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Article

More on Maps and Its Application to Soil Fertility Zoning in n-Cylindrical Neutrosophic Nano Topological Spaces

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Abstract: This paper introduces n-Cylindrical neutrosophic nano θ (resp. θS, ϵ and M) continuous maps in n-Cylindrical neutrosophic nano topological spaces, exploring their properties and characterizations alongside the corresponding irresolute maps. As a real-life illustration, we construct an n-Cylindrical neutrosophic nano topological space directly from soil-test data collected under Tamil Nadu's Soil Health Card programme: farm plots are grouped by an indiscernibility relation on pH, nitrogen, and organic-carbon bands, and a decision attribute "Fertile Soil" is used to compute the n-Cylindrical neutrosophic lower approximation, upper approximation, and boundary region, which together generate the induced nano topology. The resulting three-tier zoning – certify, develop, and verify – gives agricultural extension officers an uncertainty-aware framework for soil-fertility classification, and illustrates how the newly introduced M-open set machinery extends naturally to boundary-region decision-making in precision agriculture.
Keywords: n-Cylindrical neutrosophic nano M-closed sets, n-Cylindrical neutrosophic nano M-continuous maps and n-Cylindrical neutrosophic nano M-irresolute maps, Precision agriculture, Soil fertility zoning, Indiscernibility relation, Rough set approximation, Soil Health Card

1. Introduction

Zadeh [39] introduced the concept of fuzzy sets in 1965, generalising classical sets by allowing each element to have a membership degree in $[0,1]$. An important extension of fuzzy sets was the intuitionistic fuzzy set (IFS) introduced by Atanassov [1] in 1983, in which each element carries both a membership degree and a non-membership degree. In 1968, Chang [4] introduced fuzzy topological spaces and studied fundamental notions such as continuity, open sets, and closed sets. Following this, Lowen [14] introduced an alternative formulation of fuzzy topological spaces. Çoker introduced the notion of intuitionistic fuzzy topological spaces together with some of their basic properties [5]. The concept of the Pythagorean fuzzy set, a generalisation of fuzzy sets, was presented by Yager [36, 37, 38]. Pythagorean fuzzy topological spaces were subsequently introduced by Olgun et al. [14] following Chang's approach.

In 2013, Lellis Thivagar [10] introduced a new topology called Nano topology, which extends rough set theory. Nano topological spaces are defined in terms of approximations and the boundary region of a subset of a universe with respect to an equivalence relation. The elements of a Nano topological space are called Nano open sets and their complements are called Nano closed sets. The term "Nano" denotes something very small, and Nano topology thus literally means the study of very small surfaces. The fundamental ideas in Nano topology are approximations and the indiscernibility relation. Furthermore, nano δ-open sets in nano topological spaces were studied in [16]. Recently, Lellis Thivagar et al. [11] introduced a new concept of neutrosophic nano topology. The notions of Z-closed sets in double fuzzy topological spaces were introduced by Shiventhiradevi Sathaananthan et al. [24, 25] in 2020. Also, M-open sets in nano topological spaces were studied by Padma et al. [15], δ-open sets in nano topological spaces were investigated by Pankajam and Kavitha [16], and neutrosophic nano topological spaces were considered in [31].

Spherical fuzzy sets were proposed by Kahraman and Gündöğdu [7], requiring that the squared sum of the membership, non-membership, and hesitancy degrees be at most 1. Spherical fuzzy topological spaces were introduced by Princy and Mohana [17].

The neutrosophic set was introduced by Smarandache [22, 23] as a generalisation of the intuitionistic fuzzy set. Salama and Alblowi [19] introduced neutrosophic topological spaces, generalising intuitionistic fuzzy topological spaces by incorporating three components: membership, indeterminacy, and non-membership.

Smarandache [22] also introduced the notion of dependence and independence among the fuzzy and neutrosophic components. Arockiarani and Jency [3] extended the neutrosophic framework to the case where the sum of all three membership functions does not exceed 3. The neutrosophic topological space and its basic operations were studied by Veereswari [32].

Saranya et al. [20] introduced the concept of the n-cylindrical neutrosophic set (abbreviated as $nCy\mathcal{N}s$), characterised by α and γ as dependent components and β as an independent component. Beyond the neutrosophic set ($\mathcal{N}s$), the $nCy\mathcal{N}s$ represents a broad generalisation of fuzzy sets. In this framework, the positive membership α_A, neutral membership β_A, and negative membership γ_A satisfy $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1$ for an integer $n > 1$.

Subsequently, Saranya et al. [21] introduced the notion of $nCy\mathcal{N}$-continuity for functions between two n-cylindrical neutrosophic topological spaces ($nCy\mathcal{N}ts$), and defined the $nCy\mathcal{N}$-interior ($nCy\mathcal{N}int$) and $nCy\mathcal{N}$-closure ($nCy\mathcal{N}cl$) of subsets within $nCy\mathcal{N}ts$. Recently, Vijayalakshmi et al. [33, 34, 35] introduced $nCy\mathcal{N}$ M open sets, connectedness and separated sets in $nCy\mathcal{N}$ topological spaces. Mahalakshmi et al. [12] introduced the notion of $nCy\mathcal{N}$ nano open sets in n-cylindrical neutrosophic nano topological spaces. Recently, Kayalvizhi and Alagesan [9] introduced the notion of $nCy\mathcal{N}$ nano M-open sets in n-cylindrical neutrosophic nano topological spaces.

S. Saha [18] defined δ-open sets in fuzzy topological spaces, nano topological space by Pankajam et al. [16] and neutrosophic topological space by Vadivel et al. [28]. Recently, Lellis Thivagar et.al [11] explored a new concept of neutrosophic nano topology, intuitionistic nano topology and fuzzy nano topology. In a related development, El-Maghrabi and Al-Juhani [6] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [15] also found M-open sets in nano topological spaces. Vadivel et al. [26, 27, 29] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [8] and Vadivel et al. [30] introduced M-open sets in fuzzy and neutrosophic nano topological spaces.

Research Gap: No investigation on some stronger and weaker forms n-cylindrical neutrosophic nano operators such as n-cylindrical neutrosophic nano θ continuous maps, n-cylindrical neutrosophic nano θ-semi continuous maps, n-cylindrical neutrosophic nano e continuous maps and n-cylindrical neutrosophic nano M continuous maps on n-cylindrical neutrosophic nano topological space has been reported in the n-cylindrical neutrosophic literature.

In this paper we introduce $nCy\mathcal{N}\mathfrak{N}\theta$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta S$, $nCy\mathcal{N}\mathfrak{N}e$ and $nCy\mathcal{N}\mathfrak{N}M$) continuous maps, $nCy\mathcal{N}\mathfrak{N}\theta$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta S$, $nCy\mathcal{N}\mathfrak{N}e$ and $nCy\mathcal{N}\mathfrak{N}M$) irresolute maps and discuss their properties in n-cylindrical neutrosophic nano topological spaces.

As a real-life illustration of this framework, we construct an $nCy\mathcal{N}\mathfrak{N}ts$ directly from soil-test data collected under Tamil Nadu's Soil Health Card programme, in which farm plots are grouped by an indiscernibility relation on pH, nitrogen, and organic-carbon bands. The resulting n-Cylindrical neutrosophic nano topology, generated by the lower approximation, upper approximation, and boundary region of a "Fertile Soil" decision attribute, yields a certify–develop–verify zoning that agricultural extension officers can act on directly.

2. Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 *[39] A fuzzy set (briefly, fs) A in U is defined by a membership function $\mu_A: U \to [0,1]$, where the membership value $\mu_A(x)$ indicates the degree to which $x \in U$ belongs to the fuzzy set A, for all $x \in U$.*

Definition 2.2 *[4] A fuzzy topological space (briefly, fts) is a pair (U, τ), where U is a non-empty set and τ is a family of fuzzy sets in U satisfying the following axioms:*

1. $\emptyset, U \in \tau$,
2. If $A, B \in \tau$, then $A \cap B \in \tau$,
3. If $A_i \in \tau$ for each $i \in I$, then $\bigcup_{i \in I} A_i \in \tau$.

Definition 2.3 *[2] An intuitionistic fuzzy set (briefly, ifs) A on U is an object of the form*

where $\alpha_A(x) \in [0,1]$ is the degree of positive membership and $\gamma_A(x) \in [0,1]$ is the degree of negative membership of x in A, satisfying $\alpha_A(x) + \gamma_A(x) \leq 1$ for all $x \in U$. The set of all intuitionistic fuzzy sets on U is denoted by $\mathrm{IFS}(U)$.

Definition 2.4 *[22] A neutrosophic set (briefly, $\mathcal{N}s$) A on U is an object of the form*

where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1]$ and $0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$ for all $x \in U$. Here $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral membership, and $\gamma_A(x)$ is the degree of negative membership. The components $\alpha_A(x)$ and $\gamma_A(x)$ are dependent, while $\beta_A(x)$ is independent.

Definition 2.5 *[19] A neutrosophic topology ($\mathcal{N}t$) on a non-empty set U is a family τ_N of neutrosophic subsets of U satisfying:*

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\bigcup_{i \in J} G_i \in \tau_N$ for any sub-collection $\{G_i : i \in J\} \subseteq \tau_N$.

The pair (U, τ_N) is called a *neutrosophic topological space* (briefly, $\mathcal{N}ts$). Any neutrosophic set in τ_N is called a *neutrosophic open set* (briefly, $\mathcal{N}os$). A neutrosophic set F is *neutrosophic closed* (briefly, $\mathcal{N}c$) if and only if its complement $C(F)$ is neutrosophic open.

Definition 2.6 *[20] An n-cylindrical neutrosophic set (briefly, (briefly, $nCy\mathcal{N}s$)) A on U is an object of the form*

where

• $\alpha_A(x) \in [0,1]$ is the degree of positive membership,

• $\beta_A(x) \in [0,1]$ is the degree of neutral membership, and

• $\gamma_A(x) \in [0,1]$ is the degree of negative membership,

satisfying, for all $x \in U$,

Here $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components, while $\beta_A(x)$ is 100% independent.

For convenience, the triple $\langle \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle$ is called an n-*cylindrical neutrosophic number* (briefly, (briefly, $nCy\mathcal{N}N$)) and the set is written as $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$.

Definition 2.7 *[20] Let $\{A_i : i \in I\}$ be an arbitrary family of $nCy\mathcal{N}s$'s in U. Then:*

Definition 2.8 *[20] The n-cylindrical neutrosophic empty and whole sets are defined as:*

Definition 2.9 *[20] Let $\tau_{CyN}(U)$ denote the family of all $nCy\mathcal{N}s$'s on U. The following basic connectives are defined for any $A, B \in \tau_{CyN}(U)$.*

1. **Inclusion.** $A \subseteq B$ if and only if $\alpha_A(x) \leq \alpha_B(x)$, $\beta_A(x) \leq \beta_B(x)$, and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in U$. Equality $A = B$ holds when both $A \subseteq B$ and $B \subseteq A$.
2. **Union.**
3. **Intersection.**
4. **Complement.**

Remark 2.1 *[20] For any $nCy\mathcal{N}$'s A, B, $C \in \tau_{CyN}(U)$:*

1. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$,
2. $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (commutativity),
3. $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$ (associativity),
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ (distributivity),
5. $A \cap A = A$ and $A \cup A = A$ (idempotency),
6. De Morgan's laws: $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

Definition 2.10 *[21] An n-cylindrical neutrosophic topology (briefly, $nCy\mathcal{N}t$) on a non-empty set U is a family τ_{CyN} of $nCy\mathcal{N}s$'s in U satisfying:*

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$ for any $A_1, A_2 \in \tau_{CyN}$,
3. $\bigcup_{i \in I} A_i \in \tau_{CyN}$ for any arbitrary sub-collection $\{A_i : i \in I\} \subseteq \tau_{CyN}$.

The pair (U, τ_{CyN}) is called an n-*cylindrical neutrosophic topological space* (briefly, $nCy\mathcal{N}ts$). Any $nCy\mathcal{N}s$ belonging to τ_{CyN} is called an n-*cylindrical neutrosophic open set* (briefly, $nCy\mathcal{N}os$), and its complement is called an n-*cylindrical neutrosophic closed set* (briefly, $nCy\mathcal{N}cs$). The family $\{0_{CyN}, 1_{CyN}\}$ is the *indiscrete* $nCy\mathcal{N}ts$, and the topology containing all $nCy\mathcal{N}$ subsets is the *discrete* $nCy\mathcal{N}ts$.

Remark 2.2 *[21] Every fuzzy topological space, intuitionistic fuzzy topological space, and Pythagorean fuzzy topological space is an $nCy\mathcal{N}ts$, since any subset of those spaces can be viewed as an $nCy\mathcal{N}$ subset.*

Definition 2.11 *[21] Let A and B be two n-cylindrical neutrosophic subsets of an $nCy\mathcal{N}ts$ (U, τ_{CyN}). Then B is called a neighbourhood of A if there exists an $nCy\mathcal{N}os$ O such that $A \subset O \subset B$.*

Proposition 2.1 *[21] A subset $A \subseteq U$ is n-cylindrical neutrosophic open in (U, τ_{CyN}) if and only if it is a neighbourhood of each of its subsets.*

Definition 2.12 *[21] Let (U, τ_{CyN}) be an $nCy\mathcal{N}ts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in U\}$ be an $nCy\mathcal{N}$ subset in U. The n-cylindrical neutrosophic interior (briefly, $nCy\mathcal{N}int$) of A is defined as the $nCy\mathcal{N}$ union of all $nCy\mathcal{N}$ open subsets of A:*

Clearly, $nCy\mathcal{N}int(A)$ is the largest $nCy\mathcal{N}os$ contained in A.

Definition 2.13 *[21] Let (U, τ_{CyN}) be an $nCy\mathcal{N}ts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in U\}$ be an $nCy\mathcal{N}s$ in U. The n-cylindrical neutrosophic closure (briefly, $nCy\mathcal{N}cl$ of A is defined as the $nCy\mathcal{N}$ intersection of all $nCy\mathcal{N}$ closed sets containing A:*

Clearly, $nCy\mathcal{N}cl(A)$ is the smallest $nCy\mathcal{N}cs$ that contains A.

