International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(20s) 2026 pp. 567-575
https://doi.org/10.70917/ijcisim-2026-5199
Article

A Joint production-inventory model for stock and price dependent consumption rate with production interruptions

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Abstract: Production control plays a crucial role in economic development of an industry. In practice, it has been perceived that production rate of a manufacturing industry is often interrupted, so we developed the production inventory of deteriorating item considering stock and selling price dependent consumption rate which increases linearly with stock and decreases with price. In this paper we made an attempt to control the production in such a way that balances between overstock and shortage can be maintained. Finally, we have validated the model by set of numerical experiment under different cases to demonstrate the behavior of the proposed model.

Mathematics subject classification: 90B05, 65K10
Keywords: production, deterioration, stock, demand

1. Introduction

A production industry commonly faces the problem of fluctuation in production rate during their production period so it becomes a subject of growing interest for many researchers that how the production should be control and an economic balance can be maintained during the planning horizon. It may happen that production rate decreases suddenly due to financial crisis, loss of goodwill, strike of worker, crisis of raw material, on other hand production may be raised due to high demand of product so it's important to control the production to enhance the economy of industry. The deterioration of items in stock during the storage period constitutes a crucial role to affect the revenue of an industry or business. Deterioration is defined by Wee [1] as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness. For instance, drugs, pharmaceuticals, radioactive substances and food products etc. that deteriorates over a period and their utility decreases with time. Effect of deteriorating items on inventory had been considered firstly by Ghare and Schrader [2] to derive the EOQ model with exponential deterioration.

In manufacturing industries, it is very much difficult to decide the optimal production time and optimal replenishment time of raw materials. In this connection various author have work on production inventory model. He et al. [3] discussed an optimal production-inventory model for deteriorating items with multiple market demand in which they have incorporated the multiple-markets with different selling seasons and deterministic demands. He et al. [4] developed a production model for deteriorating inventory items with production disruptions. They have used a

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constant demand rate and constant deterioration rate to develop the production inventory model. Lee and Hsu [5] developed a two-warehouse inventory model for deteriorating items with time dependent demand. Manna and Chiang [6] established two deterministic economic production quantity models for Weibull distribution deteriorating items with ramp type demand rate. Singh et al. [7] worked on production system for time dependent decaying item and selling price dependent demand under inflationary environment. Yang et al. [8] established an economic order quantity model under inflation with shortages and partial backlogging.

Jonrinaldi and Zhang [9] proposed a model and solution method for coordinating integrated production and inventory cycles in a whole manufacturing supply chain involving reverse logistics for multiple items with finite horizon period. Jha and Shanker [10] formulated a Single vendor multi buyer integrated production-inventory model to minimize the joint total expected cost of the vendor buyers system to determine the optimal production inventory policy. Hishamuddin et al. [11] presented a newly developed disruption recovery model for a single stage production and inventory system, where the production is disrupted for a given period of time during the production up time. Chang [12] presented an improved solution procedure to determine the delivery lot size and the number of deliveries per production batch cycle that minimizes the total cost of the entire supply chain.

In this paper we have developed a production inventory model for a manufacturing firm where the normal production rate is interrupted due to various factor. Since it is practically experienced that stock and price have the significant impact on the demand rate of the consumer so we assumed that the demand rate of the product is increases with stock and decreases with selling price of product. We study the behavior under both the circumstances *i.e* for positive and negative interruption in normal production rate. Finally, we derived the appropriate production time, order time and order quantity when highly reduction occurred in production rate.

2. Assumptions and Notations:

The following assumptions and notation are used throughout the paper to develop the model.

(a)Model considers the single manufacturing facility that produces to meet the demand for a finite time horizon.

(b)Demand rate is stock and price dependent.

(c)Normal Production rate is constant and is greater than the demand rate.

(d)Deterioration rate is constant.

(e)Planning horizon is finite.

(f)Replenishment rate is infinite and lead time is zero.

(g)There is only one chance is allowed to order the product from spot market during the planning horizon if shortage occurs.

