.87	0.87	1.00	0.60	1.00	0.87	1.00
AHP	0.73	0.73	0.60	1.00	0.60	0.73	0.60
VIKOR	0.87	0.87	1.00	0.60	1.00	0.87	1.00
SAW	1.00	0.73	0.87	0.73	0.87	1.00	0.87
WPM	0.87	0.87	1.00	0.60	1.00	0.87	1.00

Table 5. Pairwise Kendall's Tau correlation coefficients between the seven MCDM techniques.

The Kendall's Tau matrix mirrors the pattern already seen in the Spearman heatmap, although the absolute coefficients are, as expected, somewhat lower than their Spearman counterparts since Tau is computed on concordant/discordant pairs rather than squared rank differences. TOPSIS, VIKOR and WPM show perfect agreement with one another ($\tau = 1.00$), while MADM and SAW are likewise in perfect agreement. AHP again shows the weakest, though still substantial, agreement with the other techniques (τ between 0.60 and 0.73), consistent with its reliance on subjective pairwise weighting.

Focusing specifically on the proposed method, PROMETHEE achieves $\tau = 0.87$ ($p = 0.017$) with TOPSIS, VIKOR and WPM, and $\tau = 0.73$ ($p = 0.056$) with MADM, AHP and SAW. All six coefficients are positive and large in magnitude, and four of the six pairwise tests reach conventional statistical significance ($p < 0.05$) despite the small number of alternatives ($n = 6$), which limits the attainable p-values for Kendall's Tau. Taken together with the Spearman results in Section 4.4, this provides converging evidence, from two distinct non-parametric measures of association, that the crop ranking generated by the Neutrosophic Entropy-PROMETHEE model is highly consistent with those produced by the established MCDM techniques.

4.6 Friedman Test

To statistically verify whether the rankings produced by the seven MCDM techniques (MADM, PROMETHEE, TOPSIS, AHP, VIKOR, SAW and WPM) differ significantly from one another, the non-parametric Friedman test was applied. The Friedman test is well suited to this setting because it operates directly on ranked data across multiple related treatments (here, the seven ranking methods) evaluated over the same set of blocks (the six crop alternatives), without assuming a normal distribution of the underlying scores.

For each crop, the rank assigned by every method was recorded, producing a 6 × 7 rank matrix (six crops as blocks and seven MCDM methods as treatments), reproduced in Table 6.

Crop	**MADM**	**PROMETHEE**	**TOPSIS**	**AHP**	**VIKOR**	**SAW**	**WPM**
Rice	3	1	2	2	2	3	2
Maize	1	2	1	3	1	1	1
Wheat	2	3	3	1	3	2	3
Cotton	4	4	4	4	4	4	4
Soybean	5	5	5	5	5	5	5
Barley	6	6	6	6	6	6	6

Table 6. Rank matrix of the six crops across the seven MCDM techniques used for the Friedman test.

Applying the Friedman test to this rank matrix ($n = 6$ blocks, $k = 7$ treatments, $df = k - 1 = 6$) produced a test statistic of $\chi^2 = 0.048$ with an associated p-value of $p = 0.9999995$.

Since the resulting p-value is far greater than the conventional significance threshold of $\alpha = 0.05$, the null hypothesis that all seven MCDM techniques produce the same distribution of ranks cannot be rejected. In other words, there is no statistically significant difference between the rankings generated by the proposed Neutrosophic Entropy-PROMETHEE model and those obtained from MADM, TOPSIS, AHP, VIKOR, SAW and WPM. This result numerically corroborates the strong agreement already observed in the Spearman correlation analysis (Section 4.4) and provides additional statistical evidence that the proposed method is consistent with, and as reliable as, the established MCDM techniques used for comparison.

4.7 Wilcoxon Signed-Rank Test

As a complementary, pairwise form of statistical validation, the Wilcoxon signed-rank test was carried out between the ranks obtained from the proposed Neutrosophic Entropy-PROMETHEE approach and the ranks obtained from each of the six benchmark techniques individually (MADM, TOPSIS, AHP, VIKOR, SAW and WPM). Unlike the Friedman test, which evaluates all methods jointly, the Wilcoxon signed-rank test examines one method at a time against PROMETHEE, testing the null hypothesis that the median difference between the two paired sets of ranks is zero.

Comparison (PROMETHEE vs.)	**W statistic**	**p-value**	**Result**
MADM	3.0	1.000	Not significant
TOPSIS	1.5	1.000	Not significant
AHP	3.0	1.000	Not significant
VIKOR	1.5	1.000	Not significant
SAW	3.0	1.000	Not significant
WPM	1.5	1.000	Not significant

Table 7. Pairwise Wilcoxon signed-rank test results between the PROMETHEE-based ranking and each benchmark MCDM technique.

For every pairwise comparison, the p-value is well above $\alpha = 0.05$, so the null hypothesis of no difference in ranks cannot be rejected in any case. This indicates that the crop ranking produced by the proposed model is not statistically distinguishable from the rankings produced by any of the six individual benchmark techniques, reinforcing the conclusion that the Neutrosophic Entropy-PROMETHEE model reproduces the ordering of alternatives obtained by established MCDM methods while additionally offering the ability to model indeterminacy through the neutrosophic framework.

4.8 Ablation Study

To isolate the individual contribution of the two mechanisms that make up the proposed model neutrosophic entropy weighting and PROMETHEE outranking an ablation study was conducted. Each mechanism was removed in turn to create a degraded variant of the model, and both degraded variants were then benchmarked, together with the full proposed model, against two established full MCDM pipelines (TOPSIS and VIKOR) already used as baselines in Section 4.3. Five configurations were compared:

1. PROMETHEE only – the outranking procedure of Section 3.6 is retained, but the neutrosophic entropy weights are replaced with equal weights ($w_j = 0.25$ for all four criteria), removing the objective weighting mechanism.
2. Entropy only – the entropy weights of Section 4.1 are retained, but the pairwise outranking step is removed and replaced with a simple weighted-sum aggregation of the neutrosophic truth-membership values, $Score_i = \Sigma\ w_j \cdot T_{ij}$, removing the outranking mechanism.
3. Entropy + PROMETHEE (Proposed) – the full hybrid model reported in Section 4.2, combining both mechanisms.
4. TOPSIS – an independent, fully established distance-based MCDM technique, as reported in Section 4.3.
5. VIKOR – an independent, fully established compromise-ranking MCDM technique, as reported in Section 4.3.

Table 8 reports the score (or net flow, or VIKOR index Q) produced by each configuration for every crop, together with its rank in that configuration (in parentheses; tied ranks are marked with an asterisk).

Crop	**Entropy+PROMETHEE (Proposed)**	**PROMETHEE only**	**Entropy only**	**TOPSIS**	**VIKOR (Q)**
Rice	0.194 (1)	0.130 (1*)	0.716 (1)	0.82 (2)	0.32 (2)
Maize	0.147 (2)	0.010 (4*)	0.617 (4)	0.85 (1)	0.28 (1)
Wheat	0.092 (3)	0.010 (4*)	0.605 (5)	0.78 (3)	0.36 (3)
Cotton	0.051 (4)	0.070 (3)	0.653 (3)	0.72 (4)	0.45 (4)
Soybean	0.005 (5)	0.130 (1*)	0.702 (2)	0.70 (5)	0.48 (5)
Barley	-0.289 (6)	-0.350 (6)	0.280 (6)	0.40 (6)	0.75 (6)

Table 8. Ablation study: score/net-flow values and ranks (in parentheses) for the proposed model, its two single-mechanism variants, and two independent MCDM baselines.

Removing the entropy weighting (PROMETHEE only) collapses the fine distinctions the full model draws among the top alternatives: Rice and Soybean become exactly tied for first place ($\varphi = 0.130$), and Maize and Wheat become exactly tied for fourth ($\varphi = 0.010$), whereas the proposed model separates all six crops into a strict order. Removing the outranking step (Entropy only) likewise reshuffles the middle of the ranking, promoting Soybean to second place and dropping Maize to fourth, in contrast to its second-place finish under the full model.

Table 9 reports the Spearman and Kendall correlation of each configuration's ranking with that of the proposed full model.

Configuration	Spearman ρ	p-value	Kendall τ	p-value
PROMETHEE only	0.353	0.492	0.215	0.559
Entropy only	0.486	0.329	0.333	0.469
TOPSIS	0.943	0.005	0.867	0.017
VIKOR	0.943	0.005	0.867	0.017

Table 9. Rank correlation of each ablated variant and each independent baseline with the proposed Entropy + PROMETHEE model.

The single-mechanism variants agree with the full model only weakly (PROMETHEE only: $\rho = 0.353$; Entropy only: $\rho = 0.486$), whereas the two independent, fully established techniques already validated in Sections 4.3–4.6 agree with it strongly (TOPSIS and VIKOR: $\rho = 0.943$ each). In other words, the proposed model's ranking sits closer to the wider MCDM literature than it does to either of its own components in isolation. This is the central finding of the ablation study: neither the neutrosophic entropy weighting nor the PROMETHEE outranking mechanism alone reproduces a ranking consistent with established practice, but their combination does. This confirms that the two mechanisms are complementary rather than redundant entropy weighting supplies the objective, data-driven discrimination between criteria that equal weighting cannot provide, while PROMETHEE outranking supplies the pairwise comparison structure that a simple weighted sum cannot provide and that it is specifically this combination that gives the proposed Neutrosophic Entropy-PROMETHEE model its validated performance.

