International Journal of Computer Information Systems and Industrial Management Applications
ISSN 2150-7988 Volume 18(22s) 2026 pp. 616-643
https://doi.org/10.70917/ijcisim-2026-5466
Article

Generalized Logarithmic Similarity Measure for Intuitionistic Fuzzy Sets and Applications for Medical Diagnosis, Pattern Recognition and Decision-Making

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Abstract: Intuitionistic fuzzy set theory is a modified version of fuzzy set theory, first introduced by Atanassov. The studies show that several similarity measures between Intuitionistic fuzzy sets are obtained, some of which are inappropriate for use in the current situation. In this study, we will create a novel and adaptable approach under the Intuitionistic fuzzy environment for investigating the decision-making process. Additionally, it suggested new logarithmic similarity and weight similarity measures under Intuitionistic fuzzy sets and clarified their validity. The proposed measure satisfies several exquisite features, allowing for implementation in various domains. The proposed similarity measure has been offered for performance in pattern recognition and medical diagnosis problem with several illustrative examples. Some unexpected scenarios have been discussed and analysed by using existing similarity measures. Using an example, illustrate the comparison analysis's effectiveness and adaptability. The study findings might offer vital information to the required stakeholders in various disciplines. A graph of the developed study is displayed to illustrate the challenges in medical diagnosis between patients and diseases. Finally, we demonstrated the MADM method for enhancing the college selection problem by drawing numerical illustration and making a graphical representation between them.
Keywords: Fuzzy sets; Intuitionistic fuzzy sets; Similarity measure; Distance measure; Pattern Recognition; MADM

1. Introduction

Initially, [1]proposed the concept of fuzzy set theory, which offered a fantastic chance to advance in the related discipline. According to the fuzzy set theory, fuzziness is performed as the membership function and arises from a person's perspective. The intuitionistic fuzzy set theory originated from the fuzzy set theory, and itis the composition of membership, non-membership functions and hesitancy index. The degrees of each of these single-valued functions range from zero to one. Intuitionistic fuzzy set theory derives information from ambiguous data regarding specific degrees of uncertainty. Although several higher-order fuzzy sets have been proposed in recent decades, [2]an intuitionistic fuzzy set is especially useful for uncleared information. An intuitionistic fuzzy set is a superset of fuzzy sets and is probably more useful than FS for dealing with inadequate information. Under it, Gau & Buehrer[3] provided the concept of a vague set, which introduced the higher degree fuzzy sets. Afterwards,[4] demonstrated how vague sets are perceived as similar to intuitionistic fuzzy sets. The concept of similarity measures on intuitionistic fuzzy sets is an innovative method for identifying similarities between two IFSs and how their perceptual values are comparable.

Based on the degree of membership and non-membership functions, the similarity measure compares several approaches to the real-world problem. The similarity measure is a prominent tool for accessing pattern recognition, decision-making, medical diagnosis problems, etc. Meanwhile, the techniques for decision-making are used in various disciplines, and these provide some efficient choices. One of the practical approaches to the decision-making problem is group decision-making. Group decision-making aims to determine the optimal solution to a problem using the information that each decision-maker offers. In the fields of decision science, management science, and operational research are prominent research topics. A group decision-making challenge involves several choices to address the problem, and numerous decision-makers often need to get preferred alternatives. The decision maker's conclusion depends on how the group members perceive the various possibilities. In recent decades, numerous authors [5], [6], [7] have been instructed to use multiple aggregation operators for precise values. The intuitionistic fuzzy similarity measure can be used to determine, to some extent, the degree of similarity between two objects. At the same time, it also has the property of partial similarity between two intuitionistic fuzzy sets. The intuitionistic fuzzy measure applies not only to medicine, pattern recognition, decision-making, and other practical fields but also has the significant advantage of dealing with uncertain information.

Unlike other methods, intuitionistic fuzzy approaches can easily describe uncertain, imprecise, and vague information. Furthermore, [8] illustrates the technique of autonomous robot systems in fuzzy environments based on local minima avoidance path planning. Ahmed et al.[9] created a fuzzy rough set for dealing with the fuzzy set-valued information system based on the correct methodology. A set-valued information system in the information system is used to compare various attribute values. Since, [10] developed the informational retrieval system of cross-language based on the fuzzy embedded bilingual dictionary model. While the success of any information retrieval operation places a high value on the relevance of the retrieved document, getting relevance is complicated by some problems that are made worse by the methods used in cross-language information retrieval. Additionally, [11] proposed a method for presenting the medical image segmentation problem based on C-mean clustering of fuzzy credibility in an IFS environment. Also, Dass et. al. [12], [13], [14] & [15] and Ozlu [16], [17], [18], [19] suggested the various information measures for improving the decision-making problem under picture type-2 hesitant fuzzy as well as interval-valued q-rung ortho pair hesitant fuzzy environment. Jan et al. [20] suggested information on complex picture fuzzy soft sets for improving the robust decision-making problem in human-computer interaction. In addition, Gwak et al. [21] developed the bipolar complex Intuitionistic fuzzy soft information measure for dealing with robotic technology. The various authors consider that the robotic study is a novel future approach. Jan et al.[22] created the complex fuzzy soft information measure to investigate the study of digital and network systems. Various authors [23], [24], [25], [26] developed the various information measures under the Pythagorean fuzzy sets.

We will study similarity and dissimilarity/divergence/distance measures based on Intuitionistic fuzzy sets. Distance and divergence measures are related to similarity measures, also known as dissimilarity measures. However, similarity measures are acceptable tools for determining the comparable information between two or more IFSs. It is utilized in various applications, including image processing, medical diagnosis, pattern recognition, machine learning, etc. Numerous authors have presented various distance and similarity measures from the literature and examined their corresponding applications and counterintuitive instances, as shown in Table 1.

In the above study, we extensively discussed the present IFS similarity measures and how these behaviours are in their various related applications. The different entropy measure has presented some counterintuitive cases, some of which have been overcome. To investigate the complementary approach, introduce pattern recognition and medical diagnosis problems using the existing similarity measures of IFSs and provide valuable findings. Additionally, a comprehensive analysis that connects IFS similarity and distance measures offers disagreement behaviour to their respective domains. Various comparative examples are provided, and the intended investigation is carried out competently. In this article, we develop an Intuitionistic fuzzy logarithmic similarity measure with mathematical formulation. The suggested similarity measure shows how two IFSs are similar and consistent. In other words, how to demonstrate the degree of similarity between two Intuitionistic fuzzy sets. The mathematical expression of the developed similarity measure is based on the logarithmic, radical and modulus difference between membership and non-membership functions. In addition, the proposed similarity measure satisfies the appropriate similarity axioms. The contrasting numerical examples demonstrate the relevance of the suggested study and its existence. Finally, we draw a figure of the proposed similarity measure between IFSs for describing the medical diagnosis problem.

Table 1. Illustrate some various types of IFSs measure and their corresponding applications

Source	Similarity measure	Distance measure	Pattern recognition	Medical diagnosis	Humming distance measure	Euclidean measure	Decision making	Remarks
Chen[27]	✓	×	×	✓	×	×	×	Proposed fuzzy reasoning algorithm with weighting
Chen & Tan [28]	✓	×	×	×	×	×	✓	Developed a method for resolving decision problems involving ambiguous sets
Szmidt &Kacprzyk[29]	×	✓	×	×	✓	✓	×	Comparing geometric interpretations of IFSs and FSs
Deng &Chuntian[30]	✓	×	✓	×	×	×	×	SuggestedIF similarity measures and regarding authenticity
Liang & Shi [31]	✓	×	✓	×	×	×	×	Li and Cheng's measure is not convincing in the study.
Mitchell [32]	✓	×	✓	×	×	×	×	Reform Dengfeng & Chuntian IF-similarity measure and identify Li and Cheng's similarity measure as in effectual.
Grzegorzewski [33]	×	✓	×	×	✓	✓	×	Improved various types of distance measures for IFSs and IVFSs
Hung & Yang [34]	✓	×	✓	×	×	×	×	Createdan IF-Hausdorff distance measure compared to the existing
Wang & Xin [35]	✓	✓	✓	×	×	×	×	Consider several illogical instances involving Szmidt and Kacperzyk's (2000) distance measure.
Chen [36]	×	✓	×	×	✓	✓	×	Construct IFSs andIVFSs distance measures and point out the shortcomings of Grzegorzewski (2004)
Li & Olson [37]	✓	×	×	×	×	×	×	Evaluate the study on IFSs and create a context between them.
Xu & Chen [38]	✓	✓	×	×	✓	✓	×	Create a chain of the similarity and distance measures in various weight compositions of Hausdorff and Euclidean distances.

