

PROPAGATION OF NONLINEAR SHOCK WAVES IN A NON-IDEAL DUSTY GAS WITH THERMAL RADIATION UNDER THE OPTICALLY THIN APPROXIMATION

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Abstract: A smooth, finite amplitude disturbance propagating and ultimately becoming a shock in a one-dimensional non-ideal radiative gas is considered. A single generalized-geometry parameter is used for planer, cylindrical and spherical. The gas is considered to be a simplified van der Waals gas, in which molecular co-volume is retained and intermolecular attraction is ignored. Thermal radiation is treated as a local source which is proportional to the difference between the fourth powers of the gas and the ambient temperature, optically thin. The basic equations for the conservation of mass, momentum, and pressure are represented as a quasilinear hyperbolic system with sources. There are three characteristic speeds: the material speed, $u-a$, and $u+a$, the two acoustic speeds. A formula based on the principle of wavefront expansion is then employed to understand the evolution of the amplitude of the leading front and when the smooth solution loses regularity, and to understand how the wavefront steepens with an initially smooth compressive disturbance. The analysis defines separate roles of the governing parameters. The non-ideal sound speed introduces the molecular co-volume to the acoustic matrix, while source terms resulting from radiation or geometry, modify the state transported along the characteristics. For admissible states, the local sound speed is increasing with the co-volume at fixed pressure and density. The radiative term provides cooling for states warmer than the equilibrium state of the atmosphere, and warming for cooler states, and the strength of the cooling or warming depends on a dimensionless radiation parameter. The cylindrical and spherical geometry have increasingly stronger radial source terms than planar flow. Together, they control the amplitude growth and decay as well as the critical propagation distance to shock formation. The formulation allows for evidence-informed future computations.

Keywords: Van der Waals gas, shock formation, radiative cooling, wavefront expansion, non-ideal gas dynamics

1. INTRODUCTION

1.1 Scientific Background and Study Context

A shock wave is a nonlinear, compressive wave in which the thermodynamic and kinematic variables may exhibit large changes across the wave. They are encountered in high-speed flight, in explosions, in propulsion, in gas dynamics in the laboratory, and in astrophysical flows, where finite-amplitude waves travel through media whose properties can be significantly different from those of an ideal gas. The effects of real-gas constitutive behavior, transport processes and geometry have also been shown to modify shock propagation and post-wave structure in recent studies (Sakamoto et al., 2024; Pandey & Patel, 2026). Analytical descriptions are still useful in helping to uncover how each of the physical mechanisms appears in the governing equations and which conditions are necessary for a smooth solution to steepen into a discontinuity. Modern investigations of converging and piston-driven shocks also



highlight the importance of geometry and constitutive closure for the wave phenomenon, not as corrections, but as a fundamental part of the dynamics itself (Jang et al., 2025; Calvo-Rivera et al., 2023). The present investigation is an investigation of a one-dimensional, finite amplitude disturbance in a radiating gas for which the following approximate van der Waals relation is used. It treats the generalized planar, cylindrical and spherical geometry along with an optically thin heat-transfer source, and considers the resulting hyperbolic characteristic structure. A situation that is of special interest is a continuous disturbance, whose speed and gradient vary with time during propagation under changing conditions due to the presence of constitutive nonlinearities and thermal exchange.

1.2 Problem Statement

Two mechanisms are not considered in ideal, non-radiative models, and can be significant in compressed, high-temperature gases: the excluded volume of molecules and thermal exchange with the surroundings. Recent studies have shown that the wave behavior of a van der Waals closure is different from classical perfect gas relationships (Calvo-Rivera et al., 2022; Prodanov, 2022), and shock attenuation and state relations are modified by caloric and real-gas effects (Civrais et al., 2023; Döntgen, 2024). This unanswered analytical problem, then, is to elucidate the role of molecular co-volume, optically thin radiation and radial geometry in propagation and steepening of a smooth nonlinear disturbance in one consistent model. The characteristic speeds and the organization of the source-term must be preserved; numerical trends must not be imposed that are not prescribed by the analytical formulation. The solution to be attained should therefore differentiate between exact deducible deductions and those that are dependent on the initial amplitude, state, and direction of propagation, and should still allow for every modeled source term to be preserved during radial propagation.

The aims and objectives of the research are stated below.

The goal is to develop and analyse the nonlinear propagation and shock formation of waves in a non-ideal radiative gas. The aims are: to obtain the governing quasilinear conservation law system for the generalized geometry; to include a simplified van der Waals equation of state and the corresponding speed of sound; express the optically thin radiation in dimensionless form; find the families of characteristics; apply wavefront expansion to the leading disturbance; and discuss the influence of co-volume, radiation, and geometry on amplitude propagation and the finite distance where regularity is lost. These objectives highlight computation that is reproducible, with interpretations that are evidence-based, instead of computation without reasons. The analysis also aims to formulate the admissibility conditions for hyperbolicity and to distinguish direct changes in the characteristic speed from indirect ones that occur as a result of source-driven evolution of the state domain over the modeled state domain.

1.3 Scope and Original Contribution

Model is inviscid, one-dimensional, and thermally radiating, and can be planar, cylindrical, or spherical. It ignores the intermolecular attraction, magnetic forces, chemical reaction, ionization, dust loading, and radiation transport beyond the optically thin local approximation. In this range of contributions the three influences have been summarized under a single characteristic interpretation: non-idealness affects the acoustic propagation directly via the sound speed, the radiation affects the transported thermodynamic state via the pressure source, while the geometry causes radial divergence. Submissions are also made into separate deductions for trends that follow algebraically from the model and parameter trends that would need numerical initial data. This distinction retains the fixed analytical evidence and also describes the physical mechanism that might lead to a smooth compression to a shock. No conclusions are drawn here based on statistics or empirical data.

2. LITERATURE REVIEW AND RESEARCH GAP

This is followed by the formation of nonlinear shock waves and their propagation. The formation of nonlinear shock waves and their propagation follows.

Courant and Friedrichs laid the foundations for the hyperbolic nature of compressible flow and the characteristics, nonlinear steepening, and discontinuity formation relations (Courant & Friedrichs, 1948). Whitham

later unified local wave evolution, breaking and geometrical shock dynamics in a single characteristic framework (Whitham, 1974). The implications of these foundations will explain that finite amplitude compression can not be treated by linear superposition and that a smooth signal can produce a discontinuity.

Numerically, Sod studied radially converging shocks, propagating in cylindrical geometry, which showed the need for cylindrical geometry (Sod, 1977). Chisnell produced an analytic description of converging shock amplification (Chisnell, 1998) and Murata was able to give Murata an exact blast-wave solution for a given density environment (Murata, 2006). The mathematical study of self-similar converging shocks has been revived by Jang, Liu and Schrecker (2025). In sum, this work separates out geometrical focus from the effects of the constituents and the sources. These two analyses provide complementary diagnostics for the issue of whether regular propagation can continue and together provide the front trajectory.