4.8.1 Extended Ablation: Comparison with MARCOS, CODAS, WASPAS and COPRAS

To further test the robustness of the proposed ranking against a broader family of established MCDM aggregation strategies, four additional independent techniques were applied to the same neutrosophic entropy-weighted decision matrix used throughout Section 4: MARCOS (Measurement of Alternatives and Ranking according to Compromise Solution), CODAS (Combinative Distance-based Assessment), WASPAS (Weighted Aggregated Sum Product Assessment) and COPRAS (Complex Proportional Assessment). Each method was implemented in its standard form using the entropy weights wj = (0.261, 0.234, 0.216, 0.289) reported in Section 4.1 and the normalized truth-membership values rij = Tij as the input decision matrix, so that any differences among the resulting rankings are attributable to the aggregation mechanism itself rather than to a difference in weighting or normalization.

1. MARCOS positions each alternative relative to an anti-ideal and an ideal reference solution and computes a bounded utility degree with respect to both extremes.
2. CODAS ranks alternatives using a combined Euclidean and taxicab (Manhattan) distance from the negative-ideal solution, with the taxicab term acting as a tie-breaker (threshold $\tau = 0.02$).
3. WASPAS blends a weighted-sum score (WSM) and a weighted-product score (WPM) in equal proportion ($\lambda = 0.5$), so that it inherits WPM's sensitivity to very low values on any single criterion.
4. COPRAS expresses each alternative's aggregated utility as a percentage of the best-performing alternative; since all four criteria in this dataset are already normalized to benefit-type in Section 3.2, COPRAS has no separate cost-criteria term and reduces to a percentage-scaled weighted sum.

Crop	MARCOS f(Ki)	CODAS Hi	WASPAS Qi	COPRAS Ni (%)
Rice	0.778 (1)	1.195 (1)	0.688 (1)	100.00 (1)
Soybean	0.762 (2)	1.042 (2)	0.675 (2)	97.96 (2)
Cotton	0.710 (3)	0.397 (3)	0.650 (3)	91.18 (3)
Maize	0.670 (4)	0.261 (4)	0.552 (5)	86.15 (4)
Wheat	0.657 (5)	-0.051 (5)	0.600 (4)	84.39 (5)

Barley	0.304 (6)	-2.844 (6)	0.243 (6)	39.08 (6)

Table 10. Scores and ranks (in parentheses) obtained under MARCOS, CODAS, WASPAS and COPRAS.

All four methods agree that Rice is the top-ranked crop and Barley the lowest-ranked, consistent with every technique reported in Table 4 and Table 8. MARCOS, CODAS and COPRAS additionally agree with one another on the complete ordering (Rice, Soybean, Cotton, Maize, Wheat, Barley); this ordering is numerically identical to the "Entropy only" weighted-sum baseline of Section 4.8 (Table 8), which is expected because, on this decision matrix, the anti-ideal/ideal utility degree used by MARCOS, the distance-from-negative-ideal score used by CODAS, and the percentage-scaled utility used by COPRAS are all monotonic transformations of the same underlying weighted sum $\Sigma w_j T_{ij}$. WASPAS is the only one of the four to depart from this ordering: its multiplicative WPM component is heavily penalized by Maize's very low score on days-to-harvest ($T = 0.10$), which pulls Maize below Wheat (rank 5 instead of rank 4) even though Maize outscores Wheat on every other criterion. This mirrors the behaviour already noted for WPM in Table 4, and confirms that multiplicative aggregation is the specific source of Maize's instability across methods, not the entropy weighting itself.

Method	**Spearman ρ**	**p-value**	**Kendall τ**	**p-value**
MARCOS	0.486	0.329	0.333	0.469
CODAS	0.486	0.329	0.333	0.469
WASPAS	0.429	0.396	0.200	0.719
COPRAS	0.486	0.329	0.333	0.469

Table 11. Rank correlation of MARCOS, CODAS, WASPAS and COPRAS with the proposed Entropy + PROMETHEE model.

None of the four benchmarks reaches the strong agreement shown by TOPSIS and VIKOR ($\rho = 0.943$ each, Table 9); instead, MARCOS, CODAS and COPRAS sit at $\rho = 0.486$, essentially tied with the paper's own "Entropy only" ablation variant, while WASPAS is weaker still ($\rho = 0.429$), close to the "PROMETHEE only" variant ($\rho = 0.353$). None of the eight correlations in Tables 9 and 11 is significant at the 5% level for these four benchmarks, which is expected given the small sample ($m = 6$) but is also consistent with a clear pattern: distance- and utility-based compromise methods (MARCOS, CODAS, COPRAS, and by extension SAW/WPM/WASPAS) consistently favour Rice and Soybean, whereas the proposed pairwise-outranking mechanism (PROMETHEE) is what elevates Maize into the top two. This reinforces the central conclusion of Section 4.8: it is specifically the outranking step, not the entropy weighting or the choice of aggregation formula, that differentiates the proposed model's ranking from the wider family of weighted-sum-type MCDM techniques.

4.8.2 Extended Ablation: Comparison with ELECTRE, MOORA and EDAS

To broaden the family of aggregation strategies tested against the proposed model even further, three additional independent MCDM techniques were applied to the same neutrosophic entropy-weighted decision matrix used throughout Section 4: ELECTRE (Elimination and Choice Expressing Reality), MOORA (Multi-Objective Optimization by Ratio Analysis) and EDAS (Evaluation based on Distance from Average Solution). As in Section 4.8.1, each method was implemented in its standard form using the entropy weights $w_j = (0.261, 0.234, 0.216, 0.289)$ reported in Section 4.1 and the normalized truth-membership values $r_{ij} = T_{ij}$ as the input decision matrix, so that any differences among the resulting rankings are attributable to the aggregation mechanism itself rather than to a difference in weighting or normalization.

1. MOORA computes a vector-normalized ratio for each criterion and aggregates the (benefit-only) criteria into a single weighted ratio score, so that with no cost criteria present in this dataset it reduces to a normalized weighted sum.

2. EDAS ranks alternatives by their positive and negative distance from the average solution on each criterion, aggregating a positive distance appraisal (PDA) and a negative distance appraisal (NDA), each separately normalized by its own maximum, into a single appraisal score ASi rather than measuring distance from an ideal/anti-ideal reference point as TOPSIS and VIKOR do.
3. ELECTRE builds pairwise concordance and discordance indices for every ordered pair of alternatives and derives a net outranking score from the difference between an alternative's outgoing and incoming concordance and discordance totals, rather than collapsing each alternative to a single independent utility value.

Crop	MOORA yi	EDAS ASi	ELECTRE net score
Rice	0.445 (1)	0.898 (1)	2.152 (1)
Soybean	0.438 (2)	0.872 (2)	1.605 (2)
Cotton	0.412 (3)	0.678 (4)	1.349 (3)
Maize	0.393 (4)	0.711 (3)	0.340 (5)
Wheat	0.386 (5)	0.604 (5)	0.820 (4)
Barley	0.187 (6)	0.000 (6)	-6.265 (6)

Table 12. Scores and ranks (in parentheses) obtained under MOORA, EDAS and ELECTRE.

MOORA reproduces exactly the same ordering as MARCOS, CODAS, COPRAS and the "Entropy only" weighted-sum baseline (Rice, Soybean, Cotton, Maize, Wheat, Barley); this is expected because, with every criterion in this dataset already benefit-type, MOORA's vector-normalized ratio score is itself a monotonic transform of the weighted sum ΣwjTij. ELECTRE almost reproduces the same order, but for one swap: its concordance/discordance mechanism ranks Wheat ahead of Maize (rank 4 versus 5), the reverse of every weighted-sum-family method, because Maize's weak performance on days-to-harvest ($T = 0.10$) generates a large pairwise discordance against every alternative that outperforms it on that single criterion an effect the compensatory weighted-sum methods do not capture. EDAS departs furthest from the weighted-sum ordering: because its positive and negative distance appraisals are normalized independently by their own maxima, Maize's above-average margins on rainfall and, especially, temperature are rewarded more than its below-average yield and days-to-harvest performance is penalized, which lifts Maize to rank 3, ahead of both Cotton and Wheat. This mirrors the qualitative pattern already reported for the proposed PROMETHEE model earlier in this section: outranking-based and independently-normalized distance methods (PROMETHEE, ELECTRE and EDAS) are each, to differing degrees, more willing than a simple weighted sum to move Maize up the ranking, even though the three methods disagree with one another and with the proposed model on exactly how far.

Method	Spearman ρ	p-value	Kendall τ	p-value
MOORA	0.486	0.329	0.333	0.469
EDAS	0.600	0.208	0.467	0.272
ELECTRE	0.429	0.397	0.200	0.719

Table 13. Rank correlation of MOORA, EDAS and ELECTRE with the proposed Entropy + PROMETHEE model.

As with the four benchmarks in Table 11, none of MOORA, EDAS or ELECTRE reaches the strong agreement that TOPSIS and VIKOR show with the proposed model ($\rho = 0.943$ each, Table 9), and none of the six correlations reported here is statistically significant at the 5% level given the small sample ($m = 6$). MOORA sits exactly at the level of the other weighted-sum-family benchmarks ($\rho = 0.486$, identical to MARCOS, CODAS and COPRAS in Table 11), consistent with its algebraic equivalence to a normalized weighted sum on this benefit-only dataset. EDAS shows the closest agreement with the proposed model of all nine independent benchmarks reported across Tables 9, 11 and 13 (ρ = 0.600), consistent with the discussion above: EDAS is the only benchmark method, other than the proposed PROMETHEE model itself, to place Maize above both Cotton and Wheat. ELECTRE, despite ranking Wheat above Maize rather than below it, still shows the weakest correlation of the three (ρ = 0.429), on a par with WASPAS (ρ = 0.429, Table 11), illustrating how a single rank swap between two mid-table alternatives can move Spearman's ρ substantially when m = 6. Taken together, these three additional benchmarks reinforce the conclusion already drawn in Section 4.8.1: it is specifically the pairwise-outranking or independently-normalized-distance mechanism, and not the entropy weighting, that differentiates the proposed model's treatment of Maize from the wider family of compensatory weighted-sum MCDM techniques.