Source	Similarity measure	Distance measure	Pattern recognition	Medical diagnosis	Humming distance measure	Euclidean measure	Decision making	Remarks
Xu & Yager [6]	✓	×	×	×	×	×	✓	To overcome the study of Szmidt and Kacprzy (2000) in the environment of IFSs & IFFSs
Xia [39]	✓	×	×	×	✓	×	✓	Develop some IFV and IVFV measures and examine how they could be used in a decision-making problem.
Ye [40]	✓	×	×	×	✓	✓	×	On the pioneering work of cosine fuzzy similarity measure, it obtained and developed IFSsimilarity and weight similarity.
Hwang et al.[41]	✓	×	✓	×	×	×	×	To develop an IFS similarity measure in terms of Sugeno integral, it implemented in robust clustering problem and comparedit with the existing one.
Papakostas et al. [42]	✓	✓	✓	✓	×	×	×	Suggested novel IFSs similarity and distance measures, created a relationship between them and compared with existing ones.
Boran & Akay [43]	✓	×	✓	✓	×	×	×	Developed the similarity measures in terms of L_p norm with some fixed parameter for introducing the level of uncertainty
Du & Hu [44]	✓	×	✓	×	×	×	×	Developed IFSs distance and similarity measures on the conception of aggregation distance, which implemented in fuzzy clustering and pattern recognition problem
Chen et al. [45]	✓	×	✓	✓	×	×	×	Proposed an IF-similarity measure on the conception of centroid points of modified right-angled triangular fuzzy integer
Song et al. [46]	✓	×	✓	✓	×	×	×	Suggested a new similarity measure

Source	Similarity measure	Distance measure	Pattern recognition	Medical diagnosis	Humming distance measure	Euclidean measure	Decision making	Remarks
								based on the direct operation of membership, non-membership, hesitation index, and upper bound of membership functions; it describes the difference of discriminating and implemented in medical diagnosis.
Garg [47]	✓	×	✓	✓	×	×	✓	An improved cosine similarity metric for IFSs
Iqbal & Rizwan [48]	×	×	✓	✓	×	×	✓	Established the distinction between distance and similarity measurement
Raji-Lawal et al. [49]	✓	×	×	✓	×	×	✓	Improved existing FSs and IFSs similarity measures
Khatibi &Montazer[50]	✓	×	✓	✓	×	✓	×	Suggested the bacteria detection problem as medical pattern recognition
Hung [51]	✓	×	×	✓	×	×	×	Introduced bacteria classification and medical diagnosis problems
Beliakov et al. [52]	✓	×	×	✓	×	×	×	Developed IFSs vector valued similarity measure for solving bacteria detection problems.
Gahlot & Saraswat [53]	✓	×	✓	×	×	×	×	Constructed IFSs similarity measure and employed in bacteria detection as well as pattern recognition process
Gora & Tomar [54]	✓	×	✓	✓	×	×	✓	Suggested logarithmic similarity measure for IFSs
Hung & Yang [55]	✓	×	✓	×	×	×	×	Constructed similarity measure on Intuitionistic fuzzy set based on L_p metric

The remainder of the paper is outlined as follows. In Section 2,we suggested some concern definitions for the proposed several existing similarity studies on IFSs and in Section 3, we present some corresponding counterexamples. The validation of proposed IFSs similarity and weight similarity measures are described along with their mathematical expression in Section 4. Then, in Section 5, several illogical situations involving both existing and proposed similarity measures are described. Some applications related to the proposed study are acquainted in Section 6 and concluded in the last section.

2. Preliminaries

The basic concept, hypotheses, and associated properties of IFSs and similarity measures are elaborated in the present sections, making it simple and practical for the present study.

Definition (i)(Zadeh [1]): Let $X = \{\theta_1, \theta_2, \dots, \theta_n\}$ be represent the Universe of discourse at which defined a fuzzy set A on Universe of discourse X. All the present elements of the set provided a definite degree of fuzziness is obtained

$$A = \{\langle \theta, \mu_A(\theta): \mu_A: X \to [0,1], \theta \in X\rangle\} \quad (1)$$

where μ_A, describes the membership function and provides a certain degree of membership between 0 and 1.

Definition (ii)(Atanassove[2]): A non-empty set $A$is said to be an Intuitionistic fuzzy A defined on the Universe of discourse X is given by

$$A = \{\langle \theta, \mu_A(\theta), \nu_A(\theta): \mu_A, \nu_A: X \to [0,1], \theta \in X\rangle\} \quad (2)$$

where μ_A, ν_A, represent the membership and non-membership functions obtained as finite degrees between 0 and 1.

Also, $\pi_A(\theta) = 1 - \mu_A(\theta) - \nu_A(\theta)$ (3)

where, π_A is known as the function of the hesitancy index corresponding to all the elements of X as

$$0 \le \mu_A(\theta) + \nu_A(\theta) \le 1, \mu_A(\theta) + \nu_A(\theta) + \pi_A(\theta) = 1, 0 \le \pi_A(\theta) \le 1 \quad (4)$$

If $\pi_A = 0$, for all elements of X, then A is a fuzzy set.

If more excellent, the value of $\pi_A(\theta)$ indicates that there is more vagueness on θ.

The formation $(\mu_A(\theta), \nu_A(\theta))$ is called an Intuitionistic fuzzy set. Let IFS(X) denote the set of all probable Intuitionistic fuzzy sets on X.

Definition (iii) Let A, B ϵ IFS(X), then following relative notions between them are defined

(P1) If B be a superset of A if and only if$\mu_A(\theta) \le \mu_B(\theta), \nu_B(\theta) \le \nu_A(\theta)$ for all $\theta \in X$.

(P2) If A and B are identical if and only if $\mu_A(\theta) = \mu_B(\theta), \nu_B(\theta) = \nu_A(\theta)$ for all $\theta \in X. A =$

(P3) $A^c = \{\langle \theta, \nu_A(\theta), \mu_A(\theta): \mu_A, \nu_A: X \to [0,1], \theta \in X\rangle\}$, where A^cbe the complement of A.

Definition (iv) Consider a map $\mathbb{D}$such that$\mathbb{D}$: $\mathrm{IFSs}(X) \times \mathrm{IFSs}(X) \to [0,1]$ obtained the degree of dissimilarity between 0 and 1. If the following postulates authenticable for $\mathbb{D}(A, B)$ as

(D1) $0 \le \mathbb{D}(A,B) \le 1$;

(D2) $\mathbb{D}(A, B) = 0$ if and only if A = B;

(D3) $\mathbb{D}(A, B) = \mathbb{D}(B, A)$;

(D4) If $A, B, C \epsilon$ $\mathrm{IFSs}(X)$such that A ⊑ B ⊑ C, then $\mathbb{D}(A, B) \le \mathbb{D}(A, C)$and $\mathbb{D}(B, C) \le \mathbb{D}(A, C)$.

Then $\text{Đ}(\text{A}, \text{B})$is a distance measure between two IFSs, A and B.

Definition (v) Consider a map Ş such that$\text{Ş}: \text{IFSs}(\text{X}) \times \text{IFSs}(\text{X}) \rightarrow [0,1]$is known as the degree of similarity measure between 0 and 1.If $\text{Ş}(\text{A}, \text{B})$ satisfies the following postulates:

(SP1) $0 \leq \text{Ş}(\text{A}, \text{B}) \leq 1$;

(SP2) $\text{Ş}(\text{A}, \text{B}) = 1$ iff A = B;

(SP3) $\text{Ş}(A, B) = \text{Ş}(\text{B}, \text{A})$;

(SP4) If A, B, C∈ IFSs(X) such that A ⊑ B ⊑ C then$\text{Ş}(A, B) \geq \text{Ş}(\text{A}, \text{C})$, and $\text{Ş}(B, C) \geq \text{Ş}(\text{A}, \text{C})$.

Then $\text{Ş}(A, B)$provided the degree of is a similarity measure between A and B.

Measures of similarity and distance between two intuitionistic fuzzy sets complement concepts; similarity measures can be used to estimate distance measures, and vice versa. We may also learn the measures of similarity and dissimilarity in the present study.

3. Existing similarity measures with counterexamples

In this section, we examine the study of existing similarity measures with critical IFSs. The mathematical expression of similarity measures and their shortcomings are presented here, nonetheless, as a result of our acquisition. Using numerical comparison, we determined the superiority of existing similarity measures.

Let $A = \{\langle \theta, \mu_A(\theta), \nu_A(\theta)\rangle\}$ and $B = \{\langle \theta, \mu_B(\theta), \nu_B(\theta)\rangle\}$ are two intuitionistic fuzzy sets on the Universe of discourse$X = \{\theta_1, \theta_2, \dots, \theta_n\}$ be the set of the Universe of discourse, then existing similarity measures on A and B are followed as.

In this work, we examined some novel counterintuitive scenarios related to the existing study. Table 1 provides several existing similarity measures and their related unexpected results. Here, we introduced various existing similarity measures between IFSs. It demonstrated that axiom 2 is not satisfied when$A = \langle 0.30, 0.30\rangle$ and $B = \langle 0.40, 0.40\rangle$ since $\text{Ş}_C(A, B) = \text{Ş}_{DC}(A, B) = \text{Ş}_Y(A, B) = 1$, cannot be proved. Therefore, if $A = \langle 0, 0\rangle$, $B = \langle 0.20, 0.80\rangle$, $C = \langle 0.40, 0.60\rangle$, and $D = \langle 0.50, 0.50\rangle$ are four IFSs, such that $\text{Ş}_{HK}(A, B) = \text{Ş}_{HK}(C, D) = 0.50$, then it occurred counter-intuitiveness nature. This can also show that it is illogical if$A = \langle 0.30, 0.30\rangle$, $B = \langle 0.40, 0.40\rangle$, $C = \langle 0.30, 0.40\rangle$, and $D = \langle 0.40, 0.30\rangle$ because $\text{Ş}_O(A, B) = \text{Ş}_O(C, D) = 0.90$ and$\text{Ş}_M(A, B)$is identical as $\text{Ş}_O(A, B)$. There is an intriguing counterintuitive case if $A = \langle 0.30, 0.30\rangle$, $B = \langle 0.40, 0.40\rangle$, $C = \langle 0.40, 0.20\rangle$, and $D = \langle 0.50, 0.30\rangle$ then $\text{Ş}_S^p(A, B) = \text{Ş}_S^p(C, D) = 0.95$ and $\text{Ş}_h^p(A, B) = \text{Ş}_h^p(C, D) = 0.93$, for p =1 providing. Given that different sets have the same degree of similarity, as shown in Table 1, such that $\text{Ş}_{HY}^c(A, B) = \text{Ş}_{HY}^c(C, D) = 0.82$, if $A = \langle 0.30, 0.30\rangle$, $B = \langle 0.40, 0.40\rangle$, $C = \langle 0.40, 0.20\rangle$, and $D = \langle 0.50, 0.30\rangle$, which is a controversial nature. As research advances, some further paradoxical cases are shown; for example, if$A = \langle 0.60, 0\rangle$, $B = \langle 0, 0.60\rangle$, $C = \langle 0.20, 0.80\rangle$, and $D = \langle 0.80, 0.20\rangle$then $\text{Ş}_L^p(A, B) = \text{Ş}_L^p(C, D) = 0.85$, for $p = 1$ while$A = \langle 0, 0\rangle$, $B = \langle 0, 1\rangle$, $C = \langle 1, 0\rangle$ then $\text{Ş}_{SY}(A, B) = \text{Ş}_{SY}(C, D) = 0.50$. According to Table 2, the similarity measure is invalid for axiom 2 because when two distinct IFSs, $A = \langle 0, 0\rangle$, $B = \langle 1, 0\rangle$are executed $\text{Ş}_{LX}(A, B) = 1$. Additionally, the existing measure$\text{Ş}_e^p(A, B) = 1$ is invalid when $A = \langle 0.20, 0.80\rangle$, $B = \langle 0.60, 0.40\rangle$ but A is not equal to B. Consequently, the current measure is not appropriate for comparing similarities.