2.1 Shock Dynamics in Non-Ideal van der Waals Gases.

Zhao et al. (2011) classified the admissible shocks and phase transitions induced by them in a van der Waals fluid. Ambika et al., 2014 demonstrated that non-ideal constitutive terms affect growth and decay by using progressive-wave analysis of the system. Nath, Gupta and Singh extracted high-frequency amplitude transport in generalized van der Waals flow (Nath et al., 2017). Prodanov clarified admissibility.

Calvo-Rivera, Huete and Velikovich investigated the stability of growing reactive shocks in a van der Waals fluid (Calvo-Rivera et al., 2022). Later, Pandey and Patel (2026) studied cylindrical shocks for the presence of viscosity and heat conduction. The models they use are different from the inviscid radiative model used here, but they support a key synthesis: Constitutive corrections should be carried out through the real closure, not through ideal gas intuition.

2.2 Thermal Radiation in Dusty Gas-Dynamic Flows

Pai was the first to formulate the coupled radiation gas dynamics problem in which radiative energy transfer is important at high temperature and low density (Pai, 1966). Gretler & Regenfelter (2008) studied strong shocks in radiative transfer of dusty gas. The studies encourage models that are radiation-aware, but also illustrate that the type of optical approximation that is chosen dictates what can be claimed.

Singh et al. (2011a, 2011b) studied weak discontinuities in non-ideal radiating gas and radiative magnetogasdynamics. Singh, Singh and Ram then gave the connection between radiative transfer with the growth/decay of an acceleration wave in generalized geometry (Singh et al., 2012). Nath studied the radiative and conductive fluxes on a magnetized dusty-gas similarity model (Nath 2021). Directly supporting this source, Gupta et al. (2020) have analyzed the weak shocks assuming optically thin approximation.

2.3 Studies on the optical characteristics of stone and concrete.

Macpherson studied the formation of shock in dense gas on molecular-dynamics grounds (Macpherson, 1971). The nonlinear wave evolution in van der Waals magnetofluid was derived by Shukla, Ghosh and Sharma (2017) while discontinuity-wave interactions were studied by Zeidan, Govekar and Pandey (2021) through characteristic compatibility. These illustrate different ways in which smooth or weakly discontinuous information can be transformed into stronger wave structure.

Devi et al., 2021 performed similarity analysis for cylindrical radiative shocks in a non-ideal gas. Shweta et al. (2022) obtained the shock conditions for planar and non-ideal relaxing gas. In the numerical shock attenuation, the real-gas and boundary-layer effects were stressed by Sakamoto, Sato, and Ohnishi (2024). The gap is an account of a wavefront that preserves co-volume, optically thin radiation, and generalized geometry, and restricts growth, decay, and conclusions on formation distance to those that the system warrants.

3. MATHEMATICAL MODELLING AND ANALYTICAL METHOD

3.1 Physical Model, Assumptions, and Generalized Geometry

The medium is a one-dimensional radially bounded compressible, inviscid, non-ideal gas. r and t are the independent variables, while the dependent variables are ρ , u , p , and T . A geometry index, m , is defined to represent planar, cylindrical and spherical flow, and is equal to $m=0$, $m=1$, and $m=2$, respectively, as summarized in a schematic in Figure 1. The undisturbed state is uniform, motionless and has ρ_0 , p_0 , and T_0 . The amplitude of the disturbance is smooth and finite prior to the formation of the shock. It excludes intermolecular attraction, viscosity, reaction, magnetic forces and heat conduction. The radiation is optically thin and hence the local emission and ambient absorption are modeled without having to solve a nonlocal transfer equation. All subsequent analytical results reported were constrained by these assumptions..

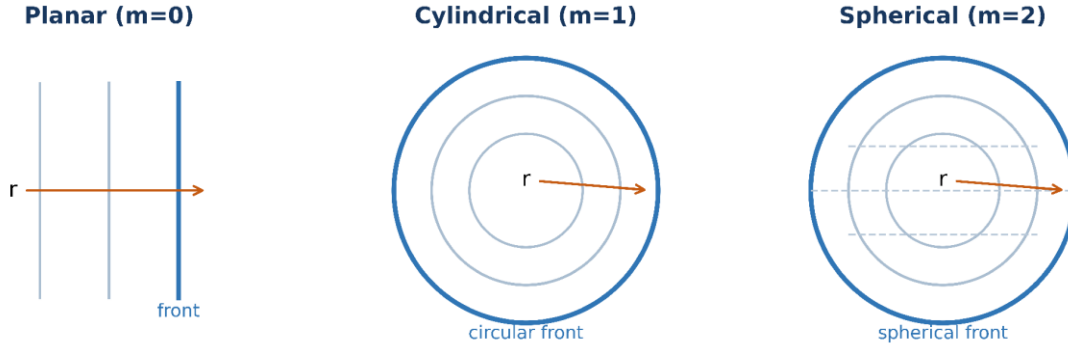


Figure 1. Generalized one-dimensional geometries represented by $m=0$, 1, and 2

Note. Original analytical schematic of the modeled symmetry classes; it is not an experimental image.

3.2 Governing Conservation Equations

Mass, radial momentum, and pressure evolution provide the closed differential system. In generalized geometry they are

$$\begin{aligned}\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{m \rho u}{r} &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} &= 0, \\ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \rho a^2 \left(\frac{\partial u}{\partial r} + \frac{m u}{r} \right) &= L(\rho, p).\end{aligned}\tag{1}$$

Here, $L(\rho, p)$ represents the radiative term of the pressure equation and a is the thermodynamic sound speed. Radial divergence is represented by the terms $m \rho u / r$ and $\rho a^2 m u / r$ and both are zero for planar flow. There is no momentum or mass dissipated and radiation is only put in through L . It is a crucial placement as it maintains the first order quasilinear hyperbolic principal part while the exchange of energy can change the thermodynamic state along a trajectory. These equations agree with the kind of treatment of compressible waves given by Courant & Friedrichs (1948) and Whitham (1974). All these geometric terms must be separated from the radiative terms for reproducibility; otherwise, they would be merged into a more empirical attenuation coefficient, which would disguise their different dependence on radius, temperature and density. The first equation represents mass conservation, the second the balance of material acceleration and the pressure gradient, and the third the relation between pressure and velocity divergence and radiation, using the material derivative $D/Dt = \partial/\partial t + u \partial/\partial r$. In collaboration they determine the value of ρ , u and p when constitutive and initial data are given, without including an equation for temperature.