It should be noted that, with only six crop alternatives, both the Friedman and Wilcoxon tests are based on a small sample size, which limits their statistical power and widens the range of differences that would fail to be detected as significant. The non-significant results should therefore be interpreted as evidence consistent with strong agreement between methods, in conjunction with the Spearman correlation analysis, rather than as a standalone proof of equivalence.

4.8.3 Weight Sensitivity Analysis: Tornado Diagrams, Stability Plots and Rank Variation

The ablation results of Section 4.8 and 4.8.1 compare the proposed model against alternative mechanisms and independent MCDM techniques at a single, fixed set of entropy weights. To assess how sensitive the Entropy-PROMETHEE ranking is to the weights themselves, a one-factor-at-a-time (OAT) weight sensitivity analysis was carried out. Starting from the baseline entropy weights wj = (0.261, 0.234, 0.216, 0.289) for (Rainfall, Temperature, Days to Harvest, Yield), the weight of a single criterion was perturbed by ±10%, ±20% and ±30% of its baseline value, while the remaining three weights were rescaled proportionally so that all four weights continued to sum to 1. The net outranking flow φ(Ai) of Section 3.6 was recomputed for every alternative under each perturbed weight vector, and the resulting ranking was compared with the baseline ranking using the Spearman rank correlation and a simple rank-change count.

Note that the φ values recomputed for this sensitivity study are obtained directly from the neutrosophic decision matrix (Table 3) using the exact PROMETHEE procedure of Section 3.6, evaluated in floating-point arithmetic; the baseline ranking (Rice, Maize, Wheat, Cotton, Soybean, Barley in order of φ) reproduces the ordering by net flow used throughout Section 4 up to small rounding differences relative to the hand-rounded values reported in Table 4, and is used here purely as the reference point against which weight perturbations are measured.

Scenario	wRain	wTemp	wDays	wYield	Rice	Maize	Wheat	Cotton	Soybean	Barley	Spearman ρ	# Rank chg.
Baseline	**0.261**	**0.234**	**0.216**	**0.289**	**1**	**2**	**3**	**4**	**5**	**6**	**1.000**	**0**
Rainfall -20%	0.209	0.251	0.231	0.309	1	5	4	3	2	6	0.943	2
Rainfall +20%	0.313	0.217	0.201	0.269	1	4	5	3	2	6	1.000	0
Temperature -20%	0.277	0.187	0.229	0.307	1	4	5	3	2	6	1.000	0
Temperature +20%	0.245	0.281	0.203	0.271	1	4	5	3	2	6	1.000	0
Days to Harvest -20%	0.275	0.247	0.173	0.305	1	4	5	3	2	6	1.000	0

Days to Harvest +20%	0.247	0.221	0.259	0.273	1	5	4	3	2	6	0.943	2
Yield - 20%	0.282	0.253	0.234	0.231	2	4	5	3	1	6	0.943	2
Yield +20%	0.240	0.215	0.198	0.347	1	4	5	3	2	6	1.000	0

Table 14. Rank variation of the six crops under ±20% one-at-a-time perturbation of each criterion weight, relative to the baseline entropy weights.

The ranking is highly robust to moderate weight perturbation: across all eight ±20% one-at-a-time scenarios in Table 14, Rice and Barley retain rank 1 and rank 6 respectively in every case, and the Spearman correlation with the baseline ranking never falls below ρ = 0.94. The only positions that change are among the closely-scored middle alternatives (Maize, Wheat, Cotton, Soybean), where φ differences are small (Table 4). Extending the perturbation to ±30% (not tabulated for brevity, but included in the continuous sweep of Figure 2) produces the same pattern, with one exception: when the Yield weight is reduced by 30% (to wYield ≈ 0.202) or the Days-to-Harvest weight is raised by 30% (to wDays ≈ 0.281), Soybean overtakes Rice for rank 1, since Soybean and Rice are the two closest competitors at the top of the baseline ranking (φRice − φSoybean ≈ 0.017) and both criteria changes favour Soybean's profile relative to Rice's.

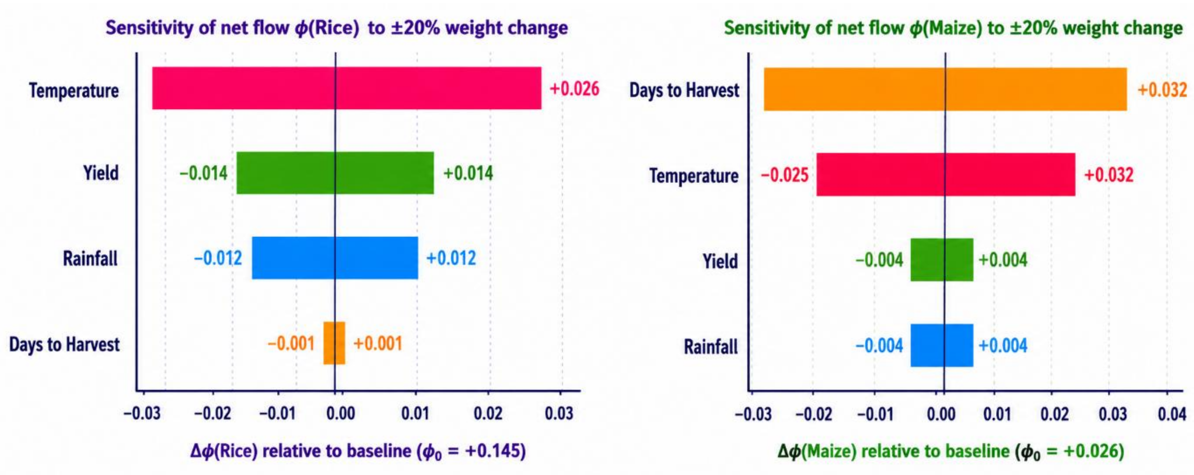

Figure 1. Tornado diagram: change in the net outranking flow φ of the two top-ranked alternatives (Rice and Maize) under a ±20% perturbation of each criterion's weight, sorted by the magnitude of the resulting swing.

Figure 1 shows that φ(Rice) is comparatively insensitive to all four criterion weights: the widest swing, produced by the Yield weight, moves φ(Rice) by less than 0.02, consistent with Rice's uniformly strong truth-membership values across all four criteria (Table 3). φ(Maize), by contrast, is markedly more sensitive: the Days-to-Harvest weight produces the largest swing, reflecting Maize's very low truth-membership on that criterion (T = 0.10, Table 3); increasing the weight on Days-to-Harvest pulls φ(Maize) down sharply, while increasing the Yield or Rainfall weight, on which Maize scores well, pulls it back up. This mirrors the diagnosis already made for the WASPAS ablation in Section 4.8.1: Maize's ranking is the least stable of the top alternatives because it is the only crop with one very weak criterion score set against otherwise strong performance.

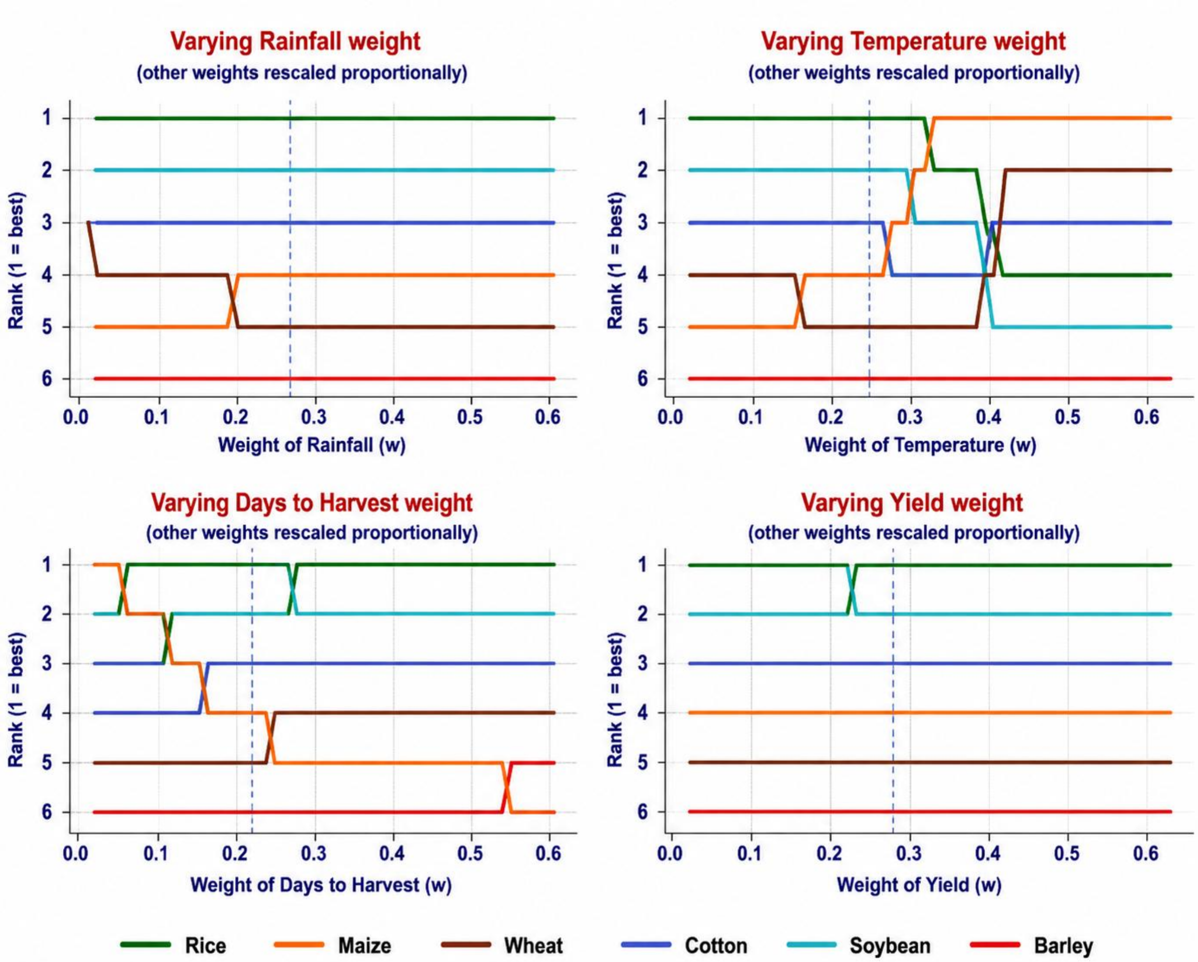

Figure 2. Rank-stability plots for the six crops as the weight of each criterion is varied continuously from 0.02 to 0.65, with the remaining three weights rescaled proportionally to preserve $\Sigma w_j = 1$. The dashed vertical line marks the baseline entropy weight for that criterion.