Table 2. Describe the existing IFS similarity measures along with counterintuitive cases

Sources	Existing similarity measures	Counterintuitive cases
Chen [56]	$\S_C(A,B) = 1 - \frac{\sum_{i=1}^{n}\lvert\left((\mu_A(\theta_i) - \nu_A(\theta_i)\right) - (\mu_B(\theta_i) - \nu_B(\theta_i))\rvert}{2n}$	$A = \{\langle\theta, 0.30, 0.$, & $B = \{\langle\theta, 0.40, 0.$ then $\S_C(A,B) = 1$
Hong & Kim [57]	$\S_{HK}(A,B) = 1 - \frac{\sum_{i=1}^{n}\lvert\left((\mu_A(\theta_i) - \mu_B(\theta_i)\right) + (\nu_A(\theta_i) - \nu_B(\theta_i))\rvert}{2n}$	$A = \{\langle\theta, 0, 0\rangle\}$, $B = \{\langle\theta, 0.20, 0.$, $C = \{\langle\theta, 0.40, 0.$, & $D = \{\langle\theta, 0.50, 0.$, then $\S_{HK}(A,B)$ $\S_{HK}(C,D)$ 0.50
Li & Xu [58]	$\S_{LX}(A,B) = 1 - \frac{\sum_{i=1}^{n}\{\left\lvert\left((\mu_A(\theta_i) - \nu_A(\theta_i)\right) - \left(\mu_B(\theta_i) - \nu_B(\theta_i)\right)\right\rvert - \lvert((\mu_A(\theta_i}{4n}$	$A = \{\langle\theta, 0, 0\rangle\}$, $B = \{\langle\theta, 1, 0\rangle\}$, $\S_{LX}(A,B)$ 1
Li &Zhongxian[59]	$\S_O(A,B) = 1 - \sqrt{\frac{\sum_{i=1}^{n}\left(\left(\mu_A(\theta_i) - \mu_B(\theta_i)\right)^2 + \left(\nu_A(\theta_i) - \nu_B(\theta_i)\right)^2\right)}{2n}}$	$A = \{\langle\theta, 0.30, 0.$, $B = \{\langle\theta, 0.40, 0.$, $C = \{\langle\theta, 0.30, 0.$, & $D = \{\langle\theta, 0.40, 0.$, then $\S_O(A,B) =$ $\S_O(C,D) =$ 0.90

Dengfeng&Chuntian[30]	$$S_{DC}(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \lvert\Phi_A(\theta_i) - \Phi_B(\theta_i)\rvert^p}{n}}$$	$A = \{\langle\theta, 0.30, 0.$, & $B = \{\langle\theta, 0.40, 0.$ then $S_{DC}(A,B) =$ 1, if $p = 1$
Mitchell [32]	$S_M(A,B) = \frac{1}{2}\left(\rho_\sigma(A,B) + \rho_\omega(A,B)\right)$ where $\rho_\sigma(A,B) = 1 - \sqrt{\sum_{i=1}^{n} \lvert\mu_A(\theta_i) - \mu_B(\theta_i)\rvert^p / n}$, and $\rho_\omega(A,B) = 1 - \sqrt{\sum_{i=1}^{n} \lvert\nu_A(\theta_i) - \nu_B(\theta_i)\rvert^p / n}$	Same as that of $S_O(A,B)$, if $p = 1$
Liang & Shi [31]	$$S_e^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \left(\omega_\mu(\theta_i) - \omega_\upsilon(\theta_i)\right)^p}{n}}$$	$A = \{\langle\theta, 0.20, 0.$, & $B = \{\langle\theta, 0.60, 0.$ then $S_e^p(A,B) =$ $1*$, if $p = 1$
	$$S_s^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n}(\chi_{s1}(\theta_i) - \chi_{s2}(\theta_i))^p}{n}}$$	$A = \{\langle\theta, 0.30, 0.$, $B = \{\langle\theta, 0.40, 0.$, $C = \{\langle\theta, 0.40, 0.$, & $D = \{\langle\theta, 0.50, 0.$, then $S_S^p(A,B) =$ $S_S^p(A,B) =$ 0.95 , if $p = 1$
	$$S_h^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n}(\sigma_1(\theta_i) + \sigma_2(\theta_i) + \sigma_3(\theta_i))^p}{3n}}$$	Same as that of $S_S^p(A,B)$, if $p = 1$
Hung & Yang [34]	$S_{HY}^a(A,B) = 1 - d_H(A,B)$ where $d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} max(\lvert(\mu_A(\theta_i) - \mu_B(\theta_i)\rvert, \lvert(\nu_A(\theta_i) - \nu_B(\theta_i)\rvert)$	$A = \{\langle\theta, 0.30, 0.$, $B = \{\langle\theta, 0.40, 0.$, $C = \{\langle\theta, 0.40, 0.$

		, & $D =$ $\{\langle\theta, 0.50, 0.$, then $Ş_{HY}^{a}(A,B) =$ $Ş_{HY}^{a}(C,D) =$ 0.90
	$Ş_{HY}^{b}(A,B) = \frac{e^{\mathrm{d}_H(A,B)} - e^{-1}}{1 - e^{-1}}$ $Ş_{HY}^{c}(A,B) = \frac{1 - \mathrm{d}_H(A,B)}{1 + \mathrm{d}_H(A,B)}$	$A =$ $\{\langle\theta, 0.40, 0.$, $B =$ $\{\langle\theta, 0.50, 0.$, & $C =$ $\{\langle\theta, 0.50, 0.$, then $Ş_{HY}^{b}(A,B) =$ $Ş_{HY}^{b}(A,C) =$ 0.85. Same as that of $Ş_{S}^{p}(A,B)$.
Liu [60]	$Ş_{L}^{p}(A,B) = 1 -$ $\sqrt[p]{\sum_{i=1}^{n} \frac{1}{2n} [\lvert\mu_A(\theta_i) - \mu_B(\theta_i)\rvert^p + \lvert\nu_A(\theta_i) - \nu_B(\theta_i)\rvert^p + \lvert\pi_A(\theta_i) -}$ where $1 < p < \infty$	$A =$ $\{\langle\theta, 0.60, 0\rangle$, $B =$ $\{\langle\theta, 0, 0.60\rangle$, $C =$ $\{\langle\theta, 0.20, 0.$, & $D =$ $\{\langle\theta, 0.80, 0.$, then $Ş_{L}(A,B) =$ $Ş_{L}(C,D) =$ 0.22 , if $p = 1$
Ye [40]	$Ş_{Y}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(\theta_i)\mu_B(\theta_i) + \nu_A(\theta_i)\nu_B(\theta_i)}{\sqrt{(\mu_A(\theta_i))^2 + (\nu_A(\theta_i))^2}\sqrt{(\mu_B(\theta_i))^2 + (\nu_B(\theta_i))^2}}$	$A =$ $\{\langle\theta, 0.30, 0.$, $B =$ $\{\langle\theta, 0.40, 0.$ then $Ş_{Y}(A,B)$ $= 1$
Ye & Shi [61]	$Ş_{SY}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(\theta_i)\mu_B(\theta_i) + \nu_A(\theta_i)\nu_B(\theta_i) + \pi_A(\theta_i)}{\sqrt{(\mu_A(\theta_i))^2 + (\nu_A(\theta_i))^2 + (\pi_A(\theta_i))^2}\sqrt{(\mu_B(\theta_i))^2 + (\nu}}$	$A =$ $\{\langle\theta, 0, 0\rangle\}$, $B =$ $\{\langle\theta, 1, 0\rangle\}$, & $C =$