3.3 Simplified van der Waals Equation of State and Sound Speed

The constitutive equation ignores attractive pressure and has finite molecular volume:

$$p(1 - b^* \rho) = \rho RT. \quad (2)$$

In this case b^* is a dimensional co-volume parameter and R is the specific gas constant. The adiabatic sound speed is equal to

$$a^2 = \frac{\gamma p}{\rho(1 - b^* \rho)}. \quad (3)$$

where γ is the ratio of specific heat. The ideal gas limit is reached as b^* tends to zero. This relation differs from a perfect-gas closure in that it explicitly depends upon the volume that is occupied. It is used in the same restricted way as the van der Waals models used in nonlinear wave studies (Ambika et al., 2014; Singh et al., 2012) but is not given any attraction dependent property. The denominator is the free volume for molecular motion that is being reduced. However, its positivity ensures that the temperature is not lost while preserving finite acoustic speed and a real characteristic structure for the closure; thus, it is a necessary condition for all physical states interpreted.

3.4 Radiative Heat-Transfer Model under Optically Thin Approximation

The optically-thin source is given by

$$L = (\gamma - 1) [-4\kappa\sigma_B T^4 + 4\kappa\sigma_B T_0^4]. \quad (4)$$

where σ_B is the Stefan–Boltzmann constant and T_0 is the ambient temperature. The selected equation of state implies that temperature is directly related to:

$$T = \frac{\rho(1 - b^* \rho)}{\rho R}. \quad (5)$$

The source is then considered to be a local emission at T and a local absorption point from an ambient field at T_0 . It is determined by the fourth power difference, so $L < 0$ at $T > T_0$, $L > 0$ at $T < T_0$, and $L = 0$ at $T = T_0$. It is not a statement that all wave amplitudes have to decrease, it is a source level statement. Both nonlinearity and geometry are present in the hydrodynamic model. This approximation is valid only in the case of small optical thickness; it is not valid for describing diffusion, spectral variation or self-absorption in a thick layer. This methodological limitation is consistent with the previous optically thin shock analysis (Gupta et al., 2020) and the general radiation-gas classification of transfer between local and nonlocal (Pai, 1966). During propagation radiative departures are computed from the thermodynamic state modeled and not from a linearized temperature law, keeping exactly the fourth power dependence.

The ambient state and parameters of the system, including the dimensionalization and control parameters, are determined.

Evaluation of the reference sound speed a_0 is carried out at the ambient state with the same non-ideal closure. The radiative source is modified as

$$L = -\Gamma \rho_0 a_0 (\mathcal{R} - 1), \quad (6)$$

$$\mathcal{R} = \left(\frac{\rho}{\rho_0}\right)^4 \left(\frac{\rho_0}{\rho}\right)^4 \left(\frac{1 - b^* \rho}{1 - b^* \rho_0}\right)^4.$$

and its strength is measured by

$$\Gamma = \frac{4\kappa\sigma_B T_0^3}{\rho_0 a_0}. \quad (7)$$

Acoustic transport is compared to radiative exchange by the dimensionless group Γ , which is related to $\rho_0 a_0$. It is convenient to use a non-idealness measure $b = b^* \rho_0$ with the ambient admissibility condition $b < 1$. The value of m is a discrete control of geometry. Constitutive, thermal-source and radial-divergence effects are represented by b , Γ and m , respectively. They are not interchangeable fitting parameters and there is no numerical specification without prescribed initial data. The use of these controls allows for comparative integration as the lowest basis.

A 3x3 matrix is given below. The following is a 3x3 matrix.

Let $U = (\rho, u, p)^T$. The equations can be written in the form of

$$\begin{aligned} \mathbf{U}_t + \mathbf{A}(\mathbf{U})\mathbf{U}_r &= \mathbf{B}(\mathbf{U}, r), \quad \mathbf{U} = (\rho, u, p)^T, \\ A_{11} &= A_{22} = A_{33} = u, \quad A_{12} = \rho, \\ A_{23} &= 1/\rho, \quad A_{32} = \rho a^2, \quad A_{ij} = 0 \text{ otherwise}, \\ \mathbf{B} &= (-mpu/r, 0, L - mpa^2 u/r)^T. \end{aligned} \quad (8)$$

with A representing spatial derivative coefficients and B geometry and radiation. The solutions to the characteristic equation of A are

$$\lambda_- = u - a, \quad \lambda_0 = u, \quad \lambda_+ = u + a. \quad (9)$$

They are the definition of the backward acoustic family C^- , the material family C_0 and the forward acoustic family C^+ . The strict hyperbolicity is guaranteed in all admissible states for positive a these speeds are real and distinct. Co-volume through a ; Γ in B , don't change the algebraic form of the eigenvalues directly. They still modify ρ , p , and u along the characteristic paths and hence, can modify the other characteristic speeds later on. The key analytical tool to which this separation is central is how the model is interpreted, namely by means of source-driven state evolution. Right and left eigenvectors project the governing equations onto these families, leading to compatibility relations which carry the state information and the derivative amplitude along their curves.

3.5 Wavefront Expansion and Shock-Formation Criterion

Think of a prominent front that is moving between an undisturbed state in front of it and a smooth disturbance in the back. The dependent variables are expanded with distance from the moving front. The substitutions of the expansions into the conservation system and the collection of equal powers produces compatibility relations along the appropriate acoustic characteristic. The leading relation determines the front motion, the next relation is the amplitude and includes nonlinear steepening terms, radiative relaxation terms and geometric divergence terms. This is the acceleration/weak-discontinuity evolution wavefront-expansion logic, which is used for the related gas models (Singh et al., 2011a; Zeidan et al., 2021).

Loss of regularity of the smooth solution is recognized to be the process of shock formation: the amplitude of the derivatives of the solution goes to infinity, but the solution itself does not. Inversely, in an amplitude representation the front is at 0. Similarly, an inverse-amplitude representation is at 0 at the front. The amplitude relation from prescribed initial data is integrated to obtain the critical time; and the front speed is integrated to obtain the corresponding distance. The formulation which is available sets this criterion, but does not provide a complete closed coefficient set or numerical initial amplitudes. It is thus interpretation that is evidence bounded.

4. RESULTS

4.1 Characteristic Wave Families and Propagation Behaviour

There are three physically distinct propagation paths summarized in Table 1 and illustrated in Figure 2 that come from the eigenstructure. The C0 family is used to carry material velocity u and contact-type information. The C− family of waves and the C+ family of waves correspond to acoustic disturbances moving relative to the moving gas at $u-a$ and $u+a$, respectively. The acoustic fronts are sent in the uniform undisturbed state, $u=0$, with the same ambient speed magnitude a_0 . C+ represents a disturbance of compression outward and C− represents a disturbance of compression inward.

Table 1. Characteristic wave families and propagation speeds

Family	Speed	Motion relative to gas	Physical interpretation
C−	$u-a$	$-a$	Backward acoustic information
C0	u	0	Material or contact information
C+	$u+a$	$+a$	Forward acoustic information; outward compressive front when applicable

Note. Speeds are exact eigenvalues of the quasilinear coefficient matrix; direction relative to the laboratory frame also depends on u .