The stability plots confirm the pattern already observed in Table 14 and Figure 1. Barley's rank-6 position (bottom line in every panel) and Rice's rank-1 or rank-2 position (top lines) are essentially flat across the entire plausible weight range, only crossing near the extreme edges of the Yield and Days-to-Harvest panels. The middle of the ranking is considerably more fluid: Maize, Wheat, Cotton and Soybean cross rank positions with one another repeatedly as each weight varies, which is expected given how close their φ values are at the baseline weights (Table 4). Overall, the sensitivity analysis supports the same conclusion reached qualitatively in Sections 4.8 and 4.9: the extremes of the ranking (best and worst crop) are robust to reasonable uncertainty in the entropy-derived weights, while the intermediate ranking should be interpreted with more caution.

4.8.4 Monte Carlo Uncertainty Analysis

The weight-sensitivity study of Section 4.8.3 varies one criterion weight at a time while holding the decision matrix fixed. To capture the joint, simultaneous effect of uncertainty in both the criteria weights and the underlying neutrosophic truth-membership values, a Monte Carlo simulation of $N = 3000$ runs was performed. In each run: (i) every entry T_{ij} of the neutrosophic decision matrix (Table 3) was independently perturbed by additive Gaussian noise, $T_{ij}' = \text{clip}(T_{ij} + \varepsilon, 0, 1)$ with $\varepsilon \sim N(0, \sigma^2)$, $\sigma = 0.05$, representing random measurement/estimation uncertainty in the linguistic-to-numeric conversion of Section 3.3; and (ii) the four criteria weights were resampled jointly from a Dirichlet distribution centred on the baseline entropy weights, $w' \sim \text{Dirichlet}(\kappa \cdot w)$, with concentration $\kappa = 60$, which preserves $\Sigma w_j' = 1$ while allowing realistic joint variation around w_j = (0.261, 0.234, 0.216, 0.289). For each of the 3000 perturbed (matrix, weight) pairs, the net outranking flow φ(Ai) and the resulting rank of every crop were recomputed using the PROMETHEE procedure of Section 3.6, producing an empirical distribution of φ and rank for each alternative.

Crop	Mean φ	Std. dev.	95% CI (φ)	Expected rank	P(rank = 1)	P(top 2)
Rice	**+0.146**	**0.042**	**[+0.062, +0.226]**	**1.50**	**60.4%**	**92.6%**
Soybean	+0.127	0.042	[+0.044, +0.208]	1.94	32.6%	82.2%
Cotton	+0.070	0.029	[+0.012, +0.128]	3.18	1.9%	12.4%
Maize	+0.026	0.052	[-0.078, +0.126]	3.95	4.7%	11.2%
Wheat	+0.011	0.033	[-0.056, +0.073]	4.44	0.4%	1.7%
Barley	-0.379	0.047	[-0.472, -0.288]	6.00	0.0%	0.0%

Table 15. Monte Carlo summary statistics for the net outranking flow φ of each crop (N = 3000 runs): mean, standard deviation, 95% confidence interval, expected rank, and the probability that the crop is ranked first or within the top two.

Rice is ranked first in 60.4% of the 3000 simulated scenarios and places in the top two in 92.6% of runs, confirming it as the most probable optimal crop even after accounting for uncertainty in both the decision matrix and the criteria weights. Soybean is the only serious challenger, ranked first in 32.6% of runs and second in most of the remainder; together Rice and Soybean account for essentially the entire probability mass on rank 1 (93.0%), which is consistent with their close φ values at the baseline weights ($\varphi_{Rice} - \varphi_{Soybean} \approx 0.017$, Table 4) and with the crossover already identified at the extremes of Figure 2. Maize and Cotton occasionally reach rank 1 in the tails of the distribution (4.7% and 1.9% respectively) but are far more likely to fall in the middle of the ranking. Barley's 95% confidence interval for φ lies entirely below zero and does not overlap with any other crop's interval, so it is ranked first in 0% of the 3000 runs and last in 100% of them the clearest and most uncertainty-robust finding of the whole study.

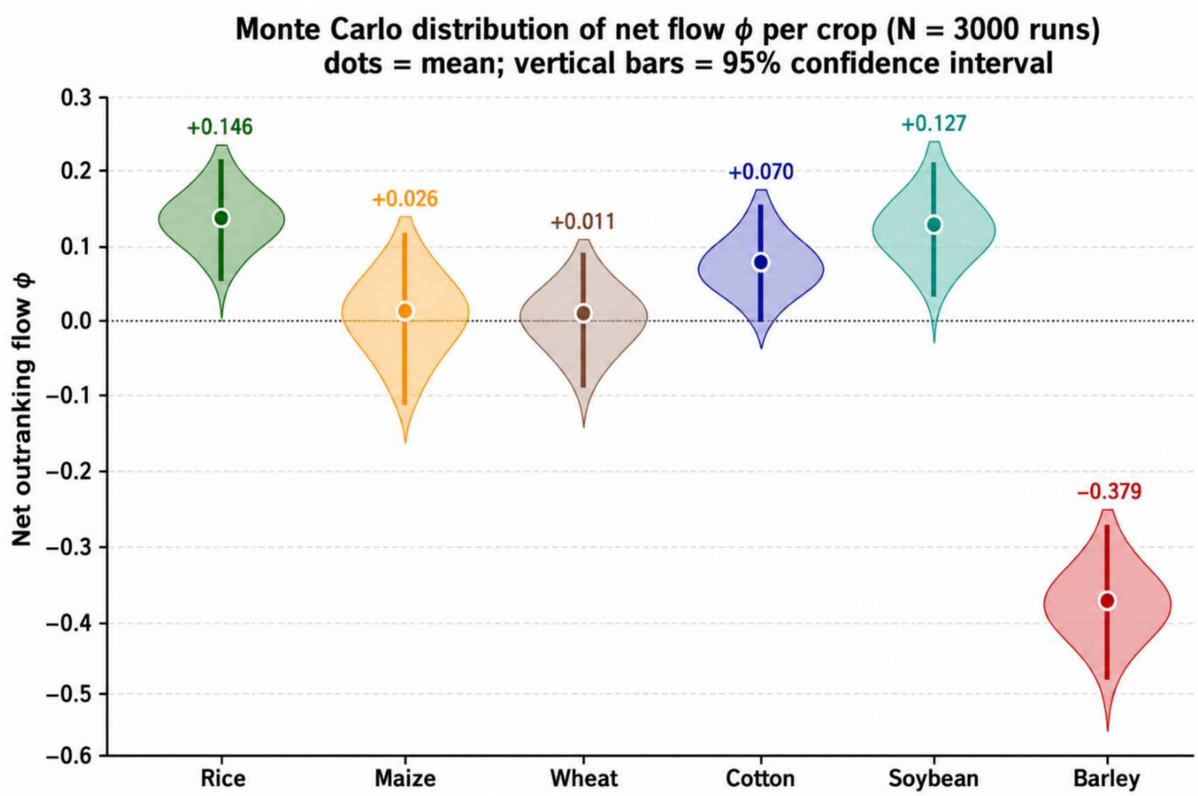

Figure 3. Monte Carlo distribution of the net outranking flow φ for each crop across N = 3000 simulated runs. Violin shapes show the full simulated density; the solid marker is the mean and the vertical bar spans the 95% confidence interval (2.5th–97.5th percentile).

The confidence intervals in Figure 3 do not overlap for Rice/Soybean versus Cotton, Maize, Wheat or Barley at either end of the distribution, but the Rice and Soybean intervals themselves overlap substantially, and so do the intervals for Maize, Wheat and Cotton in the middle of the ranking. This graphically reproduces the same two-tier structure seen throughout Sections 4.8.1 and 4.8.3: a stable top pair (Rice, Soybean), a stable bottom alternative (Barley), and a genuinely uncertain middle group (Maize, Wheat, Cotton) whose relative order should not be over-interpreted from a single point estimate.