		$\{\langle\theta, 0, 1\rangle\}$ then $\text{Ş}_{SY}(A,B) =$ $\text{Ş}_{SY}(A,C) =$ 0
Boran & Akay [43]	$\text{Ş}_t^p(A,B) = 1 - \sqrt[p]{\sum_{i=1}^{n} \frac{1}{2n(1+t)^p} \begin{Bmatrix} \lvert t(\mu_A(\theta_i) - \mu_B(\theta_i)) - (\nu_A(\theta_i) - \nu_B(\theta_i))\rvert \\ \lvert t(\nu_A(\theta_i) - \nu_B(\theta_i)) - (\mu_A(\theta_i) - \mu_B(\theta_i) \end{Bmatrix}}$	$A =$ $\{\langle\theta, 0.30, 0.$, $B =$ $\{\langle\theta, 0.40, 0.$, $C =$ $\{\langle\theta, 0.40, 0.$, & $D =$ $\{\langle\theta, 0.50, 0.$, then $\text{Ş}_t^p(A,B) =$ $\text{Ş}_t^p(A,B) =$ 0.967 , if $p = 1$
Chen et at. [45]	$\text{Ş}_{CCL}(A,B) = 1 - \begin{cases} \frac{\lvert 2((\mu_A(\theta_i) - \mu_B(\theta_i)) - (\nu_A(\theta_i) - \nu_B(\theta_i))\rvert}{3} \times \left(1 - \frac{\pi_A(\theta_i)}{}\right. \\ \frac{\lvert 2((\nu_A(\theta_i) - \nu_B(\theta_i)) - (\mu_A(\theta_i) - \mu_B(\theta_i))\rvert}{3} \times \left(\frac{\pi_A(\theta_i)}{}\right. \end{cases}$	Same as that of $\text{Ş}_t^p(A,B)$
Ye [62]	$\text{Ş}_{YC}(A,B) = \frac{1}{n}\sum_{i=1}^{n} Cos\left[\frac{\pi}{4}(\lvert\mu_A(\theta_i) - \mu_B(\theta_i)\rvert + \lvert\nu_A(\theta_i) - \nu_B(\theta_i)\rvert + \lvert\pi_A(\theta_i) - \pi_B(\theta)\rvert)\right]$	$A =$ $\{\langle\theta, 0, 0\rangle\}$, $B =$ $\{\langle\theta, 0, 1\rangle\}$, $C =$ $\{\langle\theta, 1, 0\rangle\}$, & $D =$ $\{\langle\theta, 0, 0.50\rangle$, then $\text{Ş}_{YC}(A,B) =$ $\text{Ş}_{YC}(C,D) =$ 0

4. Proposed Logarithmic Similarity Measure for Intuitionistic fuzzy sets

Let A = $\{\langle\theta, \mu_A(\theta), \nu_A(\theta)\rangle\}$ and B = $\{\langle\theta, \mu_A(\theta), \nu_A(\theta)\rangle\}$ are two IFSs on some finite Universe of discourse X = $\{\theta_1, \theta_2, \ldots, \theta_n\}$. In this scenario, we proposed an intuitionistic fuzzy similarity measure between $A, B \in IFSs(X)$. The developed study comprised the three functional components of radical, modulus and logarithmic function. This present similarity measure focuses on the level of uncertainty. As a result, the suggested Intuitionistic fuzzy similarity measure between two IFSs, A and B, is as follows:

$$Ş_{PV}(A,B) = 1 - log_2\left[1 + \frac{1}{2n}\sum_{i=1}^{n}\left(\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right)\right] \tag{5}$$

where membership & non-membership functions and hesitancy index for Intuitionistic fuzzy sets A and B are $\mu_A(\theta)$, $\mu_B(\theta)$ and $\nu_A(\theta)$, $\nu_B(\theta)$,respectively.

Theorem 4.1 Shows that $Ş_{PV}(A,B)$ is an Intuitionistic fuzzy logarithmic similarity measure between A and B.

Proof: Some postulates, from (SP1) to (SP4), must be verified.

Consider the following expressions for IFSs A and B.

$$A = \{\langle\theta_i, \mu_A(\theta_i), \nu_A(\theta_i)\rangle\} \text{ and } B = \{\langle\theta_i, \mu_B(\theta_i), \nu_B(\theta_i)\rangle\}$$

(SP1): Given $0 \le \mu_A(\theta_i), \nu_A(\theta_i), \mu_B(\theta_i), \mu_B(\theta_i) \le 1$

Therefore, $0 \le |\mu_A(\theta_i) - \mu_B(\theta_i)| \le 1$and $0 \le |\nu_A(\theta_i) - \nu_B(\theta_i)| \le 1$,

$\Rightarrow 0 \le \frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2} \le \frac{1}{2}$ and $0 \le \frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2} \le \frac{1}{2}$

$\Rightarrow 0 \le \sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} \le \frac{1}{\sqrt{2}}$ and $0 \le \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}} \le \frac{1}{\sqrt{2}}$

$$\Rightarrow 0 \le \sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}} \le \sqrt{2}$$

$$\Rightarrow 0 \le \frac{1}{2}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right] \le \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1 \le 1 + \frac{1}{2}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right] \le 1 + \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le log_2\left(1 + \frac{1}{2}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \le log_2\left(1 + \frac{1}{\sqrt{2}}\right)$$

It is also possible that,

$$0 \le log_2\left(1 + \frac{1}{2}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \le 1$$

$$\Rightarrow -1 \le -log_2\left(1 + \frac{1}{2}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \le 0$$

$$\Rightarrow 0 \leq 1 - log_2\left(1 + \frac{1}{2}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \leq n$$

$$\Rightarrow 0 \leq 1 - log_2\left(1 + \frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \leq 1$$

Thus, $0 \leq \S_{PV}(A,B) \leq 1$.

(SP2): Let A and B be two IFSs, if $\S_{PV}(A,B) = 1 \Leftrightarrow$ A = B

First, assume that A = B, then $\mu_A(\zeta_i) = \mu_B(\zeta_i)$ and $\nu_A(\zeta_i) = \nu_B(\zeta_i)$, for each i

Thus, $\S_{PV}(A,B) = 1$.

Conversely, suppose that, $\S_{PV}(A,B) = 1$

We have to prove that A = B

Let, $\S_{PV}(A,B) = 1$

$$\Rightarrow 1 - log_2\left(1 + \frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) = 1, \text{ for each } i$$

$$\Rightarrow log_2\left(1 + \frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) = 0$$

$$\Rightarrow log_2\left(1 + \frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) = log_2(1)$$

$$\Rightarrow \sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}} = 0, \text{ for all } i$$

It is possible when, $\mu_A(\theta_i) = \mu_B(\theta_i)$ and $\nu_A(\theta_i) = \nu_B(\theta_i)$, for all i

Thus, A = B.

Hence, $\S_{PV}(A,B) = 1 \Leftrightarrow$ A = B.

(SP3): Given that the above expression is commutative, so it holds

Thus, $\S_{PV}(A,B) = \S_{PV}(B,A)$.

(SP4): Consider three IFSs A, B and C are such that $A \sqsubseteq B \sqsubseteq C$,then, for each $\theta_i \epsilon X, 0 \leq \mu_A(\theta_i) \leq \mu_B(\theta_i) \leq \mu_C(\theta_i) \leq 1$ and $0 \leq \nu_C(\theta_i) \leq \nu_B(\theta_i) \leq \nu_A(\theta_i) \leq 1$.

We have to prove that,

$\S_{PV}(A,B) \geq \S_{PV}(A,C) \& \S_{PV}(A,B) \geq \S_{PV}(B,C)$.

Therefore, $\sqrt{\frac{|\mu_A(\theta_i) - \mu_C(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_C(\theta_i)|}{2}} \geq \sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}$ for all i

$$\Rightarrow log_2\left(1+\frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_C(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_C(\theta_i)|}{2}}\right]\right)\geq log_2\left(1+\frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right]\right)$$

$$\Rightarrow 1-log_2\left(1+\frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_C(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_C(\theta_i)|}{2}}\right]\right)\leq 1-log_2\left(1\frac{1}{2n}\sum_{i=1}^{n}\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right]\right)$$

Thus, $\S_{PV}(A,B)\geq \S_{PV}(A,C)$.

Similarly, $\S_{PV}(B,C)\geq \S_{PV}(A,C)$.

Therefore, all axioms of similarity from (SP1) to (SP4) holds.

Hence, the proposed Intuitionistic fuzzy similarity measure satisfies all axioms of similarity. Thus, it is an authenticated study of similarity for Intuitionistic fuzzy sets.

Let the weight function w_i for each θ_i, such that $\theta_i \in X, i$ = 1, 2, …n &$\sum_{i=1}^{n} w_i = 1$, we will define a novel intuitionistic fuzzy weight similarity measure given by

$$\S_{WPV}(A,B) = 1 - log_2\left[1+\frac{1}{2}\sum_{i=1}^{n} w_i\left(\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right)\right] \tag{6}$$

where w_i be the weight of i^{th} element of an Intuitionistic fuzzy set, A and $0\leq\theta_i\leq 1, 0\leq w_i\leq 1$, $\sum_{i=1}^{n} w_i = 1$.

Theorem 4.2 Shows that $\S_{WPV}(A,B)$, is an Intuitionistic fuzzy weight similarity measure between IFSs A and B.

Proof. It is sufficient to prove that (SP1) to (SP4) must hold in terms of weight.

(SP1):

$$0 \leq \sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\leq 1$$

$$\Rightarrow 0 \leq w_i\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right]\leq w_i$$

$$\Rightarrow 0 \leq \sum_{i=1}^{n} w_i\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right]\leq \sum_{i=1}^{n} w_i$$

$$\Rightarrow 0 \leq \frac{1}{2}\sum_{i=1}^{n} w_i\left[\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\right]\leq \frac{1}{2}$$

Also,

$$0 \leq \frac{1}{2}\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right] \leq 1$$

$$\Rightarrow 1 \leq 1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right] \leq 2$$

$$\Rightarrow 0 \leq log_2\left(1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \leq 1$$

$$\Rightarrow -1 \leq -log_2\left(1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \leq 1$$

$$\Rightarrow 0 \leq 1 - log_2\left(1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) \leq 1$$

Thus, $0 \leq Ş_{WPV}(A, B) \leq 1$.

