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Article

The Analysis of Currencies for Different Countries via Complexity and Brownian Motion

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Abstract: The foreign exchange (Forex) market is a pillar of global economic development, and the United States Dollar (USD) influences nations such as India, Britain, Mexico, South Africa, China, Brazil, and Australia. Daily exchange-rate movements against the Dollar over 2024–2025 for these seven currencies are examined here as cyclical signals, each carrying its own underlying rhythm rather than a simple straight-line trend. A periodic Laplace transform is applied to extract the periodic component from the raw time series, while Brownian motion is used to model the random, continuous-time fluctuations that characterise currency movement. Two further tools probe the shape of the data itself. The HarFA fractal (box-counting) analysis measures self-similarity and multi-scale irregularity in the exchange-rate curves, and Pixel boundary analysis tracks the distance between chosen points on the graphs to gauge how sharply each currency's path bends and swings. All computation, simulation, and plotting were carried out in Scilab. The fractal box dimension and pixel boundary distances place China, the United Kingdom, and Mexico among the most volatile and structurally complex of the seven series, while India's exchange rate follows a comparatively smooth, steadily rising path with the lowest fractal dimension and shortest pixel distance. The periodic Laplace transforms, summarised for a family of representative square-wave functions, confirm that the exchange-rate series behave as periodic signals superimposed on a broader stochastic trend. Taken together, the four methods show that exchange-rate behaviour is stochastic, fractal, and nonlinear all at once, and that it tracks the broader economic health of a nation closely. This study illustrates the power of computational tools in finance.
Keywords: USD, Seven Countries, Harfa Fractal, Pixel, Scilab, Fractal, Brownian Motion, Complexity

1. Introduction

A nation's economy affects nearly every aspect of society, including commerce, manufacturing, employment, investment, and the pace at which a community develops [7]. When an economy is healthy, markets tend to remain stable, and the country's currency holds both its value and its influence in international dealings. Currency conversion sits at the heart of global commerce, as nations depend on it for imports, exports, investment, and innovation. The worth of money is never fixed: it shifts with inflation, politics, market sentiment, policy changes, and world events, which is why currency fluctuations are directly relevant to economic behaviour and financial stability [1,7].

Financial markets, and currency exchange in particular, have grown noticeably more erratic in recent years. That unpredictability is a part which makes traditional mathematical models a poor fit here. The exchange-rate shifts are stochastic, fractal, and nonlinear all at once. Fractal geometry, complexity theory, and Brownian motion provide an alternative [6]. Fractals are well suited to picking out the rough, self-similar structure hidden in financial data. The complexity measures capture how tangled and co-dependent economic systems have become, and Brownian motion supplies a working stochastic model for the kind of random fluctuation that currency and financial markets display over time.

This study looks at seven major currencies against the US Dollar: the Indian Rupee, British Pound, Mexican Peso, South African Rand, Chinese Yuan, Brazilian Real, and Australian Dollar, chosen precisely because they sit within very different economic structures and market contexts [7]. By combining fractals, complexity measures, and Brownian motion on the exchange-rate data, the paper sets out to track how these currencies vary over time, how stochastic their behaviour really is, and what that says about the dynamics of global currency markets more broadly, since these economic shifts feed directly back into currency values and, in turn, into the health of the global financial system. The Dollar's outsized role in all of this has been well documented elsewhere: Boz, Gopinath, and PlagborgMøller discuss it directly in their paper *"Global Trade and the Dollar"* (2017) [1], which lays out how dominant the Dollar remains in global transactions and how much influence it exerts over exchangerate dynamics and economic activity across countries.

2. Methods

2.1 Brownian Motion

Brownian motion is a core method in financial modelling and continuous-time stochastic processes, making it well suited to this problem. Currency rates fluctuate constantly due to foreign trade, domestic and foreign policy, inflation, investor behaviour, and world events, and this volatility is precisely the type of behaviour Brownian motion is designed to describe [3]. For this study, exchangerate data for seven major currencies, namely the Indian Rupee, British Pound, Mexican Peso, South African Rand, Chinese Yuan, Brazilian Real, and Australian Dollar, were converted to USD using the 2024 and 2025 rates listed in Tables 1 and 2 [10,11]. Each resulting series was processed using a Brownian-motion model to examine how the currency moved over time, with all computation, simulation, and plotting carried out in Scilab; the results for both years are shown in Figures 1 and 2.

Table 1: Comparative Analysis of USD Conversion Rates for Seven Major Countries (2024) USD to India 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
83.190	83.051	82.891	83.290	83.519	83.401	83.472	83.001	83.985	83.966	84.118	84.684
83.154	82.876	83.412	83.319	83.059	83.436	83.742	83.868	83.619	84.090	84.251	85.424

USD to British 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
0.786	0.792	0.778	0.795	0.797	0.781	0.784	0.788	0.761	0.763	0.767	0.784
0.788	0.791	0.792	0.803	0.784	0.788	0.777	0.754	0.751	0.772	0.798	0.797

USD to Mexico 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
17.015	17.123	16.883	16.549	16.916	17.839	17.838	19.571	19.944	19.492	20.141	20.320
17.208	17.046	16.766	17.075	16.725	18.467	18.463	19.014	19.219	19.961	20.425	20.074

USD to China 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24

7.131	7.162	7.197	7.237	7.218	7.242	7.268	7.244	7.113	7.017	7.179	7.263
7.158	7.197	7.227	7.246	7.234	7.261	7.231	7.139	7.011	7.118	7.242	7.296

USD to South Africa 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
18.659	18.831	19.087	18.698	18.466	18.878	18.168	18.294	17.849	17.524	17.677	18.026
18.793	19.094	18.834	19.233	18.293	18.247	18.357	17.724	17.438	17.661	18.018	18.851

USD to Brazil 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
4.875	4.978	4.981	5.009	5.068	5.302	5.557	5.638	5.616	5.478	5.862	6.087
4.929	4.964	4.992	5.147	5.105	5.431	5.572	5.592	5.515	5.691	5.801	6.086

USD to Australia 2024

Jan 24	Feb 24	Mar24	Apr 24	May24	Jun 24	Jul 24	Aug24	Sep 24	Oct 24	Nov24	Dec 24
1.478	1.535	1.537	1.523	1.521	1.503	1.483	1.539	1.477	1.484	1.517	1.555
1.519	1.528	1.522	1.557	1.499	1.501	1.521	1.485	1.468	1.496	1.529	1.602

Table 2: Comparative Analysis of USD Conversion Rates for Seven Major Countries (2025) USD to India 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
85.659	86.744	87.234	85.602	84.221	85.309	86.486	87.251	87.991	88.791	88.882	89.754
86.602	87.412	86.112	85.312	85.266	86.382	87.012	87.641	88.712	88.941	89.351	90.212