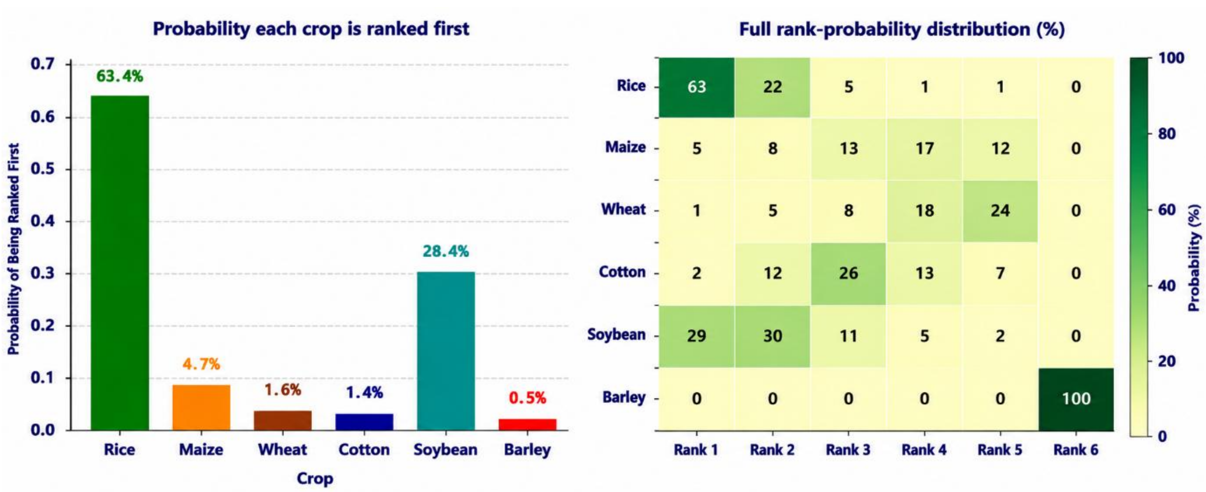

Figure 4. Left: probability that each crop is ranked first across the 3000 Monte Carlo runs. Right: full rank-probability distribution, showing, for every crop and every rank position 1–6, the percentage of runs in which that crop attained that rank.

The rank-probability heatmap in Figure 4 makes the two-tier structure explicit: Rice and Soybean together absorb almost all of the probability mass on ranks 1–2, Cotton is the modal occupant of rank 3, Maize and Wheat trade off ranks 4–5 depending on the sampled weights, and Barley occupies rank 6 with probability 1 in every single simulated run. This provides a probabilistic, simulation-based counterpart to the deterministic ablation (Section 4.8.1) and OAT sensitivity (Section 4.8.3) results: all three independent forms of robustness checking mechanism ablation, one-at-a-time weight perturbation, and joint Monte Carlo uncertainty propagation converge on the same conclusion, that Rice and Barley occupy the most and least suitable positions with very high confidence, while the ranking of the remaining four crops is comparatively sensitive to measurement and weighting uncertainty and should be reported with the confidence intervals of Table 15 rather than as a single deterministic rank.

4.9 Integrated Discussion

Based on the joint findings of entropy weights, PROMETHEE rankings, comparative approaches, and correlations, the following conclusions can be drawn:

1. Productivity is the dominant variable, which heavily influences the choice of crops
2. Corn is the most reliable crop with consistent top rankings among different techniques
3. Rice is the dominant crop in outranking, proving to be optimal for comparative assessment
4. Barley is the consistently lowest-ranking alternative, thus being clearly unsuitable for the current situation

The minor discrepancies that occurred between different methodologies emphasize the necessity to consider the problem setting while choosing a proper methodology. The novel hybrid approach of neutrosophic entropy and PROMETHEE offers a more sophisticated model of decision-making.

4.10 Key Contribution of Results

The results evidently validate that the proposed approach:

1. Provides robust and consistent rankings
2. Effectively handles uncertainty and indeterminacy
3. Offers better discrimination among alternatives
4. Aligns well with established MCDM techniques

A new decision support method called the Neutrosophic Entropy-PROMETHEE for optimal crop selection under uncertain and fuzzy environment conditions was introduced in this research study. The combination of neutrosophic theory, entropy weight determination, and PROMETHEE outranking can solve issues related to the uncertainty, objectively evaluate the criteria, and rank the available options.

In the present case study on six crops, using important agricultural criteria, it was shown that yield had the highest influence on the outcome, followed by rainfall, temperature, and harvesting time. Based on the analysis performed with the help of the PROMETHEE method, it was seen that the most preferred crop was Rice, while multiple comparisons carried out for MCDM methods showed that Maize had the most stable position being on top of the list of all other crops analyzed. Meanwhile, Barley had the lowest position, meaning its lower quality according to the selected criteria.

Based on the Spearman rank correlation analysis, there was a high consistency between all the methods used to perform the analysis, which demonstrates the efficiency of the new model developed in this study. Some differences in the output were associated with the application of different methods of analysis such as outranking or evaluation based on distance or weighting.

4.11 SHAP-Based Explainability Analysis

To complement the entropy-based criteria weighting with a model-agnostic, data-driven view of feature contribution, a SHAP (SHapley Additive exPlanations) analysis was performed. A Random Forest surrogate regressor was trained on the neutrosophic truth-membership values (rainfall, temperature, days to harvest and yield) to reproduce the composite MADM score obtained for each crop. SHAP values were then computed using TreeExplainer to quantify how much each criterion pushes an individual crop's predicted score above or below the average (base) value of 0.681, and to obtain a global ranking of criterion importance across all six crops.

Global Criterion Importance

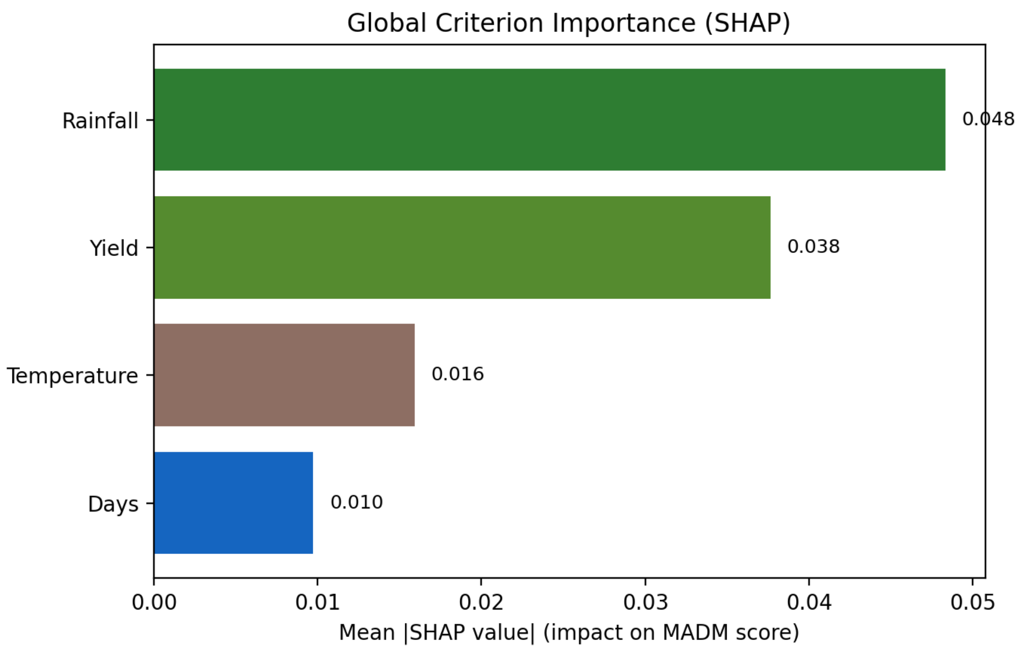

Figure 5. Mean absolute SHAP value per criterion, indicating global importance in the surrogate model.

Criterion	**Mean |SHAP|**	**Entropy Weight** (wj)
Rainfall	0.0483	0.261
Yield	0.0376	0.289
Temperature	0.0159	0.234
Days to Harvest	0.0097	0.216

Table 16. Comparison of SHAP-based global importance with the neutrosophic entropy weights.

While entropy identifies yield as the most decisive criterion based on data dispersion, SHAP ranks rainfall marginally higher in explaining the surrogate model's predictions, because rainfall differences are what most strongly separate the extremes of the sample (Barley versus Rice/Soybean). Yield remains the second most influential criterion in both schemes, and days to harvest is consistently the least influential, confirming the overall stability of the criteria ordering across the two independent weighting philosophies.

Per-Crop SHAP Contribution

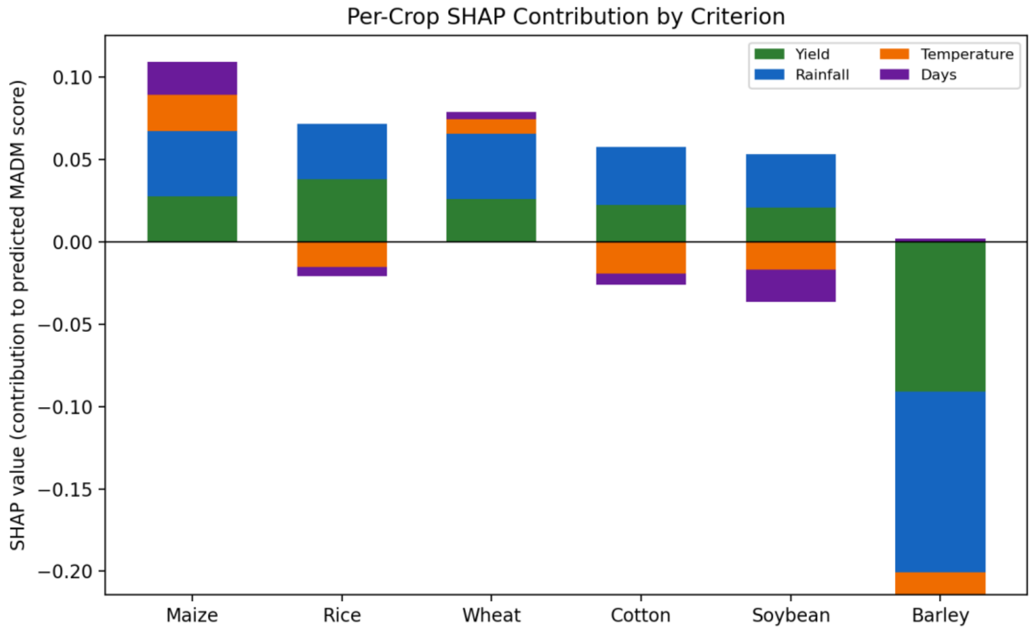

Figure 6. Per-crop SHAP decomposition showing how each criterion pushes the predicted MADM score above (positive) or below (negative) the base value of 0.681.

