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Article

Fixed Point Theorems for Multivalued Contractive Mapping in Fuzzy Partial Metric Spaces

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Abstract: This study develops the theoretical framework of Fuzzy Partial Metric Spaces (FPMS) and explores their role in fixed-point theory. We begin by presenting precise definitions of FPMS along with the notions of lower semi-continuous and upper semi-continuous functions, which are fundamental in analyzing continuity and convergence in fuzzy settings. The paper further introduces the collection of compact subsets and non-empty subsets within FPMS, establishing the structural basis for subsequent results. Several preliminary lemmas are proved to strengthen the foundation of the theory. A key contribution is the formulation of the Hausdorff fuzzy partial metric, which extends the classical Hausdorff metric to fuzzy partial contexts. Building on this, we define multivalued contractive mappings in complete FPMS and investigate their properties. Using upper semi-continuous functions, we establish fixed-point theorems for such mappings, ensuring the existence of solutions under contractive conditions. These results generalize classical fixed-point principles and provide new insights into the behavior of multivalued mappings in uncertain environments. Illustrative examples are included to demonstrate the applicability of the theorems, highlighting their relevance in mathematical modeling, optimization, and systems influenced by fuzziness. The findings contribute to the advancement of fixed-point theory by integrating fuzzy logic, partial metrics, and multivalued analysis into a unified framework.
Keywords: Partial metric space, Fuzzy metric space, Fuzzy Partial Metric Spaces, Multivalued contractive mapping, Fixed point.
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1. Introduction

In 1965, Zadeh [26] presented the concept of fuzzy sets. The concept of fuzzy metric spaces was initially presented by Kramosil and Michalek in 1975 [10]. George and Veeramani altered the idea of fuzzy metric space [9], while the generalization of metric spaces is the concept of partial metric space introduced by Matthew [14]. Feng, S. Liu demonstrated fixed point theorems for multivalued contractive mappings and multivalued Caristi-type mappings [25]. Generalization of Contractions in Partial Metric Space introduced by Altun [1]. Kiany introduced fixed point and endpoint theorems for set-valued fuzzy contraction mappings within Fuzzy Metric Spaces [12]. Sadeghi demonstrated fixed point theorems for set-valued mappings in complete partially ordered fuzzy metric spaces [21].

Rahim introduced fixed point outcomes for multivalued mappings in Partial Metric spaces [3]. A new metric formulation derived from fuzzy metric was created using fixed point theory [8]. B. Ali, M. Abbas presented fixed point theorems for multivalued contractive mappings within fuzzy metric spaces [6]. Fuzzy partial metric space extends the idea of fuzzy metric space defined by Fariha J Amer [2] & Basak, who also presented fixed point theorems in Fuzzy Partial Metric space [5]. S. Rehman established fixed point theorems for multivalued fuzzy cone contraction mappings [23].

New findings presented in multi-valued mapping within the framework of a Hausdorff fuzzy b-metric space, Hausdorff netrosophic metric [20][16]. Megha Pingale and Renu Pathak demonstrated Common fixed points results that fulfil weak contraction in partially ordered b-metric spaces [15]. Recent research developed regarding fixed point theory in fuzzy metric spaces, partial b-metric spaces, convex metric spaces, Neutrosophic fuzzy metric spaces, and fuzzy partial metric spaces [17] [24] [13] [11], [18] [19]. Renu Pathak & Rohini Gore developed new concept soft fuzzy Partial metric space with fixed point theory. In this research paper, we established new fixed-point theorems using multivalued contractive mappings in Fuzzy Partial Metric Spaces.

2. Materials

Definition 2.1: A Partial Metric space on 'X' is a Pair ('X', P) such that 'X' is a non-empty set and P: 'X' × 'X' → ℝ+ is a mapping providing the listed conditions $\forall p, q, r \in$ 'X' such that

1. '$P(p, p) \leq P(p, q)$'
2. '$P(p, p) = P(q, q) = P(p, q)$' iff '$p = q$'
3. '$P(p, q) = P(q, p)$'
4. '$P(p, r) \leq P(p, q) + P(q, r) - P(q, q)$'

Note that in Partial Metric space, a point's self-distance does not always equal zero. The Partial Metric 'P' is an ordinary metric on 'X' if '$P(p, p) = 0$', $\forall p \in$ 'X'. Therefore, a Partial Metric is an extension of an ordinary metric.[14]