(h)Shortages are not allowed.

p : Deterministic normal production rate.

θ : Constant deterioration rate.

$f(t,s)$: Demand rate of products is the function of stock level and selling price of product and is defined as follows:

$f(t,s) = a + bI(t) - cs$, where a, b, c all are positive quantity and $p > f(t,s)$

T_i : Production interruption time.

T_n : Normal production period without interruptions.

T_s : Production period with interruptions.

$I_i(t)$: Inventory level at any time t for $i = 1,2,3.$

Q : Order quantity from spot market if shortage occurs.

T_O : Order time when shortage occurs.

3. Mathematical formulation and analysis:

3.1 The production inventory model without production disruptions

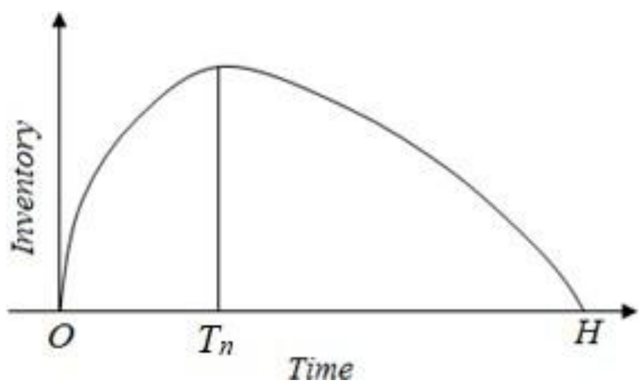


Fig.1: Inventory model with no interruptions

In this proposed model as shown in Fig.1 we assumed that a manufacturer plan to produce a same type of product during a finite planning horizon H and starts the production at time $t = 0$ after certain period of time manufacturer stop the production at T_n making the balance between the demand rate and production rate in such a way that demand would be satisfied at the end of planning horizon H.

If there is no interruption in the normal production rate p and T_n be the normal production period then the dynamic of inventory level at any time t is governed by the following differential equations.

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - f(t,s), \quad 0 \le t \le T_n \tag{3.1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -f(t,s), \quad T_n \le t \le H \tag{3.2}$$

Since from the assumption that demand rate is linearly increasing with stock and decreasing with selling price of the product hence, the differential equation (3.1) and (3.2) reduces to

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - (a + b\, I_1(t) - cs), \quad 0 \le t \le T_n \tag{3.3}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a + b\, I_2(t) - cs), \quad T_n \le t \le H \tag{3.4}$$

With boundary condition $I_1(0) = 0$and $I_2(H) = 0$

The solutions of differential equations (3.3) and (3.4) are given by

$$I_1(t) = \frac{e^{-(\theta+b)t}}{(\theta+b)}(p - a + cs)\left(e^{(\theta+b)t} - 1\right), \quad 0 \le t \le T_n \tag{3.5}$$

$$I_2(t) = \frac{(cs-a)}{(\theta+b)} e^{-(\theta+b)t}\left(e^{(\theta+b)t} - e^{(\theta+b)H}\right), \quad T_n \le t \le H \tag{3.6}$$

From the Fig. 1 considering the continuity of inventory level such that $I_1(T_n) = I_2(T_n)$, implies that

$$\frac{e^{-(\theta+b)T_n}}{(\theta+b)}(p-a+cs)\left(e^{(\theta+b)T_n}-1\right)=\frac{(cs-a)}{(\theta+b)}e^{-(\theta+b)T_n}\left(e^{(\theta+b)T_n}-e^{(\theta+b)H}\right) \quad (3.7)$$

Hence, the production period T_n can be derived from (3.7) as follows

$$T_n=\frac{1}{(\theta+b)}\log\left[1+\frac{(cs-a)\left(1-e^{(\theta+b)H}\right)}{p}\right] \quad (3.8)$$