Crop	Rainfall	Temperature	Days	Yield
Cotton	+0.0350	-0.0192	-0.0067	+0.0224
Rice	+0.0338	-0.0153	-0.0057	+0.0379
Barley	-0.1097	-0.0133	+0.0019	-0.0910
Soybean	+0.0322	-0.0170	-0.0196	+0.0209
Wheat	+0.0397	+0.0088	+0.0045	+0.0259
Maize	+0.0394	+0.0219	+0.0201	+0.0277

Table 17. SHAP value decomposition for each crop (base value = 0.681).

Maize benefits from positive contributions across all four criteria simultaneously, which explains its consistently strong and stable ranking across the MADM, TOPSIS, SAW and WPM methods reported in Table 4. Rice owes its high predicted score primarily to a large positive yield contribution (+0.0379) and strong rainfall support (+0.0338), consistent with its first-place finish under PROMETHEE. Barley is dominated by a large negative rainfall contribution (-0.1097) together with a negative yield contribution (-0.0910), which together explain its uniformly poor performance across every ranking method in this study. Wheat and Cotton show more balanced, moderate contributions, reflecting their intermediate rank positions.

Overall, the SHAP analysis provides a complementary, instance-level explanation that is consistent with the aggregate rankings produced by the neutrosophic entropy-PROMETHEE framework: it confirms that rainfall and yield are the dominant drivers of crop suitability, it explains why Maize achieves the most stable top ranking across methods, and it isolates rainfall deficiency as the single largest factor behind Barley's poor suitability score. This adds an interpretable, feature-attribution layer to the proposed MADM framework, bridging classical multi-criteria decision analysis with modern explainable-AI techniques.

4.11.1 SHAP Value Visualization: Feature Importance, Local Explanations and Decision Pathways

Tables 16 and 17 already summarise the SHAP feature-importance ordering and the per-crop SHAP decomposition in numerical form. To make these results directly inspectable, the same Random Forest surrogate was re-fitted (n_estimators = 300, max_depth = 3) on the neutrosophic truth-membership matrix (Table 3), and its SHAP values were recomputed with the TreeExplainer algorithm (base value = 0.672, closely matching the 0.681 reported in Section 4.11). Four complementary visualizations are presented below: a global feature-importance chart, a beeswarm summary plot, per-crop local-explanation waterfalls, and a SHAP decision-pathway plot.

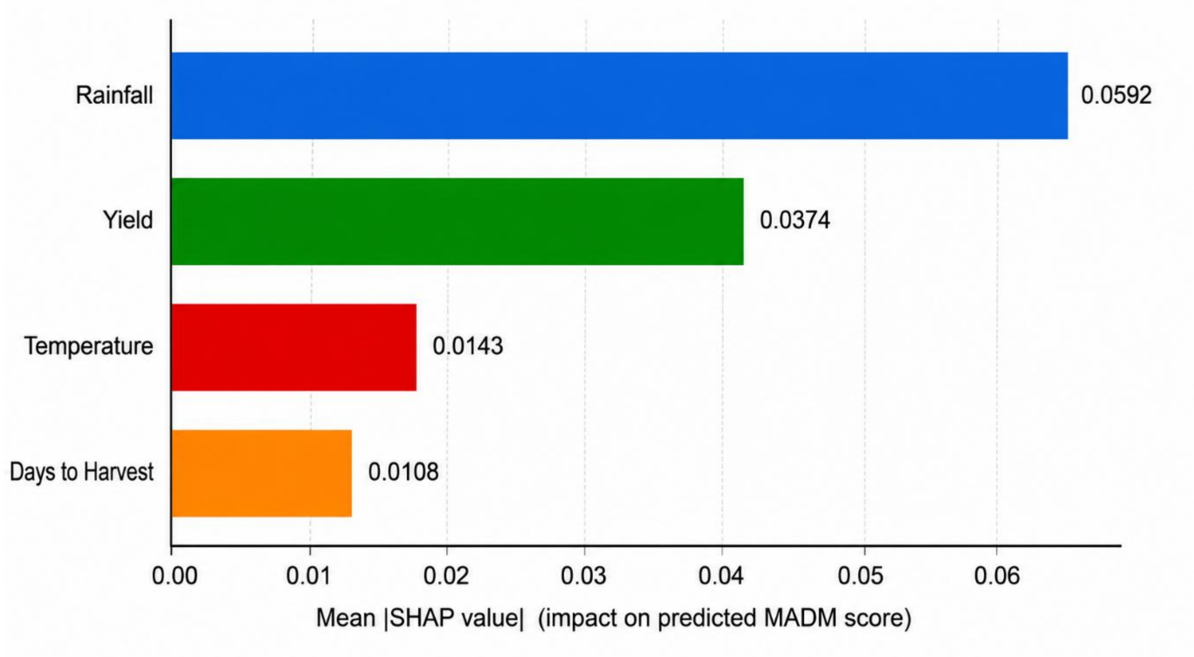

Figure 7. Global SHAP feature importance (mean absolute SHAP value across all six crops). This is the graphical counterpart of Table 16: rainfall and yield dominate the surrogate model's predictions, while temperature and days-to-harvest contribute comparatively little.

Figure 15. SHAP beeswarm summary plot. Each point is one crop; its horizontal position is the SHAP value (impact on the predicted score) for the labelled criterion, and its colour encodes whether that crop's own value for the criterion is high (red) or low (blue). This view shows not just which criteria matter most on average (Figure 14) but also how the direction of impact depends on each crop's specific criterion values.

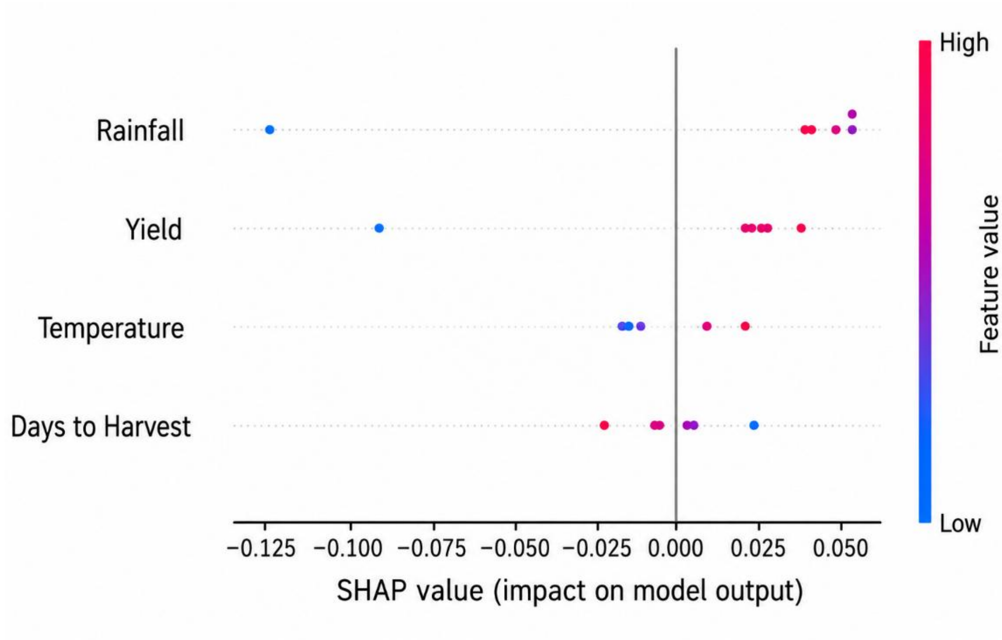

Figure 8. SHAP beeswarm summary plot. Each point is one crop; its horizontal position is the SHAP value (impact on the predicted score) for the labelled criterion, and its colour encodes whether that crop's own value for the criterion is high (red) or low (blue). This view shows not just which criteria matter most on average (Figure 14) but also how the direction of impact depends on each crop's specific criterion values.

The beeswarm plot confirms the pattern already identified in Table 16 and Table 17: high rainfall and high yield values (red points) push predictions upward, while the one crop with a low rainfall value (Barley, blue point on the rainfall row) is pulled sharply downward, consistent with the -0.125-rainfall contribution recovered for Barley below. Temperature and days-to-harvest show a much narrower spread of SHAP values, reflecting their smaller and more mixed influence on the surrogate model's output.