(SP2): let, $Ş_{PV}(A, B) = 1 \Leftrightarrow A = B$

First, consider two IFSs, A = B,

$\Rightarrow \mu_A(\theta_i) = \mu_B(\theta_i)$ and $\nu_A(\theta_i) = \nu_B(\theta_i)$, for all i

Therefore, $Ş_{WPV}(A, B) = 1$.

Conversely, let $Ş_{WPV}(A, B) = 1$

$$\Rightarrow 1 - log_2\left(1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) = 1$$

$$\Rightarrow log_2\left(1 + \tfrac{1}{2}\textstyle\sum_{i=1}^{n} w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right]\right) = 0$$

$$\Rightarrow w_i \left[\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right] = 0$$

When $\mu_A(\theta_i) = \mu_B(\theta_i)$ and $\nu_A(\theta_i) = \nu_B(\theta_i)$, for all i., then it holds

Therefore, A = B.

(SP3): Clearly, the given expression is commutative,

Thus, $Ş_{WPV}(A, B) = Ş_{WPV}(B, A)$.

(SP4): Consider three IFSs A, B and C are such that A ⊑ B ⊑ C, then, for each $\theta_i \epsilon X$, $0 \leq \mu_A(\theta_i) \leq \mu_B(\theta_i) \leq \mu_C(\theta_i) \leq 1$ and $0 \leq \nu_C(\theta_i) \leq \nu_B(\theta_i) \leq \nu_A(\theta_i) \leq 1$.

Then,

$$w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_C(\theta_i)|}{2}}\\+\sqrt{\frac{|\nu_A(\theta_i)-\nu_C(\theta_i)|}{2}}\end{bmatrix}\geq w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}\\+\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\end{bmatrix}, \text{ for all } i$$

$$\Rightarrow log_2\left(1+\frac{1}{2}\sum_{i=1}^{n}w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_C(\theta_i)|}{2}}\\+\\\sqrt{\frac{|\nu_A(\theta_i)-\nu_C(\theta_i)|}{2}}\end{bmatrix}\right)\geq log_2\left(1+\frac{1}{2}\sum_{i=1}^{n}w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}\\+\\\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\end{bmatrix}\right)$$

$$\Rightarrow 1-log_2\left(1+\frac{1}{2}\sum_{i=1}^{n}w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_C(\theta_i)|}{2}}\\+\\\sqrt{\frac{|\nu_A(\theta_i)-\nu_C(\theta_i)|}{2}}\end{bmatrix}\right)\geq 1-log_2\left(1+\frac{1}{2}\sum_{i=1}^{n}w_i\begin{bmatrix}\sqrt{\frac{|\mu_A(\theta_i)-\mu_B(\theta_i)|}{2}}\\+\\\sqrt{\frac{|\nu_A(\theta_i)-\nu_B(\theta_i)|}{2}}\end{bmatrix}\right)$$

Thus, $\S_{WPV}(A,B)\geq \S_{WPV}(A,C)$.

Similarly, $\S_{WPV}(B,C)\geq \S_{WPV}(A,C)$.

Therefore, $\S_{WPV}(A,B)\geq \S_{WPV}(A,C)$ and $\S_{WPV}(B,C)\geq \S_{WPV}(A,C)$.

Hence, the proposed similarity measure satisfies all IFS weight similarity measure axioms. Therefore, the obtained measure is a reasonable weighted similarity measure for IFSs.

5. Numerical Comparison

This section provides an example of comparing the proposed and existing similarity measures. The analysis of the current study is based on the given illustrative cases. All counterintuitive cases will be employed on the proposed measure and other existing similarity measures, showing how the proposed similarity measure is superior to others. Therefore, all cases related to the present study will be categorized into tabulated forms with individual ones. In Table 3,we provide the comparative analysis between existing and proposed similarity measures by some counterintuitive cases and the various existing measures having transverse nature in the given cases. The present measures$\S_C$, $\S_{DC}$ and $\S_Y$ do not meet the significant requirements of the similarity measure; hence, these are not appropriate for discussing the level of similarity, while the similarity measure $\S_Y$ is not suitable for cases 3 & 4. Moreover, in two or more instances, the similarity measures are identically similar and provide the same degree of similarity. The proposed IFS similarity measure can be inferred to satisfy all axiomatic properties of similarity and applies to pattern recognition and medical diagnosis problems. In this section, the pattern recognition application is the significant component of the proposed study in which we elaborate on the concept of actual pattern with the help of unknown patterns. The study of similarity is the most prominent tool for describing uncertain information regarding the degree of similarity. Therefore, we analyze the concept of existing similarity measures by their comparison with the proposed similarity measure. The comparative analysis justifies the suitable degree of similarity in all possible cases from 1 to 6.

Table 3. Discussing some counterintuitive cases on proposed and existing similarity measures

Cases → Sources ↓	Cases 1	Cases 2	Cases 3	Cases 4	Cases 5	Cases 6
	A $=\langle 0.3, 0.3\rangle$ B $= \langle 0.4, 0.4\rangle$	A $= \langle 0.3, 0.4\rangle$ B $= \langle 0.4, 0.3\rangle$	A $= \langle 0, 1\rangle$ B $= \langle 0, 0\rangle$	A $= \langle 0.5, 0.5\rangle$ B $= \langle 0, 0\rangle$	A $= \langle 0.4, 0.2\rangle$ B $= \langle 0.5, 0.3\rangle$	A $= \langle 0.4, 0.2\rangle$ B$= \langle 0.5, 0.2\rangle$
$\mathcal{S}_C$[56]	1	0.90	0.50	1	1	0.95
$\mathcal{S}_{HK}$[57]	0.90	0.90	0.50	0.50	0.90	0.95
$\mathcal{S}_{LX}$[58]	0.95	0.90	0.50	0.75	0.95	0.95
$\mathcal{S}_O$[59]	0.90	0.90	0.30	0.50	0.90	0.93
$\mathcal{S}_{DC}$[30]	1	0.90	0.50	1	1	0.95
$\mathcal{S}_M$[32]	0.90	0.90	0.50	0.50	0.90	0.95
$\mathcal{S}_e^p$[31]	0.90	0.90	0.50	0.50	0.90	0.95
$\mathcal{S}_s^p$[31]	0.95	0.90	0.50	0.75	0.95	0.95
$\mathcal{S}_h^p$[31]	0.93	0.93	0.50	0.67	0.93	0.95
$\mathcal{S}_{HY}^a$[34]	0.90	0.90	0	0.50	0.90	0.90
$\mathcal{S}_{HY}^b$[34]	0.85	0.85	0	0.38	0.85	0.85
$\mathcal{S}_{HY}^c$[34]	0.82	0.82	0	0.33	0.82	0.82
$\mathcal{S}_L^p$[60]	0.83	0.90	0	0.13	0.83	0.90
$\mathcal{S}_Y$[40]	1	0.96	NA	NA	0.99	0.99
$\mathcal{S}_{SY}$[61]	0.91	0.97	0	0	0.92	0.97
$\mathcal{S}_t^p$[43]	0.97	0.90	0.50	0.83	0.97	0.95
$\mathcal{S}_{CCL}$[45]	0.97	0.90	0.50	0.83	0.97	0.94
$\mathcal{S}_{YC}$[62]	0.95	0.99	0	0	0.40	0.99
Proposed similarity measure, $\mathcal{S}_{PV}(A, B)$	0.93	0.93	0.41	0.41	0.93	0.97

Note: "**NA**" stands for degree not defined due to "the division by zero problem".

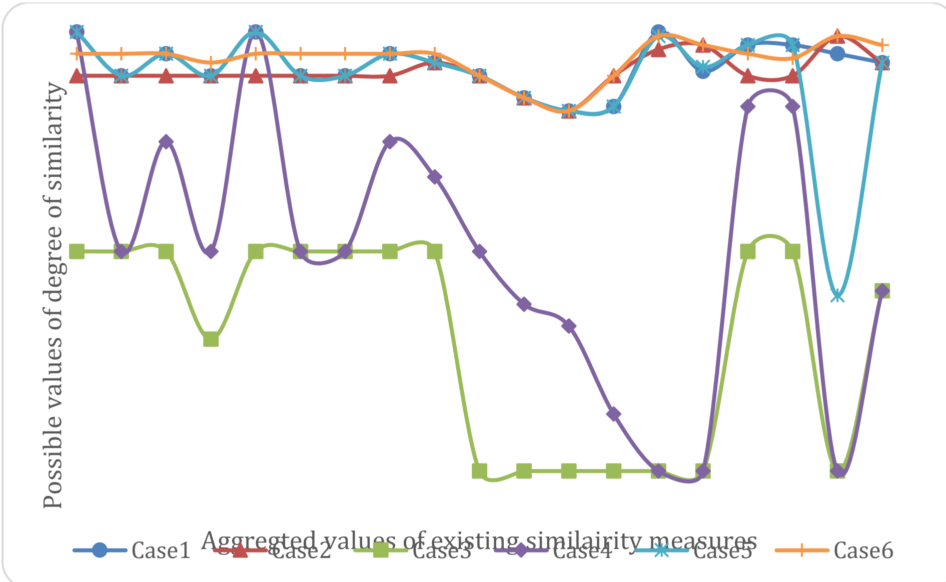

Figure 1. The graph between all possible aggregated values of existing similarity measures and the proposed similarity measure

Depict the graph between all possible aggregated values of existing similarity measures. In the present figure, the dark symbol represents the degree of similarity corresponding to each existing and proposed similarity measure for each case.