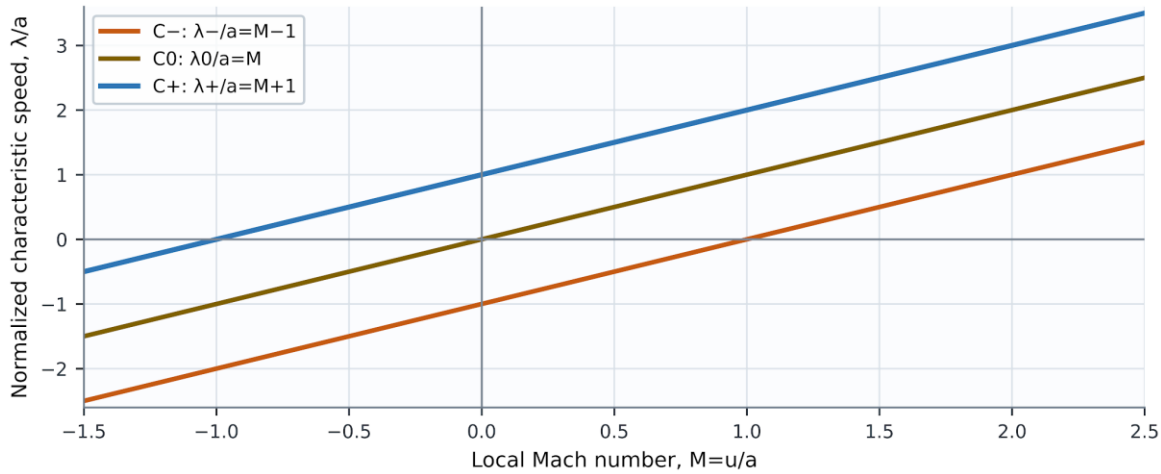


Figure 2. Normalized characteristic speeds as exact functions of local Mach number

Note. The curves plot $\lambda-/a=M-1$, $\lambda_0/a=M$, and $\lambda+/a=M+1$ from the characteristic eigenvalues; no measured observations are used.

This structure is exactly the same in all three modeled geometries. Table 2 separates each modeled mechanism by its mathematical location. Geometry and radiation are not in the characteristic polynomial. They affect each other over time, as a front moves forward they continually change the local state, and therefore the results for u and a . Co-volume has a direct action since it is already present in a . The effect is to separate propagation kinematics from amplitude dynamics, and to avoid the radiative parameter being confused with another local wave speed.

Table 2. Mathematical location of the three governing mechanisms

Mechanism	Control	Location in system	Supported direct effect
Molecular co-volume	b^*	Principal matrix A through a ; closure for T	Changes acoustic speed and couples into L through temperature

Optically thin radiation	Γ	Source vector B through L	Changes pressure and the subsequent state; introduces no additional eigenvalue
Generalized geometry	m	Source vector B through terms proportional to μ/r	Produces radial divergence or convergence; introduces no additional eigenvalue

Note. The separation between A and B distinguishes instantaneous characteristic kinematics from source-driven state evolution.

4.2 Wavefront Amplitude-Evolution Relation

The wavefront expansion transforms the partial differential system in the vicinity of the leading acoustic front into an ordinary differential evolution propagation problem for the first derivative amplitude. It can be written in a generic Riccati form as

$$\frac{d\alpha}{ds} + Q(s)\alpha^2 + P(s)\alpha = S(s). \quad (10)$$

In this equation, α is the amplitude to be selected on the front, s represents propagation along the front, and Q , P and S are evaluated from the base state, the constitutive derivatives, geometry, and radiative source. The supported role of each term is identified in Table 3 and Figure 3 shows the analytical contribution of the terms to the amplitude derivative. The self-steepening nonlinearity is represented by the quadratic term, $Q\alpha^2$. Linear terms sum up the amplification or attenuation due to background variation, radial divergence, and the derivative of the radiative-source. The inhomogeneous term is dependent upon the specification of the front variable and the ambient state chosen.

Table 3. Terms in the generic wavefront amplitude relation

Term	Principal dependence	Analytical role	Evidence-bounded interpretation
$Q(s)\alpha^2$	Characteristic projection and constitutive derivatives	Nonlinear self-interaction	Can produce steepening; sign requires evaluation of Q and α
$P(s)\alpha$	Base-state variation, geometry, and derivatives of L	Linear amplification or attenuation	May reinforce or oppose nonlinear growth
$S(s)$	Selected front variable and ambient specification	Inhomogeneous forcing	Form depends on the complete compatibility relation

Note. The available formulation establishes the Riccati structure but does not provide numerical coefficient histories.

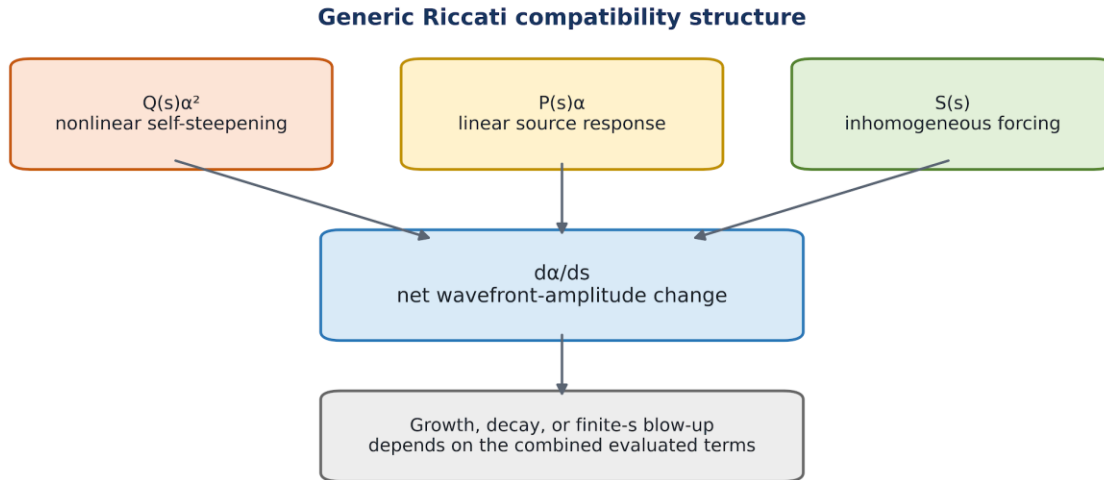


Figure 3. Mechanism map for the generic wavefront amplitude equation

Note. The diagram records the analytical dependencies of $Q\alpha^2$, $P\alpha$, and S ; arrow directions denote contribution pathways, not numerical magnitude.