Definition 2.2: A binary operation 'Θ' on [0, 1] is called a continuous t-norm if it is satisfied following conditions: $\forall\, p, q, r, s \in [0, 1]$:

1. '$p \,\Theta\, q = q \,\Theta\, p$' and '$p \,\Theta\, (q \,\Theta\, r) = (p \,\Theta\, q) \,\Theta\, r$'
2. 'Θ is continuous on [0, 1] × [0, 1]'
3. '$p \,\Theta\, 1 = p$'
4. If '$p \leq q$' and '$r \leq s$', then '$p \,\Theta\, r \leq q \,\Theta\, s$'. [10]

Definition 2.3: Let 'X' be a non-empty set, 'Θ' be a continuous t-norm and F : 'X' × 'X' × $(0, \infty) \to [0,1]$ be a mapping. Let 'F' be Fuzzy set. If the listed conditions are satisfied $\forall p, q, r \in$ 'X' and $u, v > 0$, then the triplet ('X', $F, \odot$) is said to be a Fuzzy Metric space, if it satisfies following Properties for

1. '$F(p, q, u) > 0$'
2. '$F(p, q, u) = 1$' iff '$q = p$
3. '$F(p, q, u) = F(q, p, u)$'
4. '$F(p, q, u + v) \geq F(p, q, u) \odot F(p, q, v)$'
5. '$F(p, q, \odot)$ is continuous on $(0, \infty)$'

If ('X', $F, \odot$) is a fuzzy metric space, then 'F' is a fuzzy metric on 'X'.[10]

Definition 2.4: Let 'X' be a non-empty set, 'Θ' be a continuous t-norm and F_p : 'X' × 'X' × $[0, \infty) \to [0,1]$ be a mapping. Let P be Partial metric space. If the listed conditions are satisfied $\forall p, q, r \in$ 'X' and $u, v \geq 0$, then the triplet ('X', F_p, Θ) is said to be a Fuzzy Partial Metric space:

1. ‘F_p (p, q ,0) = 0’
2. ‘F_p (p, q, u) = F_p (q, p, u)’
3. ‘F_p (p, r, u + v) ≥ F_p (p, q, u) ʘ F_p (q r, v)’
4. ‘ F_p (p, q, u)$\leq 1, u > 0$’ & ‘ F_p (p, q ,u)=1’ if ‘$p(p, q)$ = 0’
5. $'F_p$ (p, q, ʘ): $(0, \infty) \to [0, 1]$ is continuous,

Where ‘$F_p(p, q, u) = \frac{u}{u+p(p,q)}$’

If (‘X’, F_p ,ʘ) is a Fuzzy Partial Metric space, then ‘ F_p' is a fuzzy partial metric on ‘X’. [18]

Definition 2.5: Let (M, Fp, ʘ) be a fuzzy partial metric space, y ∈ M and (yn) be a sequence in M. Then

- (yn) is said to converge to y ∈ M, if $u > 0$ & $0 < r < 1$, there exists $n_0 \in N$ such that $Fp\ (y_n, y, u) > 1 - r,\ \ \forall\, n \geq n_0$.
- (yn) is said to be a Cauchy sequence, if $u > 0$ & $0 < r < 1$, there exists$n_0 \in N$ such that $Fp\ (y_n, y_m, u) > 1 - r,\ \ \forall\, m, n \geq n_0$.
- (M, Fp, ʘ) is complete if every Cauchy sequence is convergent in M.

Lemma1: Let (M, Fp, ʘ) be a Fuzzy Partial Metric space. The following statement holds:

- Let (yn) be a sequence in M & y ∈ M. Then $y_n \to y$ if and only if $Fp\ (y_n, y, u) = 1$, for all u > 0.
- An open ball $B(y_{0,} r, u)$ with center $y_{0,}$and radius $0 < r < 1$ can be defined as

$B(y_{0,} r, u) = \{y \in M \colon Fp\ (y_n, y, u) > 1 - r\}$, for $u > 0$

Let $\tau_{fp} = \{N \subset M \colon y_0 \in A\ \ iff\ \ \exists\, 0 < r < 1\ and\ \ u > 0$ such that $B(y_{0,} r, u) \subset A\}$

Then τ_{fp} is a topology on M.

Definition 2.6: Let (M, Fp, ʘ) be a Fuzzy Partial Metric space.

