

Triangles in Space: The Design of Gravitational-Wave Observatories for the Millihertz Universe

Aaryan Tanna

Aditya Birla World Academy, Mumbai
aaryantanna208@gmail.com

Abstract: Ground-based interferometers detect gravitational waves down to only a few hertz; below that threshold, seismic motion on Earth makes further sensitivity impossible. Yet the loudest and most scientifically valuable sources in the Universe—merging supermassive black holes, extreme-mass-ratio inspirals, and the vast population of compact binaries within our own Galaxy—radiate at millihertz frequencies, well below what any Earth-bound detector can reach. Observing this band requires leaving the Earth entirely. This paper reviews the observatories, operational, proposed, and conceptual, that are designed to close this gap, and argues that the reach of any gravitational-wave observatory is fixed at the design stage by two architectural choices: its orbit and the length of its interferometer arms. Longer arms convert a passing wave's tiny strain into a larger, more measurable displacement, while the same arm length sets an upper bound on the frequencies a detector can sense. These two relationships explain why heliocentric missions with arms of millions of kilometres reach the millihertz band, while detectors with shorter, geocentric or decihertz-scale arms trade low-frequency sensitivity for higher-frequency reach. Atom-interferometric, lunar-surface, and pulsar-timing concepts extend the picture further still, each occupying a niche the interferometric missions cannot. Ten architectures are classified by orbit and compared against shared physical metrics; the resulting picture is not one dominant design but a complementary network, each observatory suited to a different slice of the gravitational-wave spectrum.

Keywords: gravitational waves, space-based interferometry, LISA, TianQin, DECIGO, pulsar timing arrays, detector architecture, millihertz astronomy, time-delay interferometry, multi-band astronomy

1. INTRODUCTION

Gravitational waves were a theoretical prediction for a full century before an instrument was sensitive enough to detect one. A gravitational wave is a travelling distortion of spacetime itself: as it propagates, it stretches space along one transverse direction while compressing it along the perpendicular direction, and an interferometer measures the resulting fractional change in length, known as strain. That principle was finally confirmed in September 2015, when the two Advanced LIGO detectors jointly registered the passage of a wave produced by a pair of merging black holes [1], launching the field of gravitational-wave astronomy. Advanced LIGO senses strain by bouncing laser light along two perpendicular four-kilometre arms and looking for a differential length change smaller than one part in 10^{21} [2].

This sensitivity, however, has a floor set not by the instrument but by the ground beneath it. Below roughly ten hertz, seismic tremor, ocean-driven microseism, and everyday human activity move the ground by amounts far larger than any gravitational wave, and no isolation system can subtract a noise source of that size [2]. Unfortunately, the Universe's loudest and scientifically richest sources sit well below this floor: supermassive black-hole binaries at galactic centres, stellar-mass black holes long before they merge, extreme-mass-ratio inspirals, and millions of white-dwarf binaries scattered through our own Galaxy all radiate in the millihertz band, roughly between 10^{-4} and 10^{-1} Hz [3], [4]. The only way to hear them is to build a detector that never touches the ground.



Once an observatory leaves Earth, its designer must resolve several interdependent choices: a heliocentric or a geocentric orbit, laser or atom interferometry, three spacecraft or four, arms measured in thousands of kilometres or in millions. None of these choices can be optimised in isolation—each closes off some parts of the gravitational-wave spectrum while opening others, and together they determine the science an observatory can perform before it is ever launched. The argument developed here is that an observatory's architecture, specifically its orbital choice and arm length, sets its reach: the band of frequencies, and therefore the population of astrophysical sources, that it will ever be able to observe. Section II reviews prior work on space-based and alternative gravitational-wave detection concepts; Section III develops the underlying physical relations; Sections IV–VIII survey the proposed and operating architectures grouped by design; Section IX compares them against shared metrics; Section X discusses their complementarity; Section XI notes the limitations of this survey; and Section XII concludes.

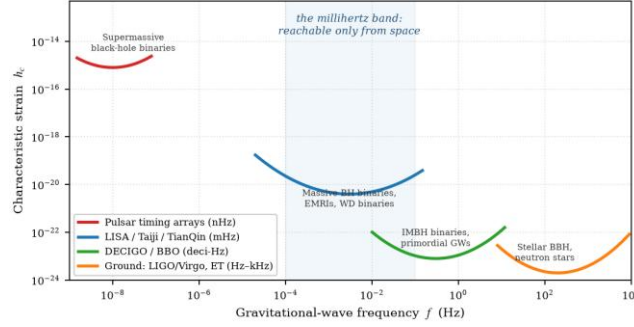


Fig. 1. Sensitivity and frequency coverage of major gravitational-wave detector classes.

Fig. 1. Schematic gravitational-wave spectrum showing characteristic strain h_c against frequency for the principal detector classes; lower curves indicate greater sensitivity. Ground-based interferometers cover only the high-frequency end of the spectrum, leaving the millihertz band, home to the most massive and numerous sources, accessible only from space, following the conventions of [5], [6].

