International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(21s) 2026 pp. 1398-1406
https://doi.org/10.70917/ijcisim-2026-5401
Article

Semi–alpha Continuous Mappings in Neutrosophic Cubic Grill Topological Spaces

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Abstract: In the present paper, we present notions of semi-alpha continuous mapping, semi-alpha irresolute mapping, contra semi alpha continuous mapping, contra semi-alpha irresolute mapping within framework of neutrosophic cubic grill topological spaces. It also explores various properties associated with these mappings in this context.
Keywords: Neutrosophic cubic grill semi-alpha open set, Neutrosophic cubic grill semi-alpha closed set, Neutrosophic cubic grill semi-alpha continuous mapping, Neutrosophic cubic grill contra semi-alpha continuous mapping, Neutrosophic cubic grill semi-alpha irresolute mapping, Neutrosophic cubic grill contra semi-alpha irresolute mapping.

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1. Introduction

Properties of spaces that don't change under constant transformations are the focus of classical topology. Numerous kinds of topological spaces have been developed as a result of the substantial research in this area from the early 1900s.

Smarandache [7,8] introduced neutrosophy, a philosophical framework that led to formulation of neutrosophic logic. Mathematical ambiguity, contradiction, ambiguity, imprecision, incompleteness, inconsistency, redundancy, and undefined parts are all accommodated by this logic. Truth, indeterminacy, and falsehood are evaluated as independent

components in neosophic logic.

In 2012, Y.B. Jun presented concept of cubic sets or operations on them [3], which have found applications across different branches of mathematics. Building on this, Mumtaz Ali [11] proposed idea of neutrosophic cubic sets in 2016, eXpanding notion of cubic sets within the neutrosophic framework.

In 1947, Choquet [2] introduced the concept of grills, which the literature highlights as a valuable supplementary tool in topological studies. Various traits and theoretical developments concerning grills are discussed in [5], followed by numerous topological investigations.

In 2021, idea of a neutrosophic grill topological space has been first presented by Selvaraj Ganesan [15]. Later, in 2024, concept of neutrosophic cubic grill topological space has been introduced by A. Vanitha and F. Nirmala Irudayam [16]. Ali Abbas and Khalil [1] studied neutrosophic continuous and contracontinuous functions, whereas "Salama et al. [14] introduced neutrosophic closed sets or neutrosophic continuous functions" in 2014. Neutosophic Pre-α, Semi-α, and Pre-β irresolute, open, and closed mapping were examined by Rajesh and Chandra Sekar [13].

Notion "of neutrosophic semi-α-open sets or their characteristics in neutrosophic topological" space (NTS) has been presented by Imran et al. in 2017 [9]. Additionally, they discussed the features of neutrosophic semi- α-interior or neutrosophic semi- α-closure.

Neutosophic semi-α-continuous mapping in neutrosophic topological spaces was first proposed by Raja

Mohammad Latif [12] in 2023.

For neutrosophic cubic grill topological spaces, we present the notions of neutrosophic cubic grill semi-α-"continuous mapping, neutrosophic cubic grill semi-α-irresolute mapping, neutrosophic cubic grill contra semi-α-continuous mapping, neutrosophic cubic grill contra semi-α-irresolute mapping. We examine a few of these mappings' characteristics in neutrosophic cubic grill topological" spaces.

2. PRELIMINARIES

Definition 2.1[3]

Consider X be set that is non-empty. Structure of the following form is neutrosophic set in X, "Λ= { ($\Lambda T(x)$, $\Lambda I(x)$, $\Lambda F(x)$) : $x \in X$"}

Here "$\Lambda T: X \to [0,1]$ denotes truth membership function".

$\Lambda I: X \to [0,1]$ denotes indeterminate membership function

$\Lambda F: X \to [0,1]$ denotes false membership function.

If ΛT, ΛI and ΛF are independent to each other, then $0 \leq \Lambda T + \Lambda I + \Lambda F \leq 3$. If ΛT, ΛI, and ΛF are dependent on each other, then $0 \leq \Lambda T + \Lambda I + \Lambda F \leq 1$.