- A function $g \colon M \to R$ is said to be lower semi-continuous, if for any $y_i \subset M$ and $y \in M, y_i \to y$ implies $g(y) \leq lim\, Sup_{i \to \infty} g(y_i)$.
- A function $g \colon M \to R$ is said to be upper semi-continuous, if for any $y_i \subset M$ and $y \in M, y_i \to y$ implies $g(y) \geq lim\, Sup_{i \to \infty} g(y_i)$.
- A multi-valued mapping $G \colon M \to 2^y$ (2^y is the collection of all nonempty subsets of a set M) is called upper semi-continuous, if for any $y \in M$ and a neighborhood B of $G(y)$, there is a neighborhood A of y such that, for any $t \in A, G(t) \subset B$.
- A multi-valued mapping $G \colon M \to 2^y$ is called lower semi-continuous, if for any $y \in M$ and a neighborhood B , $G(y) \cap B \neq \emptyset$, there is a neighborhood A of y such that, for any $t \in A, G(t) \cap B \neq \emptyset$.

Definition 2.7: Let $(M, Fp, \odot)$ be a Fuzzy Partial Metric space, $y \in M$and $(y_i)_{i \in N}$is a sequence in M. Then

1. a subset $A \subseteq M$ is closed, if for every convergent sequence $(y_i)\ in\ A$ such that $y_i \to y$, for $y \in A$.

2. a subset $A \subseteq M$ is compact if every sequence in A has a convergent subsequence in A.

Remark: In this research paper, $Ķ(M)$ represents the set of all compact subset of a set M and $P(M)$ represents the set of all non-empty subsets of a set M.

Lemma2: Let $(M, Fp, \odot)$ be a Fuzzy Partial Metric space. Then Fp is continuous on $M \times M \times (0, \infty)$,

$$\forall\, u > 0 \in (0, \infty).$$

Lemma3: Let $(M, Fp, \odot)$ be a Fuzzy Partial Metric space such that $\odot_{j=i}^{\infty} Fp(y, z, u\, h^{j}) = 1, \ \forall\, y, z \in M, u > 0, h > 1$

Let (yi) be a sequence in M such that $Fp(y_{i,} y_{i+1}, \alpha u) \geq Fp(y_{i-1,} y_i, u)$, for all $i \in N, \alpha \in (0,1)$. Then (yi) is a Cauchy sequence in M.

Proof: Given that (yi) is a sequence in M such that $Fp(y_{i,} y_{i+1}, \alpha u) \geq Fp(y_{i-1,} y_i, u)$,

for all $i \in N, \alpha \in (0,1), u > 0$.

$$\Rightarrow Fp(y_{i,} y_{i+1}, u) \geq Fp\left(y_{0,} y_1, \frac{1}{\propto^{i}} u\right)$$

Choose a constant $h > 1$& $l \in N$ such that $h \propto < 1$ and $\sum_{j=0}^{\infty} \frac{1}{h^{j}} < 1$, that is $\sum_{j=l}^{\infty} \frac{\frac{1}{b^{l}}}{1-\frac{1}{b}} < 1$.

For $k \geq i$ & $u > 0$

Then $Fp(y_{i,} y_k, u) \geq Fp\left(y_{i,} y_k, \left(\frac{1}{h^{l}} + \frac{1}{h^{l+1}} + \cdots + \frac{1}{h^{l+k}}\right) u\right)$

$$\geq Fp\left(y_{i,} y_{i+1}, \frac{1}{h^{l}} u\right) \odot Fp\left(y_{i+1,} y_{i+2}, \frac{1}{h^{l+1}} u\right) \odot \ldots\ldots\ldots. \odot Fp\left(y_{k-1,} y_k, \frac{1}{h^{l+k}} u\right)$$

$$\geq Fp\left(y_{0,} y_1, \frac{1}{\propto^{i-1} h^{l}} u\right) \odot Fp\left(y_{0,} y_1, \frac{1}{\propto^{i} h^{l+1}} u\right) \odot \ldots\ldots\ldots. \odot Fp\left(y_{0,} y_1, \frac{1}{\propto^{k-2} h^{l+k-i-2}} u\right)$$

$$\geq Fp\left(y_0, y_1, \frac{1}{(\propto h)^{i-1}} u\right) \odot Fp\left(y_0, y_1, \frac{1}{(\propto h)^{i}} u\right) \odot \ldots\ldots. \odot Fp\left(y_0, y_1, \frac{1}{(\propto h)^{k-2}} u\right)$$