2. LITERATURE REVIEW

The development of gravitational-wave (GW) observatories has progressively extended observational coverage from the high-frequency regime accessible to ground-based interferometers toward the millihertz, decihertz, and nanohertz bands. Advanced LIGO established the observational capability of kilometre-scale laser interferometry [1], while third-generation ground-based concepts such as the Einstein Telescope seek substantially improved sensitivity at frequencies above the seismic wall [7]. The remaining low-frequency gap is therefore addressed through space-based interferometry, long-baseline atom interferometry, lunar observatories, and pulsar timing arrays. Sensitivity-curve studies provide a common basis for relating detector noise to frequency-dependent GW response [5], [6], while the literature on individual missions demonstrates that orbital configuration, arm length, interferometer geometry, and instrumental noise jointly influence their scientific reach.

The most mature space-based architecture is the Laser Interferometer Space Antenna (LISA), which consists of three drag-free spacecraft in a heliocentric triangular constellation with approximately 2.5 million km arms [4]. LISA is designed to probe the millihertz regime and sources including massive black-hole binaries, extreme-mass-ratio inspirals, and Galactic compact binaries [4]. Its architecture establishes the basic design principle underlying many subsequent space-based concepts: extending the interferometric baseline shifts the detector response toward lower frequencies while placing stringent requirements on drag-free control, optical metrology, laser stabilisation, and time-delay interferometry. LISA therefore provides an important reference architecture against which alternative space-based observatories can be evaluated.

Taiji adopts a closely related heliocentric configuration, with approximately 3 million km arms and a constellation leading Earth in its solar orbit [8]. Subsequent studies have considered Taiji both as an independent

observatory and as part of a network with LISA [9], [10]. The LISA–Taiji literature is particularly important because it demonstrates that the scientific benefit of multiple observatories cannot be assessed through arm length or individual sensitivity curves alone. The large separation between the two constellations produces complementary antenna responses and can improve source localisation and parameter estimation [9]. Taiji therefore illustrates how constellation geometry can provide scientific information beyond that obtained by simply increasing the interferometric baseline.

TEGO represents a different modification of the conventional space-interferometer geometry. Rather than using three spacecraft in a triangular configuration, it proposes a four-spacecraft tetrahedral constellation [11]. The additional geometry provides six interferometric links and is designed to enable sensitivity to the six gravitational-wave polarisation modes allowed in general metric theories of gravity [11]. Consequently, TEGO extends the role of a space-based observatory from source detection toward tests of the gravitational interaction itself. Its significance within the present comparison is therefore not limited to arm length; constellation dimensionality and the number of independent interferometric observables introduce an additional architectural variable.

TianQin adopts a contrasting geocentric architecture. Its three spacecraft are designed to form a constellation in Earth orbit with an arm length of approximately 1.7×10^5 km [12]. Early studies established its orbital configuration and scientific objectives, while subsequent work examined orbital optimisation under solar and lunar perturbations [13]. More recent work reports continued technological development and progress toward the staged implementation of the TianQin programme [14]. Compared with heliocentric concepts such as LISA and Taiji, TianQin provides a different balance between frequency coverage, orbital dynamics, constellation stability, communication requirements, and mission operations. It therefore demonstrates that the choice of orbit is a fundamental architectural decision rather than simply an implementation detail.

A separate group of concepts targets the decihertz regime between space-based millihertz observatories and ground-based interferometers. DECIGO was proposed as a space-based detector with shorter interferometric arms and high sensitivity in the decihertz band, with scientific objectives including compact-binary observations and searches for primordial gravitational waves [15], [16]. The Big Bang Observer (BBO) extends the decihertz concept through multiple LISA-like constellations, with cross-correlation between detector outputs proposed as a means of suppressing uncorrelated instrumental noise and improving sensitivity to a stochastic cosmological background [17]. These concepts demonstrate that shorter arms are not intrinsically inferior to LISA-scale baselines; rather, they are deliberately selected to shift the detector response toward higher frequencies and to access a different population of sources.

ALIA provides another intermediate architecture between LISA-class millihertz observatories and decihertz concepts. It was proposed as a shorter-arm, higher-frequency evolution of the LISA concept, with an approximately 5×10^5 km heliocentric baseline and enhanced instrument-performance requirements [26]. Its relevance to the present study lies in demonstrating that arm length cannot be considered independently of instrumental noise. Reducing the baseline shifts the interferometric response toward higher frequencies, but maintaining competitive strain sensitivity requires corresponding improvements in displacement noise, acceleration noise, optical power, or telescope performance [26]. ALIA therefore provides an important intermediate case for examining the relationship between baseline length, frequency response, and instrument requirements.