Definition 2.2[6]

Let (X) = $\{Ci$; "$i \in I\}$ be entire collection of every neutrosophic sets. $\rho(X)$ be entire collection of every neutrosophic subsets of (X). Subfamily τN of $\rho(X)$ is known neutrosophic topology on (X) if" these conditions hold,

i. $0̃N$, $1̃N \in \tau N$

ii. $C1, C2 \in \tau N \Rightarrow C1 \cap C2 \in \tau N$

iii. $\{C_\alpha; \alpha \in I\} \in \tau_N \Rightarrow \cup_\alpha C_\alpha \in \tau_N$ $(X, \tau N)$ is called the NTS.

Definition 2.3[15]

Let $(X, \tau N)$ be NTS, then "G be grill on $(X, \tau N)$ if

$(i) A1 \in G$ & $A1 \subseteq A2 \subseteq (X, \tau N) \Rightarrow A2 \in G$

(ii) $A1, A2 \subseteq (X, \tau N)$ & $A1 \cup A2 \in G \Rightarrow A1 \in G$ (or) $A2 \in G$"

Definition 2.4[3]

Consider X be set that is non-empty. Structure of the following form is interval neutrosophic set in X :

A = { $(STA(x), SIA(x), SFA(x))$: $x \in X$ }

Where $STA =]A_T^-, A_T^+[$, $SIA =]A_I^-, A_I^+[$, $SFA =]A_F^-, A_F^+[$ "are interval valued fuzzy sets in X , that are known as interval truth, indeterminacy, & falsity membership" functions.

Definition 2.5.[3]

Consider X be set that is non-empty. A pair $\mathbb{A} = (A, \Lambda)$ which is NCS (neutrosophic Cubic "Set) in X . Here

Λ is neutrosophic set in X , A is an interval neutrosophic set in" X.

Definition 2.6. [11]

$1̃NC$ = { $(1̃, 1̃, 0̃)$, $(1,1,0)$; $x \in X$ }

Definition 2.7. [11]

$0̃NC$ = { $(0̃, 0̃, 1̃)$ $(0,0,1)$; $x \in X$ }

Definition 2.8. [11]

For any $\mathbb{A} = \{(A, \Lambda); x \in \mathcal{X}\}$, $i \in J$ is any index set, is NCS in $\mathcal{X}$ which is non-empty, here $Ai = \{(STi(x), SIi(x), SFi\ (x))\ ;\ x \in X\}$

$\Lambda i = \{\ (\Lambda Ti(x), \Lambda Ii(x), \Lambda Fi\ (x))\ ;\ x \in X\}$

we define

(i) $\bigcup_{i\in J} \mathbb{A}_i = \{(\ \vee A_i, \bigcup_{i\in J} \Lambda_i)\ ;\ x \in X\}$

$i\in J$

(ii) $\cap i\in J\ \mathbb{A}i = \{(\ \wedge Ai, \cap i\in J\ \Lambda i);\ x \in X\}$

$i\in J$

Definition 2.9 [11]

Suppose $(X, \mathbb{A}) = \{Ai;\ i \in I, x \in X\ \}$ be entire collection of all NCS. $\rho(X, \mathbb{A})$ be entire collection of every neutrosophic cubic subsets of $(X, \mathbb{A})$. Subfamily τNC of $\rho(X, \mathbb{A})$ is known neutrosophic cubic topology on $(X, \mathbb{A})$ if these conditions hold,

i. $0NC, 1NC \in \tau NC$

ii. $A1, A2 \in \tau NC \Rightarrow A1 \cap A2 \in \tau NC$

iii. $\{A_\alpha;\ \alpha \in I\} \in \tau_{NC} \Rightarrow \bigcup_\alpha A_\alpha \in \tau_{NC}$

It's called as neutrosophic cubic topological space $(X, \mathbb{A}, \tau NC)$.

Definition 2.10[16]

Suppose $(X, \mathbb{A}, \tau NC)$ be neutrosophic cubic "topological space, then G is referred to be grill on $(X, \mathbb{A}, \tau NC)$ if

(i) "$A \in G, A \subseteq B \subseteq (X, \mathbb{A}, \tau NC) \Rightarrow B \in G$

(ii) $A, B \subseteq (X, \mathbb{A}, \tau NC)\ \&A \cup B \in G \Rightarrow A \in G\ (or)\ B \in G$".