3.2 The production inventory model with production interruptions

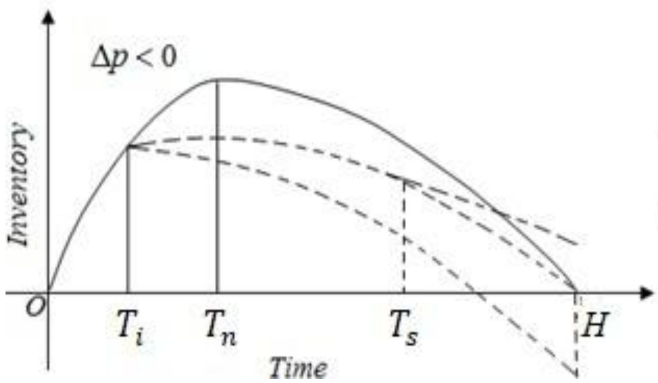


Fig.2(a): Inventory level with negative production interruptions

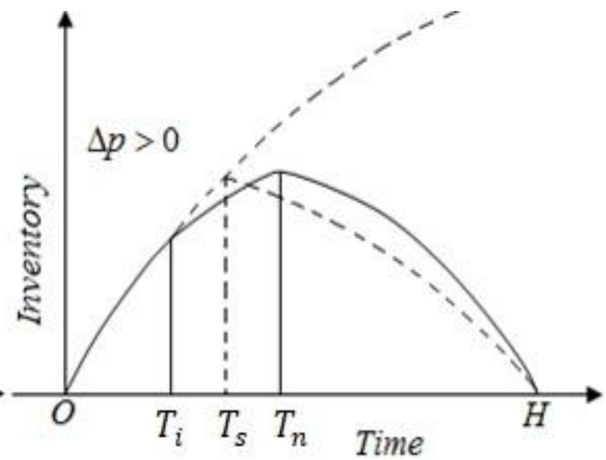


Fig.2(b): Inventory level with positive production interruptions

Since it has been practically experienced that the production rate of a manufacturing industries often affected by various factor. We assume that T_i is the time when production rate gets interrupted. For instant production rate may reduce due to some scarcity of workers, row material or unplanned activity and on the other hand production rate may be increases if demand of the product increases. We consider that $p+\Delta p$ be the new production rate where $\Delta p>0$ if production rate increase and $\Delta p<0$ if production rate decreases. Now it's important to decide that what should be the appropriate production time T_i and how much change in production rate is admissible in order to satisfy all the demand at the end of planning horizon H. Fig 2. represents the various situations of production inventory when interruption occurred.

Proposition: 3.1

If $\Delta p \geq (p-a+cs)\left(1-e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H}-e^{(\theta+b)T_i}\right)$,then manufacturer can still satisfy the demand even after production interruptions where as if $-p \leq \Delta p < (p-a+cs)\left(1-e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H}-e^{(\theta+b)T_i}\right)$ then system will face the shortage due to more interruption in production.

Proof: We assume that production is being continued up to the planning horizon H then the equations of variation of inventory level is represented by following differential equations.

$$\frac{dI_1(t)}{dt}+\theta I_1(t)=p-(a+b\,I_1(t)-cs), \qquad 0\leq t\leq T_i \quad (3.9)$$

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=p+\Delta p-(a+b\,I_2(t)-cs), \quad T_i\leq t\leq H \quad (3.10)$$

with boundary conditions $I_1(0) = 0$ and $I_2(T_i) = I_1(T_i)$ then the solutions of (3.9) and (3.10) are

$$I_1(t) = \frac{e^{-(\theta+b)t}}{(\theta+b)}(p - a + cs)\left(e^{(\theta+b)t} - 1\right), \quad 0 \le t \le T_i \tag{3.11}$$

$$I_2(t) = \frac{e^{-(\theta+b)t}}{(\theta+b)}\left[(p - a + cs)\left(e^{(\theta+b)t} - 1\right) + \Delta p\left(e^{(\theta+b)t} - e^{(\theta+b)T_i}\right)\right], \quad T_i \le t \le H \tag{3.12}$$

From equation (3.12) at time $t = H$

$$I_2(H) = \frac{e^{-(\theta+b)H}}{(\theta+b)}\left[(p - a + cs)\left(e^{(\theta+b)H} - 1\right) + \Delta p\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)\right], \ T_i \le t \le H \tag{3.13}$$