Atom-interferometric observatories introduce a fundamentally different sensing technology. SAGE proposes the use of ultracold atoms, atomic clocks, and atom interferometry for gravitational-wave detection together with fundamental-physics objectives [18]. AION similarly targets the frequency region between space-based laser interferometers and ground-based detectors through a staged atom-interferometry programme [19]. Unlike conventional laser interferometers, these concepts use atomic matter waves as inertial references, potentially enabling long-baseline measurements with different systematic-noise characteristics. However, the technology remains substantially less mature at the large baselines required for a space-based observatory. The SAGE and AION concepts

therefore represent a complementary technology pathway rather than a direct replacement for LISA-class laser interferometry.

GLOC provides a further alternative by locating the observatory on the lunar surface rather than in free space. The proposed Gravitational-wave Lunar Observatory for Cosmology targets the decihertz-to-hertz frequency range and seeks to exploit the lunar environment for gravitational-wave observations [20]. A lunar architecture could avoid some of the formation-flying and propellant requirements associated with free-flying spacecraft while introducing its own challenges involving lunar infrastructure, environmental disturbances, optical stability, and deployment. GLOC consequently represents a conceptual intermediate-frequency architecture whose ultimate performance remains dependent on substantial technological development.

At the lowest frequencies considered in the present comparison, pulsar timing arrays (PTAs) use naturally occurring astrophysical systems as the timing reference. PTAs monitor the arrival times of pulses from millisecond pulsars and search for correlated timing residuals induced by gravitational waves. The Hellings–Downs correlation provides the characteristic angular correlation expected from an isotropic stochastic gravitational-wave background [21]. The astrophysical interpretation and expected sources of the nanohertz signal have been extensively studied [22], while the NANOGrav 15-year data set provided evidence for a common-spectrum process with spatial correlations consistent with the expected gravitational-wave signature [23]. PTAs therefore demonstrate a fundamentally different form of detector architecture in which Earth–pulsar distances provide effective baselines of thousands of light-years without the deployment of purpose-built spacecraft.

Two additional areas of literature underpin the comparison across these architectures. First, sensitivity-curve methodologies provide the common framework for translating instrumental noise into frequency-dependent detector sensitivity [5], [6]. Second, technological developments such as drag-free control and time-delay interferometry are essential for converting the theoretical advantages of long-baseline space interferometers into practical measurement capability. LISA Pathfinder provided an important in-flight demonstration of the drag-free and inertial-sensing technologies required for future space-based interferometers [24], while time-delay interferometry provides the signal-processing framework required to suppress residual laser-frequency noise in unequal-arm space interferometers [25].

Taken together, the literature establishes the scientific and engineering rationale for the major gravitational-wave observatory architectures, but the existing studies generally evaluate individual missions according to their own science objectives, orbital configurations, and instrument noise budgets. A common comparison spanning heliocentric laser interferometers, geocentric constellations, decihertz observatories, atom interferometers, lunar concepts, and pulsar timing arrays remains comparatively limited. The present study addresses this gap by organising these architectures around orbit, arm length, transfer frequency, and dominant low-frequency noise constraints. These variables are treated as first-order architectural determinants rather than as substitutes for complete mission-specific sensitivity models, allowing the different observatory concepts to be compared on a common physical basis.

3. RESEARCH METHODOLOGY

A. *The Physics that Fixes the Reach*

Two quantities that follow directly from how an interferometer measures a passing wave are sufficient to explain the entire landscape of architectures that follows. The first is the definition of strain itself. A gravitational wave of

amplitude h passing a detector whose arms have length L changes each arm's length by

$$h = \frac{\Delta L}{L}, \quad \text{so} \quad \Delta L = h L. \quad (1)$$

Because h is of order 10^{-21} , the absolute displacement ΔL that must be measured is minuscule, but it scales in direct proportion to arm length. Advanced LIGO's four-kilometre arms yield $\Delta L \sim 10^{-18}$ m; a space-based

interferometer with arms of two and a half million kilometres turns an identical strain into a displacement of order picometres [4], a far more tractable target. This is the first design lesson: longer arms convert an unchanged spacetime strain into a larger, more measurable length change.

A gravitational wave's two independent polarisations, h_+ ('plus') and $h_\times$ ('cross'), act in the plane transverse to its direction of travel as the traceless strain patterns

$$h_+ \propto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad h_\times \propto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

the same symmetric, traceless tidal pattern that governs ocean tides, propagating at the speed of light. How well a detector separates these two patterns depends on its geometry, which is exactly why concepts such as TEGO add a fourth spacecraft [11], and why networks of detectors [9] add information a single instrument cannot recover.

The second relation sets the high-frequency edge of a detector's response. An interferometer only responds cleanly to waves whose period exceeds the light travel time along an arm; once a gravitational wave's wavelength becomes comparable to the arm length itself, a crest and a trough occupy the arm at the same instant and the signal averages away. This crossover defines the *arm transfer frequency*

$$f_* = \frac{c}{2\pi L}, \quad (2)$$

above which sensitivity falls away. Equation (2) is the organising relation of this paper: it states that the accessible frequency band is set by arm length alone. Advanced LIGO's four kilometres give $f_* \sim 10^4$ Hz, safely above its seismic floor; DECIGO's thousand-kilometre arms place f_* near tens of hertz, opening the deci-hertz window [16]; and LISA's arms of millions of kilometres pull f_* down to roughly twenty millihertz [6]. Longer arms therefore reach lower frequencies at the cost of the higher ones—arm length alone decides which sources a mission can, in principle, ever observe.