USD to British 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
0.805	0.802	0.782	0.785	0.749	0.738	0.736	0.743	0.747	0.749	0.761	0.751
0.801	0.794	0.771	0.751	0.738	0.743	0.740	0.739	0.742	0.753	0.763	0.741

USD to Mexico 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
20.631	20.517	20.647	20.437	19.687	19.241	18.655	18.893	18.716	18.399	18.383	18.235
20.386	20.412	20.048	19.635	19.407	18.892	18.675	18.654	18.405	18.464	18.308	17.976

USD to China 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
7.332	7.288	7.265	7.271	7.225	7.189	7.175	7.212	7.142	7.119	7.114	7.072
7.272	7.247	7.248	7.292	7.184	7.171	7.178	7.184	7.112	7.126	7.108	7.042

USD to South Africa 2025

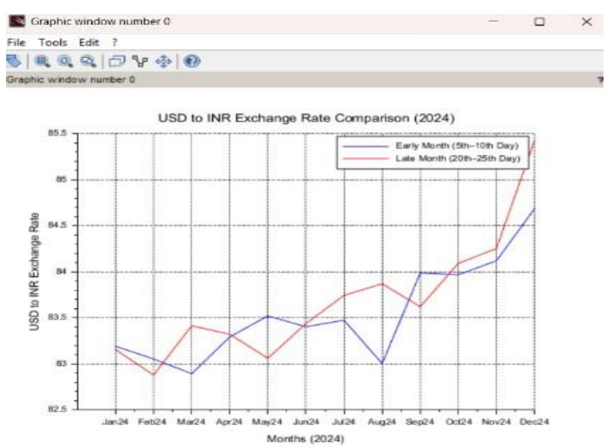
Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
18.939	18.547	18.253	19.423	18.556	17.857	17.514	17.904	17.613	17.234	17.319	17.040
18.512	18.422	18.284	18.780	18.048	18.059	17.764	17.441	17.243	17.427	17.365	16.770

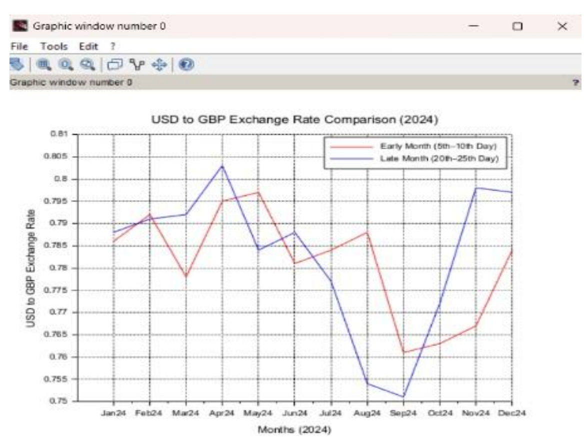
USD to Brazil 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
6.117	5.842	5.904	5.846	5.746	5.675	5.582	5.505	5.469	5.379	5.358	5.308
6.071	5.690	5.672	5.682	5.669	5.513	5.518	5.475	5.365	5.400	5.402	5.544

USD to Australia 2025

Jan 25	Feb 25	Mar25	Apr 25	May25	Jun 25	Jul 25	Aug25	Sep 25	Oct 25	Nov25	Dec 25
1.611	1.628	1.609	1.671	1.537	1.534	1.537	1.533	1.534	1.514	1.542	1.509
1.614	1.562	1.588	1.568	1.538	1.533	1.532	1.549	1.503	1.525	1.545	1.489





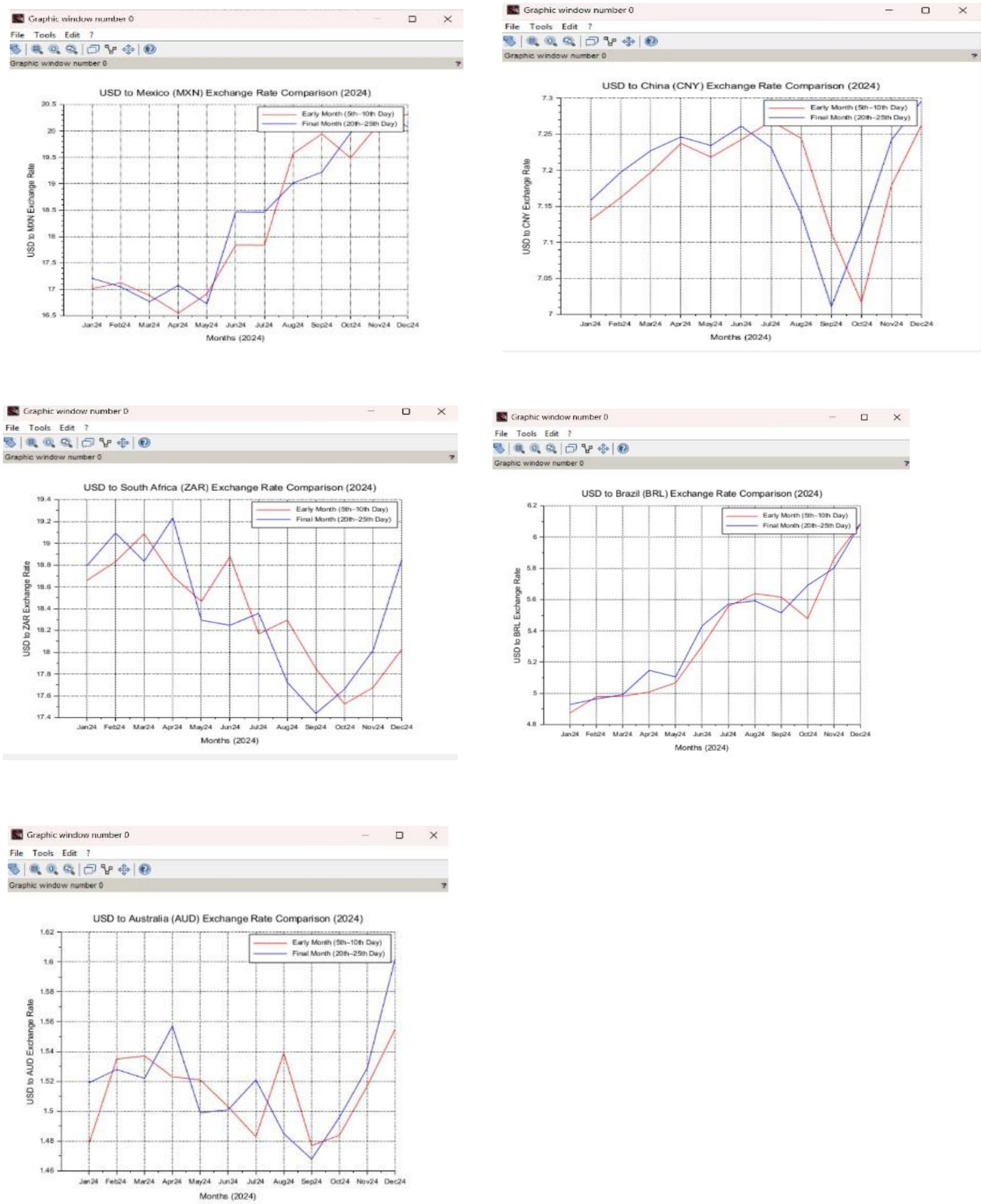
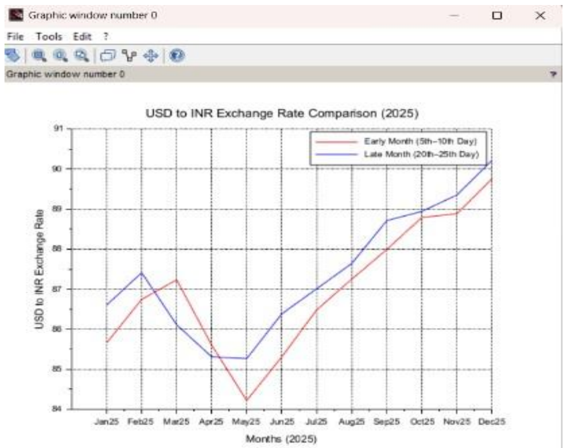