Manufacturer will satisfy all the demand even after disruption in production if $I_2(H) \ge 0$ this implies that

$$\frac{e^{-(\theta+b)H}}{(\theta+b)}\left[(p - a + cs)\left(e^{(\theta+b)H} - 1\right) + \Delta p\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)\right] \ge 0 \tag{3.14}$$

Thus, from the inequality (3.14)

$$\Delta p \ge \frac{(p-a+cs)\left(1-e^{(\theta+b)H}\right)}{\left(e^{(\theta+b)H}-e^{(\theta+b)T_i}\right)} \tag{3.15}$$

Hence, if $\Delta p \ge (p - a + cs)\left(1 - e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)$ then manufacturer will still satisfy the demand even after production interruptions.

On other hand if $I_2(H) < 0$ *i.e.* $-p \le \Delta p < (p - a + cs)\left(1 - e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)$ then shortage will occur due to highly reduction in production rate. Right side of the inequality is the limit by which if we decrease the production rate below this limit then system will face shortage. Hence this proves the proposition.

Since we know that to increase in production rate or continuous production may lead to over stock which increases the holding cost of the system so if change in production is more than the prescribed limit as discussed above *i.e.* $\Delta p \ge (p - a + cs)\left(1 - e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)$ then to overcome from overstock we shut down the production at production shut down time T_s in such a way that at the end of planning horizon inventory level reaches to zero satisfying all the demand during this planning horizon.

Proposition 3.2

If $\Delta p \ge (p - a + cs)\left(1 - e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)$, then optimal production time T_s with production interruptions is

$$T_s = \frac{1}{(\theta + b)}\log\left[\frac{(p - a + cs) - (cs - a)e^{(\theta+b)H} + \Delta p e^{(\theta+b)T_i}}{p + \Delta p}\right]$$

Proof: since, $\Delta p \ge (p - a + cs)\left(1 - e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)$then we have to find the optimal production time T_s such that $T_s \in [T_i, H]$. The inventory levels for this case are depicted in all situations as given in Fig.2 and their behavior after the production interruption time can be represented by the following differential equations.

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - (a + b\,I(t) - cs), \quad T_i \le t \le T_s \tag{3.16}$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -(a + b\,I(t) - cs), \quad T_s \le t \le H \tag{3.17}$$

With the boundary conditions $I_2(T_i) = I_1(T_i)$ and $I_3(H) = 0$, then, we have,

$$I_2(t) = \frac{e^{-(\theta+b)t}}{(\theta+b)}\left[(p-a+cs)\left(e^{(\theta+b)t}-1\right)+\Delta p(e^{(\theta+b)t}-e^{(\theta+b)t_i}\right],\ T_i \le t \le T_s \qquad (3.18)$$

$$I_3(t) = \frac{(cs-a)e^{-(\theta+b)t}}{(\theta+b)}\left((e^{(\theta+b)t}-e^{(\theta+b)H}\right), \qquad T_s \le t \le H \qquad (3.19)$$

Using the boundary condition $I_2(T_s) = I_3(T_s)$ we have,

$$T_s = \frac{1}{(\theta+b)} log\left[\frac{(p-a+cs)-(cs-a)e^{(\theta+b)H}+\Delta p e^{(\theta+b)t_i}}{p+\Delta p}\right] \qquad (3.20)$$

Hence, this proves the proposition.