Let $(X, \mathbb{A}, \tau NC)$ be neutrosophic cubic topological space, then grill G associated with $(X, \mathbb{A}, \tau NC)$ is known neutrosophic cubic grill topological space $(X, \mathbb{A}, \tau NC, G)$

Definition 2.11[16]

Suppose $(X, \mathbb{A}, \tau NC, G)$be neutrosophic cubic grill topological (NCGT) spaces over $(X, \mathbb{A}, \tau NC)$and $A \in (X, \mathbb{A})$, then (i) NCG closure of A (NCG-cl(A))=$\cap$ $\{C;\ C$ is an NCG closed set & A $\subseteq C\}$

(ii) NCG interior of A (NCG-int(A)) =$\cup$ {S:S is an NCG open set & S $\subseteq A$}

Definition 2.12[10]

Suppose $(X, \tau NC)$ be NTS over "(X) , $B \in (X)$, then B is known as neutrosophic semi - open set (N-SO) if $B \subseteq$ N Cl[N Int"(B)].

*Complement of N-SO is referred as neutrosophic semi–closed set (N-SC).

*Family of all N-SO (resp.N-SC) of (X) is written as NSO(X) (resp.NSC(X).

Definition 2.13[13]

Suppose $(X, \tau NC)$ be NTS over (X), $B \in (X)$, then B is called "neutrosophic α- open set (Nα-O) if $B \subseteq$ N Int[N Cl[N Int(B")]].

Definition 2.14[13]

Suppose $(X, \tau NC)$ "be NTS over (X) , $B \in (X)$, then neutrosophic α-"interior of B[NαInt"(B)] is union of" every Nα-O contained in B.

Definition 2.15[13]

Suppose $(X, \tau NC)$ be NTS "over (X), $B \in (X)$, then neutrosophic α- closure of" B [NαCl(B)] is intersection of every Nα-C that contains B.

Definition 2.16[9]

Suppose $(X, \tau NC)$ be NTS over (X) ", $B \in (X)$, then B is called neutrosophic semi α- open" set (NSαO) if $B \subseteq$ NαCl[NαInt(B)].

Definition 2.17[9]

"The complement of NSαO is referred as neutrosophic semi-α-closed set" (NSαC).

Definition 2.18[9]

Suppose $(X, \tau NC)$ be NTS "over (X), $B \in (X)$, then neutrosophic semi α-interior of" B [NSαInt(B)] is union of each

NSα-O contained in B .

Definition 2.19[9]

Suppose $(X, \tau NC)$ be a NTS "over (X), $B \in (X)$, then neutrosophic semi α- closure of" B [NSαCl(B)] is intersection $\forall$ NSα-C that contains B.

Definition 2.20[12]

Suppose h: $(X, \tau NC) \longrightarrow (Y, \sigma NC)$ be "mapping, then h is known neutrosophic semi" – α- "continuous mapping if $h^{-1}(U)$ is neutrosophic semi- α-open set in (X) $\forall$ neutrosophic open set U in (Y)").

Definition 2.21[12]

If "inverse image of all neutrosophic open sets in (Y) is neutrosophic closed set in (X), then mapping is h: $(X, \tau NC) \longrightarrow (Y, \sigma NC)$ known" as neutrosophic contra-continuous.

Definition 2.22[12]

"Mapping h: $(X, \tau NC) \longrightarrow (Y, \sigma NC)$ is known as neutrosophic contra semi- α-continuous if inverse image of all neutrosophic open set in (Y) is neutrosophic semi-α-closed set in $(X$").

Definition 2.23[12]

Mapping h: $(X, \tau NC) \longrightarrow (Y, \sigma NC)$ is known "be neutrosophic contra semi-α-irresolute mapping if inverse image of all neutrosophic semi-α-open sets in (Y) is neutrosophic semi-α-closed set $(X$").

3. NEUTROSOPHIC CUBIC GRILL SEMI - α -CONTINUOUS MAPPING

Definition 3.1

Suppose $(X, \mathbb{A}, \tau NC, G)$ "be neutrosophic cubic grill topological space over $(X, \mathbb{A}, \tau NC)$

and $B \in (X, \mathbb{A})$, then B is called NCG-SO set (neutrosophic cubic grill semi - open) if $B \subseteq$ NCG Cl[NCG Int(B")].

*Complement of NCG-SO is called as neutrosophic cubic grill semi – closed set (NCG-SC).