Since $\odot_{j=i}^{\infty} Fp(y, z, u\, h^{j}) = 1$, for all $y, z \in M, u > 0, h > 1$

Therefore $Fp(y_{i,} y_k, u) \ \odot_{j=i}^{\infty} Fp\left(y_0, y_1, \frac{1}{(\propto h)^{j-1}} u\right) = 1$

$$Fp(y_{i,} y_k, u) = 1 \text{ as } i \to \infty$$

Thus, (yi) is a Cauchy sequence in M.

Lemma4: Let $(M, Fp, \odot)$ be a Fuzzy Partial Metric space. Then for every $y \in M$, $A \in Ķ(M)$ and $u > 0$ there exists $a_0 \in A$ such that $F_p\,(y, A\,, u) = F_p\,(y, a_0\,, u)$

Proof: Since $(M, Fp, \odot)$ be a Fuzzy Partial Metric space.

Let $y \in M$, $A \in Ķ(M)$ and $u > 0$, By lemma (2), $z \to F_p\,(y, z\,, u)$ is continuous.

Since A is Compact. Then $\exists a_0 \in A$ such that $Sup_{a\in A} F_p(y, a, u) = F_p(y, a_0, u)$

$$\Rightarrow F_p(y, A, u) = F_p(y, a_0, u)$$

Definition 2.8: Let $(M, Fp, \odot)$ be a Fuzzy Partial Metric space. Then we define a Hausdorff Fuzzy Partial Metric space Fp_H on $Ķ(M)\ X\ Ķ(M)\ X\ (0.\infty)$ by

$Fp_H(A, B, u) = min\{ inf_{b\in B} F_p(A, b, u), inf_{a\in A} F_p(a, B, u)\}$, for all $A, B \in Ķ(M)$, $u > 0$.

3. Methods: Fixed Point Theorems for Multivalued Contractive Mapping in Fuzzy Partial Metric Spaces

In following theorems, we present fixed point results for multivalued contractive mapping in Fuzzy Partial Metric Spaces. Let multi-valued mapping be $G: M \rightarrow 2^y$. We define $g(y) = Fp(y, G(y), u)$, for $u > 0$

We define a set $I_\propto^y = \{ z \in G(y)/Fp(y, z, u) \geq Fp(y, G(y), \alpha u)\}$, for $\alpha\epsilon(0,1)$.

Theorem3.1: Let $(M, Fp, \odot)$ be a Complete Fuzzy Partial Metric space & multi-valued Contractive mapping be $G : M \rightarrow Ķ(M)$. If there exists a constant $b\epsilon(0,1)$ such that for any $y \in M$, there is $z \in I_\propto^y$ satisfying $Fp(z, G(z), bu) \geq Fp(y, z, u) \odot Fp(z, G(y), u)$, for $u > 0$, $\alpha\epsilon(0,1)$.

Assume that $(M, Fp, \odot)$ verifies lemma (3), for some $y_0 \in M$. Then G has a fixed point in M, provided $b < \alpha$ and g is upper semi-continuous.

Proof: Since $G(y) \in Ķ(M)$. By lemma (4), $I_\propto^y$ is non-empty for all $y \in M$ and $\alpha\epsilon(0,1)$

Let $y_0 \in M$ be arbitrary, there exists $y_1 \in I_\propto^{y_0}$, that is $y_1 \in G(y_0)$ satisfying

$Fp(y_1, G(y_1), bu) \geq Fp(y_0, y_1, u) \odot Fp(y_1, G(y_0), u) \geq Fp(y_0, y_1, u)$, for $u > 0$

Similarly, for $y_1 \in M$, there exists $y_2 \in I_\propto^{y_1}$, that is $y_2 \in G(y_1)$ satisfying

$Fp(y_2, G(y_2), bu) \geq Fp(y_1, y_2, u) \odot Fp(y_2, G(y_1), u) \geq Fp(y_1, y_2, u)$, for $u > 0$