Figure 1: Scilab-Based Analysis of International Currency Exchange Rates in 2024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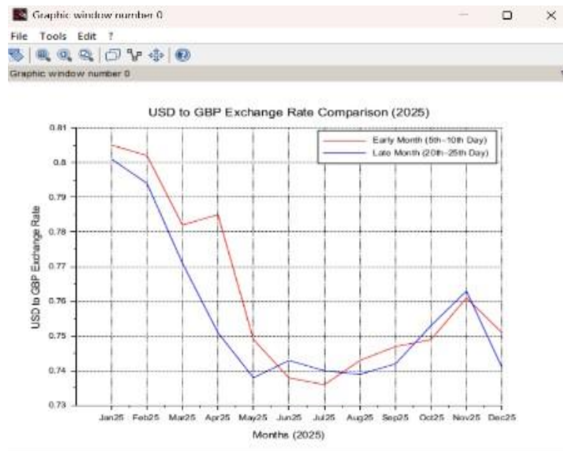
USD to INR Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Late Month (20th–25th Day)
USD to INR Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)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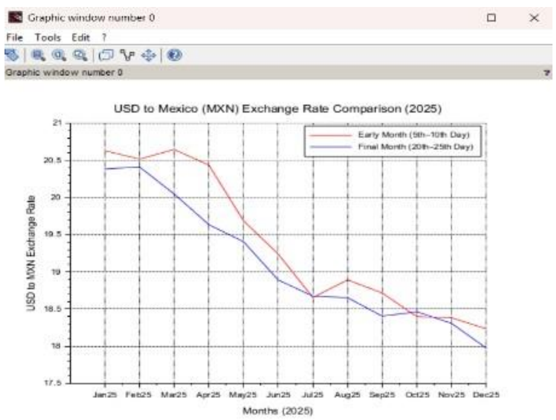
USD to GBP Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Late Month (20th–25th Day)
USD to GBP Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)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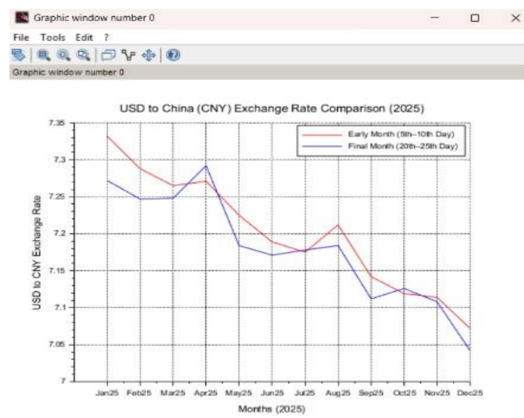
USD to Mexico (MXN) Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Final Month (20th–25th Day)
USD to MXN Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)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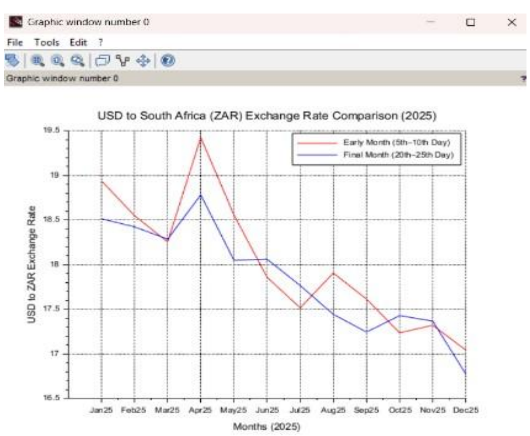
USD to China (CNY) Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Final Month (20th–25th Day)
USD to CNY Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)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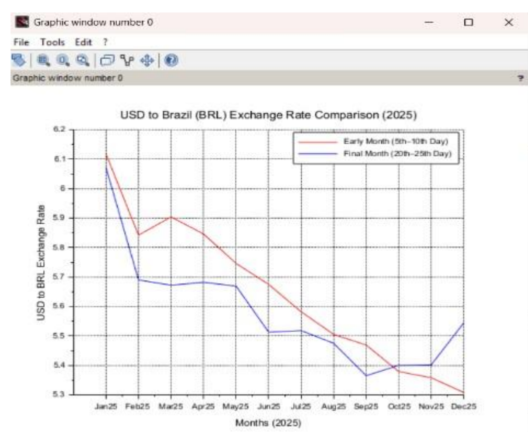
USD to South Africa (ZAR) Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Final Month (20th–25th Day)
USD to ZAR Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)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USD to Brazil (BRL) Exchange Rate Comparison (2025)
Early Month (5th–10th Day)
Final Month (20th–25th Day)
USD to BRL Exchange Rate
Jan25 Feb25 Mar25 Apr25 May25 Jun25 Jul25 Aug25 Sep25 Oct25 Nov25 Dec25
Months (2025)

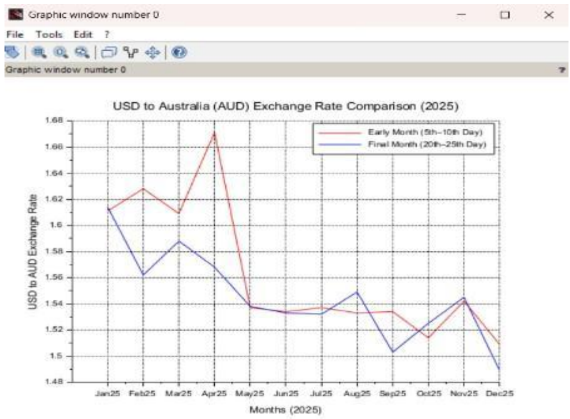


Figure 2: Scilab-Based Analysis of International Currency Exchange Rates in 2025

2.2 Laplace Transformation

The Laplace transform is one of the more versatile tools in applied mathematics: it converts a function from the time domain into the frequency domain [2], and it turns up across engineering, economics, and financial analysis largely because it makes unwieldy differential equations far more tractable. In currency analysis specifically, it offers a way to separate periodic variation, temporal change, and continuous behaviour that would otherwise stay buried in the raw exchange-rate series [4].