Definition 2.14 *[12] Let U be a non-empty set, R an equivalence relation on U, and A an n-cylindrical neutrosophic set in U with positive membership function μ_A, indeterminacy function σ_A, and non-membership function λ_A. The n-cylindrical neutrosophic lower approximation, n-cylindrical neutrosophic upper approximation, and n-cylindrical neutrosophic boundary of A in (U, R), denoted $\underline{nCy\mathcal{N}}(A)$, $\overline{nCy\mathcal{N}}(A)$, and $B_{nCy\mathcal{N}}(A)$ respectively, are defined as follows:*

1. $\underline{nCy\mathcal{N}}(A) = \{\langle x,\ \mu_{\underline{R}(A)}(x),\ \sigma_{\underline{R}(A)}(x),\ \lambda_{\underline{R}(A)}(x)\rangle : x \in U\}$,
2. $\overline{nCy\mathcal{N}}(A) = \left\{\left\langle x,\ \mu_{\overline{R}(A)}(x),\ \sigma_{\overline{R}(A)}(x),\ \lambda_{\overline{R}(A)}(x)\right\rangle : x \in U\right\}$,
3. $B_{nCy\mathcal{N}}(A) = \overline{nCy\mathcal{N}}(A) \backslash \underline{nCy\mathcal{N}}(A) \ = \ \overline{nCy\mathcal{N}}(A) \cap (\underline{nCy\mathcal{N}}(A))^c$,

where the component functions are given by

Here $[x]_R$ denotes the equivalence class of x under R.

Definition 2.15 *[12] Let U be a universe, R an equivalence relation on U, and A an n-cylindrical neutrosophic set in U. If the collection*

forms a topology on U, then it is called the *n-cylindrical neutrosophic nano topology* on U with respect to A. The pair $(U, \tau_R^{nCy\mathcal{N}}(A))$ is called an *n-cylindrical neutrosophic nano topological space* (briefly, $nCy\mathcal{N}ts$). The elements of $\tau_R^{nCy\mathcal{N}}(A)$ are called *n-cylindrical neutrosophic nano open sets* (briefly, $nCy\mathcal{N}os$).

Definition 2.16 *[12] Let $(U, \tau_R^{nCy\mathcal{N}}(A))$ be a $nCy\mathcal{N}\mathfrak{N}ts$ with respect to A where A is a n-cylindrical neutrosophic subset of U. Let S be a n-cylindrical neutrosophic subset of U. Then n-cylindrical neutrosophic nano*

1. interior of S (briefly, $nCy\mathcal{N}\mathfrak{N}int(S)$) is defined by

$nCy\mathcal{N}\mathfrak{N}int(S) = \cup \{I : I \subseteq S \ \& \ I$ is a $nCy\mathcal{N}\mathfrak{N}$oset in $U\}$,

2. closure of S (briefly, $nCy\mathcal{N}\mathfrak{N}cl(S)$) is defined by

$nCy\mathcal{N}\mathfrak{N}cl(S) = \cap \{A: S \subseteq A \text{ \& } A \text{ is an } nCy\mathcal{N}\mathfrak{N}c \text{ set in } U\}$.

Definition 2.17 *[9] Let $(U, \tau_R^{nCy\mathcal{N}}(A))$ be a $nCy\mathcal{N}\mathfrak{N}ts$ with respect to A where A is a n-Cylindrical neutrosophic subset of U. Then a n-Cylindrical neutrosophic subset S in U is said to be n-Cylindrical neutrosophic nano regular open (briefly, $nCy\mathcal{N}\mathfrak{N}ro$) set if $S = nCy\mathcal{N}\mathfrak{N}int(nCy\mathcal{N}\mathfrak{N}cl(S))$ and n-Cylindrical neutrosophic nano regular closed (briefly, $nCy\mathcal{N}\mathfrak{N}rc$) set if $S = nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}int(S))$.*

Definition 2.18 *[9] Let $(U, \tau_R^{nCy\mathcal{N}}(A))$ be an $nCy\mathcal{N}\mathfrak{N}ts$ and $S = \{< s, \mu_S(s), \nu_S(s), \lambda_S(s) > | s \in U\}$ be an $nCy\mathcal{N}s$ in U. Then the $nCy\mathcal{N}\mathfrak{N}\delta$-interior and the $nCy\mathcal{N}\mathfrak{N}\delta$-closure of S are denoted by $nCy\mathcal{N}\mathfrak{N}\delta int(S)$ and $nCy\mathcal{N}\mathfrak{N}\delta cl(S)$ and are defined as follows. $nCy\mathcal{N}\mathfrak{N}\delta int(S) = \cup \{G | G$ is an $nCy\mathcal{N}\mathfrak{N}ros$ and $G \subseteq S\}$, $nCy\mathcal{N}\mathfrak{N}\delta cl(S) = \cap \{K | K$ is an $nCy\mathcal{N}\mathfrak{N}rcs$ and $S \subseteq K\}$.*

Definition 2.19 *[9] Let $(U, \tau_R^{nCy\mathcal{N}}(A))$ be a $nCy\mathcal{N}\mathfrak{N}ts$ and $S = \{< s, \mu_S(s), \nu_S(s), \lambda_S(s) > | s \in U\}$ be an $nCy\mathcal{N}s$ in U. A set S is said to be $nCy\mathcal{N}\mathfrak{N}$*

1. δ-open set (briefly, $nCy\mathcal{N}\mathfrak{N}\delta os$) if $S = nCy\mathcal{N}\mathfrak{N}\delta int(S)$,
2. δ-pre open set (briefly, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$) if $S \subseteq nCy\mathcal{N}\mathfrak{N}int(nCy\mathcal{N}\mathfrak{N}\delta cl(S))$,
3. semi open set (briefly, $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$) if $S \subseteq nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}int(S))$,
4. δ-semi open set (briefly, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$) if $S \subseteq nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}\delta int(S))$,
5. e open set (briefly, $nCy\mathcal{N}\mathfrak{N}eos$) if $S \subseteq nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}\delta int(S)) \cup nCy\mathcal{N}\mathfrak{N}int(nCy\mathcal{N}\mathfrak{N}\delta cl(S))$,
6. δ (resp. δ-pre, δ-semi and e) dense if $nCy\mathcal{N}\mathfrak{N}\delta cl(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cl(S), nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cl(S)$ and $nCy\mathcal{N}\mathfrak{N}ecl(S)) = 1_{nCy\mathcal{N}}$.

The complement of an $nCy\mathcal{N}\mathfrak{N}\delta os$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os, nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$ $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ and $nCy\mathcal{N}\mathfrak{N}eos$) is called an $nCy\mathcal{N}\mathfrak{N}\delta$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}, nCy\mathcal{N}\mathfrak{N}\mathcal{S}$ $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}$ and $nCy\mathcal{N}\mathfrak{N}e$) closed set (briefly, $nCy\mathcal{N}\mathfrak{N}\delta cs$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cs, nCy\mathcal{N}\mathfrak{N}\mathcal{S}cs$ $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cs$ and $nCy\mathcal{N}\mathfrak{N}ecs$)) in U.

The family of all $nCy\mathcal{N}\mathfrak{N}\delta os$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta cs, nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os, nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cs, nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}\mathcal{S}cs, nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cs, nCy\mathcal{N}\mathfrak{N}eos$ and $nCy\mathcal{N}\mathfrak{N}ecs$) of U is denoted by $nCy\mathcal{N}\mathfrak{N}\delta OS(U)$, (resp. $nCy\mathcal{N}\mathfrak{N}\delta CS(U), nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}OS(U), nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}CS(U), nCy\mathcal{N}\mathfrak{N}\mathcal{S}OS(U), nCy\mathcal{N}\mathfrak{N}\mathcal{S}CS(U), nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}OS(U), nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}CS(U)$, $nCy\mathcal{N}\mathfrak{N}eOS(U)$ and $nCy\mathcal{N}\mathfrak{N}eCS(U)$).