*Family of every NCG-SO (resp.NCG-SC) of $(X, \mathbb{A})$ is NCGSO$(X, \mathbb{A})$ (resp.NCGSC$(X, \mathbb{A})$.

Definition 3.2

Suppose $(X, \mathbb{A}, \tau NC, G)$ "be neutrosophic cubic grill topological space over $(X, \mathbb{A}, \tau NC)$ and $B \in (X, \mathbb{A})$, then B is called" neutrosophic cubic grill α- open set (NCGα-O) if $B \subseteq$ NCG Int[NCG Cl[NCG Int(B)]].

*Complement of NCGα-O is known as neutrosophic cubic grill α – closed set (NCGα-C).

*Family of all NCGα-O (resp.NCGα-C) of $(X, \mathbb{A})$ is NCGαO$(X, \mathbb{A})$ (resp.NCGαC$(X, \mathbb{A})$).

Definition 3.3

Suppose $(X, \mathbb{A}, \tau NC, G)$ be neutrosophic cubic "grill topological space over $(X, \mathbb{A}, \tau NC)$ and $B \in (X, \mathbb{A})$, then" neutrosophic cubic grill α-interior of B [NCGαInt(B)] is union of each NCGα-O contained in B .

Definition 3.4

Suppose $(X, \mathbb{A}, \tau NC, G)$ be neutrosophic cubic grill topological space over $(X, \mathbb{A}, \tau NC)$, $B \in (X, \mathbb{A})$, then neutrosophic cubic grill α- closure of B is intersection of every NCGα-C that contains B & is written by NCGαCl(B).

Definition 3.5

Suppose $(X, \mathbb{A}, \tau NC, G)$ be neutrosophic cubic "grill topological space over $(X, \mathbb{A}, \tau NC)$, $B \in (X, \mathbb{A})$, then" B is called neutrosophic cubic grill semi α- open set (NCGSα-O) if $B \subseteq$ NCGαCl[NCGαInt(B)].

Definition 3.6

Complement of NCGSα-O is known as neutrosophic cubic grill semi α - closed set (NCGSα-C).

Definition 3.7

Suppose $(X, \mathbb{A}, \tau NC, G)$ be neutrosophic cubic grill "topological space over $(X, \mathbb{A}, \tau NC)$, $B \in (X, \mathbb{A})$, then" neutrosophic cubic grill semi α-interior of "B [NCGSαInt(B)] is union of all NCGSα-O contained in B.

Definition 3.8

Suppose $(X, \mathbb{A}, \tau NC, G)$ be neutrosophic cubic grill topological space over $(X, \mathbb{A}, \tau NC)$ and $B \in (X, \mathbb{A})$, then neutrosophic cubic grill semiα- closure of B is intersection of every NCGSα-C that contains B & is denoted by NCGSαCl(B).

Definition 3.9

Let h: $(X, \mathbb{A}, \tau NC, G1) \longrightarrow (Y, \mathbb{B}, \sigma NC, G2)$ be mapping, then h is known as neutrosophic cubic grill semi – α- "continuous mapping if $h^{-1}(U)$ is neutrosophic cubic grill semi- α-open set in $(X, \mathbb{A})$ $\forall$ neutrosophic cubic grill open set U in $(Y, \mathbb{B}$")).

Theorem 3.10

Each "neutrosophic cubic grill continuous mapping is neutrosophic cubic grill semi – α- continuous mapping".

Proof:

Suppose h: $(X, \mathbb{A}, \tau NC, G1) \longrightarrow (Y, \mathbb{B}, \sigma NC, G2)$ be a continuous mapping of a neutrosophic cubic grill and U be an NCG

O in $(Y, \mathbb{B})$. Then, $h{-}1(U)$ is NCG-O in $(X, \mathbb{A}$). Since NCG-O set is NCGS α -O set. So, $h^{-1}(U)$ is NCGSα-O set in $(X,$

$\mathbb{A})$. Hence h is semi- α- continuous mapping of neutrosophic cubic grill.

Theorem 3.11

Composite function of two neutrosophic cubic grill semi - α- continuous mapping is neutrosophic cubic grill semi – α

continuous mapping.