The Laplace transform of a function $f(t)$is defined as

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)_{dt} \tag{1}$$

For example, if

$$f(t) = e^{at} \tag{2}$$

then the Laplace transform is

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \tag{3}$$

where $s > a$. This transformation is useful for analyzing continuous and varying data patterns in financial and currency systems.

Given Function

$$f(t) = \begin{cases} 1, & 0 < t < 86 \\ -1, & 86 < t < 90 \end{cases} \tag{4}$$

$$T = 90$$

Use periodic Laplace formula

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt \tag{5}$$

$$= \frac{1}{1-e^{-90s}} \left[\int_0^{86} e^{-st} dt + \int_{86}^{90} (-e^{-st}) dt \right] \tag{6}$$

$$\int_0^{86} e^{-st} dt = \frac{1-e^{-86s}}{s} \tag{7}$$

$$\int_{86}^{90} (-e^{-st}) dt = \frac{e^{-90s} - e^{-86s}}{s} \tag{8}$$

$$L[f(t)] = \frac{1}{1-e^{-90s}} \cdot \frac{1}{s} [(1 - e^{-86s}) + (e^{-90s} - e^{-86s})] \tag{9}$$

$$= \frac{1}{s(1-e^{-90s})} [1 - 2e^{-86s} + e^{-90s}] \tag{10}$$

$$L[f(t)] = \frac{1-2e^{-86s}+e^{-90s}}{s(1-e^{-90s})} \tag{11}$$

Table 3 . Mathematical Representation of Periodic Functions and Their Laplace Transforms

S.No	Function $f(t)$	Period T	Laplace Transform $L[f(t)]$
1	$f(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \end{cases}$	2	$\frac{1-e^{-s}}{s(1+e^{-s})}$

2	$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$	2	$\frac{1}{s(1 + e^{-s})}$
3	$f(t) = \begin{cases} 1, & 83.154 < t < 83.619 \\ -1, & 83.619 < t < 84.084 \end{cases}$	0.93	$\frac{1 - e^{-0.465s}}{s(1 + e^{-0.465s})}$
4	$f(t) = \begin{cases} 1, & 0 < t < 83.154 \\ 0, & 83.154 < t < 83.619 \end{cases}$	83.619	$\frac{1 - e^{-83.154s}}{s(1 - e^{-83.619s})}$
5	$f(t) = \begin{cases} 1, & 0 < t < 83 \\ -1, & 83 < t < 85 \end{cases}$	85	$\frac{1 - 2e^{-83s} + e^{-85s}}{s(1 - e^{-85s})}$
6	$f(t) = \begin{cases} 1, & 0 < t < 86 \\ -1, & 86 < t < 90 \end{cases}$	90	$\frac{1 - 2e^{-86s} + e^{-90s}}{s(1-e^{-90s})}$
7	$f(t) = \begin{cases} 1, & 0 < t < 4 \\ -1, & 4 < t < 26 \end{cases}$	26	$\frac{1 - 2e^{-4s} + e^{-26s}}{s(1 - e^{-26s})}$

2.3 Complexity

Complexity is a key feature of financial time-series data, with exchange-rate fluctuations being irregular, non-linear and dynamic [9]. Currency values fluctuate constantly with economic growth, inflation, trade, demand, policy and global financial conditions. Owing to these interactions, exchangerate time series tend to display intricate structure and fractal temporal scaling [6]. As a result, exchangerate series are characterised by intricate temporal structure and self-similarity. This study explores the multifractal nature of the USD exchange-rate data for seven countries in 2024-2025 by HarFA fractal analysis in Figure 3 [5,8] and Pixel analysis in Figure 4 [8]. HarFA assesses the fractal nature of the exchange-rate series, and Pixel computes distances between boundary-to-boundary points in the produced currency graphs. It reveals the geometrical and statistical properties of the exchange-rate time series and sheds light on the complexity and dynamics of foreign exchange markets [6,9].

2.4 HarFA–Fractal Analysis of Currency Exchange Data

The Box Counting technique was implemented using the HarFA software package developed at the Institute of Physical and Applied Chemistry, Brno University of Technology, Czech Republic. In this study, the method was applied to analyse the fractal characteristics of USD exchange-rate data for seven countries during the years 2024 and 2025. The dimension obtained through this procedure is referred to as the fractal box dimension (DB).[8]

The Box Counting approach is based on a simple geometric concept. A grid consisting of square cells of different sizes is superimposed on the graphical representation of the currency data. For each grid size, the number of squares intersecting the graph is determined. By repeating this process for several scales, the variation between the box size and the number of occupied boxes can be examined.[5] HarFA mainly works with binary images consisting of black and white regions. In the Box Counting analysis, the fractal structure is covered using square grids, and the following quantities are evaluated as MK, MT, and MKT, where [8]

- MK– count of black grid cells,
- MT– count of white grid cells,
- MKT– count of mixed grid cells containing both black and white regions,
- $MKTK$– count of mixed cells dominated by black regions,

- $MKTT$– count of mixed cells dominated by white regions.