Definition 2.20 *[9] Let $(U, \tau_R^{nCy\mathcal{N}}(A))$ be an $nCy\mathcal{N}\mathfrak{N}ts$ and $S = \{< s, \mu_S(s), \nu_S(s), \lambda_S(s) > | s \in U\}$ be an $nCy\mathcal{N}s$ in U. Then the $nCy\mathcal{N}\mathfrak{N}\delta$-pre (resp. $nCy\mathcal{N}\mathfrak{N}\delta$-semi and $nCy\mathcal{N}\mathfrak{N}e$)-interior and the $nCy\mathcal{N}\mathfrak{N}\delta$-pre (resp. $nCy\mathcal{N}\mathfrak{N}\delta$ -semi and $nCy\mathcal{N}\mathfrak{N}e$) -closure of S are denoted by $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}int(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}int(S)$ and $nCy\mathcal{N}\mathfrak{N}eint(S)$) and the $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cl(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cl(S)$ and $nCy\mathcal{N}\mathfrak{N}ecl(S)$) and are defined as follows:*

$nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}int(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}int(S)$ and $nCy\mathcal{N}\mathfrak{N}eint(S)$) $= \cup \{G | G$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ and $nCy\mathcal{N}\mathfrak{N}eos$)

and $G \subseteq S\}$ and $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cl(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cl(S)$ and $nCy\mathcal{N}\mathfrak{N}ecl(S)$) $= \cap \{K | K$ is an $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cs$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cs$ and $nCy\mathcal{N}\mathfrak{N}ecs$) and $S \subseteq K\}$.

Definition 2.21 *[9] Let* $(U, \tau_R^{nCy\mathcal{N}}(A))$ *be a* $nCy\mathcal{N}\mathfrak{N}ts$ *and* S *be a* $nCy\mathcal{N}s$ *in* U. *A set* S *is said to be* $nCy\mathcal{N}\mathfrak{N}$

1. θ-interior of S (briefly, $nCy\mathcal{N}\mathfrak{N}\theta int(S)$) is defined by

$nCy\mathcal{N}\mathfrak{N}\theta int(S) = \cup \quad \{nCy\mathcal{N}\mathfrak{N}int(T): T \subseteq S \,\&\, T$ isa $nCy\mathcal{N}\mathfrak{N}cs$ in $U\}$,

2. θ-closure of S (briefly, $nCy\mathcal{N}\mathfrak{N}\theta cl(S)$) is defined by

$nCy\mathcal{N}\mathfrak{N}\theta cl(S) = \cap \quad \{nCy\mathcal{N}\mathfrak{N}cl(T): T \supseteq S \,\&\, T$ isa $nCy\mathcal{N}\mathfrak{N}os$ in $U\}$,

3. θ-open set (briefly, $nCy\mathcal{N}\mathfrak{N}\theta os$) if $S = nCy\mathcal{N}\mathfrak{N}\theta int(S)$,
4. θ -semi open set (briefly, $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$) if $S \subseteq nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}\theta int(S))$,
5. M-open set (briefly, $nCy\mathcal{N}\mathfrak{N}Mos$) if

$S \subseteq nCy\mathcal{N}\mathfrak{N}cl(nCy\mathcal{N}\mathfrak{N}\theta int(S)) \cup nCy\mathcal{N}\mathfrak{N}int(nCy\mathcal{N}\mathfrak{N}\delta cl(S))$.

The complement of a $nCy\mathcal{N}\mathfrak{N}Mos$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta os$ & $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$) is called an $nCy\mathcal{N}\mathfrak{N}M$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta$ & $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}$) closed set (briefly, $nCy\mathcal{N}\mathfrak{N}Mcs$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta cs$ & $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cs$)) in U.

The family of all $nCy\mathcal{N}\mathfrak{N}\theta os$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta cs, nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os, nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cs, nCy\mathcal{N}\mathfrak{N}Mos$ and $nCy\mathcal{N}\mathfrak{N}Mcs$) of U is denoted by $nCy\mathcal{N}\mathfrak{N}\theta OS(U)$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta CS(U), nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}OS(U), nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}CS(U), nCy\mathcal{N}\mathfrak{N}MOS(U)$ and $nCy\mathcal{N}\mathfrak{N}MCS(U)$).

Definition 2.22 *[9] Let* $(U, \tau_R^{nCy\mathcal{N}}(A))$ *be a* $nCy\mathcal{N}\mathfrak{N}ts$ *and* S *be a* $nCy\mathcal{N}s$ *in* U. *Then the* $nCy\mathcal{N}\mathfrak{N}$

1. M-interior (resp. $nCy\mathcal{N}\mathfrak{N}\theta$-semi interior) of S (briefly, $nCy\mathcal{N}\mathfrak{N}Mint(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}int(S)$)) is defined by $nCy\mathcal{N}\mathfrak{N}Mint(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}int(S)$) $=\cup \{T: T \subseteq S$ and T is a $nCy\mathcal{N}\mathfrak{N}Mos$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$)$\}$ in U.

2. M-closure (resp. θ-semi closure) of S (briefly, $nCy\mathcal{N}\mathfrak{N}Mcl(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cl(S)$)) is defined by $nCy\mathcal{N}\mathfrak{N}Mcl(S)$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cl(S)$) $=\cap \{T: S \subseteq T$ and T is a $nCy\mathcal{N}\mathfrak{N}Mcs$ (resp. $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cs$)$\}$ in U.

Definition 2.23 *Let* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$ *and* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be any two* $nCy\mathcal{N}\mathfrak{N}ts$ *'s. A mapping* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is said to be an* n*-Cylindrical neutrosophic nano (i) continuous (briefly,* $nCy\mathcal{N}\mathfrak{N}Cts$ *) if the inverse image of every* $nCy\mathcal{N}\mathfrak{N}os$ *in* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is a* $nCy\mathcal{N}\mathfrak{N}os$ *in* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$*, (ii) irresolute (briefly,* $nCy\mathcal{N}\mathfrak{N}Irr$*) if the inverse image of every* $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$ *in* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is a* $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$ *in* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$.

3. -Cylindrical neutrosophic nano -continuous mapsnM

Definition 3.1 *Let* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$ *and* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be any two* $nCy\mathcal{N}\mathfrak{N}ts$ *'s. A mapping* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is said to be an* n*-Cylindrical neutrosophic nano* δ *(resp.* $\delta\mathcal{P}$, $\delta\mathcal{S}$, e, θ, $\theta\mathcal{S}$ *and* M*) - continuous (briefly,* $nCy\mathcal{N}\mathfrak{N}\delta Cts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ *and* $nCy\mathcal{N}\mathfrak{N}MCts$*)) if the inverse image of every* $nCy\mathcal{N}\mathfrak{N}os$ *in* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is a* $nCy\mathcal{N}\mathfrak{N}\delta os$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}eos$, $nCy\mathcal{N}\mathfrak{N}\theta os$, $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ *and* $nCy\mathcal{N}\mathfrak{N}Mos$*) in* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$.