6. Applications of Proposed Intuitionistic Fuzzy Similarity Measure

In this scenario, we will illustrate some applications of the proposed study with suitable examples. The study of similarity measures can include decision-making, pattern recognition, medical diagnosis, and cluster analysis. However, we will mention pattern analysis and medical diagnosis problems mainly from the above.

6.1 Medical Diagnosis

Various difficulties arose in the medical field, but medical diagnosis is the biggest issue for an expert. It is a fascinating study of IFSs to which [40], [43], [63], [64] are extensively told. They provided some numerical examples related to this study by using different approaches and getting the solution to medical diagnosis and pattern recognition problems. The medical diagnosis problem is a significant challenge in the medical field, and without a diagnosis, we can never find the diseases and cannot provide proper treatment. This study will define a different approach to solving medical diagnosis problems.

(1) Consider there are four patients, Jack, Harry, Thomas, & James, denoted as P = {Jack, Harry, Thomas, James}. The patients' symptoms are S = {Temperature, Cough, Throat pain, Headache, Body pain}. The diseases are denoted as a set D = {Viral fever, Tuberculosis, Typhoid, Throat disease}. IFSs relations from patient to symptoms and symptoms to diseases are indicated by P → S in Table 4 and S → D in Table 5, respectively. The related data to the study are represented in tabulated form with IFV, the ordered paired as membership and non-membership values. All patient diseases will be identified with their corresponding similarity values. To make a proper diagnosis for every patient, we calculate the similarity values in the context of each patient and each diagnosis. The maximum similarity value will judge the appropriate diagnosis. The similarity values of each patient and diagnosis are represented in Table 6. According to values of similarity in Table 6, we concluded that Jack and James suffer from typhoid, Harry suffers from tuberculosis, and Thomas suffers from throat disease. We analyzed that in Table 6, no patient had been

suffering from viral fever. Therefore, the proposed IFS similarity measure has obtained a valid and suitable measure.

Table 4. Patients versus Symptoms

Symptoms / Patient	Jack	Harry	Thomas	James
Temperature	$\langle 0.65, 0.15\rangle$	$\langle 0.35, 0.45\rangle$	$\langle 0.15, 0.65\rangle$	$\langle 0.45, 0.40\rangle$
Cough	$\langle 0.35, 0.45\rangle$	$\langle 0.65, 0.2\rangle$	$\langle 0.25, 0.3\rangle$	$\langle 0.35, 0.40\rangle$
Throat pain	$\langle 0.15, 0.70\rangle$	$\langle 0.55, 0.30\rangle$	$\langle 0.75, 0.05\rangle$	$\langle 0.15, 0.65\rangle$
Headache	$\langle 0.55, 0.35\rangle$	$\langle 0.45, 0.50\rangle$	$\langle 0.25, 0.65\rangle$	$\langle 0.55, 0.35\rangle$
Body Pain	$\langle 0.25, 0.5\rangle$	$\langle 0.75, 0.15\rangle$	$\langle 0.35, 0.55\rangle$	$\langle 0.45, 0.50\rangle$

Table 5. Diseases versus Symptoms

Symptoms / Diseases	Viral fever	Tuberculosis	Typhoid	Throat disease
Temperature	$\langle 0.80, 0.10\rangle$	$\langle 0.20, 0.70\rangle$	$\langle 0.50, 0.30\rangle$	$\langle 0.10, 0.70\rangle$
Cough	$\langle 0.20, 0.70\rangle$	$\langle 0.90, 0\rangle$	$\langle 0.30, 0.50\rangle$	$\langle 0.30, 0.60\rangle$
Throat pain	$\langle 0.30, 0.50\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.20, 0.70\rangle$	$\langle 0.80, 0.10\rangle$
Headache	$\langle 0.50, 0.30\rangle$	$\langle 0.60, 0.30\rangle$	$\langle 0.20, 0.60\rangle$	$\langle 0.10, 0.80\rangle$
Body Pain	$\langle 0.50, 0.40\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.40, 0.40\rangle$	$\langle 0.10, 0.80\rangle$

Table 6. Patients versus Diseases

Patient / Disease	Jack	Harry	Thomas	James
Viral fever	0.6731	0. 5439	0.5179	0.6537
Tuberculosis	0.4941	**0.6551**	0.5800	0.5386
Typhoid	**0.7023**	0. 5598	0.6045	**0.7092**
Throat disease	0. 4932	0. 4953	**0.6859**	0.5014

Figure 2. The graph of proposed similarity measure for medical diagnosis problems along with classified values of patients and diseases

The graph of the developed study is shown with the values of the patients and their respective diseases. Dotted values represent the prescribed diseases for the patients based on their symptoms. As a result, the proposed similarity measure is a suitable and well-prescribed approach.

The proposed measure is obtained as an operational-based technique, and the formation involves summation, subtraction, and modulus in terms of logarithmic function. Its implementation depends on some non-parametric measure with membership and non-membership functions, coinciding with [48].

6.2 Pattern Recognition

(2) Let three known patterns be represented in terms of Intuitionistic fuzzy sets $\breve{P}_1$, $\breve{P_2}$ and $\breve{P_3}$. It is possible to write each one as IFS in $X = \{\theta_1, \theta_2, \theta_3, \theta_4\}$.

`$\breve{P}_1 = \{\langle\theta_1, 0.5, 0.3\rangle, \langle\theta_2, 0.7, 0\rangle, \langle\theta_3, 0.4, 0.5\rangle, \langle\theta_4, 0.7, 0.3\rangle\}$

$$\breve{P_2} = \{\langle\theta_1, 0.5, 0.2\rangle, \langle\theta_2, 0.6, 0.1\rangle, \langle\theta_3, 0.2, 0.7\rangle, \langle\theta_4, 0.7, 0.3\rangle\}$$

$\breve{P_3} = \{\langle\theta_1, 0.5, 0.4\rangle, \langle\theta_2, 0.7, 0.1\rangle, \langle\theta_3, 0.4, 0.6\rangle, \langle\theta_4, 0.7, 0.2\rangle\}$.

On the Universe of discourse $X = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, we want to classify the unknown pattern from the following known patterns$\breve{P}_1$, $\breve{P_2}$ and $\breve{P_3}$which is denoted by an IFS $\breve{B}$.

where, $\breve{B} = \{\langle\theta_1, 0.4, 0.3\rangle, \langle\theta_2, 0.7, 0.1\rangle, \langle\theta_3, 0.3, 0.6\rangle, \langle\theta_4, 0.7, 0.3\rangle\}$.

Table 7 compares the classified findings of the proposed measure with some existing similarity measures. Additionally, different similarity measures that correlate to similarity values are assigned in Table 7 and explain their behaviour. The degree of similarity of the proposed similarity measure concerning pattern$\breve{B}$is obtained as, $\text{Ş}_{PV}(\breve{P}_1, \breve{B}) = 0.8184$, $\text{Ş}_{PV}(\breve{P_2}, \breve{B}) = 0.7902$, $\text{Ş}_{PV}(\breve{P_3}, \breve{B}) = 0.8764$.

The goal of the proposed study is to determine the correct pattern Q among one of the classifications $\breve{P}_1$, $\breve{P_2}$, and $\breve{P_3}$. Pattern recognition process assigning $\breve{B}$ to $\breve{P}_\iota$ which performed as the maximum value of degree similarity measures between IFSs is given by

$$\text{Ş}_{IFS,i} = arg \max_{1 \le i \le 3} \text{Ş}\,(\breve{P}_\iota, \breve{B})$$

Table 7. The comparative analysis of proposed and existing similarity measures, along with their obtained degree of similarity

Similarity Measures	$\text{Ş}\,(\breve{P}_1, \breve{B})$	$\text{Ş}\,(\breve{P_2}, \breve{B})$	$\text{Ş}\,(\breve{P_3}, \breve{B})$	Corresponding Results
Ş_C[56]	0.78	0.78	0.85	$\breve{P_3}$
Ş_{HK}[57]	0.78	0.78	0.85	$\breve{P_3}$
Ş_{LX}[58]	0.78	0.78	0.85	$\breve{P_3}$
Ş_O[59]	0.73	0.76	0.84	$\breve{P_3}$
Ş_{DC}[30]	0.78	0.78	0.85	$\breve{P_3}$
Ş_M[32]	0.78	0.78	0.85	$\breve{P_3}$
Ş_e^p[31]	0.78	0.78	0.85	$\breve{P_3}$
Ş_s^p[31]	0.78	0.78	0.85	$\breve{P_3}$
Ş_h^p[31]	0.84	0.84	0.89	$\breve{P_3}$
Ş_{HY}^a[34]	0.73	0.73	0.83	$\breve{P_3}$
Ş_{HY}^b[34]	0.63	0.63	0.76	$\breve{P_3}$

$\mathcal{S}^{c}_{HY}$[34]	0.58	0.58	0.71	$\breve{P}_3$
$\mathcal{S}^{p}_{L}$[60]	0.72	0.74	0.84	$\breve{P}_3$
$\mathcal{S}_{Y}$[40]	0.93	0.95	0.97	$\breve{P}_3$
$\mathcal{S}_{SY}$[61]	0.24	0.25	0.24	$\breve{P}_2$
$\mathcal{S}^{p}_{t}$[43]	0.78	0.78	0.85	$\breve{P}_3$
$\mathcal{S}_{CCL}$[45]	0.77	0.77	0.84	$\breve{P}_3$
$\mathcal{S}_{YC}$[62]	0.22	0.22	0.20	Not suitable
Proposed measure, $\mathcal{S}_{PV}$	0.82	0.79	0.88	$\breve{P}_3$