Nonlinear acoustic-speed variation results in portions of the profile moving at different rates for a compressive disturbance. The gradient could be increased by a faster compressed part overtaking the leading part. That evolution, or change, can be opposed, or aided, indirectly by radiation, which can alter the pressure and temperature, and the redistribution of the disturbance can take place by radial spreading in an expanding area. Consequently, the conclusion here is conditional: α increases for a combined steepening while source-mediated decays increase. If one of the coefficients, Γ , b , m changes, then the trajectory of the state changes, and it is not always possible to ensure that the sign of $d\alpha/ds$ is always positive for an admissible initial condition. As a consequence the initial amplitude given is still needed for each shock prediction.

4.3 A criterion for shock formation is based on the critical distance. A criterion for the formation of shock is the critical distance.

The amplitude equation is not only the basis for the gradient-catastrophe criterion but also for the possibility that the initial jump will be discontinuous. For finite s_c , if the quadratic steepening forces $|\alpha|$ to grow to infinity, the description of the wavefront by a smooth surface becomes irregular and a shock has to take the place of the multivalued classical continuation of the wavefront. If α is nonzero, then the blow up test can be modified to $z(s_c)=0$. The exact conditions that are relevant for each decision are given in Table 4; Figure 4 describes the analytical workflow without being insinuating about a computed trajectory. The distance from the front of the shock to the location where the shock forms is called the

$$r_c - r_{\text{initial}} = \int_0^{s_c} \lambda_+(s) ds. \quad (11)$$

where r_c and r_{initial} are the front positions at s_c and the initial point, and λ_+ is the forward characteristic speed evaluated along the evolving front. Both amplitude history and front trajectory are required.

Table 4. Shock-formation diagnostics and required information

Diagnostic	Exact condition	Meaning	Required information
Regular front	$ \alpha $ remains finite	Smooth solution persists locally	State and amplitude history

Gradient catastrophe	$ \alpha \rightarrow \infty$ at finite sc	Shock replaces the nonregular continuation	Initial amplitude and coefficient functions
Inverse-amplitude test	$z(sc)=0, z=1/\alpha$	Equivalent blow-up criterion for nonzero α	Integrated amplitude relation
Critical distance	$rc - r_{initial} = \int \lambda_+(s) ds$	Front distance accumulated to sc	Trajectory $u(s)$, $a(s)$, and sc
No shock in interval	$z \neq 0$ and $ \alpha $ bounded	No loss of regularity on that interval	Solution over the tested interval

Note. No numerical critical distance is implied because the required initial profile and coefficient histories are unspecified.

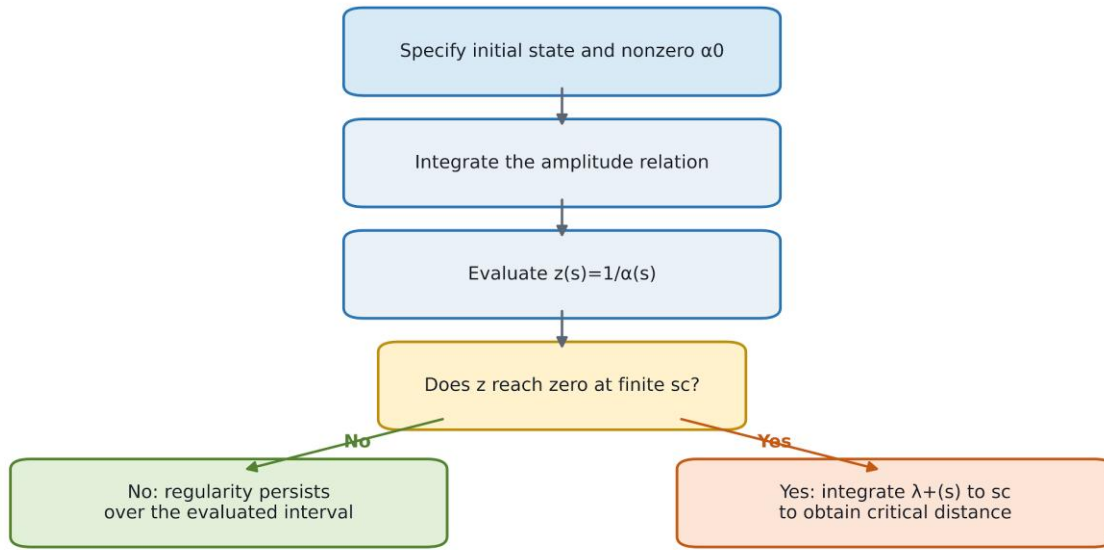


Figure 4. Evidence-bounded workflow for determining shock formation and critical distance

Note. The decision structure follows the inverse-amplitude criterion and trajectory integral; it does not represent a computed case.

The criterion sets out a number of precise criteria. First, the formation of finite shocks is not automatic: a rarefactive amplitude or a sufficiently weak one can stay bounded. Second, b , Γ , and m are not enough to compute a critical distance, and initial amplitude and thermodynamic state are necessary. Thirdly, distance is affected by both steepening time and propagation speed. The state and amplitude coefficients evolve as a function of time, but they can be changed by co-volume through a . There is no numerical initial profile given, hence the evidence does not support the criterion and mechanisms, but rather a numerical value. This is in good agreement with the classical breaking analysis (Whitham, 1974) and mathematical study of converging shock formation (Jang et al., 2025).

4.4 Section 4.4 is on the influence of Gas Co-Volume Parameter.

The acoustic propagation is modified by the simplified van der Waals term. For given positive values of p and ρ ,

$$\left. \frac{\partial a^2}{\partial b^*} \right|_{p, \rho} = \frac{\gamma p}{(1 - b^* \rho)^2} > 0. \quad (12)$$

The factor $1/(1 - b^* \rho)$ is exactly monotonically increasing, and is the same for each of the acoustic characteristics, as is also shown in figure 5, which plots this factor against the separation between the acoustic eigenvalues u - a and

$u+a$, normalized to the eigenvalue u . The conclusion is valid assuming the values of p and ρ are fixed; actually, p and ρ change along the trajectory, and the net front speed change has to be computed along the actual trajectory.

Table 5. *Exact fixed-state effects of molecular co-volume*

Quantity	Exact relation	Effect of increasing b^*	Condition
Sound speed squared	$a^2 = \gamma p / [\rho(1 - b^* \rho)]$	Increases at fixed p and ρ	$p > 0, \rho > 0, 0 \leq b^* \rho < 1$
Temperature	$T = p(1 - b^* \rho) / (\rho R)$	Decreases at fixed p and ρ	Same fixed-state comparison
Acoustic separation	$\lambda_+ - \lambda_- = 2a$	Increases at fixed p and ρ	Real positive a
Admissibility margin	$1 - b^* \rho$	Decreases toward zero	The state must remain below $b^* \rho = 1$

Note. These derivatives hold with p and ρ fixed; a coupled wave trajectory need not preserve either variable.