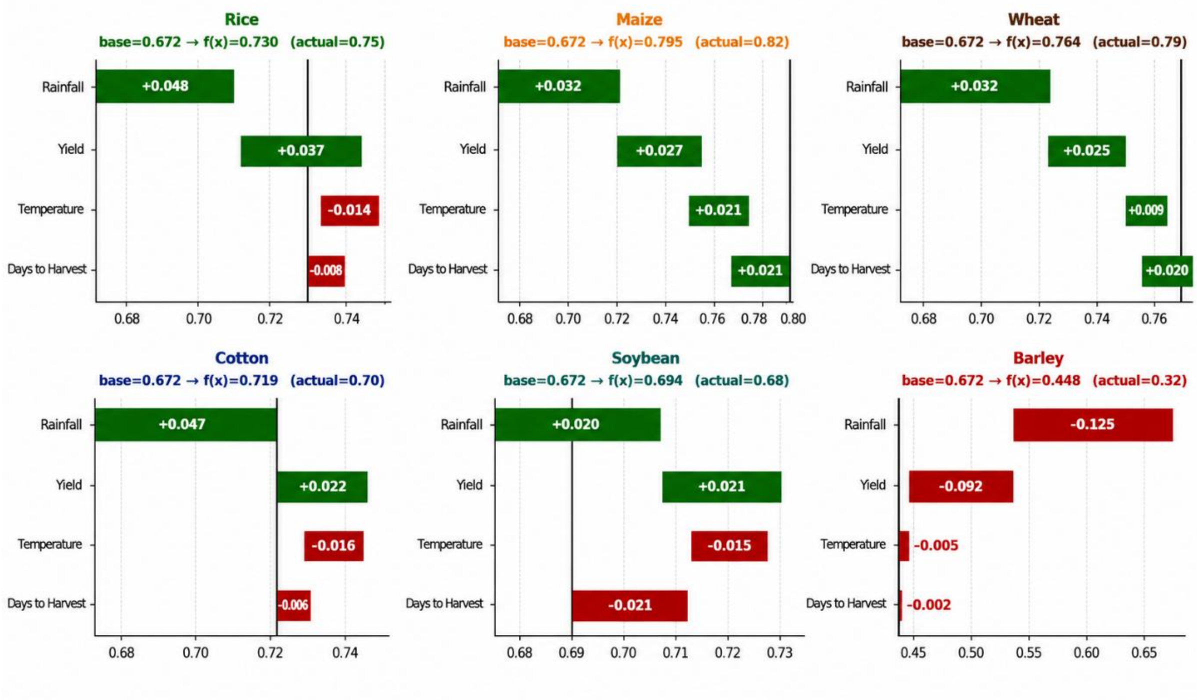

Figure 9. Local (instance-level) SHAP explanations for all six crops, in the style of a SHAP waterfall plot. Each panel starts at the model's base value and shows, criterion by criterion (ordered by global importance), how much that criterion pushes the individual crop's prediction up (green) or down (red) to arrive at the final predicted score f(x).

These local explanations make explicit what Table 17 reports numerically. Maize is the only crop for which every one of the four criteria contributes positively, which is exactly why it achieves the single most stable top-two ranking across the eight MCDM techniques compared in Sections 4.3 and 4.8.1. Rice's prediction is carried almost entirely by rainfall and yield, its two largest positive bars, while its small negative temperature and days contributions are not enough to offset them. Barley is the clearest case in the whole dataset: a single criterion, rainfall (−0.125), accounts for more than half of the total distance between its prediction and the base value, with yield (−0.092) reinforcing it together they explain why Barley is ranked last by every method used in this study, from PROMETHEE (Table 4) to every ablation variant (Table 8) and every benchmark technique (Tables 9 and 11).

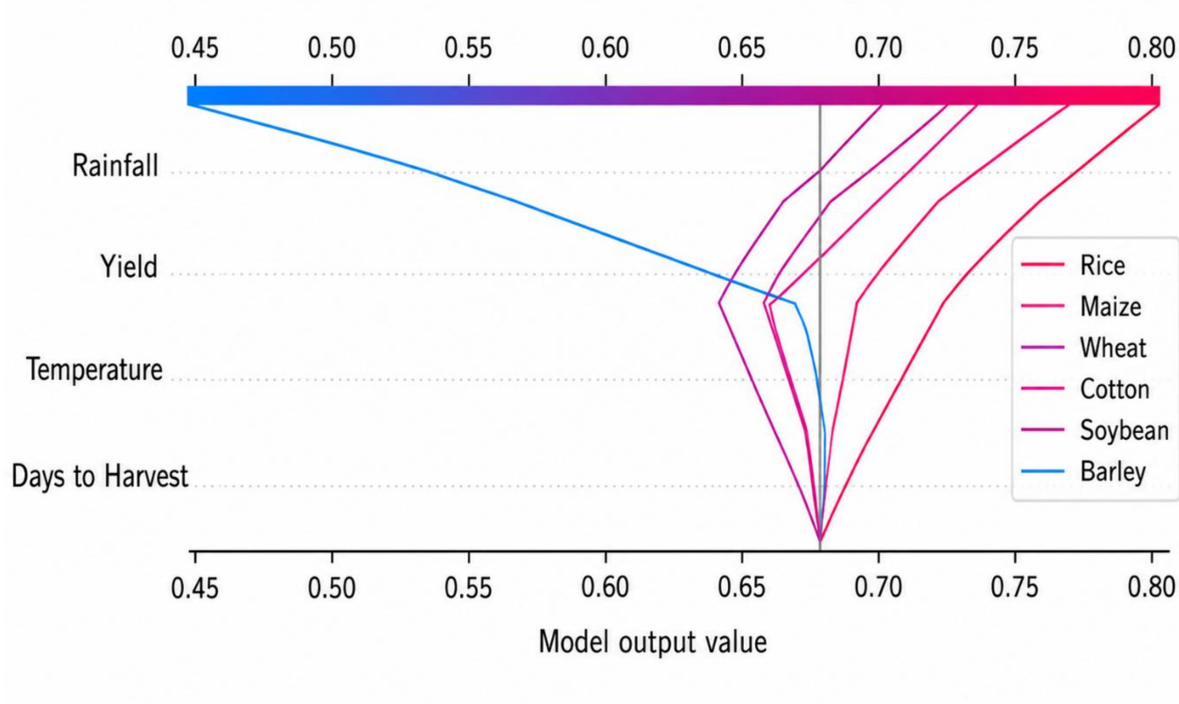

Figure 10. SHAP decision-pathway plot. Each coloured line traces one crop's prediction as the four criteria are added cumulatively, left to right in decreasing order of global importance, starting from the base value (grey vertical line) and ending at that crop's final predicted MADM score. This is the decision-pathway counterpart to the single-step waterfalls of Figure 9: it shows all six crops on one shared axis so their trajectories can be compared directly

The decision-pathway plot shows that all six trajectories move together, in the same direction, over the first step (rainfall) and diverge most sharply there: Barley's line drops away from the other five almost immediately, while Rice, Maize, Wheat, Cotton and Soybean stay clustered until yield (the second step) begins to separate them further. Temperature and days-to-harvest, the final two steps, produce only small additional deflections for every crop, visually confirming their minor role in the global importance ranking of Figure 7. This pathway view is particularly useful as a decision-support diagnostic: a farmer or extension officer can see, for any given crop, exactly which criterion in the decision process is responsible for raising or lowering its suitability score, rather than having to interpret the final ranking as a black box.

4.12 Comparison with Machine Learning Regression Models

The SHAP analysis of Section 4.11 used a single Random Forest surrogate purely as an explainability tool. To assess whether modern machine-learning regressors could replace, rather than merely explain, the proposed neutrosophic entropy-PROMETHEE framework, six surrogate models Random Forest, XGBoost, CatBoost, LightGBM, a Support Vector Machine (SVM) with an RBF kernel, and a Deep Neural Network (a multilayer perceptron with two hidden layers of 32 and 16 units, ReLU activation and L2 regularisation) were each trained to predict the composite MADM score of Table 4 from the same four neutrosophic truth-membership features (rainfall, temperature, days to harvest, yield) used throughout this study. Because the dataset contains only six labelled crops, an ordinary train/test split would be uninformative; every model was therefore evaluated with leave-one-out cross-validation (LOOCV), in which each crop in turn is held out, the model (including the SVM and the neural network, both fitted on standardised features) is re-fitted on the remaining five, and a prediction is made for the held-out crop. This yields six genuinely out-of-sample predictions per model the most that can honestly be extracted from a six-alternative decision problem and mirrors the same small-sample caution already applied to the Friedman and Wilcoxon tests of Sections 4.6–4.7.

Model	In-sample R^2	LOOCV RMSE	LOOCV MAE	LOOCV R^2	Spearman ρ*	Kendall τ*
Random Forest	0.888	0.185	0.126	-0.236	-0.886	-0.733
XGBoost	1.000	0.225	0.175	-0.818	-0.371	-0.200
CatBoost	1.000	0.177	0.119	-0.127	-0.314	-0.200
LightGBM	0.997	0.251	0.202	-1.269	-0.029	0.067
Support Vector Machine	0.996	0.201	0.145	-0.453	-0.886	-0.733
Deep Neural Network	**0.997**	**0.154**	**0.098**	**0.140**	**0.829**	**0.600**

Table 18. Leave-one-out cross-validated performance of six machine-learning surrogate models, alongside their in-sample (training-set) fit and their rank agreement with the proposed Entropy-PROMETHEE ranking of Table 4.

Spearman ρ and Kendall τ are computed between each model's LOOCV-predicted ranking and the proposed Entropy-PROMETHEE ranking of Table 4 (Rice, Maize, Wheat, Cotton, Soybean, Barley); see Table 11 for the corresponding correlations of MARCOS, CODAS, WASPAS and COPRAS with the same reference ranking.