Two further effects close the band from below and complete the picture. At the lowest frequencies, the dominant noise source is spurious acceleration of the test masses; writing S_a for the residual acceleration noise, its contribution to strain noise scales as

$$h_n(f) \sim \frac{\sqrt{S_a(f)}}{(2\pi f)^2 L}, \quad (3)$$

so a long arm L again helps, but only if the test masses are held in near-perfect free fall. The LISA Pathfinder mission demonstrated exactly this, achieving sub-femto- g residual acceleration and validating the drag-free control that every space mission surveyed below depends on [24]. Because a single laser link in space cannot form a physically rigid arm, the outputs of multiple links must instead be combined in software to synthesise an interferometer and cancel otherwise overwhelming laser-frequency noise, the technique of time-delay interferometry (TDI) [25], whose flexibility itself depends on constellation geometry. With strain, the high-frequency limit, free fall, and TDI established, every architecture surveyed below reduces to two design choices: where the test masses sit, and how far apart they fall.

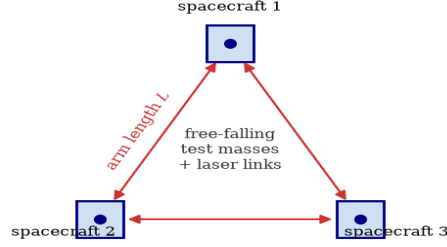


Fig. 2. Gravitational-wave interferometer

Fig. 2. The interferometer geometry shared by LISA, Taiji, and TianQin: three drag-free spacecraft in an equilateral triangle, each carrying free-falling test masses linked by laser beams of arm length L . Length changes are recovered by combining the six one-way links through time-delay interferometry [25].

B. Heliocentric Architectures: The Millihertz Band

A. LISA and Taiji, and the Value of a Network

The Laser Interferometer Space Antenna (LISA) and China's Taiji mission are the flagship concepts for the millihertz band. Each places three drag-free spacecraft in an equilateral triangle trailing or leading the Earth in a heliocentric orbit, using laser interferometry between free-falling test masses to locate and characterise massive black-hole binaries, stellar-mass binaries, and black-hole–neutron-star systems [4], [10]. LISA's arms measure roughly 2.5 million kilometres; Taiji's are somewhat longer, near 3 million, giving it marginally greater sensitivity at the lowest frequencies [8], exactly as Equation (2) predicts. LISA trails the Earth by about 20° ; Taiji leads it by a comparable angle, the two separated by roughly 0.7 AU.

A single triangular detector localises a source on the sky only poorly. Combined, however, LISA and Taiji form a network whose wide baseline sharply improves the angular localisation of massive black-hole binaries, in some cases by orders of magnitude over either mission alone [9]. The principle that shapes a single mission's design also shapes how missions can be combined.

A heliocentric orbit brings its own costs and benefits. Solar radiation pressure and the solar wind continuously disturb the test masses, requiring active drag-free compensation; the distance from Earth imposes long communication delays and demands a high degree of spacecraft autonomy; and propellant depletion and thruster wear ultimately limit mission lifetime, with no possibility of refuelling. Against these costs, the triangular formation is largely self-maintaining, since the spacecraft orbits are chosen so the constellation keeps its shape almost passively through the year [4]. The stable deep-space thermal environment reduces solar disturbance, and only a heliocentric orbit provides the room for arms spanning millions of kilometres, a scale impossible to achieve around the Earth.

B. TEGO: A Tetrahedron for Six Polarisations

The Tetrahedron Constellation Gravitational-Wave Observatory (TEGO) replaces the triangle with four spacecraft in a tetrahedral formation, a stable triangular base with a fourth craft held above it, in a heliocentric orbit with arms near 3 million kilometres [11]. The extra spacecraft buys two capabilities a planar mission cannot reach. First, its three-dimensional geometry makes TEGO sensitive to all six gravitational-wave polarisations permitted under generalised metric theories of gravity, not merely the two predicted by general relativity, turning the mission into a direct test of gravity itself. Second, the larger number of independent laser links substantially expands the combinations available for time-delay interferometry, suppressing laser-frequency noise more deeply than a triangular constellation allows [25].