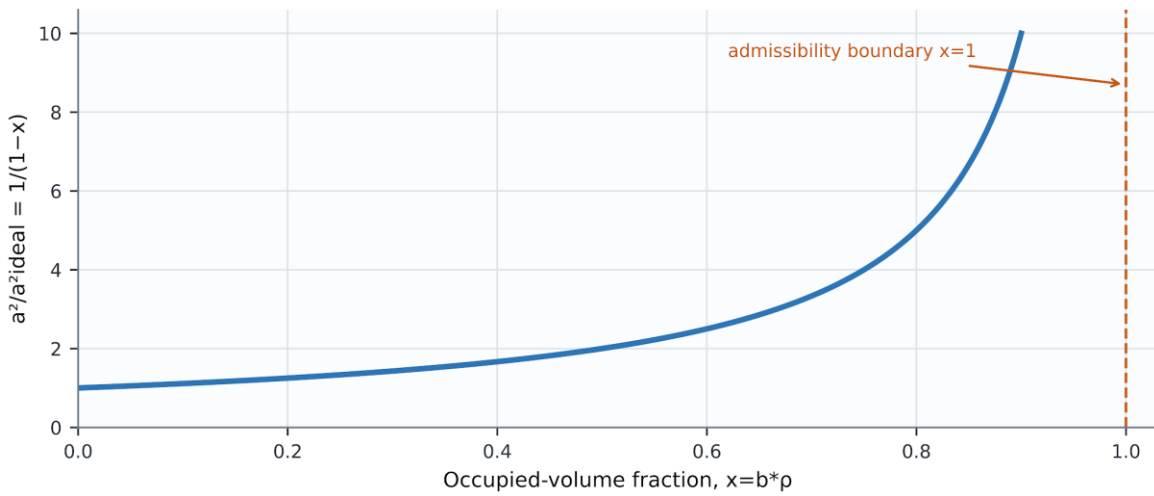


Figure 5. *Exact normalized sound-speed factor versus occupied-volume fraction*

Note. The analytical curve is $a^2/a^2_{\text{ideal}} = 1/(1-x)$, with $x = b^* \rho$ plotted over $0 \leq x \leq 0.9$ inside the admissible domain $x < 1$.

Co-volume also varies temperature recovered from p and ρ . T is a decreasing function of b^* at fixed p and ρ . Non-idealness couples into radiation even though b^* and Γ are different physical processes, since the radiative source is a function of $T^4 - T_0^4$. This coupling can change the linear coefficient of the amplitude equation and the time for steepening. The model sets up that co-volume affects the dynamics of waves and the critical formation distance, but it does not set up that the distance always increases or decreases. This ordering depends on fixed initial data and coupled coefficients evaluation similar to what has been observed for van der Waals shocks (Zhao et al., 2011; Calvo-Rivera et al., 2022).

4.5 The influence of the radiation parameter is discussed in 4.5.

The dimensionless radiation parameter Γ scales out the deviation $(T/T_0)^4$ from unity. In the inviscid model, the pressure equation is adiabatic for the case of $\Gamma = 0$. If $\Gamma > 0$, a hotter state than the ambient field will have $L < 0$ (a negative index), which means it will lose energy locally, while a colder state will have $L > 0$ (a positive index) and gain energy locally. Source disappears at $t = T_0$. These are the main exact sign regimes recorded in Table 6. The normalized source factor $1 - (T/T_0)^4$ is plotted in figure 6 for the three phases: heating, equilibrium, and cooling with no fitted observations.

Table 6. *Sign of the optically thin radiative source*

Thermal regime	Normalized factor $1-(T/T_0)^4$	Sign of L for $\Gamma>0$	Local action
$T<T_0$	Positive	$L>0$	Heating toward ambient equilibrium
$T=T_0$	Zero	$L=0$	Radiative equilibrium
$T>T_0$	Negative	$L<0$	Cooling toward ambient equilibrium
$\Gamma=0$	Not activated	$L=0$	Adiabatic pressure evolution in this inviscid model

Note. The sign statement concerns the radiative source alone and does not determine the sign of the complete wavefront amplitude derivative.

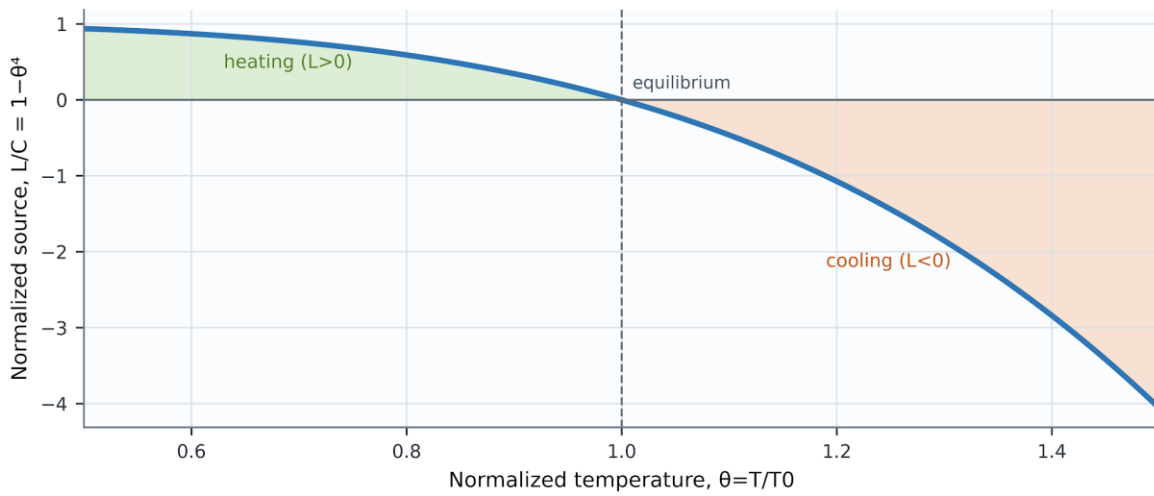


Figure 6. Exact normalized optically thin radiative-source factor

Note. The analytical curve is $L/C = 1 - \theta^4$, where $\theta = T/T_0$ and $C = 4(\gamma - 1)\kappa\sigma BT_0^4 > 0$; the plotted range is illustrative, not empirical.

Radiation is not directly included in the instantaneous eigenvalue formula, but it will modify the pressure and the sound speed that vary with the progress of the wave. It also adds derivatives of L to the wavefront compatibility relation. Near the equilibrium, the fourth-power law provides a restoring tendency, which results from the cooling that occurs due to a positive temperature perturbation and the heating that occurs due to a negative temperature perturbation. This can dampen up the thermal departures, but the overall front amplitude can increase if nonlinear compression and/or geometric focusing overwhelm the effect. The radiation strength is increased by increasing Γ for a given nondimensional state and does not assure the overall sign of the amplitude derivative. The outcome was consistent with optically thin weak-shock investigations (Gupta et al., 2020), and was found to explain radiation-dependent growth, decay, and shock distance without the need to import numerical trends from another initial-value problem. The neutral state of the source is therefore a state of thermal equilibrium.