Remark 3.1 *Thus from the definitions of some stronger and weaker forms of continuous functions in n-Cylindrical neutrosophic topological spaces, we can assure that throughout this paper all the properties and examples also hold good when it is possible for n-Cylindrical neutrosophic nano continuous functions.*

Proposition 3.1 *Let* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$ *and* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be two* $nCy\mathcal{N}\mathfrak{N}ts$*'s. Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a mapping. Then the following statements hold, but not conversely.*

1. Every $nCy\mathcal{N}\mathfrak{N}\theta Cts$ is a $nCy\mathcal{N}\mathfrak{N}Cts$,
2. Every $nCy\mathcal{N}\mathfrak{N}\theta Cts$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$,
3. Every $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ is a $nCy\mathcal{N}\mathfrak{N}MCts$,
4. Every $nCy\mathcal{N}\mathfrak{N}\delta Cts$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$,
5. Every $nCy\mathcal{N}\mathfrak{N}\delta Cts$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$,
6. Every $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$ is a $nCy\mathcal{N}\mathfrak{N}eCts$,
7. Every $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$ is a $nCy\mathcal{N}\mathfrak{N}MCts$,
8. Every $nCy\mathcal{N}\mathfrak{N}MCts$ is a $nCy\mathcal{N}\mathfrak{N}eCts$,
9. Every $nCy\mathcal{N}\mathfrak{N}\delta Cts$ is a $nCy\mathcal{N}\mathfrak{N}Cts$.

Proof.

1. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\theta os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\theta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}Cts$.

2. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\theta os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\theta os$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$.

3. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ is a $nCy\mathcal{N}\mathfrak{N}Mos$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$.

4. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\delta os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$.

5. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\delta os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$.

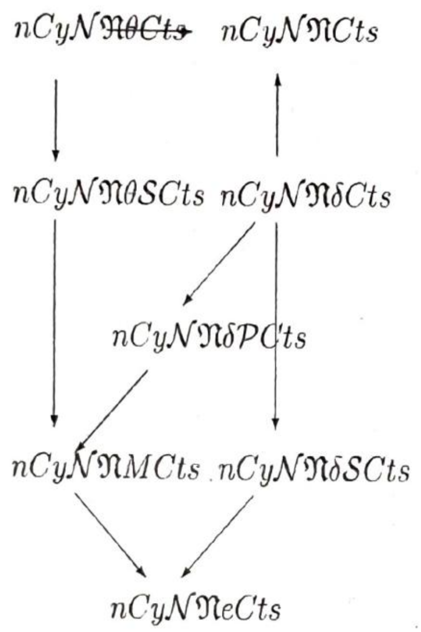
6. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ is a $nCy\mathcal{N}\mathfrak{N}eos$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}eos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}eCts$.

7. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$ is a $nCy\mathcal{N}\mathfrak{N}Mos$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$.

8. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}Mos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}Mos$ is a $nCy\mathcal{N}\mathfrak{N}eos$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}eos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}eCts$.

9. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}\delta os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}Cts$.

Remark 3.2 *We obtain the following diagram from the results are discussed above.*



Note: $A \rightarrow B$ denotes A implies B, but not conversely.

Example 3.1 *Assume* $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ *be the universe set and the equivalence relation is* $U/R = \{\{s_1, s_2\}, \{s_3, s_4\}\}$. *Let*

$\mathcal{P} = \left\{ \left\langle \frac{s_1}{0.8,0.5,0.2} \right\rangle, \left\langle \frac{s_2}{0.5,0.3,0.5} \right\rangle, \left\langle \frac{s_3}{0.7,0.4,0.3} \right\rangle, \left\langle \frac{s_4}{0.4,0.2,0.6} \right\rangle \right\}$ *be a* n*-Cylindrical neutrosophic subset of* U. *Here* $n = 4$,

Thus $\tau_R^{nCy\mathcal{N}1}(\mathcal{P}) = \tau_R^{nCy\mathcal{N}2}(\mathcal{P}) = \tau_R^{nCy\mathcal{N}}(\mathcal{P}) = \{0_{nCy\mathcal{N}}, 1_{nCy\mathcal{N}}, \underline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})\}$.

Let $h_{\mathcal{C}}: (U, \tau_R^{nCy\mathcal{N}1}(\mathcal{P})) \rightarrow (U, \tau_R^{nCy\mathcal{N}}(\mathcal{P}))$ be an identity function, Then $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}Cts$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$ and $nCy\mathcal{N}\mathfrak{N}Cts$) but not $nCy\mathcal{N}\mathfrak{N}\delta Cts$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta Cts$ and $nCy\mathcal{N}\mathfrak{N}\theta Cts$). Since,

$\overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is a $nCy\mathcal{N}\mathfrak{N}o$ set in U_2 but $h_{\mathcal{C}}^{-1}(\overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})) = \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is not $nCy\mathcal{N}\mathfrak{N}\delta o$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta o$ and $nCy\mathcal{N}\mathfrak{N}\theta o$) set in U_1.

Example 3.2 *Assume* $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ *be the universe set and the equivalence relation is* $U/R = \{\{s_1, s_2\}, \{s_3, s_4\}\}$.

Let $\mathcal{P} = \left\{\left\langle\frac{s_1}{0.9,0.7,0.3}\right\rangle, \left\langle\frac{s_2}{0.6,0.8,0.5}\right\rangle, \left\langle\frac{s_3}{0.8,0.5,0.4}\right\rangle, \left\langle\frac{s_4}{0.7,0.5,0.6}\right\rangle\right\}$ *be a* n*-Cylindrical neutrosophic subset of* U. *Here* $n = 4$,

Thus $\tau_R^{nCy\mathcal{N}1}(\mathcal{P}) = \tau_R^{nCy\mathcal{N}2}(\mathcal{P}) = \tau_R^{nCy\mathcal{N}}(\mathcal{P}) = \{0_{nCy\mathcal{N}}, 1_{nCy\mathcal{N}}, \underline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})\}$. Let $h_{\mathcal{C}}: (U, \tau_R^{nCy\mathcal{N}1}(\mathcal{P})) \to (U, \tau_R^{nCy\mathcal{N}}(\mathcal{P}))$ be an identity function, Then $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$ (resp. $nCy\mathcal{N}\mathfrak{N}eCts$) but not $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$). Since, $\overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is a $nCy\mathcal{N}\mathfrak{N}o$ set in U_2 but $h_{\mathcal{C}}^{-1}(\overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})) = \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is neither $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}o$ nor $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}o$ set in U_1.

Theorem 3.1 *A map* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is* $nCy\mathcal{N}\mathfrak{N}MCts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ *) iff the inverse image of each* $nCy\mathcal{N}\mathfrak{N}cs$ *in* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is* $nCy\mathcal{N}\mathfrak{N}Mcs$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta cs$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}cs$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}cs$, $nCy\mathcal{N}\mathfrak{N}\theta cs$, $nCy\mathcal{N}\mathfrak{N}ecs$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}cs$ *) in* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$.

Proof. Let B be a $nCy\mathcal{N}\mathfrak{N}cs$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. This implies B^c is $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$, $h_{\mathcal{C}}^{-1}(B^c)$ is $nCy\mathcal{N}\mathfrak{N}Mos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since, $h_{\mathcal{C}}^{-1}(B^c) = h_{\mathcal{C}}^{-1}(B)^c$, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$.

Conversely, suppose that the inverse image of each $nCy\mathcal{N}\mathfrak{N}cs$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ is $nCy\mathcal{N}\mathfrak{N}Mcs$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. Then, B^c is a $nCy\mathcal{N}\mathfrak{N}cs$ in $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$. By hypothesis, $h_{\mathcal{C}}^{-1}(B^c)$ is $nCy\mathcal{N}\mathfrak{N}Mcs$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Since, $h_{\mathcal{C}}^{-1}(B^c) = (h_{\mathcal{C}}^{-1}(B))^c$, $(h_{\mathcal{C}}^{-1}(B))^c$ is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Therefore, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$. Hence, $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$. The proof of other cases are similar.