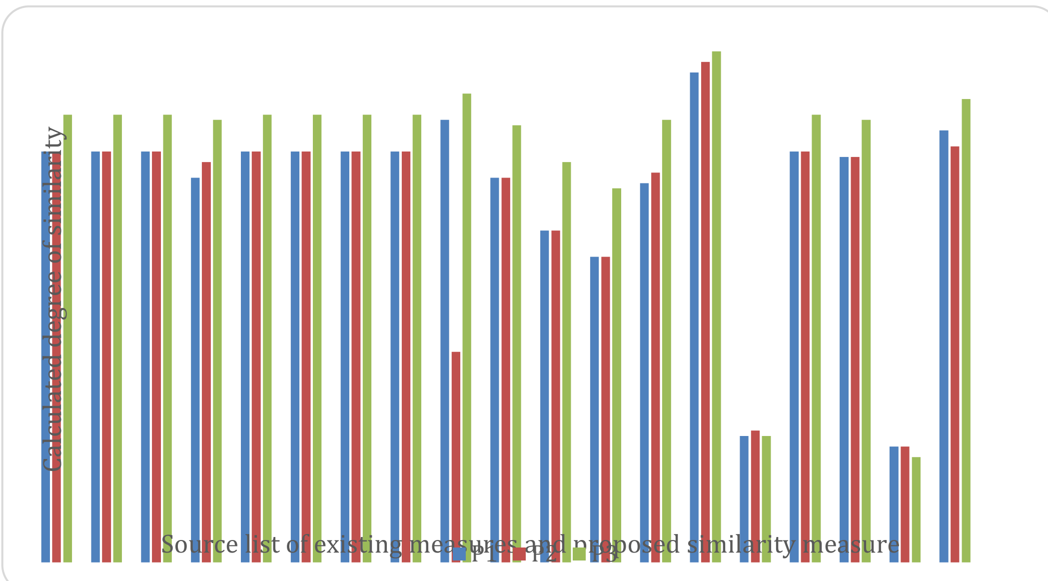

Figure 3.The classical representation of existing measures and proposed similarity measure

In this section, we introduced the degree of similarity of each existing measure and proposed similarity measures in tabulated form along with their graphical representation. The degree of similarity between two or more Intuitionistic fuzzy sets has been depicted in the present graph. Additionally, distinct similarity measures provided distinct similarity between two Intuitionistic fuzzy sets.

Except $\mathcal{S}_{SY}$, all other existing similarity measures deduced the unknown pattern represented by the Intuitionistic fuzzy set $\breve{B}$ into the pattern $\breve{P}_3$. It is impossible to identify the actual value of a specific pattern using the similarity measure $\mathcal{S}_{YC}$. As a result, the proposed measure performs the best compared to other similarity measures.

(3) Let it suppose that there exist three known patterns $\breve{P}_1$, $\breve{P}_2$ and $\breve{P}_3$ represented as intuitionistic fuzzy sets $\breve{P}_1$, $\breve{P}_2$ and $\breve{P}_3$. Each one can be expressed as IFS in X = $\{\theta_1, \theta_2, \theta_3\}$

$\breve{P}_1 = \{\langle\theta_1, 0.50, 0.45\rangle, \langle\theta_2, 0.45, 0.50\rangle, \langle\theta_3, 0.50, 0.50\rangle\}$,

$\breve{P}_2 = \{\langle\theta_1, 0.425, 0.475\rangle, \langle\theta_2, 0.45, 0.55\rangle, \langle\theta_3, 0.45, 0.475\rangle\}$,

$\breve{P}_3 = \{\langle\theta_1, 0.375, 0.525\rangle, \langle\theta_2, 0.40, 0.60\rangle, \langle\theta_3, 0.40, 0.525\rangle\}$.

On the Universe of discourse X = $\{\theta_1, \theta_2, \theta_3, \theta_4\}$, we want to classify the unknown pattern from the following known patterns $\breve{P}_1$, $\breve{P}_2$ and $\breve{P}_3$ which is denoted by an IFS $\breve{B}$.

where, $\breve{B} = \{\langle\theta_1, 0.40, 0.50\rangle, \langle\theta_2, 0.425, 0.075\rangle, \langle\theta_3, 0.425, 0.5\rangle\}$.

Table 8 compares the classified findings of the proposed measure with some existing similarity measures. Additionally, different similarity measures that correlate to similarity values are assigned in Table 8 and explain their behaviour. The degree of similarity of the proposed similarity measure concerning the pattern $\breve{B}$ is obtained as, $Ş_{PV}(\breve{P}_1, \breve{B})$ = 0.8184, $Ş_{PV}(\breve{P}_2, \breve{B})$ = 0.7902, $Ş_{PV}(\breve{P}_3, \breve{B})$ = 0.8764.

Our goal is to categorize the pattern Q into one of the classifications $\breve{P}_1$, $\breve{P}_2$, and $\breve{P}_3$. According to the principle of pattern recognition, the process of assigning $\breve{B}$ to $\breve{P}_\iota$ is presented by the maximum value of similarity measures between IFSs is provided as

$$Ş_{IFS,i} = arg \max_{1 \le i \le 3} Ş\,(\breve{P}_\iota, \breve{B})$$

Table 8. The comparative analysis of proposed and existing similarity measures, along with the corresponding obtained results

Similarity Measures	$Ş_{PV}\ (\breve{P}_1, \breve{B})$	$Ş_{PV}\ (\breve{P}_2, \breve{B})$	$Ş_{PV}\ (\breve{P}_3, \breve{B})$	Corresponding Results
$Ş_{C}$[56]	0.95	0.94	0.97	$\breve{P}_3$
$Ş_{HK}$[57]	0.95	0.94	0.95	Not suitable
$Ş_{LX}$[58]	0.95	0.94	0.96	$\breve{P}_3$
$Ş_{O}$[59]	0.93	0.92	0.93	Not suitable
$Ş_{DC}$[30]	0.95	0.94	0.97	$\breve{P}_3$
$Ş_{M}$[32]	0.95	0.94	0.95	Not suitable
$Ş_{e}^{p}$[31]	0.95	0.94	0.95	Not suitable
$Ş_{s}^{p}$[31]	0.95	0.94	0.96	$\breve{P}_3$
$Ş_{h}^{p}$[31]	0.96	0.95	0.96	Not suitable
$Ş_{HY}^{a}$[34]	0.93	0.93	0.93	Not suitable
$Ş_{HY}^{b}$[34]	0.88	0.88	0.88	Not suitable
$Ş_{HY}^{c}$[34]	0.86	0.86	0.86	Not suitable
$Ş_{L}^{p}$[60]	0.91	0.91	0.89	Not suitable
$Ş_{Y}$[40]	0.99	0.99	1	$\breve{P}_3$
$Ş_{SY}$[61]	0.27	0.25	0.27	Not suitable
$Ş_{t}^{p}$[43]	0.95	0.94	0.97	$\breve{P}_3$

$\S_{CCL}$[45]	0.95	0.93	0.97	$\widetilde{P_3}$
$\mathcal{S}_{YC}$[62]	0.20	0.22	0.29	$\widetilde{P_3}$
Proposed measure, $\S_{PV}$	0.81	0.83	0.85	$\widetilde{P_3}$

The present similarity measures $\S_{HK}$, $\S_O$, $\S_M$, $\S_e^p$, $\S_h^p$, $\S_{HY}^a$, $\S_{HY}^b$, $\S_{HY}^c$, $\S_L^p$ and $\S_{SY}$ cannot provide the appropriate degree of similarity, so these are unauthorized for the pattern recognition problem. The rest of the degrees of similarity are well classified, while an IFS represents the unrecognized pattern $\breve{B}$ into the pattern $\widetilde{P_3}$. These similarity measures are suitable for solving prescribed pattern recognition problems. As a result, the proposed measure performs the best compared to other similarity measures.

6.3 Application of MADM of Symmetric Divergence Measure for Solving Decision-Making Problem

The uncertainty concept is the most appropriate approach for finding the best decision in the relevant field. This section will classify the MCDM problem using the generalized Intuitionistic fuzzy symmetric logarithmic divergence measure. With the help of the proposed method, we can find the most appropriate decision from the given criteria.

Consider $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \dots, \mathcal{M}_m\}$ be the arrangement of options corresponding to the problem while $\mathbb{C} = \{\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3, \dots, \mathbb{C}_n\}$ be the arrangement of specific criteria under the problem. For multicriteria problems, each strategy has a certain degree of membership and non-membership concerning the cost criteria. Now, define the characteristic set $\mathcal{M}_j$ in the form of a set of criteria under the problem $\mathbb{C}$ be an IFS:

$$\mathcal{M}_j = \{(\mathbb{C}_k, \mu_{jk}, \nu_{jk}) : \mathbb{C}_k \epsilon \mathbb{C}\}, \text{ where } j = 1\ to\ m\ \&\ k = 1\ to\ n.$$

Also, μ_{jk} represent the degree of membership with which the option $\mathcal{M}_j$ satisfies the present criteria $\mathbb{C}_k$ while ν_{jk} represent the degree of non-membership with which the option $\mathcal{M}_j$ do not satisfy the present criterion $\mathbb{C}_k$.

Using the proposed method (7), we will implement the present method by solving the Intuitionistic fuzzy MCDM problem to find the best solution.