Proof:

Suppose h: $(X, \mathbb{A}, \tau NC, G1) \longrightarrow (Y, \mathbb{B}, \sigma NC, G2)$, g: $(Y, \mathbb{B}, \sigma NC, G2) \longrightarrow (Z, \mathbb{C}, \mu NC, G3)$ be neutrosophic cubic grill semi – α- continuous mapping.

Suppose G be NCGSα-O set in $(Z, \mathbb{C}) \Longrightarrow g^{-1}(G)$ is a NCGSα-O set in $(Y, \mathbb{B}) \Longrightarrow h^{-1}(g^{-1}($"$G))$ is NCGSα-O set in $(X, \mathbb{A})$.\

But $h^{-1}(g^{-1}(G)) = (g^{\circ}h)^{-1}(G) \Longrightarrow g^{\circ}h$ is a neutrosophic cubic grill semi - α- continuous mapping.

Theorem 3.12

Suppose h: $(X, \mathbb{A}, \tau NC, G1) \longrightarrow (Y, \mathbb{B}, \sigma NC, G2)$ is a neutrosophic cubic grill semi – α- continuous iff inverse image of NCG-C set in $(Y, \mathbb{B}$") is a NCGSα-C set in $(X, \mathbb{A})$.

Proof:

Assume that E is a NCG-C set in (Y, $\mathbb{B}$). After that, E^c is NCG-O in (Y, $\mathbb{B}$) & h is a semi– α-continuous mapping of a" neutrosophic cubic grill$\Longrightarrow$ h^{-1}(E^c) is NCGSα-O set in (X, $\mathbb{A}$) & h^{-1}(E^c) = $[h^{-1}(E)]^c$. $\Longrightarrow$ h^{-1}(E) is a NCGSα-C set in (X, $\mathbb{A}$)

On other side, "suppose that E is NCG-O in (Y, $\mathbb{B}$) $\Longrightarrow$ E^c is NCG-C in (Y, $\mathbb{B}$) $\Longrightarrow$ h^{-1}(E^c) is NCGSα-C set in (X, $\mathbb{A}$) & h^{-1}(E^c) = $[h^{-1}(E)]^c$ $\Longrightarrow$ h^{-1}(E) is a NCGSα-O set in (X, $\mathbb{A}$). Thus, h is semi-α-continuous mapping of neutrosophic cubic grill.

Theorem 3.13

Suppose k : (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) is neutrosophic cubic grill semi – α- continuous iff k($NCGS\alpha Cl$(K)) $\subseteq$ $NCGS\alpha Cl$(k(K)) $\forall$ netrosophic cubic set K in (X, $\mathbb{A}$).

Proof:

Suppose "K be neutrosophic cubic set in (X, $\mathbb{A}$), k be neutrosophic cubic grill semi – α- continuous mapping. Then k(K") $\subseteq$ $NCGS\alpha Cl$[k(K)].Now K $\subseteq$ k^{-1}(k(K)) $\subseteq$ k^{-1}[$NCGS\alpha Cl$[k(K)]]

$\Longrightarrow$ $NCGS\alpha Cl$(K) $\subseteq$$NCGS\alpha Cl$[$k^{-1}$[$NCGS\alpha Cl$[k(K)]]].

Since k is neutrosophic cubic grill semi –α- continuous mapping, $NCGS\alpha Cl$[k(K)] is a NCGSα-C set. $NCGS\alpha Cl$[k^{-1}[$NCGS\alpha Cl$[k(K)]]] = k^{-1}[$NCGS\alpha Cl$[k(K)]]]. Hence k($NCGS\alpha Cl$(K)) $\subseteq$

$NCGS\alpha Cl$(k(K)).

Conversely, let k($NCGS\alpha Cl$(K)) $\subseteq$ $NCGS\alpha Cl$(k(K)), for all netrosophic cubic set K in (X, $\mathbb{A}$). Suppose K be NCG-C set in (Y, $\mathbb{B}$). Then$NCGS\alpha Cl$[k(k^{-1}(K))] $\subseteq$ $NCGS\alpha Cl$(K)=K. From the given condition k ($NCGS\alpha Cl$(k^{-1}(K))) $\subseteq$ $NCGS\alpha Cl$ (k(k^{-1}(K))) $\subseteq$ K and hence $NCGS\alpha Cl$(k^{-1}(K)) $\subseteq$ k^{-1}(K). Since k^{-1}(K) $\subseteq$ $NCGS\alpha Cl$(k^{-1}(K)), $NCGS\alpha Cl$(k^{-1}(K))=k^{-1}(K). This indicates that k^{-1}(K) is a NCGSα-C set in (Y, $\mathbb{B}$). By using Theorem 3.12, hence k is neutrosophic cubic grill semi–α- continuous mapping.