Definition 3.2 *A* $nCy\mathcal{N}\mathfrak{N}ts$ $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$ *is said to be an* n*-Cylindrical neutrosophic nano* $MU_{1/2}$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\theta U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}eU_{1/2}$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}U_{1/2}$ *)-space, if every* $nCy\mathcal{N}\mathfrak{N}Mos$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$, $nCy\mathcal{N}\mathfrak{N}\theta os$, $nCy\mathcal{N}\mathfrak{N}eos$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ *) in* U_1 *is a* $nCy\mathcal{N}\mathfrak{N}os$ *in* U_1.

Theorem 3.2 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}MCts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ *), then* $h_{\mathcal{C}}$ *is a* $nCy\mathcal{N}\mathfrak{N}Cts$ *if* U_1 *is a* $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\theta U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}eU_{1/2}$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}$ $U_{1/2}$*)-space.*

Proof. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. Then, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1, by hypothesis. Since U_1 is a $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$-space, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}Cts$. The proof of other cases are similar.

Theorem 3.3 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}MCts$ *map and* $g_{\mathcal{C}}: (U_2, \tau_R^{nCy\mathcal{N}}(A_2)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *be a* $nCy\mathcal{N}\mathfrak{N}Cts$ *map, then* $g_{\mathcal{C}} \circ h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *is a* $nCy\mathcal{N}\mathfrak{N}MCts$ *map.*

Proof. Let K be a $nCy\mathcal{N}\mathfrak{N}os$ in U_3. Then, $g_{\mathcal{C}}^{-1}(K)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2, by hypothesis. Since $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$ map, $h_{\mathcal{C}}^{-1}(g_{\mathcal{C}}^{-1}(K))$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Since $(g_{\mathcal{C}} \circ h_{\mathcal{C}})^{-1}(K) = h_{\mathcal{C}}^{-1}(g_{\mathcal{C}}^{-1}(K))$, $(g_{\mathcal{C}} \circ h_{\mathcal{C}})^{-1}(K)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Hence $g_{\mathcal{C}} \circ h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$ map. The proof of other cases are similar.

Theorem 3.4 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}MCts$ *map. Then, the following conditions are hold.*

1. $h_{\mathcal{C}}(nCy\mathcal{N}\mathfrak{N}Mcl(A)) \subseteq nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A))$, for all $nCy\mathcal{N}s$ A in U_1.
2. $nCy\mathcal{N}\mathfrak{N}Mcl(h_{\mathcal{C}}^{-1}(B)) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$, for all $nCy\mathcal{N}s$ B in U_2.

Proof.

1. Since $nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A))$ is a $nCy\mathcal{N}\mathfrak{N}cs$ in U_2 and $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$, then $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A)))$ is $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_1. Now, since $A \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A)))$ and $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A)))$ is $nCy\mathcal{N}\mathfrak{N}Mcs$, and $nCy\mathcal{N}\mathfrak{N}Mcl(A)$ is the smallest $nCy\mathcal{N}\mathfrak{N}Mcs$ containing A, we get $nCy\mathcal{N}\mathfrak{N}M\ cl(A) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A)))$. Therefore, $h_{\mathcal{C}}\ (nCy\mathcal{N}\mathfrak{N}Mcl(A)) \subseteq nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(A))$.

2. By replacing A with $h_{\mathcal{C}}^{-1}(B)$ in (i), we obtain $h_{\mathcal{C}}(nCy\mathcal{N}\mathfrak{N}Mcl(h_{\mathcal{C}}^{-1}(B))) \subseteq nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}(h_{\mathcal{C}}^{-1}(B))) \subseteq nCy\mathcal{N}\mathfrak{N}cl(B$.

Hence, $nCy\mathcal{N}\mathfrak{N}Mcl(h_{\mathcal{C}}^{-1}(B)) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$.

Theorem 3.5 $h_{\mathcal{C}}$ *is* $nCy\mathcal{N}\mathfrak{N}MCts$ *iff*

$h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B)) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$, *for all* $nCy\mathcal{N}s$ B *in* U_2.

Proof. Let $h_{\mathcal{C}}$ be $nCy\mathcal{N}\mathfrak{N}MCts$ and $B \in U_2$. $nCy\mathcal{N}\mathfrak{N}int(B)$ is $nCy\mathcal{N}\mathfrak{N}os$ in U_2 and hence, $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B))$ is $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1 . Therefore, $nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B))) = h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B))$. Also, $nCy\mathcal{N}\mathfrak{N}int(B) \subseteq B$ implies that $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B)) \subseteq h_{\mathcal{C}}^{-1}(B)$.

Therefore, $nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B))) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$. That is, $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B)) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$.

Conversely, let $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B)) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$, for all subset B of U_2 . If B is $nCy\mathcal{N}\mathfrak{N}os$ in U_2, then $nCy\mathcal{N}\mathfrak{N}int(B) = B$. By assumption, $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}int(B)) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$. Thus, $h_{\mathcal{C}}^{-1}(B) \subseteq nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B))$. But $nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B)) \subseteq h_{\mathcal{C}}^{-1}(B)$. Therefore, $nCy\mathcal{N}\mathfrak{N}Mint(h_{\mathcal{C}}^{-1}(B)) = h_{\mathcal{C}}^{-1}(B)$. That is, $h_{\mathcal{C}}^{-1}(B)$ is $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1, for all $nCy\mathcal{N}\mathfrak{N}os(B)$ in U_2. Therefore, $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$ on U_1.

Remark 3.3 *Theorems 3.3, 3.4 and 3.5 are true for* $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$.

4. - Cylindrical neutrosophic nano - irresolute maps in$\boldsymbol{nMnCy\mathcal{N}\mathfrak{N}ts}$

In this section, we introduce $nCy\mathcal{N}\mathfrak{N}MIrr$ (resp. $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ and $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$) maps and study some of its characterizations.

Definition 4.1 *A map* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *is called a* n*-Cylindrical neutrosophic* M *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}$, $nCy\mathcal{N}\mathfrak{N}\theta$, $nCy\mathcal{N}\mathfrak{N}e$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}$*) -irresolute (briefly,* $nCy\mathcal{N}\mathfrak{N}MIrr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$*)) map if* $h_{\mathcal{C}}^{-1}(B)$ *is a* $nCy\mathcal{N}\mathfrak{N}Mos$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$, $nCy\mathcal{N}\mathfrak{N}\theta os$, $nCy\mathcal{N}\mathfrak{N}eos$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$*) in* $(U_1, \tau_R^{nCy\mathcal{N}}(A_1))$ *for every* $nCy\mathcal{N}\mathfrak{N}Mos$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$, $nCy\mathcal{N}\mathfrak{N}\theta os$, $nCy\mathcal{N}\mathfrak{N}eos$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$*)* B *of* $(U_2, \tau_R^{nCy\mathcal{N}}(A_2))$.

Theorem 4.1 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}Irr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}MIrr$, $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$*), then* $h_{\mathcal{C}}$ *is a* $nCy\mathcal{N}\mathfrak{N}\mathcal{S}Cts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}MCts$, $nCy\mathcal{N}\mathfrak{N}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$*) map. But not conversely.*

Proof. (i) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}os$ is a $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$, B is a $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\mathcal{S}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\mathcal{S}Cts$ map.

(ii) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}MIrr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}os$ is a $nCy\mathcal{N}\mathfrak{N}Mos$, B is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$ map.

(iii) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}\delta Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}\delta os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, B is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}Cts$ map.