C. DECIGO and BBO: Bridging to the Deci-Hertz

DECIGO, the Deci-hertz Interferometer Gravitational-Wave Observatory, targets the 0.1–10 Hz band, chosen to sit above the millihertz confusion noise produced by Galactic white-dwarf binaries and to search for primordial gravitational waves from shortly after the Big Bang [15], [16]. Its architecture differs from LISA's in two ways dictated by that higher band. By Equation (2), reaching tens of hertz requires short arms, and DECIGO's are only about 1000 km. Rather than free-floating proof masses, each cluster of three spacecraft forms Fabry–Pérot Michelson interferometers between large mirrors, lowering environmental noise at these frequencies but requiring the arm length to be held nearly constant so the optical cavities stay resonant, active control of a kind LISA-class missions do not need. The Big Bang Observer (BBO) is a proposed successor aimed at the primordial background, using four LISA-like heliocentric constellations; overlapping two of them allows their independent noise to be cross-correlated away, exposing the cosmological signal beneath [17].

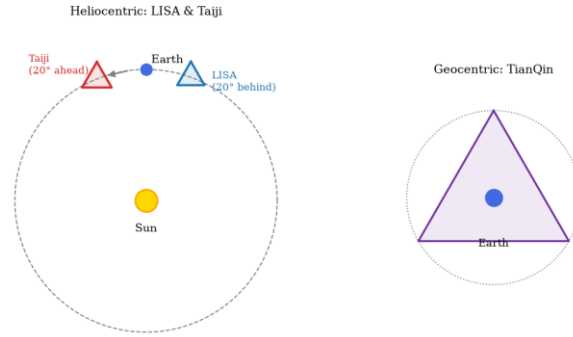


Fig. 3. Comparison of heliocentric and geocentric gravitational-wave detector architectures.

Fig. 3. Two orbital architectures. Left: the heliocentric missions LISA and Taiji trail and lead the Earth by about 20° , their million-kilometre triangles maintained passively by solar orbit. Right: TianQin's geocentric triangle orbits the Earth, with far shorter arms but easier communication and launch [12], [13].

C. A Geocentric Architecture: Tianqin

TianQin takes the opposite orbital choice, placing its triangular constellation in a high geocentric orbit [12]. It was originally conceived to observe a specific verification source, the white-dwarf binary RX J0806.3+1527, before the first direct gravitational-wave detection; subsequent work showed it could also reach double white dwarfs, massive black-hole binaries, stellar-mass binaries, and extreme-mass-ratio inspirals [14]. The geocentric choice reshapes every engineering trade-off. Because the orbital plane is fixed in space, the Sun crosses it every half year; to protect the optics, TianQin follows a 'three months on, three months off' observing schedule, trading duty cycle for optical stability.

Solar and lunar gravity perturb the triangle, so its orbit is chosen to minimise deviation from the equilateral configuration [13]; the orbit further avoids lunar eclipses outright and keeps Earth eclipses outside the observing windows, sparing the spacecraft thermal and power transients, while lunar and terrestrial tidal forcing on the test masses, a potential noise source, is suppressed by the same orbital choice. Solar-wind plasma interacting with the lasers was assessed to remain within acceptable bounds. In exchange for these constraints, a geocentric orbit offers concrete advantages: more mature and less costly technology, far shorter launch and transfer times, and rapid data return with minimal communication delay. Its price is paid in arm length: shorter arms mean a higher-frequency band and reduced sensitivity at the lowest frequencies relative to the heliocentric flagships, precisely the trade that Equation (2) makes unavoidable.

D. Atom-Interferometric Architectures: Sage and Aion

A different sensing technology answers the same underlying problem. The Space Atomic Gravity Explorer (SAGE) and the Atom Interferometer Observatory and Network (AION) replace mirrored or gold proof masses with clouds of ultra-cold atoms, strontium in SAGE's design, read out through atom interferometry, ultra-precise atomic clocks, and lasers [18], [19]. Atomic proof masses are exceptionally well characterised and largely immune to the charging events and some of the laser noise that affect macroscopic test masses, relaxing certain laser stability requirements; their effective sensitivity can also be tuned in flight through applied optical fields, allowing a detector to track a binary as its frequency sweeps upward.

SAGE targets roughly 0.001–10 Hz using two complementary strain measurements: differential phase across light-pulse atom interferometers, which amplifies the phase signal and favours shorter baselines, and differential frequency measurement with lattice clocks, which favours very long baselines of up to about 10 million kilometres and offers continuous tunability as binary frequencies rise [18]. AION applies closely related technology to intermediate-mass black-hole mergers and the supermassive black holes they may seed [19]. Because the atoms themselves suppress laser-frequency noise, an atom interferometer can operate with only two spacecraft rather than three, cutting cost, simplifying orbital dynamics, and enabling a linear, more easily controlled configuration.

E. Lunar Architectures: Gloc

A gravitational-wave detector need not orbit at all; it can instead stand on the lunar surface. Concepts such as the Gravitational-wave Lunar Observatory for Cosmology (GLOC) target the deci-hertz to 1 Hz range, a band too high for most orbiting missions, whose sensitivity is limited by residual spacecraft acceleration, and inaccessible to ground-based detectors because of seismic noise [20]. The Moon is seismically far quieter than the Earth, and a surface instrument escapes the finite fuel and thruster lifetime that caps orbiting missions; repair and upgrade, essentially impossible for a deep-space spacecraft, become at least conceivable. GLOC could observe black-hole binaries across the lower and intermediate mass range over much of the observable Universe, where space-based detectors are often limited to within roughly a gigaparsec. The trade-off remains geometric: a lunar instrument cannot match the million-kilometre arms of a heliocentric mission, so its low-frequency sensitivity is correspondingly reduced, and the low-frequency white-dwarf foreground must still be subtracted to isolate individual binaries.