Repeating the procedure for different box sizes $r$produces a logarithmic relationship between the box size and the number of boxes $M(r)$required to cover the fractal pattern completely. The slopes of the linear relations [8]

$$Gk = \frac{\log M_k(r)}{\log(1/r)} \tag{12}$$

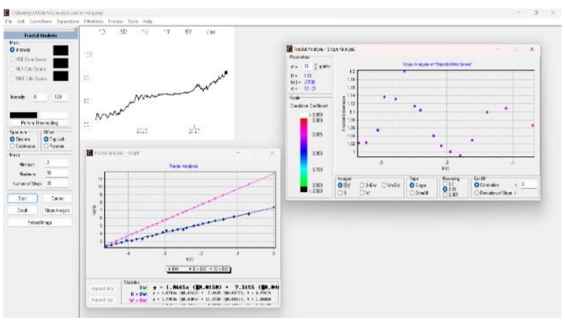
$$Gt = \frac{\log M_t(r)}{\log(1/r)} \tag{13}$$

$$Gkt = \frac{\log M_{kt}(r)}{\log(1/r)} \tag{14}$$

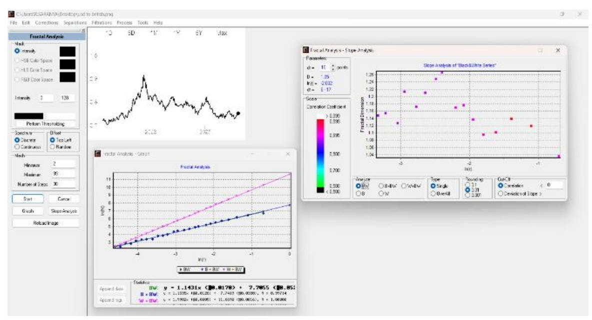
determine the corresponding fractal dimensions, Gkrepresents the boundary characteristics of the fractal structure, Gtdescribes the properties of the pattern on a white background, and Gkt characterises the fractal pattern on a black background.

Using this procedure, the fractal dimensions of USD exchange-rate data for India, China, Brazil, Australia, Mexico, South Africa, and the United Kingdom were obtained for both discrete and continuous processes. These values help in understanding the complexity, fluctuation, and irregular behaviour of international currency movements during 2024 and 2025 in Figure 3.

(a) India

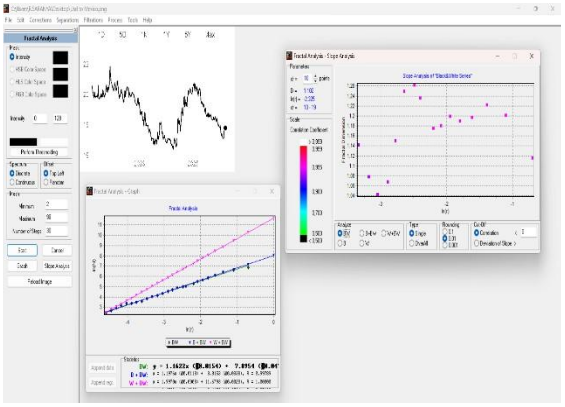


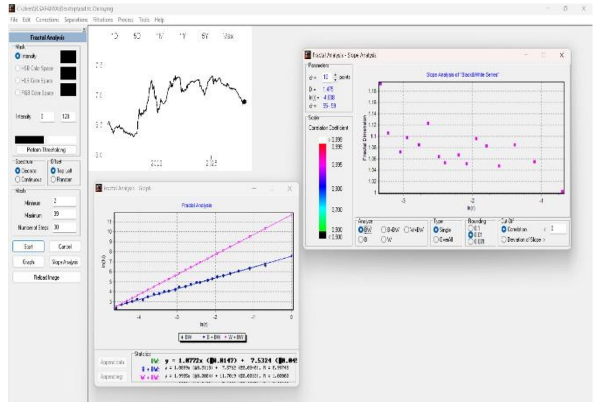
(b) British



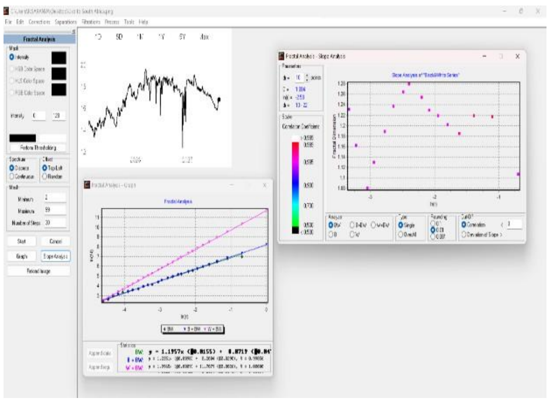
(c) Mexico

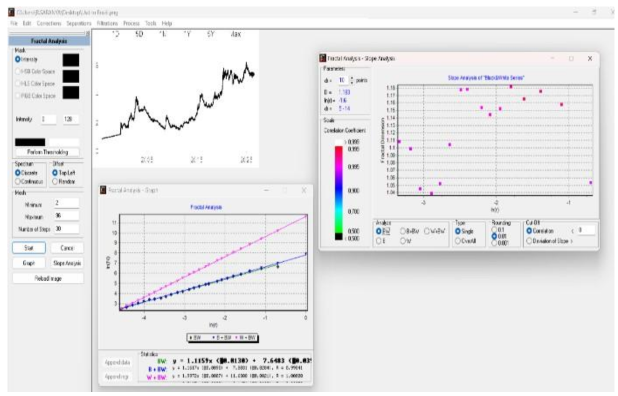
(d) China





(e) South Africa (f) Brazil





(g) Australia

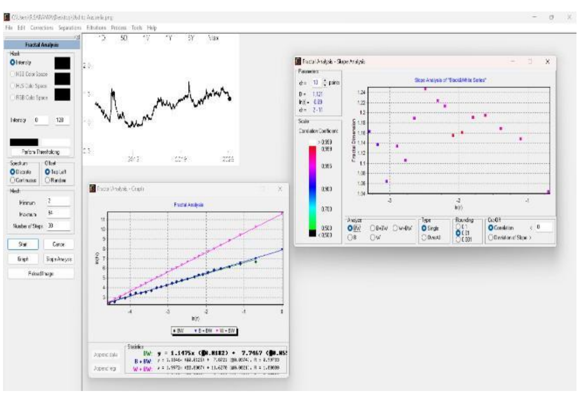


Figure 3: Original Graphical Patterns Used for HarFA Fractal Analysis for Seven Countries

2.5 Pixel analysis for the seven countries

For the analysis of USD exchange-rate data of seven countries during 2024 and 2025, the PIXEL program was used to study the boundary structure and complexity of the currency graphs. The monthly USD exchange-rate values were converted into graphical images, and the perimeter of each graph was considered an important feature for boundary analysis [8]. The boundary representation is given by

$$Z(t) = (x(t), y(t)) \tag{15}$$

where $Z(t)$denotes the boundary curve of the graph at a given point t. The functions $x(t)$and $y(t)$represent the horizontal and vertical coordinates of the boundary points, respectively. Together, these coordinates describe the shape and outline of the exchange-rate graph.