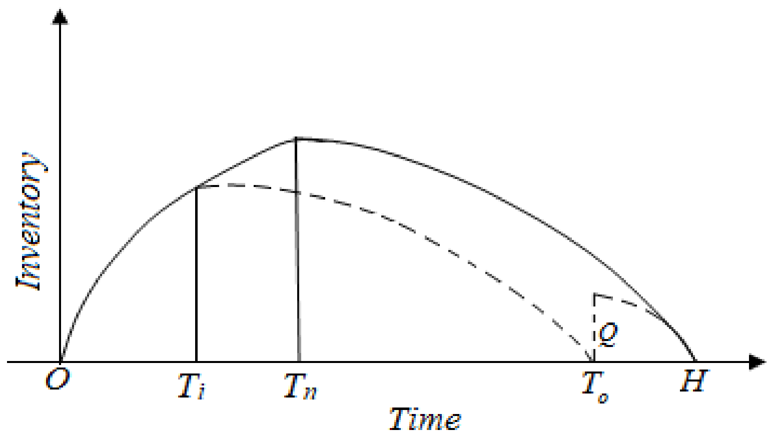


Fig.3: Model when production decreases excessively

If reduction in production rate is more. $-p \le \Delta p < (p-a+cs)\left(1-e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H}-e^{(\theta+b)T_i}\right)$, then manufacturer will face the shortage even production is continue up to complete planning horizon$H$. Fig.3 reflects this situation graphically. To overcome this manufacture will have to order the optimal order quantity from spot market to meet all the demand of the consumer. So problem is converted to find optimal order quantity Q and optimal order time T_O in such a way that inventory get vanished at the end of the planning horizon$H$. Since the production is being done throughout the planning horizon so for this case $T_s = H$.

Proposition 3.3

If $-p \le \Delta p < (p-a+cs)\left(1-e^{(\theta+b)H}\right)/\left(e^{(\theta+b)H}-e^{(\theta+b)T_i}\right)$ then order time and order quantity are obtained by following expressions.

$$T_O = \frac{1}{(\theta+b)}\log\left[\frac{\Delta p e^{(\theta+b)T_i}+(p-a+cs)}{(p+\Delta p-a+cs)}\right]$$

$$Q = \frac{(p+\Delta p-a+cs)\left(1-e^{(\theta+b)(H-T_O)}\right)}{(\theta+b)}$$

Proof: since T_O be the order point where inventory get vanished and hence shortage occurs ,so order point can be obtained by the relation $I_2(T_o) = 0$ using (3.18), which implies that

$$T_O = \frac{1}{(\theta+b)}\log\left[\frac{\Delta p e^{(\theta+b)T_i}+(p-a+cs)}{(p+\Delta p-a+cs)}\right] \qquad (3.21)$$

Variation of inventory level after this order point T_O to planning horizon H can be described by the following differential equation.

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = p + \Delta p - (a + bI_3(t) - cs), T_o \leq t \leq H \tag{3.22}$$

Using the boundary condition $I_3(H) = 0$, hence solution of above equation is

$$I_3(t) = \frac{(p+\Delta p-a+cs)}{(\theta+b)}\left[1 - e^{(\theta+b)(H-t)}\right]T_0 \leq t \leq H \tag{3.23}$$

Replacing t by order time T_O, we have,

$$Q = \frac{(p+\Delta p-a+cs)\left(1-e^{(\theta+b)(H-T_O)}\right)}{(\theta+b)} \tag{3.24}$$

Hence, this completes the proof of proposition.

4. Numerical results and discussion:

Following are the various parameters used to illustrate the developed model numerically and based on these parameters we plotted the various graph representing the result graphically with the help of Mathematica. For calculation purpose we define the limit K from proposition 3.2.

$$K = \frac{(p - a + cs)\left(1 - e^{(\theta+b)H}\right)}{\left(e^{(\theta+b)H} - e^{(\theta+b)T_i}\right)}$$

(a)Model without disruptions:

$p = 500, b = 1, a = 100, c = 2, s = 10, \theta = 0.04, H = 10.$ Using the equation (3.8), we get

Normal production time $T_n = 8.2380$.