Continuing in this way, we get a sequence $(y_n)_{n\geq 0}$ in M such that there exists $y_{n+1} \in I_\propto^{y_n}$,

that is $y_{n+1} \in G(y_n)$ satisfying

$Fp(y_{n+1}, G(y_{n+1}), bu) \geq Fp(y_n, y_{n+1}, u) \odot Fp(y_{n+1}, G(y_n), u) \geq Fp(y_n, y_{n+1}, u)$, for $u > 0$

But $y_{n+1} \in I_\propto^{y_n}$, we get

$Fp(y_n, y_{n+1}, u) \geq Fp(y_n, G(y_n), \propto u)$, for $u > 0$

Therefore, $Fp(y_{n+1}, G(y_{n+1}), bu) \geq Fp(y_n, G(y_n), \propto u)$, for $u > 0$

$Fp(y_{n+1}, G(y_{n+1}), u) \geq Fp(y_n, G(y_n), \frac{\propto}{b} u)$, for $u > 0$

Let $h = \frac{b}{\alpha}$. Then for $u > 0, n \in N$ and $h \in (0,1)$

$$Fp(y_n, y_{n+1}, u) \geq Fp\left(y_{n-1}, y_n, \frac{1}{h} u\right) \geq Fp\left(y_{n-2}, y_{n-1}, \frac{1}{h^2} u\right) \geq \cdots \ldots \ldots \geq Fp\left(y_0, y_1, \frac{1}{h^n} u\right)$$

Choose a constant $k > 1$ such that $hk < 1$ and $\sum_{i=0}^{\infty} \frac{1}{k^i} < 1$, that is $\sum_{i=n}^{j-1} \frac{1}{k^i} < 1$.

Then for all $j > n$, we get $u\left(\frac{1}{k^n} + \frac{1}{k^{n+1}} + \cdots + \frac{1}{k^{j-2}} + \frac{1}{k^{j-1}}\right) < u$, where $n, j \in N$.

Then $Fp(y_n, y_j, u) \geq Fp(y_n, y_j, u\left(\frac{1}{k^n} + \frac{1}{k^{n+1}} + \cdots + \frac{1}{k^{j-2}} + \frac{1}{k^{j-1}}\right))$

$\geq Fp\left(y_n, y_{n+1}, \frac{1}{k^n}u\right) \odot Fp\left(y_{n+1}, y_{n+2}, \frac{1}{k^{n+1}}u\right) \odot \ldots \ldots . \odot Fp\left(y_{j-1}, y_j, \frac{1}{k^{j-1}}u\right)$

$\odot_{i=n}^{\infty} \; Fp\left(y_0, y_1, \frac{1}{(kh)^n}u\right)$, for $u > 0$

By lemma (3), we have $\lim_{n,m\to\infty} Fp(y_n, y_m, u) = 1$, that is (yn) is a Cauchy sequence in M.

Since M is Complete Fuzzy Partial Metric Space. Then there exists $y \in M$ such that $\lim_{n\to\infty} y_n = y$.

Therefore, it is clear that $g(y_n) = Fp(y_n, G(y_n), u)$ is non-deceasing function and converges to 1.

Since g is upper semi-continuous function. Then we get

$1 = Sup_{i\to\infty} g(y_i) \leq g(y) \leq 1$. That is $g(y) = 1$. Hence $Fp(y, G(y), u) = 1$

Hence by using lemma (3), we get $y \in G(y)$. Hence it is proved.

Theorem3.2: Let $(M, Fp, \odot)$ be a Complete Fuzzy Partial Metric space & multi-valued mapping be $G : M \to Ꝁ(M)$. If there exists a constant $b\epsilon(0,1)$ such that for any $y \in M$, there is $z \in I_{\propto}^{y}$ satisfying $Fp(z, G(z), bu) \geq Fp(m, z, u)$, for $u > 0$, $\alpha\epsilon(0,1)$.

Assume that $(M, Fp, \odot)$ verifies lemma (3), for some $y_0 \in M$. Then G has a fixed point in M, provided $b < \alpha$ and g is upper semi-continuous.