Step 1. To make a decision-making matrix $D_{m\times n}$, on the behalf of preference of the decision-maker for each alternative $\mathcal{M}_j (j = 1\ to\ m)$ concerning cost set $\mathbb{C}_k (k = 1\ to\ n)$ are as followed

$$D_{mxn}(c_{ij}) = \begin{bmatrix} (\mu_{11}, \nu_{11})(\mu_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21})(\mu_{22}, \nu_{22}) & \cdots & (\mu_{2n}, \nu_{2n}) \\ \vdots \qquad \vdots & \ddots & \vdots \\ (\mu_{m1}, \nu_{m1})(\mu_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \nu_{mn}) \end{bmatrix}, \text{ where } c_{ij} = (\mu_{jk}, \nu_{jk}), j = 1\ to\ m\ and\ k = 1\ to\ n.$$

Step 2. Determine the optimum solution, $\mathcal{M}^*$ as followed

$$\mathcal{M}^* = \{(\mu_{1^*}, \nu_{1^*}), (\mu_{2^*}, \nu_{2^*}), (\mu_{3^*}, \nu_{3^*}), \dots, (\mu_{n^*}, \nu_{n^*})\}$$

where for each $k = 1\ to\ n$,

$$(\mu_{k^*}, \nu_{k^*}) = \left(\max_j \mu_{jk}, \min_j \nu_{jk}\right)$$

Step 3. Compute $J(\mathcal{M}_j || \mathcal{M}^*)$ are given by

$$\S_{PV}(A,B) = 1 - log_2\left[1 + \frac{1}{2n}\sum_{i=1}^{n}\left(\sqrt{\frac{|\mu_A(\theta_i) - \mu_B(\theta_i)|}{2}} + \sqrt{\frac{|\nu_A(\theta_i) - \nu_B(\theta_i)|}{2}}\right)\right] \quad (7)$$

Step 4. The calculated value will justify the rank of the desirable choice on behalf of the estimated value. So, we have to choose the smallest value of $\S_{PV}(\mathcal{M}_j||\mathcal{M}^*)$ for selecting the alternative $\mathcal{M}_j$.

To classify multicriteria decision-making problems using the proposed method, we consider a numerical example to select the best technical institute in Haryana for engineering purposes.

7. Numerical Example: In the present time, it is a difficult task to select the best studying institute, so we take the help of decision-making techniques to choose the best institute. All decision-making techniques have a bit of uncertainty; therefore, we try to improve it and make the right decision. We define various decision-making methods to measure these uncertainties and get the best one. Suppose a problem regarding Intuitionistic fuzzy multicriteria for selecting the best technical institute. We will justify the problem by giving a numerical example regarding the preference for admission to a technical institute. There are five technical institutes in Haryana: $\mathcal{M}_1$= GJU Hisar, $\mathcal{M}_2$= NIT Kurukshetra, $\mathcal{M}_3$= DCRUST Murthal, $\mathcal{M}_4$= RPS Mahendergarh, $\mathcal{M}_5$= YMCA Faridabad.

The following institutes are the most valuable institutes for engineering purposes. All the institutes are rivals to each other for technical education. Based on the following criteria, the students will choose the best technical institute for engineering purposes: $\mathbb{C}_1$= Ranking, $\mathbb{C}_2$= Placement, $\mathbb{C}_3$= Faculty, $\mathbb{C}_4$= Fee. Based on five present alternatives $\mathcal{M}_j(j = 1,2,3,4,5)$, the decision-maker will characterize five characteristic IFSs have formed corresponding to the following:

$$\mathcal{M}_1 = \{(\mathbb{C}_1, 0.5,0.4), (\mathbb{C}_2, 0.6,0.3), (\mathbb{C}_3, 0.3,0.6), (\mathbb{C}_4, 0.2,0.7)\}$$

$$\mathcal{M}_2 = \{(\mathbb{C}_1, 0.7,0.3), (\mathbb{C}_2, 0.7,0.2), (\mathbb{C}_3, 0.7,0.2), (\mathbb{C}_4, 0.4,0.5)\}$$

$$\mathcal{M}_3 = \{(\mathbb{C}_1, 0.6,0.4), (\mathbb{C}_2, 0.5,0.4), (\mathbb{C}_3, 0.5,0.3), (\mathbb{C}_4, 0.6,0.3)\}$$

$$\mathcal{M}_4 = \{(\mathbb{C}_1, 0.8,0.1), (\mathbb{C}_2, 0.6,0.3), (\mathbb{C}_3, 0.3,0.4), (\mathbb{C}_4, 0.2,0.6)\}$$

$$\mathcal{M}_5 = \{(\mathbb{C}_1, 0.6,0.2), (\mathbb{C}_2, 0.4,0.3), (\mathbb{C}_3, 0.7,0.1), (\mathbb{C}_4, 0.5,0.3)\}$$

Step1. The decision matrix preference of the decision maker for each alternative $\mathcal{M}_j$ follow as:

$$D_{m\times n}(c_{ij}) = \begin{array}{c c} & \begin{array}{cccc} \mathbb{C}_1 & \mathbb{C}_2 & \mathbb{C}_3 & \mathbb{C}_4 \end{array} \\ \begin{array}{c} \mathcal{M}_1 \\ \mathcal{M}_2 \\ \mathcal{M}_3 \\ \mathcal{M}_4 \\ \mathcal{M}_5 \end{array} & \begin{bmatrix} (0.5,0.4) & (0.6,0.3) & (0.3,0.6) & (0.2,0.7) \\ (0.7,0.3) & (0.7,0.2) & (0.7,0.2) & (0.4,0.5) \\ (0.6,0.4) & (0.5,0.4) & (0.5,0.3) & (0.6,0.3) \\ (0.8,0.1) & (0.6,0.3) & (0.3,0.4) & (0.2,0.6) \\ (0.6,0.2) & (0.4,0.3) & (0.7,0.1) & (0.5,0.3) \end{bmatrix} \end{array}.$$

Step 2. We obtain the optimum solution, $\mathcal{M}^*$

$$\mathcal{M}^* = \{(\mathbb{C}_1, 0.8,0.1), (\mathbb{C}_2, 0.7,0.2), (\mathbb{C}_3, 0.7,0.1), (\mathbb{C}_4, 0.6,0.3)\}$$

Step 3. Now, calculate the value of the proposed logarithmic similarity measure, $\S_{PV}(\mathcal{M}_j||\mathcal{M}^*)$ w.r.t to each corresponding alternative $\mathcal{M}_j$.

Table 9. Calculated values of $\S_{PV}(\mathcal{M}_j||\mathcal{M}^*)$ corresponding to each $\mathcal{M}_j$.

Sources	Calculated Values	Rank of Strategy
$\S_{PV}(\mathcal{M}_1\|\|\mathcal{M}^*)$	0.6145	5th
$\S_{PV}(\mathcal{M}_2\|\|\mathcal{M}^*)$	0.8115	2nd
$\S_{PV}(\mathcal{M}_3\|\|\mathcal{M}^*)$	0.7408	3rd
$\S_{PV}(\mathcal{M}_4\|\|\mathcal{M}^*)$	0.7230	4th
$\S_{PV}(\mathcal{M}_5\|\|\mathcal{M}^*)$	0.08142	1st

Because of the above discussion, Table 9 represents the divergence values with their corresponding ranks of the alternatives. We depict the graph of the proposed similarity measure obtained in Table 9, which shows the degree of similarity and their respective rank of alternatives along with aggregated measure in a dark circle solid line in Fig. 4.

Figure 4. The graph of the proposed logarithmic similarity measure is given in Table 9with their respective rank of alternatives corresponding to the similarity value.

The order of alternatives with their Ranking is as follows:

$$\mathcal{M}_5 \succ \mathcal{M}_2 \succ \mathcal{M}_3 \succ \mathcal{M}_4 \succ \mathcal{M}_1.$$

Step 4. From the above table, we concluded that $\S_{PV}(\mathcal{M}_5||\mathcal{M}^*)$ having the most negligible value among all of them, so it can easily justify that the best choice or alternative is $\mathcal{M}_5$. Therefore, on that behalf, the best technical institute for engineering study is NIT Kurukshetra in Haryana, so the student would like to be admitted to NIT Kurukshetra.

Hence, the proposed logarithmic similarity measure is the most desirable approach to solving multicriteria decision-making problems.

7. Conclusion

In this article, we have developed a novel Intuitionistic fuzzy sets similarity measure based on the conception of logarithmic function. The proposed study includes the logarithmic, radical and modulus differences between membership and non-membership functions. Moreover, we suggested a weight similarity measure corresponding to the proposed similarity measure for some fixed weight and its validity. The authenticity of the proposed measure is presented in brief and satisfies all axioms of similarity by their proof. Additionally, the proposed study yields significant behavioural in pattern recognition and medical diagnosis problems. The validity of the suggested measure is presented in Table 2-9, which includes several numerical examples, and it does not occupy any controversy as compared to the others. The proposed IFS similarity measure and the existing measures are interpreted graphically in Figure 1-4. Finally, we demonstrated the MADM method for enhancing the college selection problem by a numerical illustration and making a graphical representation between them. The proposed similarity measure can extend in future scope as the Pythagorean fuzzy, hesitant fuzzy and picture fuzzy similarity measure. It would be more advantageous for vital applications in weather forecasting, facility location selection, decision-making problems, etc.

Conflict of Interest

The authors declared no conflict of interest

Requesting Data Availability Statement

The data cited in this research that support the study's findings are accessible without restriction in [43], [45], [65] and [48]. The information came from the publicly accessible sources listed above.

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