(iv) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}\delta os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, B is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$ map.

(v) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}\delta os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}\delta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, B is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$ map.

(vi) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}\theta Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}\theta os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}\theta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, B is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\theta os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\theta Cts$ map.

(vii) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}eIrr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}os$ is a $nCy\mathcal{N}\mathfrak{N}eos$, B is a $nCy\mathcal{N}\mathfrak{N}eos$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}eos$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}eCts$ map.

(viii) Let $h_{\mathcal{C}}$ be a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$ map. Let B be any $nCy\mathcal{N}\mathfrak{N}\theta os$ in U_2. Since every $nCy\mathcal{N}\mathfrak{N}\theta os$ is a $nCy\mathcal{N}\mathfrak{N}os$, B is a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. By hypothesis $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$ map.

Example 4.1 *Assume* $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ *be the universe set and the equivalence relation is* $U/R = \{\{s_1, s_2\}, \{s_3, s_4\}\}$. *Here* $n = 4$, *let* $\mathcal{P} = \left\{\left\langle\frac{s_1}{0.8,0.5,0.2}\right\rangle, \left\langle\frac{s_2}{0.5,0.3,0.5}\right\rangle, \left\langle\frac{s_3}{0.7,0.4,0.3}\right\rangle, \left\langle\frac{s_4}{0.4,0.2,0.6}\right\rangle\right\}$ *be a* n*-Cylindrical neutrosophic subset of* U.

Thus $\tau_R^{nCy\mathcal{N}}(\mathcal{P}) = \{0_{nCy\mathcal{N}}, 1_{nCy\mathcal{N}}, \underline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})\}$.

Let $\mathcal{P} = \left\{\left\langle\frac{s_1}{0.9,0.7,0.3}\right\rangle, \left\langle\frac{s_2}{0.6,0.8,0.5}\right\rangle, \left\langle\frac{s_3}{0.8,0.5,0.4}\right\rangle, \left\langle\frac{s_4}{0.7,0.5,0.6}\right\rangle\right\}$ be a n-Cylindrical neutrosophic subset of U.

Thus $\tau_R^{nCy\mathcal{N}}(\mathcal{P}) = \{0_{nCy\mathcal{N}}, 1_{nCy\mathcal{N}}, \underline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), \overline{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P}), B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})\}$.

Let $h_{\mathcal{C}}: (U, \tau_R^{nCy\mathcal{N}}(\mathcal{P})) \to (U, \tau_R^{nCy\mathcal{N}}(\mathcal{P}))$ be an identity function, Then $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MCts$ but not $nCy\mathcal{N}\mathfrak{N}MIrr$. Since, $B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ set in U_2 but $h_{\mathcal{C}}^{-1}(B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})) = B_{nCy\mathcal{N}\mathfrak{N}}(\mathcal{P})$ is not $nCy\mathcal{N}\mathfrak{N}Mos$ set in U_1.

Theorem 4.2 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}MIrr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$*), then* $h_{\mathcal{C}}$ *is a* $nCy\mathcal{N}\mathfrak{N}Cts$ *map if* U_1 *is a* $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}\theta U_{1/2}$, $nCy\mathcal{N}\mathfrak{N}eU_{1/2}$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}U_{1/2}$ *)-space.*

Proof. Let B be a $nCy\mathcal{N}\mathfrak{N}os$ in U_2. Then, B is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. Therefore $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1, by hypothesis. Since U_1 is a $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$ -space, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}os$ in U_1. Hence, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}Cts$ map. The proof of other cases is similar.

Theorem 4.3 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *and* $g_{\mathcal{C}}: (U_2, \tau_R^{nCy\mathcal{N}}(A_2)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *be a* $nCy\mathcal{N}\mathfrak{N}MIrr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$ *) maps, then* $g_{\mathcal{C}} \circ h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *is a* $nCy\mathcal{N}\mathfrak{N}MIrr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$*) map.*

Proof. Let K be a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_3. Since $g_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map, $g_{\mathcal{C}}^{-1}(K)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. Since $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map, $h_{\mathcal{C}}^{-1}(g_{\mathcal{C}}^{-1}(K))$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Hence $g_{\mathcal{C}} \circ h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map. The proof of other cases is similar.

Theorem 4.4 *Let* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$ *be a* $nCy\mathcal{N}\mathfrak{N}MIrr$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$*) map and* $g_{\mathcal{C}}: (U_2, \tau_R^{nCy\mathcal{N}}(A_2)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *be a* $nCy\mathcal{N}\mathfrak{N}MCts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$*) map, then* $g_{\mathcal{C}} \circ h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_3, \tau_R^{nCy\mathcal{N}}(A_3))$ *is a* $nCy\mathcal{N}\mathfrak{N}MCts$ *(resp.* $nCy\mathcal{N}\mathfrak{N}\delta Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Cts$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Cts$, $nCy\mathcal{N}\mathfrak{N}\theta Cts$, $nCy\mathcal{N}\mathfrak{N}eCts$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Cts$*) map.*

Proof. Let K be a $nCy\mathcal{N}\mathfrak{N}os$ in U_3. Since $g_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$ map, $g_{\mathcal{C}}^{-1}(K)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. Since $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map, $h_{\mathcal{C}}^{-1}(g_{\mathcal{C}}^{-1}(K))$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Hence, $g_{\mathcal{C}} \circ h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MCts$ map. The proof of other cases is similar.

Theorem 4.5 *Let a map* $h_{\mathcal{C}}: (U_1, \tau_R^{nCy\mathcal{N}}(A_1)) \to (U_2, \tau_R^{nCy\mathcal{N}}(A_2))$*. Then the following conditions are equivalent if* U_1 *and* U_2 *are* $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$ *-spaces.*

1. $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map,
2. $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1, for each $nCy\mathcal{N}\mathfrak{N}Mos(B)$ in U_2,

3. $nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(B)) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$, for each $nCy\mathcal{N}s$ B of U_2.

Proof. (i) → (ii): Let B be any $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. Then, B^c is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_2. Since $h_{\mathcal{C}}$ is $nCy\mathcal{N}\mathfrak{N}MIrr$, $h_{\mathcal{C}}^{-1}(B^c)$ is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_1. But $h_{\mathcal{C}}^{-1}(B^c) = (h_{\mathcal{C}}^{-1}(B))^c$. Therefore, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1.

(ii) → (iii) : Let B be any $nCy\mathcal{N}s$ in U_2 and $B \subseteq nCy\mathcal{N}\mathfrak{N}cl(B)$. Then, $h_{\mathcal{C}}^{-1}(B) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$. Since $nCy\mathcal{N}\mathfrak{N}cl(B)$ is a $nCy\mathcal{N}\mathfrak{N}cs$ in U_2, $nCy\mathcal{N}\mathfrak{N}cl(B)$ is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_2. Therefore, $(nCy\mathcal{N}\mathfrak{N}cl(B))^c$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_2. By hypothesis, $h_{\mathcal{C}}^{-1}((nCy\mathcal{N}\mathfrak{N}cl(B))^c)$ is a $nCy\mathcal{N}\mathfrak{N}Mos$ in U_1. Since, $h_{\mathcal{C}}^{-1}((nCy\mathcal{N}\mathfrak{N}cl(B))^c) = (h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B)))^c$, $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$ is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_1. Since, U_1 is a $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$ -space, $h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$ is a $nCy\mathcal{N}\mathfrak{N}cs$ in U_1. Hence, $nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(B)) \subseteq nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))) = h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$. That is, $nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(B)) \subseteq h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}cl(B))$.