4.6 Combined effects of non-idealness, radiation and flow geometry

These three controls are working at different mathematical positions and are to be interpreted as a whole. Co-volume changes the acoustic matrix with $a^2 = \gamma p / [\rho(1 - b^*\rho)]$ and changes the temperature of the radiative source. The impact of radiation on the pressure equation is provided by L . Geometry provides terms proportional to μ/r in the mass and pressure equations. The divergence multipliers of the three geometry indices are presented in Table 7, and

the exact divergence multipliers are shown in Figure 7. The spherical coefficient is 2 times the cylindrical coefficient at the same local thermodynamic state & radius; Planar flow eliminates radial divergence.

Table 7. *Geometry-dependent contributions to the source vector*

Geometry	m	Continuity contribution	Pressure contribution	Interpretation at fixed state and r
Planar	0	0	0	No radial divergence term
Cylindrical	1	$-\rho u/r$	$-\rho a^2 u/r$	Divergence for $u>0$; convergence for $u<0$
Spherical	2	$-2\rho u/r$	$-2\rho a^2 u/r$	Twice the cylindrical coefficient at the same local state

Note. Only the geometric part of the pressure source is shown; the full pressure source also contains L.

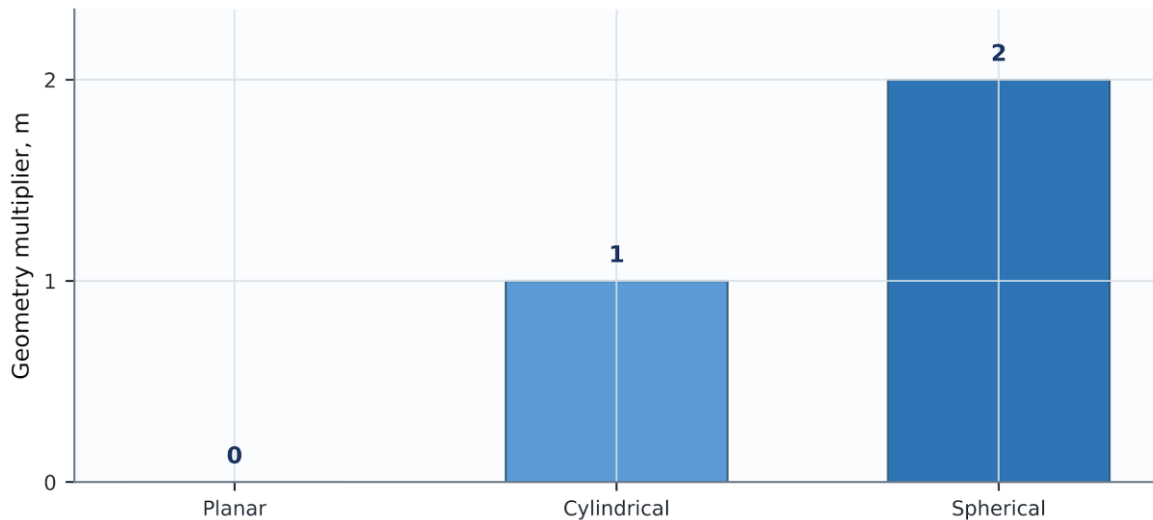


Figure 7. *Exact radial-divergence multiplier for the three modeled geometries*

Note. Bar heights are the discrete geometry index m; they are model coefficients, not sample statistics.

If a disturbance is outwards ($u>0$), density tends to decrease by continuity. The accompanying pressure-divergence term changes the thermodynamic state. This is a geometric spreading, the actual amplitude trend being governed by competition with nonlinear compression & radiative exchange. Inward motion, the u sign changes and the same geometry would indicate a convergence, so blanket attenuation would be wrong. Radial convergence also strengthens a front as demonstrated via classical studies (Chisnell, 1998; Calvo-Rivera et al., 2023).

Table 8 distinguishes between model consequences and conditional outcomes and unavailable quantities. The hierarchy is evident: characteristic families define the propagation of information, the closure of the characteristic families determines the acoustic speed and the temperature, the radiation and the geometry impose the state along the characteristic families, and the nonlinear compatibility determines whether the gradient is finite or not. This hierarchy is integrated to give predicted critical shock distance, which is not a predicted result set by itself.

Table 8. *Hierarchy of exact, conditional, and unavailable outcomes*

Outcome	Evidence status	Basis	What further quantification requires
Characteristic speeds	Exact	Eigenvalues $u-a$, u , $u+a$	No additional input for their local form
Strict hyperbolicity	Exact under admissibility	$p>0$, $p>0$, $0\leq b^*p<1$	Local thermodynamic state
Co-volume sensitivity	Exact at fixed p and ρ	$\partial a^2/\partial b^*>0$	Coupled trajectory for net front-speed change
Radiative source sign	Exact	Sign of $1-(T/T_0)^4$	Local temperature for magnitude
Geometry multiplier	Exact	$m=0, 1, 2$	Direction u and radius r for physical effect
Amplitude growth or decay	Conditional	Competition among $Q\alpha^2$, $P\alpha$, and S	Initial amplitude and coefficient histories
Critical shock distance	Not numerically determined	Integral of λ^+ to unknown sc	Complete initial-value problem and integration

Note. The final row records an evidential limit, not a zero or infinite shock distance.

5. DISCUSSION

5.1 Nonlinear Steepening and Wave Growth–Decay Mechanisms

The main conclusion is that the formation of shocks is most easily understood as an interplay of the wavefront amplitude equation. Propagation speed is a function of the local thermodynamic state, so nonlinear acoustics provides a quadratic steepening mechanism. A smooth compressive profile may thus be sharpen up progressively, despite the continuity of all the state variables involved, prior to the critical point. This interpretation agrees with the classical characteristic description of breaking (Courant & Friedrichs, 1948; Whitham, 1974) and with analyses of breaking in terms of progressive waves in non-ideal gases (Ambika et al., 2014).

A parameter alone does not have growth and decay. They are results of the combination of sign and size of the assembled coefficients at a specific front. Radiative restoration can decrease the temperature deviations, outward geometric spreading can weaken a disturbance, inward convergence can strengthen a disturbance, and co-volume can affect the travel speed and source sensitivity of a disturbance. As long as α can be bounded by attenuation no finite shock will be obtained within the modeled interval. In the case of nonlinear steepening, the inverse amplitude is zero and a discontinuity is needed. This conditional structure is the reason for not transferring the results of similarity solutions, numerical attenuation study, or magnetogasdynamic interaction numerically to the present problem (Shukla et al., 2017; Sakamoto et al., 2024). Evolution is locally and state dependent, mechanistically.