The radial distance from the centre of the graph to the boundary points was determined using

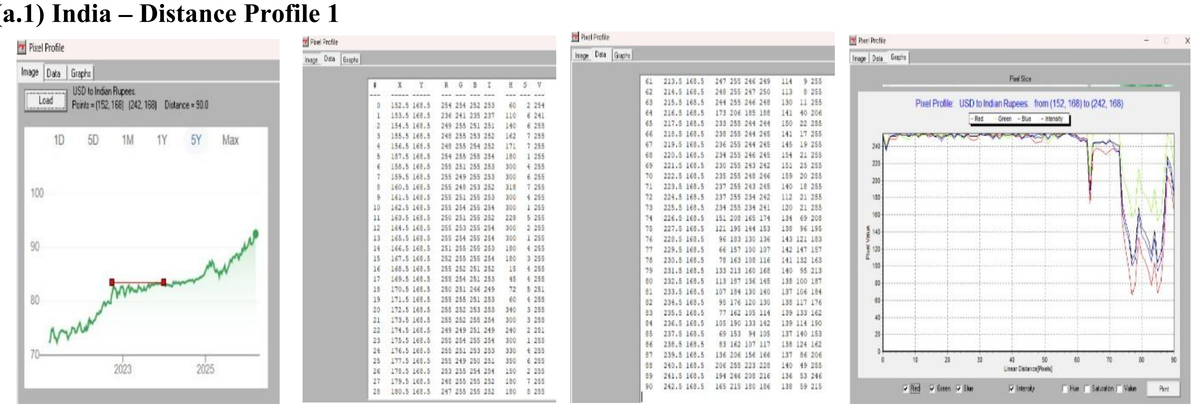
$$R_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \tag{16}$$

where:

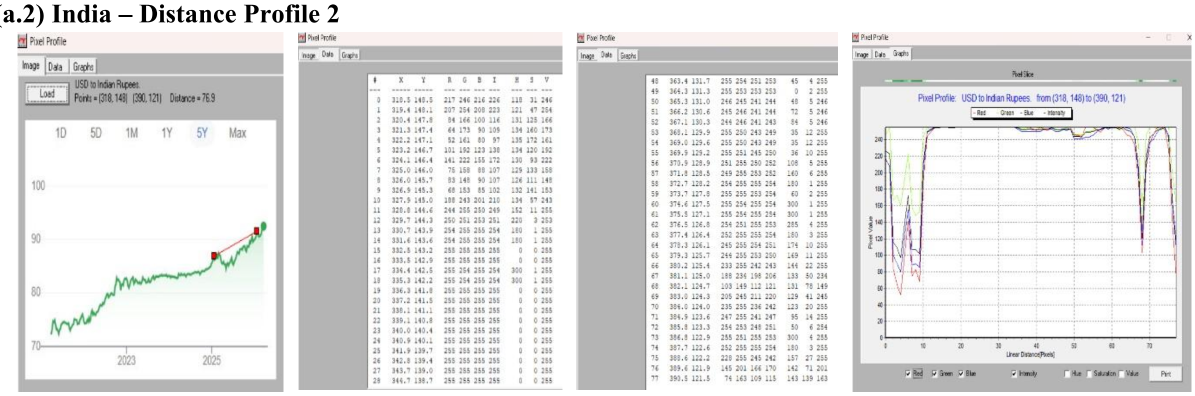
- R_i is the radial distance of the i^{th}boundary point from the centre.
- x_i and yi are the coordinates of the i^{th}boundary point.
- x_c and yc are the coordinates of the centroid (centre) of the graph.
- i represents the index of the boundary point being analysed.

The radial distance measures how far each boundary point lies from the centre of the graph and provides information about the shape and irregularity of the boundary. Larger variations in radial distance indicate greater fluctuations and complexity in the exchange-rate pattern, whereas smaller variations suggest a more regular and stable structure. Through these measurements, the PIXEL program helps identify differences in the geometric behaviour of USD exchange-rate movements among India, Britain, Mexico, South Africa, China, Brazil, and Australia, thereby providing insights into the variability and structural complexity of currency fluctuations during the study period of 2024–2025 in Figure 4. [8]