F. The Earth as the Array: Pulsar Timing

The final architecture surveyed here builds no spacecraft at all. A pulsar timing array (PTA) treats the Galaxy itself as a detector: the distances between the Earth and an ensemble of millisecond pulsars act as its arms, with the Earth as the hub [21], [22]. Millisecond pulsars are among the most stable natural clocks known; a gravitational wave passing between the Earth and a pulsar perturbs the arrival times of its pulses in a characteristic quadrupolar pattern, and monitoring the timing residuals of many pulsars simultaneously allows that strain to be extracted. The angular signature, the Hellings–Downs correlation between pulsar pairs as a function of their angular separation, is the fingerprint distinguishing a genuine gravitational-wave background from other sources of timing noise [21].

With arms measured in thousands of light-years, a PTA reaches the nanohertz band: the gravitational-wave background produced by the population of supermassive black-hole binaries across cosmic history, entirely beyond the reach of any Earth- or space-based interferometer. Early analyses compared only a handful of pulsars; today's arrays time close to a hundred, and in 2023 several collaborations reported evidence consistent with exactly the Hellings–Downs-correlated background a PTA is designed to find [23]. A PTA thus converts naturally occurring celestial bodies, the Earth and the Galaxy's own pulsars, into an observatory that no other architecture surveyed here needs to launch.

millisecond pulsars act as the 'arms'; Earth is the hub

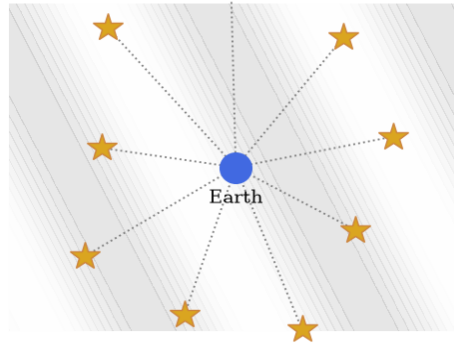


Fig. 4. Pulsar timing array configuration for detecting nanohertz gravitational waves.

Fig. 4. A pulsar timing array uses the Galaxy itself as a detector: Earth–pulsar distances act as arms thousands of light-years long, granting access to the nanohertz band. The Hellings–Downs correlation among pulsar pairs is the signature of a stochastic gravitational-wave background [21], [23].

4. COMPARATIVE ANALYSIS

TABLE I places the ten surveyed architectures side by side, and Fig. 5 plots them against the single relation that organises them all, the arm transfer frequency of Equation (2). The trend is unambiguous: as arm length increases from the kilometre scale of ground detectors, through the thousand kilometres of DECIGO and the hundred thousand of TianQin, to the millions of kilometres of LISA, Taiji, and TEGO, the accessible frequency band shifts steadily downward. Despite differing technologies and orbits, the missions fall along a single arm-length–frequency curve, exactly as the physics of Section III predicts.

TABLE I

TEN GRAVITATIONAL-WAVE OBSERVATORY ARCHITECTURES CLASSIFIED BY ORBIT AND PLATFORM

Observatory	Orbit	Arm Len.	Band	Sources	Status
LISA	Heliocentric, trailing $\sim 20^\circ$	$\sim 2.5 \times 10^6$ km	~ 0.1 – 100 mHz	Massive BH binaries, EMRIs, Galactic binaries	Adopted
Taiji	Heliocentric, leading $\sim 20^\circ$	$\sim 3 \times 10^6$ km	~ 0.1 – 100 mHz	Massive BH binaries; LISA–Taiji network	In dev. (China)
TianQin	Geocentric	$\sim 1.7 \times 10^5$ km	~ 0.1 mHz– 1 Hz	White-dwarf & BH binaries, EMRIs	In dev. (China)
TEGO	Heliocentric, tetrahedral	$\sim 3 \times 10^6$ km	~ 0.1 – 100 mHz	All six GW polarisations; tests of gravity	Concept
DECIGO	Heliocentric, 4 clusters	$\sim 10^3$ km	~ 0.1 – 10 Hz	Primordial GWs; bridges mHz–Hz gap	Concept (Japan)

BBO	Heliocentric, 4 const.	LISA-like	deci-Hz	Primordial / Big-Bang background	Concept
ALIA	Heliocentric, LISA-like (shorter)	< LISA	mHz–deci-Hz	Intermediate-mass black holes	Concept
SAGE / AION	Atom interferometry (2 s/c)	up to $\sim 10^7$ km	~ 1 mHz–10 Hz	IMBH mergers; tunable sources	Concept
GLOC	Lunar surface	(surface)	~ 0.1 –1 Hz	Stellar & intermediate-mass BH binaries	Concept
Pulsar timing arrays	Galaxy (Earth + pulsars)	$\sim 10^3$ ly	~ 1 –100 nHz	Supermassive BH-binary background	Operating

Arm length and frequency band move together exactly as Equation (2) requires; bands are approximate, with sources given in Sections IV–VIII.