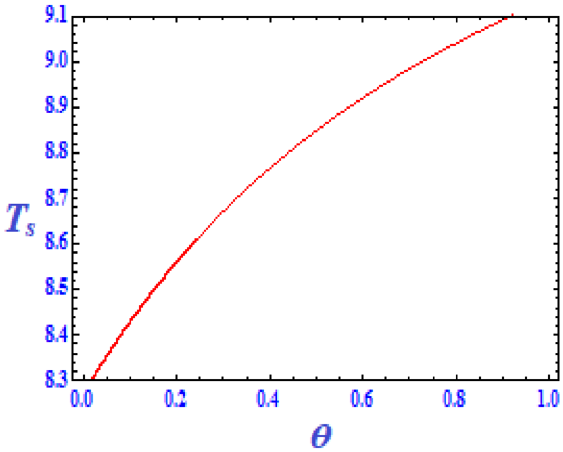
(b) Model with disruptions:

Case 1: $T_i = 4$, $\Delta p = +100$, when production rate increases suddenly and also change in production remains above the limit K ($100 > -420.808$) then the output for this case is $T_s = 8.0650$, The output indicates that manufacturer has to stop the production before the normal production time $T_n = 8.2380$, where the limit$K = -420.808$.

Case 2: $T_i = 4$, $\Delta p = -50$, when production rate decreases suddenly but change in production remains above the limit K *i.e* $\Delta p \geq K$ then the production time we obtained $T_s = 8.3381$. This shows that manufacturer has to continue the production up to $T_s = 8.3381$ which is more than the normal production time $T_n = 8.2380$, where the limit$K = -420.808$.

Case 3: $T_i = 4$, $\Delta p = -425$, this is the situation when production rate decreases drastically and change in production becomes below to the limit ($-425 < K$).We get the result as follows.

$T_s \equiv H = 10,\ T_O = 8.2568,\ Q = 24.6546$.Numerical result for this case revels that shortage will occur at $T_O = 8.2568$and manufacturer has to place an order of quantity $Q = 24.6546$ to satisfy the demand of the consumer even that production will remain continue during whole planning horizon H



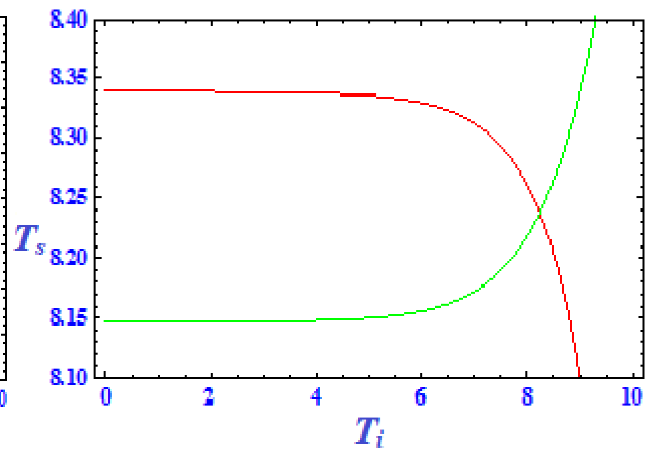


Fig.4: Variation of production time with respect to deterioration rate

Fig.5: Variation of production time with Respect to interruption time

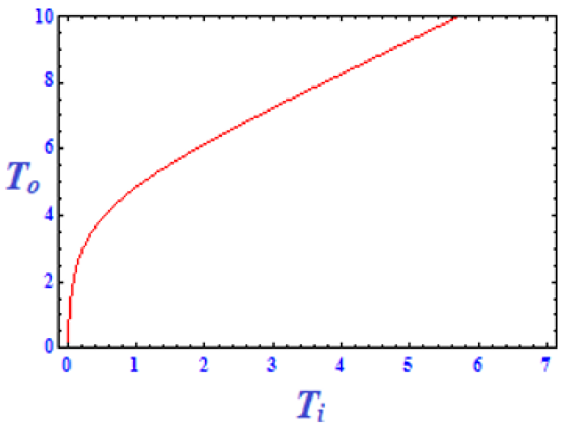


Fig.6: Variation of order time with respect to interruption time.