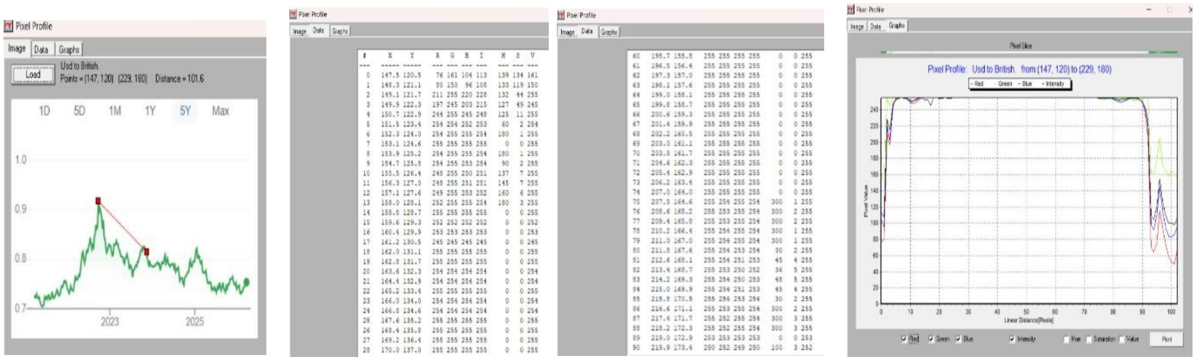
(a.1) India – Distance Profile 1



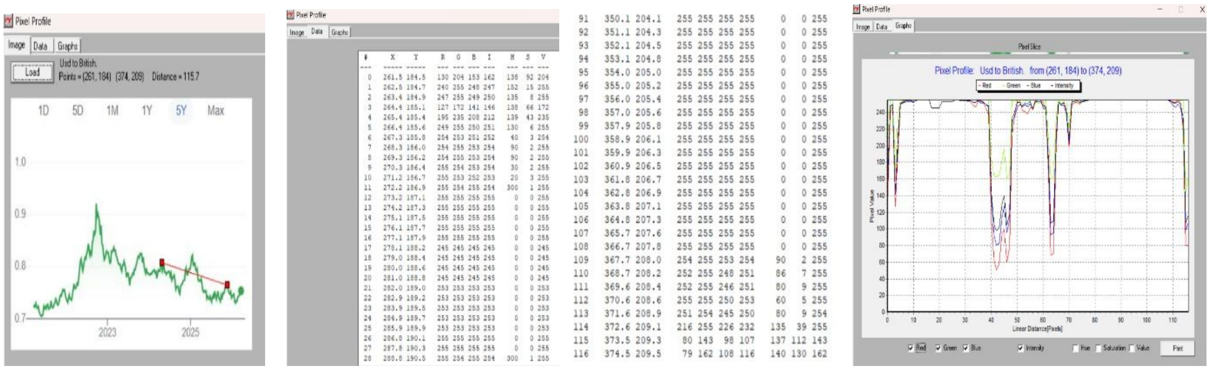
(a.2) India – Distance Profile 2



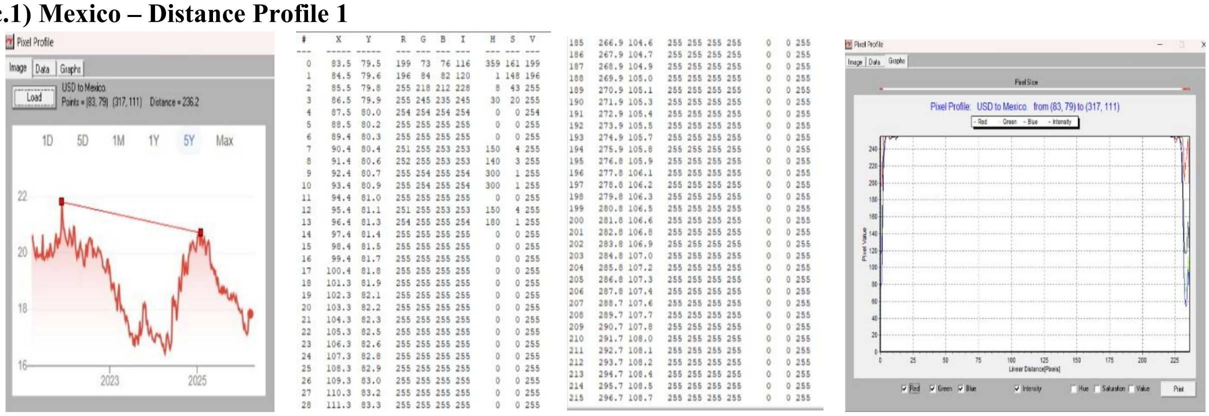
(b.1) British – Distance Profile 1



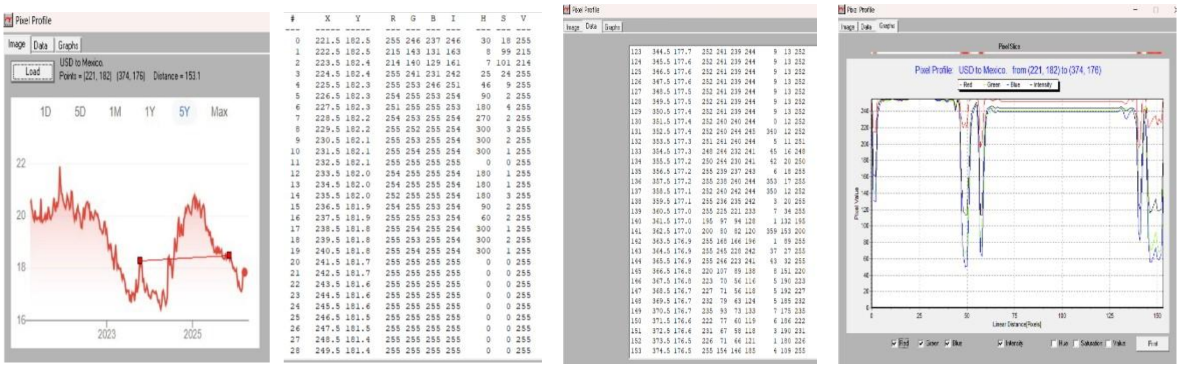
(b.2) British – Distance Profile 2



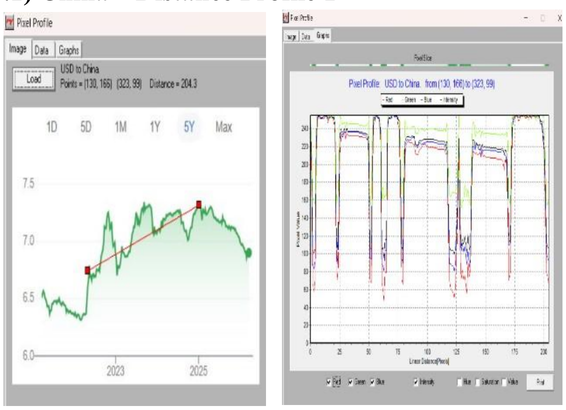
(c.1) Mexico – Distance Profile 1



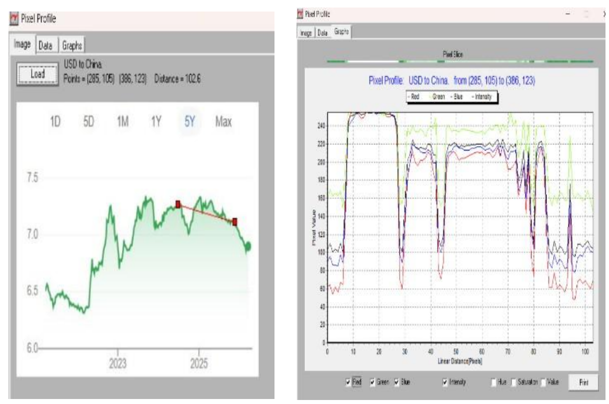
(c.2) Mexico – Distance Profile 2



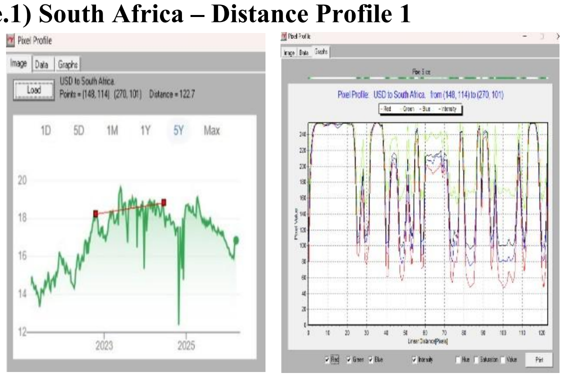
(d.1) China – Distance Profile 1



(d.2) China – Distance Profile 2

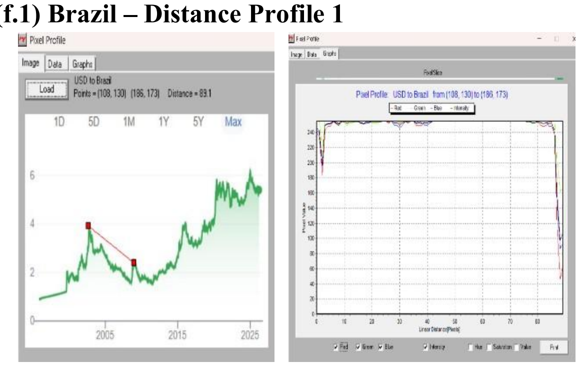


(e.1) South Africa – Distance Profile 1

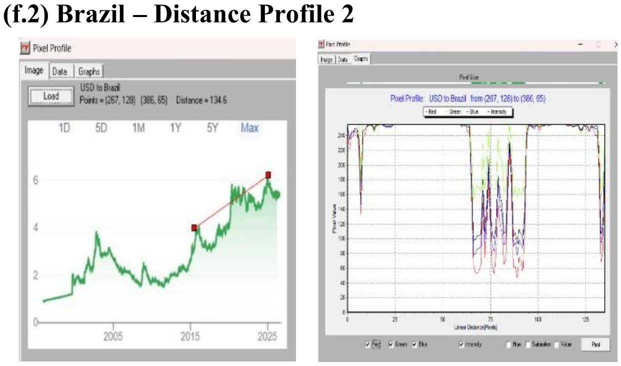


(e.2) South Africa – Distance Profile 2

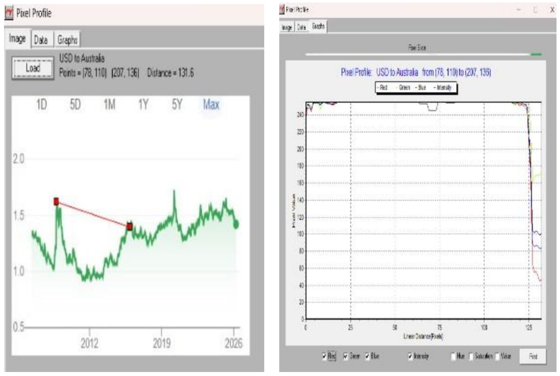
(f.1) Brazil – Distance Profile 1



(f.2) Brazil – Distance Profile 2



(g.1) Australia – Distance Profile 1



(g.2) Australia – Distance Profile 2

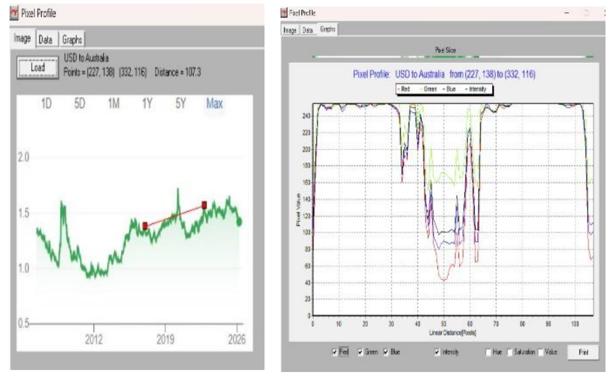


Figure 4: Pixel Profile Graphs for USD Exchange-Rate Analysis of Seven Countries.

3. Results

The USD exchange-rate data of seven major countries—India, the United Kingdom, Mexico, South Africa, China, Brazil, and Australia, for 2024 and 2025 were analysed using the Brownian Motion method. The comparative exchange-rate data are presented in Tables 1 and 2. The Scilab-generated Brownian Motion paths shown in Figures 1 and 2 for 2024 and 2025 exhibit stochastic behaviour with irregular and non-linear trajectories. The variation in these trajectories indicates different levels of complexity among the selected countries and highlights the changes in exchange-rate dynamics between the two years.

The HarFA-based complexity analysis in Figure 3 confirms that the USD exchange rate follows an irregular, non-linear path for every country, indicating a fractal rather than a smooth deterministic structure. The resulting fractal box dimensions (D) are summarised in Table 4. China recorded the highest dimension, indicating the most complex exchange-rate path, while India recorded the lowest, consistent with its comparatively smooth, steadily rising trend against the Dollar. The United Kingdom and South Africa show the next-highest complexity, while Mexico and Australia are comparatively more regular.

The Pixel profile in Figure 4 measured the boundary-to-boundary distance between selected points on each exchange-rate graph as a further indicator of volatility, with larger distances corresponding to sharper movements and smaller distances reflecting more stable behaviour. The measured distances for both profiles of each country are given in Table 4. Mexico and China showed the most pronounced swings, while India remained the most stable. The Brazil and Australia profiles, however, were taken over their full available price history rather than the five-year window used for the other five countries, so these two sets of values should be read with that difference in mind rather than compared directly against the rest.