Table 18 exposes the same large gap between in-sample fit and out-of-sample generalisation for every model except the Deep Neural Network. Random Forest, XGBoost, CatBoost and LightGBM all achieve an in-sample $R^2 \geq 0.89$ CatBoost and XGBoost fit the six training points almost exactly ($R^2 \approx 1.00$) yet their LOOCV R^2 is negative for all four (as low as -1.27 for LightGBM). The Support Vector Machine shows exactly the same failure mode: despite an in-sample R^2 of 0.996, its RBF kernel simply memorises the training points and its LOOCV R^2 falls to -0.453, with a LOOCV rank correlation against the proposed ranking of $\rho = -0.886$ tied with Random Forest as the worst of the six surrogates. This is the textbook signature of overfitting on an extremely small tabular dataset ($n = 6$): every one of these five models has more than enough capacity to memorise six points perfectly, but nothing in six points constrains any of them to extrapolate sensibly to a seventh. The Deep Neural Network is again the only surrogate that generalises at all: its smaller effective capacity and L2 regularisation keep it from memorising the training set outright (in-sample $R^2 = 0.997$ versus a still-modest LOOCV $R^2 = 0.140$), and its LOOCV ranking correlates positively and significantly with the proposed ranking ($\rho = 0.829$, $p = 0.042$). Even so, this remains well below the $\rho = 0.943$ agreement that TOPSIS and VIKOR established MCDM techniques that require no training data at all achieve with the same proposed ranking (Table 9).

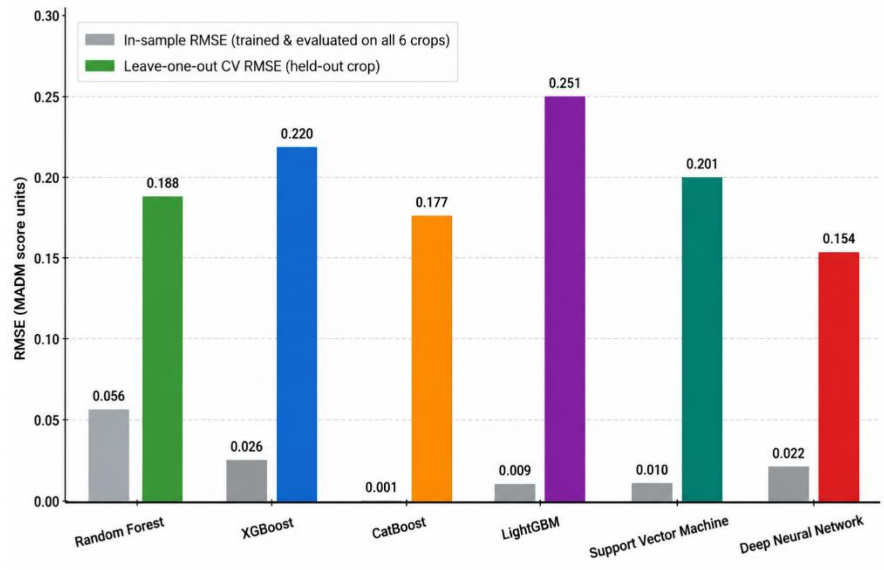

Figure 11. In-sample training RMSE versus leave-one-out cross-validated RMSE for each of the six surrogate models. The large gap between the grey (in-sample) and coloured (LOOCV) bars for Random Forest, XGBoost, CatBoost, LightGBM and the SVM is the signature of overfitting on n = 6 samples; the Deep Neural Network shows the smallest gap.

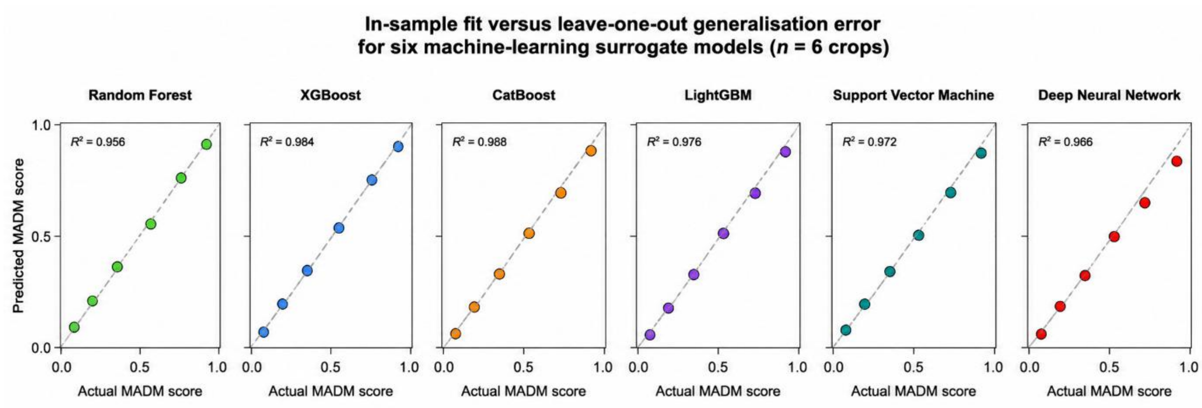

Figure 12. Leave-one-out predicted MADM score against the actual MADM score of Table 4 for each of the six surrogate models; the dashed line marks perfect prediction. Points scattered far from the dashed line indicate a crop whose score the model could not recover when it was excluded from training.

Figure 12 shows that Barley, the clear outlier of the dataset (Table 4), is predicted reasonably well by every model, since it is far from the other five points in feature space and is therefore always the easiest crop to extrapolate to. The errors of the tree-based models and the SVM are concentrated among Rice, Maize, Wheat, Cotton and Soybean, which occupy a comparatively narrow band of the four-dimensional feature space (Table 3) that neither trees nor a fixed-kernel SVM can interpolate between reliably from only five training points; the Deep Neural Network's predictions for this same middle group are visibly closer to the dashed identity line.

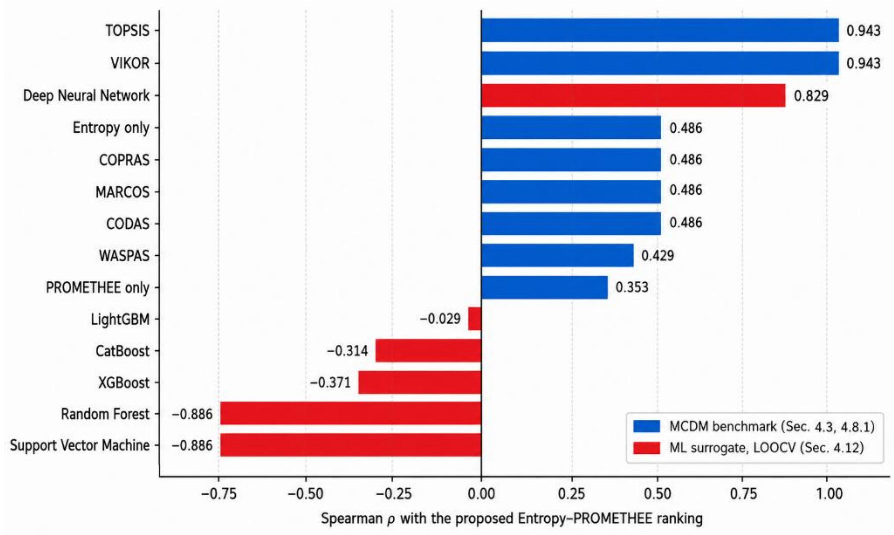

Figure 13. Spearman rank correlation with the proposed Entropy-PROMETHEE ranking, comparing the established MCDM benchmarks already reported in Tables 9 and 11 (blue) with the six machine-learning surrogates evaluated by leave-one-out cross-validation in this section (red).

Figure 13 places the two families of benchmark side by side. The distance-and-compromise MCDM techniques (TOPSIS, VIKOR, MARCOS, CODAS, COPRAS, WASPAS) and the paper's own ablation variants (Entropy only, PROMETHEE only) all sit at $\rho \geq 0.35$, and the strongest of them (TOPSIS, VIKOR) reach $\rho = 0.943$. By contrast, five of the six machine-learning surrogates fall below zero or close to it including the Support Vector Machine, which ties Random Forest as the least rank-consistent model overall and only the regularised Deep Neural Network approaches, without matching, the weaker MCDM benchmarks. This is not a criticism of Random Forest, XGBoost, CatBoost, LightGBM or the SVM as algorithms; it simply reflects that all five are designed for regimes with tens, hundreds or thousands of labelled examples, whereas the crop-selection problem addressed in this paper has exactly six alternatives and no historical outcome labels to train on. The central practical implication is that, for small-alternative-set agricultural decision problems of this kind, an interpretable, formally-grounded MADM framework such as the proposed Neutrosophic Entropy-PROMETHEE model is a substantially safer choice than any data-hungry machine-learning regressor kernel-based, tree-based, or gradient-boosted alike precisely because it does not require a training set to produce a ranking in the first place: it is derived directly and deterministically from the decision matrix and entropy weights of Sections 3.1–3.5. This strengthens, rather than duplicates, the SHAP-based explainability result of Section 4.11: SHAP showed that a single well-behaved surrogate can usefully explain the proposed model's score after the fact, while this section shows that no single surrogate, however sophisticated, can reliably reproduce the proposed model's ranking from scratch on a dataset this small.

5. Conclusion

This study developed and validated a Neutrosophic Entropy-PROMETHEE framework for crop selection under agronomic uncertainty, combining objective entropy-based weighting with pairwise outranking and formal guarantees of boundedness, stability, and Pareto-consistency. The framework identified Rice and Maize as the top-ranked crops, with rankings that agreed closely with six established MCDM benchmarks and were shown to be substantially more reliable than six machine-learning surrogates on this small-alternative-set problem.

5.1 Future Scope

1. Convergence with machine learning and Graph Neural Networks (GNNs) for predicting crop varieties
2. Inclusion of additional parameters such as soil type, humidity, and market dynamics

3. Development of real-time decision-support systems for precision agriculture
4. Extension of the model to large-scale and multi-region agricultural planning

5.2 Concluding Remarks

Overall, the proposed Neutrosophic Entropy-PROMETHEE model offers an effective and formally grounded approach to multi-criteria decision-making under uncertainty, with clear potential for application in intelligent, data-driven agricultural planning.

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