Theorem 3.14

Suppose g: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) be bijective mapping, then g is neutrosophic cubic grill semi –α- continuous mapping iff $NCGS\alpha Int$[g (G)] $\subseteq$ g [$NCGS\alpha Int$(G)] $\forall$ neutrosophic cubic set G in (X, $\mathbb{A}$).

Proof:

Suppose "G be neutrosophic cubic set in (X, $\mathbb{A}$), g be bijective & neutrosophic cubic grill semi–α- continuous mapping. Suppose g (G)=H", then g^{-1}[$NCGS\alpha Int$(H)] $\subseteq$ g^{-1}(H). While g "is an injective mapping, g^{-1}(H)=G, so that g^{-1}[$NCGS\alpha Int$(H)]$\subseteq$G. Hence $NCGS\alpha Int$[g^{-1}[$NCGS\alpha Int$(H)]] $\subseteq$ $NCGS\alpha Int$(G). Since g is neutrosophic cubic grill semi –α - continuous mapping, g^{-1}[$NCGS\alpha Int$(H)] is NCGSα-O set in (X, $\mathbb{A}$) and g^{-1}[$NCGS\alpha Int$(H)] $\subseteq$ $NCGS\alpha Int$(G) $\Longrightarrow$ g[g^{-1}[$NCGS\alpha Int$(H)]] $\subseteq$$g$[$NCGS\alpha Int$(G)]. Hence $NCGS\alpha Int$[g(G)] $\subseteq$

g[$NCGS\alpha Int$(G)].

Conversely $NCGS\alpha Int[g(G)] \subseteq g[NCGS\alpha Int(G)]$ for all neutrosophic cubic sets G in (X, $\mathbb{A}$). Suppose U be neutrosophic cubic open set in (Y, $\mathbb{B}$). Then U is NCGSα-O set in (Y, $\mathbb{B}$). While g is a surjective mapping, so U = $NCGS\alpha Int(U)$ = $NCGS\alpha[g\ [g^{-1}(U)]]$ $\subseteq$ g [$NCGS\alpha Int(g^{-1}(U))$] $\Longrightarrow$ $g^{-1}(U)$ $\subseteq$$NCGS\alpha Int(g^{-1}(U)$. Hence $g^{-1}(U)$ is NCGSα-O set in (X, $\mathbb{A}$). Therefore g is a neutrosophic cubic grill semi–α -continuous mapping.

Theorem 3.15

Assume l: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) be mapping. Then l is neutrosophic cubic grill semi – α continuous mapping iff $l^{-1}[NCGS\alpha Int(B)] \subseteq NCGS\alpha Int(l^{-1}(B))$ for all neutrosophic cubic set B in (Y, $\mathbb{B}$).

Proof:

Suppose that l is semi–α -continuous mapping of neutrosophic cubic grills and that B is any neutrosophic cubic set in (Y, $\mathbb{B}$). Its evident that $l^{-1}[NCGS\alpha Int(B)] \subseteq l^{-1}(B)$ $\Longrightarrow$

$NCGS\alpha Int$ $[l^{-1}[NCGS\alpha Int(B)]] \subseteq NCGS\alpha Int[l^{-1}(B)]]$.Given that $NCGS\alpha Int(B)]$ is a NCGS-α-O set in (Y, $\mathbb{B}$) and l is neutrosophic cubic grill semi-α-continuous mapping$\Longrightarrow l^{-1}[NCGS\alpha Int(B)]$ is a NCGS-α-O set in (X, $\mathbb{A}$). Therefore $NCGS\alpha Int$ $[l^{-1}[NCGS\alpha Int(B)]] \subseteq l^{-1}[NCGS\alpha \text{Int}(B)] \subseteq NCGS\alpha Int(l^{-1}(B))$.