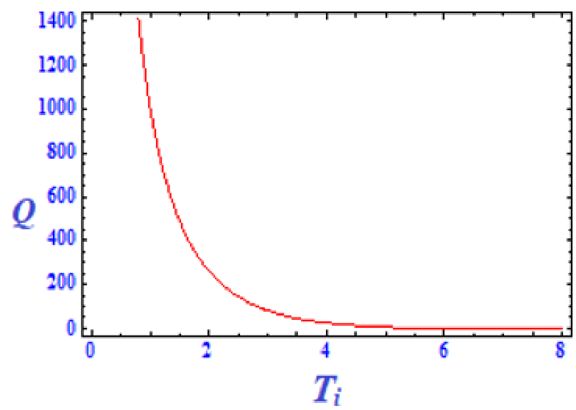
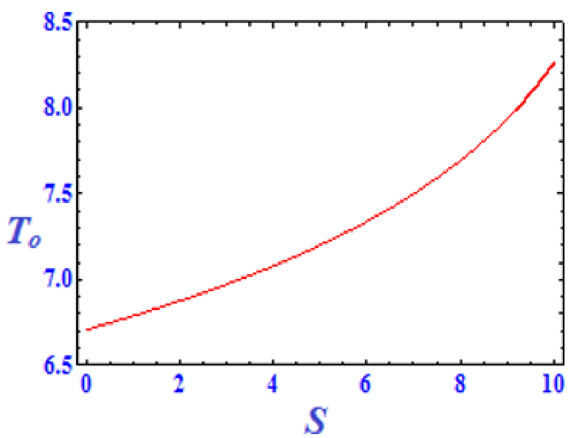


Fig.7: Variation of order quantity with respect to interruption time



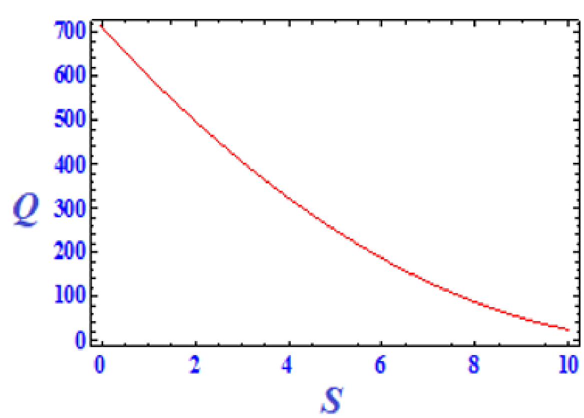


Fig.8: Variation of order time with respect to selling price.

Fig.9: Variation of order quantity with respect to selling price.

From the Fig.4 it is observed that T_s is increasing with respect to the deterioration parameter θ this indicates that if deterioration rate increases then manufacturer has to increase their production time to meet the demand of the consumer. Fig.5 revels that on increasing the production interruption time T_i for positive change in production then the manufacturer has to run the production for some time more in comparison to earlier production interruption time T_i that has been shown by green curve. While the negative effect has been observed for negative change in production exhibited by red curve in Fig.5 which shows that manufacturer has to stop the production earlier in comparison to earlier interruption time T_i.

Fig.6 and Fig.7 are plotted for case when highly reduction in production is happened and hence shortage occurs. Here Fig.6 reflects that if we disrupt the production earlier we have to place an order soon that results more order quantity has to be order or equivalently we can say that order quantity is decreases with increasing production interruption time T_i depicted in Fig.7.

Effects of selling price on order time and order quantity are depicted in Fig.8 and Fig.9 respectively. It has been found from these figures that as manufacture increases the selling price of product then time to place an order increases due to reduction in the demand of product as a consequence less amount of order quantity are required to satisfy the demand of consumer.

5. Conclusions:

This paper describes a production inventory model for determining a production policy for a manufacturing industry. In this paper we admitted that production is being interruption due to various factor like strike of some worker, financial crisis etc. We developed the model assuming demand rate of product is stock and price dependent with constant deterioration rate. We also analyses the behavior of production time, interruption time, order time, order quantity with respect to various parameter.

Moreover, we have shown the various situations depending upon production interruption and discussed that how the decision can be made during such situations that will helpful for the decision maker, so the developed model has a nice application for a manufacturing firm. The model can be extended incorporating some more realistic feature like variable production rate, discount policy etc. for further research.

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