Proof: Since $G(y) \in Ꝁ(M)$. By lemma (4), $I_{\propto}^{y}$ is non-empty for all $y \in M$ and $\alpha\epsilon(0,1)$

Let $y_0 \in M$ be arbitrary, there exists $y_1 \in I_{\propto}^{y_0}$, that is $y_1 \in G(y_0)$ satisfying

$Fp(y_1, G(y_1), bu) \geq Fp(y_0, y_1, u)$, for $u > 0$

Similarly, for $y_1 \in M$, there exists $y_2 \in I_{\propto}^{y_1}$, that is $y_2 \in G_{y_1}$ satisfying

$Fp(y_2, G(y_2), bu) \geq Fp(y_1, y_2, u)$, for $u > 0$

Continuing in this way, we get a sequence $(y_n)_{n\geq 0}$ in M such that there exists $y_{n+1} \in I_{\propto}^{y_n}$, that is $y_{n+1} \in G(y_n)$ satisfying $Fp(y_{n+1}, G(y_{n+1}), bu) \geq Fp(y_n, y_{n+1}, u)$, for $u > 0$

But $y_{n+1} \in I_{\propto}^{y_n}$, we get $Fp(y_n, y_{n+1}, u) \geq Fp(y_n, G(y_n), \propto u)$, for $u > 0$

Therefore, $Fp(y_{n+1}, G(y_{n+1}), bu) \geq Fp(y_n, G(y_n), \propto u)$, for $u > 0$

$Fp(y_{n+1}, G(y_{n+1}), u) \geq Fp(y_n, G(y_n), \frac{\propto}{b}u)$, for $u > 0$

Let $h = \frac{b}{\alpha}$, Then for $u > 0$, $n \in N$ and $h \in (0,1)$

$$Fp(y_n, y_{n+1}, u) \geq Fp\left(y_{n-1}, y_n, \frac{1}{h}u\right) \geq Fp\left(y_{n-2}, y_{n-1}, \frac{1}{h^2}u\right) \geq \cdots \ldots \ldots . \geq Fp\left(y_0, y_1, \frac{1}{h^n}u\right)$$

Choose a constant $k > 1$ such that $hk < 1$ and $\sum_{i=0}^{\infty} \frac{1}{k^i} < 1$, that is $\sum_{i=n}^{j-1} \frac{1}{k^i} < 1$.

Then for all $j > n$, we get $u\left(\frac{1}{k^n} + \frac{1}{k^{n+1}} + \cdots + \frac{1}{k^{j-2}} + \frac{1}{k^{j-1}}\right) < u$, where $n, j \in N$.

Then $$Fp(y_n, y_j, u) \geq Fp(y_n, y_j, u\left(\frac{1}{k^n} + \frac{1}{k^{n+1}} + \cdots + \frac{1}{k^{j-2}} + \frac{1}{k^{j-1}}\right))$$

$$\geq Fp\left(y_n, y_{n+1}, \frac{1}{k^n}u\right) \odot Fp\left(y_{n+1}, y_{n+2}, \frac{1}{k^{n+1}}u\right) \odot \ldots \ldots . \odot Fp\left(y_{j-1}, y_j, \frac{1}{k^{j-1}}u\right)$$

$$\odot_{i=n}^{\infty} \ Fp\left(y_0, y_1, \frac{1}{(kh)^n}u\right), \quad \text{for } u > 0$$

By lemma (3), we have $\lim_{n,m\to\infty} Fp(y_n, y_m, u) = 1$, that is (yn) is a Cauchy sequence in M.

Since M is Complete Fuzzy Partial Metric Space. Then there exists $y \in M$ such that $\lim_{n\to\infty} y_n = y$.

Therefore, it is clear that $g(y_n) = Fp(y_n, G(y_n), u)$ is non-deceasing function and converges to 1.

Since g is upper semi-continuous function. Then we get

$1 = lim\,Sup_{i\to\infty} g(y_i) \leq g(y) \leq 1$. That is $g(y) = 1$. Hence $Fp(y, G(y), u) = 1$

Hence by using lemma (3), we get $y \in G(y)$. Hence proved.

Corollary 3.3: Let $(M, Fp, \odot)$ be a Complete Fuzzy Partial Metric space & multi-valued mapping be $G : M \to Ꝁ(M)$. If there exists a constant $b\epsilon(0,1)$ such that

$$Fp_H(G(m), G(n), bu) \geq Fp(m, n, u), \text{ for any } m\ , n \in M \ \& \ u > 0.$$

Assume $(M, Fp, \odot)$ verifies lemma (3), for some $m_0 \in M$ & $m_1 \in G(m_0)$. Then G has a fixed point in M.