5.2 Physical interpretation of Co-Volume and Radiation effects

Co-volume is the finite volume of the molecules. As density reaches the excluded-volume limit, the $1-b^*p$ goes down and the simulated sound speed becomes more compression sensitive. This increases the inherent nonlinearity of the acoustic subsystem but the bound also constitutes the limit of admissibility of the simplified equation. Prodanov has mathematically analyzed the van der Waals equation, highlighting the need to respect its state domain (Prodanov, 2022); Riemann and stability studies show that non-ideal wave behavior occurs with the choice of initial state (Gupta et al., 2022; Calvo-Rivera et al., 2022).

The physical role of radiation is different. The power law exchange locally drives the temperature towards the ambient radiative equilibrium. It has a strong non-linear dependence on temperature and no independent momentum forcing. This is because a hotter compressed region cools, changing pressure and the speed of sound which affect

propagation. A colder area is warmed. Singh et al. (2011a) has shown that radiative terms also modify the evolution of discontinuities in non-ideal gas models and Singh et al. (2012) has linked the co-volume to the development of acceleration waves with radiation. The current formulation clears up their coupling: b^* alters the temp that is inferred from the same p and ρ ; while Γ scales the exchange that is generated by the temp. Therefore the parameters cannot be thought of as independent sliders when the wave is out of equilibrium.

5.3 Planar, Cylindrical and Spherical Geometries

Generalized geometry filters out radial effects without altering the acoustic eigenvalues. Planar flow provides a reference where divergence is null for m -dependent flow. To increase the source coefficients progressively, cylindrical and spherical flow are introduced at the same r and u . These terms represent area expansion for outward motion and convergence for inward motion. Geometry becomes increasingly important near the origin, due to the $1/r$ dependence. This is shown by the cylindrical calculations of Sod and by the converging-shock analysis of Chisnell (Sod, 1977; Chisnell, 1998). The present result shows that the effect of radiation and co-volume is simultaneous on state, therefore it is recommended that only geometry under common initial and boundary data should be compared. The sign of the wave function changes when comparing diverging and converging waves.

5.4 A comparison of the analytical findings to the previous analytical findings is provided below.

The characteristic result preserves the usual material and acoustic families with heat transfer in compatibility source terms, which is in line with the established radiating-gas studies (Singh et al., 2011a; Gupta et al., 2020). It is also in agreement with the van der Waals progressive-wave work which shows that: The thermodynamic derivatives of the constitutive non-idealness modify the steepening behavior. The wavefront method is different from self-similar radiative-shock analysis, which assumes a strong discontinuity, or shock, (Devi et al., 2021) to follow its local evolution. It does not include any particulate inertia or interphase exchange as is the case in dusty-gas models (Carrier, 1958; Marble, 1970; Steiner & Hirschler, 2002). Such differences account for the significance and the boundaries of comparison. Nath's similarity analysis (Nath, 2021) considers also the particulate, magnetic, conductive, and radiative effects but does not provide a numerical closure of this source-only model; Shweta and co-workers (Shweta et al., 2022) consider also the relaxation but also without a numerical closure of this source-only model.

5.5 Implications for High-Temperature and High-Pressure Flows

The framework is applicable for optically thin exchange and excluded volume is important when the compressed gases are sufficiently dense. Some possible applications are in high-temperature laboratory pulses, a part of hypersonic-flow fields, and rapidly heated gaseous media. Pai's formulation reveals the high-temperature, low-density importance of radiation (Pai, 1966) and recent real-gas simulations indicate that the influence of thermodynamic closure on attenuation should be considered (Sakamoto et al., 2024). However, case-specific opacity, state data, and initial amplitude are needed for application, and the current model is not a validated design correlation.

5.6 In this chapter, the validity of the model, assumptions, and limitations are discussed.

The conclusions were limited by several factors. The gas is inviscid and one-dimensional, and shock thickness, boundary layers, multi-dimensional instability and viscous dissipation are not considered. The van der Waals closure ignores molecular attraction and is only valid below the excluded-volume singularity. Chemical reaction, dissociation, ionization, magnetic field, dust and particulate relaxation are not included. Interphase inertia can significantly alter the propagation of dusty shocks (Igra et al., 1987; Ishii et al., 1989; Miura et al., 1986) and radiative magnetogasdynamic work introduces dusty shock additional characteristic modes (Singh et al., 2011b; Zeidan et al., 2021).

Thus, the paper can calculate eigenvalues, signs of sources, admissibility conditions, local derivatives, and present a shock criterion, but it cannot claim to correctly present a numerical shock distance, an error bar, nor significance tests, nor could it present plotted response curves. Optical thinness is also a condition for radiative validity.

6. CONCLUSION

6.1 Principal Findings

Smooth finite amplitude wave propagation in a simplified van der Waals gas and optically thin radiation is formulated across planar, cylindrical and spherical geometries. The governing system is strictly hyperbolic for admissible positive states with characteristic speeds $u-a$, u , and $u+a$. Co-volume changes the fundamental acoustics by $a^2 = \gamma p / [\rho(1-b^*\rho)]$, and at constant pressure and density will lead to higher sound speeds locally. The fourth power temperature difference of hot to cool causes radiation to enter into the pressure source, colder states become hotter, hotter states become cooler, and the ambient state is an equilibrium state. The introduction of m -dependent radial divergence brings in the elements of geometry. These ingredients are transformed into an amplitude evolution problem by wavefront expansion where nonlinear steepening plays a role in competition with radiative and geometric changes of the state. The formation of shock is conditional and occurs when the derivative amplitude has a finite propagation distance where it becomes unbounded or equivalently when the derivative's inverse has a zero value. Without the initial disturbance and the integrated state trajectory there is no universal numerical distance or unconditional parameter ordering for the critical distance. All of the findings are within the scope of the analytical evidence and help meet the objectives. The outcome is an analytical narrative based on evidence which clarifies what is known and what is not.

6.2 How Well Do You Know the Science? (That is, the Practical Part)

The contribution is a single mechanism map correlating constitutive non-idealness, thermal radiation, geometry, characteristic propagation and gradient catastrophe. It supplies a clear guideline for the choice of parameters and diagnostics for further numerical or experimental studies and can be used for the benchmarking of current investigations of real-gas shocks (Sakamoto et al., 2024; Pandey & Patel, 2026). It also guards against quality sensitivity mistakenly being reported as quantified predictive performance.

6.3 Future Research Directions

In the future, complete amplitude coefficients for specific initial profiles should be derived and evaluated, as well as the characteristic system integrated and the critical distances examined for values of b , Γ and m under control. Extensions will involve testing optically thick transport, viscosity, multidimensional stability and more complete real-gas equations of state. The experimental validation should be pursued.

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