(iii) → (i) : Let B be any $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_2. Since U_2 is a $nCy\mathcal{N}\mathfrak{N}MU_{1/2}$-space, B is a $nCy\mathcal{N}\mathfrak{N}cs$ in U_2 and $nCy\mathcal{N}\mathfrak{N}cl(B) = B.$ Hence, $h_{\mathcal{C}}^{-1}(B) = h_{\mathcal{C}}^{-1}(nCy\mathcal{N}\mathfrak{N}Mcl(B)) \subseteq nCy\mathcal{N}\mathfrak{N}Mcl(h_{\mathcal{C}}^{-1}(B))$. But clearly, $h_{\mathcal{C}}^{-1}(B) \subseteq nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(B))$. Therefore, $nCy\mathcal{N}\mathfrak{N}cl(h_{\mathcal{C}}^{-1}(B)) = h_{\mathcal{C}}^{-1}(B)$. This implies, $h_{\mathcal{C}}^{-1}(B)$ is a $nCy\mathcal{N}\mathfrak{N}cs$ and hence, it is a $nCy\mathcal{N}\mathfrak{N}Mcs$ in U_1. Thus, $h_{\mathcal{C}}$ is a $nCy\mathcal{N}\mathfrak{N}MIrr$ map

Remark 4.1 *Theorem 4.5 is true for* $nCy\mathcal{N}\mathfrak{N}\delta Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{S}Irr$, $nCy\mathcal{N}\mathfrak{N}\delta\mathcal{P}Irr$, $nCy\mathcal{N}\mathfrak{N}\theta Irr$, $nCy\mathcal{N}\mathfrak{N}eIrr$ *and* $nCy\mathcal{N}\mathfrak{N}\theta\mathcal{S}Irr$.

5. Formation of an -Cylindrical neutrosophic nano topology: A Precision Agriculture Illustration$n$

We now illustrate, with a representative data set, how an n-Cylindrical neutrosophic nano topology itself (Definition 2.15) arises from field measurements. Under Tamil Nadu's Soil Health Card programme, agricultural extension officers periodically test farm plots for soil pH, available nitrogen, and organic carbon, and classify plots as indiscernible whenever these three conditional attributes fall in the same graded band. This indiscernibility relation is precisely the equivalence relation R required by the nano topology construction.

5.1 Universe and indiscernibility relation

Let $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ denote six plots in a farm block. Soil-test bands for pH, nitrogen, and organic carbon group the plots into three indiscernibility classes reflecting a common irrigation-and-drainage zone:

5.2 Decision attribute: Fertile Soil

Let $n = 3$. Combining the soil-test readings with the agronomist's assessment, define the n-Cylindrical neutrosophic set A ("Fertile Soil") on U:

$$A = \left\{\left\langle\frac{P_1}{0.80,0.60,0.50}\right\rangle, \left\langle\frac{P_2}{0.90,0.60,0.30}\right\rangle, \left\langle\frac{P_3}{0.50,0.50,0.10}\right\rangle, \left\langle\frac{P_4}{0.60,0.50,0.30}\right\rangle, \left\langle\frac{P_5}{0.60,0.50,0.30}\right\rangle, \left\langle\frac{P_6}{0.50,0.50,0.40}\right\rangle\right\}.$$

Each triple satisfies $0 \leq \beta_A(x) \leq 1$ and $\alpha_A^3(x) + \gamma_A^3(x) \leq 1$; e.g., for P_1, $0.80^3 + 0.50^3 = 0.637 \leq 1$, and similarly for $P_2, \ldots, P_6$.

5.3 Lower and upper approximations, and the induced nano topology

Applying the operators of Definition 2.15 class-by-class (taking the componentwise minimum of α, β and maximum of γ for the lower approximation, and the reverse for the upper approximation) gives

One can verify directly, zone by zone, that this five-set family is closed under (pairwise, hence arbitrary) union and intersection – every union or intersection of two members again equals one of $0_{nCy\mathcal{N}}, 1_{nCy\mathcal{N}}, \underline{nCy\mathcal{N}\mathfrak{N}}(A), \overline{nCy\mathcal{N}\mathfrak{N}}(A), B_{nCy\mathcal{N}\mathfrak{N}}(A)$ – so, by Definition 2.15,

is indeed the n-Cylindrical neutrosophic nano topology on U with respect to A, and $(U, \tau_R^{nCy\mathcal{N}}(A))$ is the corresponding $nCy\mathcal{N}\mathfrak{N}ts$.

5.4 Agronomic interpretation

The three graded sets have a direct field meaning:

• $\underline{nCy\mathcal{N}\mathfrak{N}}(A)$ – the *conservative* fertility estimate for each irrigation zone (the weakest reading within the zone). Plots meeting this level qualify for full Soil Health Card certification without further testing.

• $\overline{nCy\mathcal{N}\mathfrak{N}}(A)$ – the *optimistic* estimate (the strongest reading within the zone), used to flag zones with fertility potential worth developing through targeted fertigation.

• $B_{nCy\mathcal{N}\mathfrak{N}}(A)$ – the *boundary* region separating the two, identifying zones $\{P_1, P_2\}$, $\{P_3, P_4\}$, and $\{P_5, P_6\}$ as needing supplementary field verification (soil sampling at finer resolution) before a fertility grade is finalised.

This three-tier structure – certify, develop, verify – gives the extension department an actionable, uncertainty-aware zoning of the farm block directly from the topology generated by A, rather than from a single crisp threshold. In practice, however, individual field readings taken between certification rounds – a single agronomist's spot assessment, say, rather than the class-wise lower, upper, or boundary approximation itself – need not be $nCy\mathcal{N}\mathfrak{N}os$ with respect to $\tau_R^{nCy\mathcal{N}}(A)$; such a reading that still satisfies the weaker $nCy\mathcal{N}\mathfrak{N}Mos$ condition of Definition 2.21 can be treated as provisionally actionable pending verification, which is exactly the role the $nCy\mathcal{N}\mathfrak{N}M$-continuous framework of Sections 3 and 4 is designed to support when successive soil-testing rounds (i.e., successive maps between $nCy\mathcal{N}\mathfrak{N}ts$'s) are compared over time.

6. Conclusion

In this paper, using $nCy\mathcal{N}\mathfrak{N}Mos$, we defined $nCy\mathcal{N}\mathfrak{N}MCts$ maps and investigated some of their properties. We also compared n-Cylindrical neutrosophic continuity maps with $nCy\mathcal{N}\mathfrak{N}M$-continuity maps, and further extended the discussion to $nCy\mathcal{N}\mathfrak{N}M$-irresolute maps.

In addition, we illustrated the formation of an n-Cylindrical neutrosophic nano topology from real soil-test data collected under Tamil Nadu's Soil Health Card programme. Grouping farm plots by an indiscernibility relation on pH, nitrogen, and organic-carbon bands, we computed the lower approximation, upper approximation, and boundary region of a "Fertile Soil" decision attribute, generating a nano topology that yields a certify–develop–verify zoning of the farm block.

The study demonstrates that the proposed approach offers a transparent, uncertainty-aware way to translate field measurements directly into an actionable topological structure, rather than a single crisp threshold. Although the method depends on data quality and is context-specific, it improves the reliability and interpretability of soil-fertility classification for precision agriculture. Future work may include applying the $nCy\mathcal{N}\mathfrak{N}M$-continuous framework to compare successive soil-testing rounds over time, and extending the illustration to larger multi-block data sets.

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