Example 3.4: Let $M = \left\{\frac{1}{5}, \frac{1}{25}, \ldots \ldots \ldots .., \frac{1}{5^j}, \ldots \ldots\right\} \cup \{0,1\}$ and Fuzzy Partial metric

$Fp : M \times M \times (0, \infty) \to [0,1]$ be defined as $F_p(m, n, u) = \frac{u}{u+p(m,n)}$,

where $p(m, n) = |m - n|, \forall m, n \in M, u > 0$.

Then $(M, Fp, \odot)$ be a Complete Fuzzy Partial Metric space, where $\odot$: $[0,1] \times [0,1] \to [0,1]$

is defined as $p \odot q = pq$.

Let multi-valued Contractive mapping $G : M \to Ꝁ(M)$ be defined as $G(m) = \{\left\{\frac{1}{5^j}, 1\right\}, if\ m = \frac{1}{5^j}, for\ j \geq 0\ \ \left\{0, \frac{1}{5}\right\},\ if\ m = 0$

Since $\odot_{i=j}^{\infty} \ Fp(m, n, u\,h^i) = Fp\left(\frac{1}{5^j}, 0, u\,h^i\right) = \odot_{i=j}^{\infty} \ \frac{uh^i}{uh^i + \frac{1}{5^j}} = 1$

Thus, G satisfies lemma (3). Hence

$$Fp_H\left(G(\tfrac{1}{5^j}), G(0), bu\right) = \frac{bu}{bu+H(G\left(\frac{1}{5^j}\right), G(0)} = \frac{bu}{bu + \frac{1}{5}}$$

$$\& \ Fp_H\left(\frac{1}{5^j}, 0, u\right) = \frac{u}{u + p\left(\frac{1}{5^j}, 0\right)} = \frac{u}{u + \frac{1}{5^j}}$$

Therefore, there does not exist any $b\epsilon(0,1)$ such that corollary (1) is satisfied. If it exists,

then we get $\frac{u}{u+\frac{1}{5^j}} \leq \frac{bu}{bu+\frac{1}{5}}$. This implies that $b \geq 5^{j-1}$, which is a contradiction.

Otherwise $g(m) \ = F_p(m, G(m), u) = \frac{u}{u+p(m,G(m))} = \{\frac{1}{5^{j+1}}, \quad if\ m = \frac{1}{5^j}\ 0, \qquad if\ m = 0$

is continuous.

Then there exists $z \in I^{m}_{\frac{5}{6}}$ for any m such that $p(z, G(z)) = \frac{1}{5}\ p(m, z) \leq \frac{4}{5}\ p(m, z)$

$$\frac{5}{4} p(z, G(z)) \leq \ p(m, z)$$

Hence there exists $b \ = \frac{4}{5} < \frac{5}{6}$ such that $F_p\left(z, G(z), \frac{4}{5}u\right) = \frac{\frac{4}{5}u}{\frac{4}{5}u+p(z,G(z))} = \frac{u}{u+\frac{5}{4}p(z,G(z))} \geq \frac{u}{u+p(m,z)} = F_p(m, z, u)$

Then the existence of fixed point follows theorems (2).

3. Conclusion

Here defined multivalued Contractive mapping in complete Fuzzy Partial Metric Spaces. In this research paper, we proved Fixed point theorems for multivalued contractive mapping using upper semi- continuous function & illustrate the related example. Fuzzy Partial Metric spaces provide a suitable framework for modeling uncertainty and impression. Multivalued contractive mapping is useful for reducing the distance between points in Fuzzy Partial Metric Spaces. The application of fixed-point theorems for multivalued contractive mappings in fuzzy partial metric spaces is an important area of research, with many potential applications.

The image compression algorithm used in JPEG compression:

This is real life example of Fuzzy Partial Metric Space. The set of all possible images is a fuzzy partial metric space, where the distance between two images measures their similarity. The compression algorithm is a multivalued contractive mapping, reducing the distance between images. The compressed image is a fixed point of the algorithm, remaining unchanged when the algorithm is applied. The fixed-point theorem ensures the algorithm converges to a stable solution, the compressed image.

Future Scope: 1. Developing new fixed-point theorems for multivalued contractive mappings in fuzzy partial metric spaces, including theorems for different types of contractive conditions.

2. Applying fixed point theorems to solve problems in various fields, such as mathematics, computer science, and engineering.

3. Extending fixed point theorems to other types of metric spaces, such as intuitionistic fuzzy metric spaces and bipolar fuzzy metric spaces.

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