Table 4: Summary of HarFA Fractal Dimensions and Pixel Boundary Distances (2024–2025)

Country	**Fractal Dimension (D)**	**Pixel Distance – Profile 1**	**Pixel Distance – Profile 2**
India	1.010	50.0	76.9
United Kingdom	1.350	101.6	115.7
Mexico	1.102	236.2	153.1
China	1.475	204.3	102.6
South Africa	1.204	122.7	100.2

Brazil	1.193	89.1	134.6
Australia	1.121	131.6	107.3

The periodic Laplace transformation in Table 3 fits the exchange-rate data as a periodic square-wave function, with periods ranging from 2 to 90 time units across the cases examined. This confirms that the exchange-rate series carries a periodic component superimposed on the broader stochastic trend, consistent with both the Brownian-motion paths and the HarFA fractal dimensions.

Taken together, the countries with higher fractal dimensions and larger pixel distances, namely China, the United Kingdom, and Mexico, display visibly rougher Brownian paths, while India's low dimension and short pixel distance align with its comparatively smooth long-term appreciation trend against the Dollar.

4. Conclusion

This study examined the exchange-rate dynamics of seven major currencies, namely the Indian Rupee, British Pound, Mexican Peso, South African Rand, Chinese Yuan, Brazilian Real, and Australian Dollar, against the US Dollar across 2024 and 2025, using Brownian motion, HarFA fractal analysis, Pixel analysis, and a periodic Laplace transformation. Across all four methods, Forex movements were found to be irregular and non-linear, confirming that currency markets are stochastic and carry a fractal character rather than following smooth, predictable curves. The Brownian-motion simulations built in Scilab captured this randomness effectively, and the HarFA fractal dimensions quantified it directly: values ranged from 1.010 for India to 1.475 for China, with the United Kingdom (1.350) and South Africa (1.204) close behind, and Mexico (1.102), Australia (1.121), and Brazil (1.193) closer to the smoother end of the scale. The Pixel analysis supported this pattern, with Mexico and China producing the largest boundary-to-boundary distances and India consistently the smallest, indicating that China and Mexico's currencies moved most erratically over the study period while India followed a comparatively steady, upward path against the Dollar. The periodic Laplace transformation added a

result the other methods do not directly provide: treating each currency's data as a periodic square-wave function returned periods ranging from 2 to 90 time units, showing that a recurring, cyclical structure exists beneath the stochastic noise. Together, these four methods, namely Brownian motion for the randomness, HarFA and Pixel for the complexity and roughness, and the Laplace transform for the periodicity, form a complete framework for understanding how currency markets behave. This approach could be extended to other countrie's currencies, or to other financial time series such as stock markets, commodities, and crypto assets, as a direction for future work.

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