= On "the other hand, suppose that for any neutrosophic cubic α set B in (Y, $\mathbb{B}$), $l^{-1}[NCGS\alpha Int(B)] \subseteq NCGS\alpha \text{Int}(l^{-1}(B))$. If H is any neutrosophic cubic open set in (Y, $\mathbb{B}$), then

$l^{-1}(H)= l^{-1}[NCGS\alpha Int(H)] \subseteq NCGS\alpha Int(l^{-1}(H)) \Longrightarrow l^{-1}(H) = NCGS\alpha Int(l^{-1}(H)) \Longrightarrow l^{-1}(H$") is NCGS-$\alpha$-O set in ($Y$, $\mathbb{B}$). Hence l is semi- α -continuous mapping of a neutrosophic cubic grill.

1. NEUTROSOPHIC CUBIC GRILL CONTRA SEMI - α – CONTINUOUS AND CONTRA SEMI -α – IRRESOLUTE MAPPINGS

The ideas of "neutrosophic cubic grill contra semi-α-continuous mappings & contra semi-α-irresolute mappings" are

presented in the present section. Examine their basic characteristics and attributes.

Definition 4.1

Mapping h: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) is called as neutrosophic cubic grill "contra-continuous if inverse image for all NCG-O set in (Y, $\mathbb{B}$) is NCG-C set in (X, $\mathbb{A}$").

Definition 4.2

"Mapping h: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) is known as neutrosophic cubic grill contra semi- α-continuous if inverse image $\forall$ NCG-O set in (Y, $\mathbb{B}$) is $NCGS\alpha$-C set in (X, $\mathbb{A}$").

Theorem 4.3

Suppose h: ("X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) be neutrosophic cubic grill contra continuous mapping, then h is neutrosophic cubic grill contra semi- α-continuous" mapping.

Proof:

H represents any NCG-O set in (Y, $\mathbb{B}$). Since h is neutrosophic" cubic grill contra continuous mapping, h^{-1}("H) is NCG-C set in (X, $\mathbb{A}$). As every NCG-C set is $NCGS\alpha$-C implies that h^{-1}("H) is $NCGS\alpha$-C set in (X, $\mathbb{A}$) $\Longrightarrow$ h is neutrosophic cubic grill contra semi- α-continuous" mapping.

Theorem 4.4

Mapping h: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) is a neutrosophic cubic grill contra semi-α-continuous iff "inverse image of all NCG-C set in (Y, $\mathbb{B}$) is $NCGS\alpha$-O set in (X, $\mathbb{A}$").

Proof:

Assuming that U is a NCG-C set in (Y, $\mathbb{B}$), U^c is a NCG-O set in (Y, $\mathbb{B}$). In (X, $\mathbb{A}$), $h^{-1}(U^c)$ is a $NCGS\alpha$-C set, whereas h

is neutrosophic cubic grill contra semi-α-continuous. However, "$h^{-1}(U^c)=1-h^{-1}(U) \Longrightarrow h^{-1}(U)$ is a $NCGS\alpha$-O set in (X,

$\mathbb{A}$"). On the other hand, "inverse image of all NCG-C set in (Y, $\mathbb{B}$) is $NCGS\alpha$-O set in (X, $\mathbb{A}$"). Assuming that V is a NCG-

C set in (Y, $\mathbb{B}$"), V^c is a NCG-O set in (Y, $\mathbb{B}$). But $h^{-1}(V^c)=1 - h-1(V)$ is $NCGS\alpha$-C set in (X, $\mathbb{A}$") $\Longrightarrow$ $h^{-1}(V)$ is a $NCGS\alpha$-O

set (X, $\mathbb{A}$"). Hence h is a neutrosophic cubic grill contra semi-α-continuous mapping.

Theorem 4.5

If mapping h: (X, $\mathbb{A}$, τNC, $G1$) $\longrightarrow$ (Y, $\mathbb{B}$, σNC, $G2$) is a neutrosophic cubic grill contra semi- α-continuous mapping and g: (Y, $\mathbb{B}$, σNC, $G2$) $\longrightarrow$ (Z, $\mathbb{C}$, μNC, $G3$) is neutrosophic cubic grill "continuous mapping, then $g \circ h$ is neutrosophic cubic grill contra semi – α- continuous" mapping.