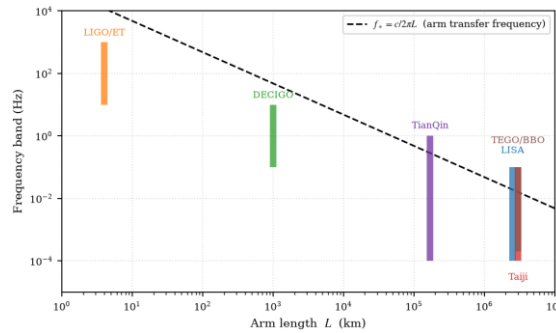


Fig. 5. Comparison of detector arm length and corresponding frequency coverage.

Fig. 5. Each observatory's frequency band (coloured bar) plotted against arm length, with the arm transfer frequency $f_* = c/2\pi L$ of Equation (2) shown as a dashed guide. Shorter arms reach higher frequencies; longer arms reach lower ones.

5. DISCUSSION: A NETWORK OF COMPLEMENTARY ARCHITECTURES

Considered individually, the missions surveyed in Sections IV–VIII can look like competing proposals for similar triangles in similar orbits. Viewed through Equation (2), however, they instead divide the gravitational-wave spectrum between them. Pulsar timing arrays already occupy the nanohertz band [23]; the heliocentric flagships will occupy the millihertz [4], [8]; DECIGO, BBO, and lunar observatories target the deci-hertz bridge between space and ground [15], [20]; and the existing ground-based network covers tens to thousands of hertz [2], [7]. No single architecture spans even two of these bands, because the arm length that opens one closes the others.

Complementarity also operates within a single band. The LISA–Taiji network demonstrates that two millihertz detectors with a wide baseline localise sources far better than either alone [9]; TEGO's fourth spacecraft and BBO's overlapped constellations both trade additional hardware for access to information, polarisation content and the cosmological background respectively, that geometry alone would otherwise hide [11], [17]. The same logic motivates multi-band astronomy: a stellar-mass binary observed by LISA years before it sweeps into the ground-based band could be predicted, localised, and later caught at merger, with each detector contributing exactly what its architecture

allows [3]. The diversity of designs surveyed in this paper is therefore not redundancy but necessity: only a range of architectures can cover a spectrum that no single instrument spans.

6. LIMITATIONS

This survey has several limitations that should be kept in mind when interpreting its conclusions. First, the comparison in Section IX relies on published, mission-level estimates of arm length and frequency band rather than a uniform sensitivity-curve calculation performed under a single, common noise model; different missions' figures were derived under different assumptions, and a quantitative side-by-side comparison using the methods of [5], [6] remains future work. Second, several of the architectures reviewed here — TEGO, SAGE, AION, GLOC, BBO, and ALIA — remain at the concept or pre-formulation stage, so their published parameters may change substantially before, or in the absence of, adoption; the survey should be read as a snapshot of current proposals rather than a forecast of what will ultimately fly. Third, the paper's central relations, Equations (1)–(3), capture the dominant design trade-offs but omit second-order effects such as orbit-dependent thermal noise, mission-specific optical design, and detailed noise budgets, all of which shift a real mission's sensitivity curve away from the simple arm-length scaling used here. Fourth, this is a literature-based survey rather than an independent simulation or original data analysis; its conclusions depend on the accuracy and currency of the source publications cited throughout, several of which describe missions still under active revision. Finally, the ten architectures classified in TABLE I are not exhaustive; other proposed concepts exist and were excluded for reasons of scope rather than relevance.

7. CONCLUSION AND FUTURE SCOPE

Starting from a single idea, that orbit and arm length fix a gravitational-wave observatory's reach, this paper has surveyed ten architectures spanning heliocentric and geocentric orbits, laser and atom interferometry, the lunar surface, and the Galaxy itself. Two relations carry the argument: longer arms turn an unchanged strain into a larger displacement, $\Delta L = hL$, while the arm transfer frequency $f_* = c/2\pi L$ caps the band a given architecture can reach. Every mission examined is a different resolution of this trade-off, and the comparison in Section IX shows them falling, as predicted, along a single arm-length–frequency curve.

The implication is that the future of gravitational-wave astronomy is not one ultimate detector but a network of complementary observatories, each suited to a different region of the spectrum. Several questions remain open for future work: a quantitative sensitivity-curve comparison using a common noise model [5], [6]; the science return of specific multi-band detections; and the engineering maturity of drag-free control [24], time-delay interferometry [25], and atom-interferometric proof masses [18] on which the next decade of missions depends. What is already clear is that an observatory's reach is largely fixed the moment its orbit and arm length are chosen.

REFERENCES

1. B. P. Abbott et al. (LIGO Scientific & Virgo Collaborations), “Observation of gravitational waves from a binary black hole merger,” *Phys. Rev. Lett.*, vol. 116, p. 061102, 2016.
2. J. Aasi et al., “Advanced LIGO,” *Classical and Quantum Gravity*, vol. 32, no. 7, p. 074001, 2015.
3. B. S. Sathyaprakash and B. F. Schutz, “Physics, astrophysics and cosmology with gravitational waves,” *Living Rev. Relativity*, vol. 12, p. 2, 2009.
4. P. Amaro-Seoane et al., “Laser Interferometer Space Antenna,” arXiv:1702.00786, 2017.
5. C. J. Moore, R. H. Cole, and C. P. L. Berry, “Gravitational-wave sensitivity curves,” *Classical and Quantum Gravity*, vol. 32, no. 1, p. 015014, 2015.
6. T. Robson, N. J. Cornish, and C. Liu, “The construction and use of LISA sensitivity curves,” *Classical and Quantum Gravity*, vol. 36, no. 10, p. 105011, 2019.
7. M. Punturo et al., “The Einstein Telescope: a third-generation gravitational wave observatory,” *Classical and Quantum Gravity*, vol. 27, no. 19, p. 194002, 2010.
8. Y. Gong, J. Luo, and B. Wang, “Concepts and status of Chinese space gravitational wave detection projects,” *Nature Astronomy*, vol. 5, no. 9, pp. 881–889, 2021.
9. W.-H. Ruan, C. Liu, Z.-K. Guo, Y.-L. Wu, and R.-G. Cai, “The LISA–Taiji network,” *Nature Astronomy*, vol. 4, no. 2, pp. 108–109, 2020.

10. W.-H. Ruan, Z.-K. Guo, R.-G. Cai, and Y.-Z. Zhang, “Taiji program: gravitational-wave sources,” arXiv:1807.09495, 2018.
11. G. Wang et al., “TEGO: tetrahedron constellation gravitational-wave observatory,” arXiv:2405.03492, 2024.
12. J. Luo et al., “TianQin: a space-borne gravitational wave detector,” *Classical and Quantum Gravity*, vol. 33, no. 3, p. 035010, 2016.
13. B. Ye et al., “Optimizing orbits for TianQin,” *Int. J. Mod. Phys. D*, vol. 28, no. 9, p. 1950121, 2019.
14. J. Luo et al., “Progress of the TianQin project,” arXiv:2502.11328, 2025.
15. S. Kawamura et al., “Current status of space gravitational wave antenna DECIGO and B-DECIGO,” *Prog. Theor. Exp. Phys.*, vol. 2021, no. 5, p. 05A105, 2021.
16. N. Seto, S. Kawamura, and T. Nakamura, “Possibility of direct measurement of the acceleration of the universe using 0.1 Hz band laser interferometer gravitational wave antenna in space,” *Phys. Rev. Lett.*, vol. 87, p. 221103, 2001.
17. J. Crowder and N. J. Cornish, “Beyond LISA: exploring future gravitational wave missions,” *Phys. Rev. D*, vol. 72, no. 8, p. 083005, 2005.
18. G. M. Tino et al., “SAGE: a proposal for a space atomic gravity explorer,” *Eur. Phys. J. D*, vol. 73, no. 11, p. 228, 2019.
19. L. Badurina et al., “AION: an atom interferometer observatory and network,” *J. Cosmol. Astropart. Phys.*, vol. 2020, no. 05, p. 011, 2020.
20. K. Jani and A. Loeb, “Gravitational-wave lunar observatory for cosmology,” *J. Cosmol. Astropart. Phys.*, vol. 2021, no. 06, p. 044, 2021.
21. R. W. Hellings and G. S. Downs, “Upper limits on the isotropic gravitational radiation background from pulsar timing analysis,” *Astrophys. J. Lett.*, vol. 265, pp. L39–L42, 1983.
22. S. Burke-Spolaor et al., “The astrophysics of nanohertz gravitational waves,” *Astron. Astrophys. Rev.*, vol. 27, p. 5, 2019.
23. G. Agazie et al. (NANOGrav Collaboration), “The NANOGrav 15 yr data set: evidence for a gravitational-wave background,” *Astrophys. J. Lett.*, vol. 951, no. 1, p. L8, 2023.
24. M. Armano et al., “Sub-femto-g free fall for space-based gravitational wave observatories: LISA Pathfinder results,” *Phys. Rev. Lett.*, vol. 116, p. 231101, 2016.
25. M. Tinto and S. V. Dhurandhar, “Time-delay interferometry,” *Living Rev. Relativity*, vol. 24, p. 1, 2021.
26. W.-T. Ni, “Gravitational wave detection in space,” *Int. J. Mod. Phys. D*, vol. 25, no. 14, Art. no. 1630019, 2016.