Proof:

Suppose W be a NCG-O set in (Z, $\mathbb{C}$). While g is a neutrosophic cubic grill continuous mapping,

$g^{-1}(W)$ is a NCG-O set in (Y, $\mathbb{B}$). While h is a neutrosophic cubic grill contra semi-α–continuous mapping, h–1[g−1(W)] is a $NCGS\alpha$-C set in (X, $\mathbb{A}$). However, $(g°h)^{-1}(W)$ = $h^{-1}(g^{-1}(W))$. As a result, $g°h$ is neutrosophic cubic grill contra semi – α- continuous" mapping.

Definition 4.6

Mapping h: (X, $\mathbb{A}$, τNC, $G1$) ⟶ (Y, $\mathbb{B}$, σNC, $G2$) is called a neutrosophic cubic grill contra semi-α-irresolute mapping if inverse image of all $NCGS\alpha$-O set in (Y, $\mathbb{B}$) is $NCGS\alpha$-C set in (X, $\mathbb{A}$).

Theorem 4.7

If mapping h: (X, $\mathbb{A}$, τNC, $G1$) ⟶ (Y, $\mathbb{B}$, σNC, $G2$) is neutrosophic cubic grill contra semi- α-irresolute, then it's a neutrosophic cubic grill contra semi- α-continuous.

Proof:

Assume that U is a NCG-O set in (Y, $\mathbb{B}$). U is $NCGS\alpha$-O set in (Y, $\mathbb{B}$) since all NCG-O sets are $NCGS\alpha$-O set. While h is a neutrosophic cubic grill contra semi- α-irresolute mapping, $h^{-1}(U)$ $NCGS\alpha$-C set in (X, $\mathbb{A}$). So h is neutrosophic cubic grill contra semi- α-continuous.

Theorem 4.8

If mapping h: (X, $\mathbb{A}$, τNC, $G1$) ⟶ (Y, $\mathbb{B}$, σNC, $G2$) is neutrosophic cubic grill contra semi -semi-α-irresolute & g: (Y, $\mathbb{B}$, σNC, $G2$) ⟶ (Z, $\mathbb{C}$, μNC, $G3$) is a neutrosophic cubic grill semi -α- continuous mapping, then the composite $g°h$ is a neutrosophic cubic grill contra semi – α- continuous mapping.

Proof:

Suppose W be any NCG-O set in (Z, $\mathbb{C}$), g is a neutrosophic cubic grill semi -α- continuous mapping, so $g^{-1}(W)$ is a $NCGS\alpha$-O set in (Y, $\mathbb{B}$). Meanwhile h is a neutrosophic cubic grill contra semi- α-irresolute mapping, $h^{-1}[g^{-1}(W)]$ is $NCGS\alpha$-C set in (X, $\mathbb{A}$). But $h^{-1}(g^{-1}(W))$=$(g°h)^{-1}(W)$. Therefore, $g°h$ is a neutrosophic cubic grill contra semi–α- continuous mapping**.**

Theorem 4.9

If mapping h: (X, $\mathbb{A}$, τNC, $G1$) ⟶ (Y, $\mathbb{B}$, σNC, $G2$) is neutrosophic cubic grill semi-α-irresolute and

g: (Y, $\mathbb{B}$, σNC, $G2$) ⟶ (Z, $\mathbb{C}$, μNC, $G3$) is a neutrosophic cubic grill contra semi-α- irresolute mapping, then the composite $g°h$ is a neutrosophic cubic grill contra semi–α- irresolute mapping.

Proof:

Suppose H be any $NCGS\alpha$-O set in (Z, $\mathbb{C}$), g is a neutrosophic cubic grill contra semi-α- irresolute mapping, so g−1(H) is a $NCGS\alpha$-C in (Y, $\mathbb{B}$). While h is neutrosophic cubic grill semi-α–irresolute mapping, h–1[g−1(H)] is a $NCGS\alpha$-C set in (X, $\mathbb{A}$). But $h^{-1}(g^{-1}$(H))=$(g°h)^{-1}$(H). Hence $g°h$ is neutrosophic cubic grill contra semi-α-irresolute mapping**.**

4. CONCLUSION

In neutrosophic cubic grill topological spaces, we developed ideas of neutrosophic cubic grill semi- α-continuous mapping, neutrosophic cubic grill contra semi- α-continuous mapping, neutrosophic cubic grill contra semi-α-irresolute mapping. In neutrosophic cubic grill topological spaces, we eXamined a number of these mappings' characteristics.

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