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Article

DESIGN AND OPTIMIZATION OF A FUZZY CHAIN SAMPLING PLAN FOR TEXTILE QUALITY CONTROL

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Abstract: Quality control is essential in the textile industry, as defects in fabrics, yarns, dyeing, and finishing directly affect production efficiency and customer satisfaction. This study proposes a fuzzy chain sampling plan to handle the variability commonly found in textile processes. A case study has been carried over the data obtained from original inspection reports by applying fuzzy chain sampling techniques for varying: sample sizes, number of defectives, acceptance number and chain lengths. The defect proportion is modeled as a triangular fuzzy number, and α-cut decomposition is used to generate fuzzy acceptance intervals. By using single-sampling plan acceptance probabilities are computed through the Poisson distribution for fuzzy acceptance intervals. A decision rule is chosen by selecting an appropriate defuzzification or fuzzy decision method. One option is to require that the centroid value of the chain acceptance probability exceeds a chosen threshold T. Chain acceptance is determined based on consecutive acceptable samples. The plan is optimized by adjusting the chain length and acceptance number to balance producer's and consumer's risks while minimizing inspection effort. The highest probability of acceptance when the defect proportion lies roughly between 5% and 8% is also determined. The resulting plan offers a more flexible and reliable approach to quality assessment in textile manufacturing.

Keywords: Sampling Plan, Fuzzy Probability Theory, Operating Characteristic Curve, Quality Control

1. Introduction

Quality control in the textile industry is critical, because defects (in fabrics, yarns, dyeing, finishing etc.) can strongly affect downstream processes and end customer satisfaction. Combining chain sampling plan with fuzzy probability theory allows more realistic, flexible quality control, especially in textile manufacturing where materials and processes have inherent variability. In this paper a case study using chain sampling plan and fuzzy probability theory have been carried over a number of inspection reports which has used textile quality control standards for inspection.

To design a fuzzy chain sampling plan, the sampling units are obtained from the inspection report and the acceptance rule for a single sample is being considered. For each sample, the acceptance probability is calculated using the Poisson cumulative distribution $P(\text{defects} \leq c)$ [1].

The chain rule is specified to determine how historical information is used, to accept a lot only when both the current sample and the previous $'i'$samples each contain no more than $'c'$ defectives. Assuming independence, the chain acceptance probability for a defect proportion p is obtained because the plan requires $i + 1$ consecutive acceptable samples.

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The defect proportion is then fuzzified by defining it as a triangular fuzzy number $\tilde{p} = (a1, a2, a3)$. Using α-cut decomposition, the fuzzy parameter is converted into an interval for each $\alpha \in [0,1]$[13].

For each value of p in this interval, the single-sample acceptance probability is computed, and the chain acceptance probability is obtained as $[P\text{sample } (p)]i+1$, producing for each α-level a fuzzy acceptance interval $[Pa,\min(\alpha), Pa,\max(\alpha)]$ [4].

A decision rule is chosen by selecting an appropriate defuzzification or fuzzy decision method. One option is to require that the centroid (or modal) value of the chain acceptance probability exceeds a chosen threshold T(such as 0.4 or 0.9). A more conservative option is to require that the lower bound at a selected α-level (such as $\alpha = 0.5$) meets the threshold. Alternatively, other fuzzy decision rules, such as maxi-min criteria or α-cut acceptance rules, may be used.

Finally, the plan is optimized by varying the chain length $'i'$ and possibly the acceptance number $'c'$ to satisfy producer's and

consumer's risk requirements or to minimize inspection cost. By computing the expected number inspected or the associated inspection cost for different combinations of (i, c), the most efficient and balanced plan can be selected.Alsothe highest probability of acceptance when the defect proportion lies roughly between 5% and 8%is determined.

2. Literature Review:

Ezzatallah Baloui Jamkhaneh and Bahram Sadegpour Gildeh (2011) have argued on acceptance sampling plan when the fraction of defective products is a fuzzy number and it is being modeled based on the fuzzy Poisson distribution [1] [2] [3] [4] [5]. V.Sangeetha and K.S Karunya (2022) studied and compared the two stage chain sampling plan's operating characteristics curve to a fuzzy two stage chain sampling curve [15]. An attribute chain sampling

plan design process is presented in this article by K. Rebecca Jebaseeli Edna and V.Jemmy Joyce (2013) [12].Nandhni Devi and G.Uma (2018), used a Poisson distribution with OC curves of various sample sizes and a set acceptance number to examine the effects of fuzzy on Quick Switching Systems [11]. In the year (2020), V. Jemmy Joyce K. Rebbeca Jebaseeli Edna developed an acceptance modified chain sampling plan , when the fraction defective (p) is a fuzzy number [13]. Julia T. Thomas and Mahesh Kumar (2022), developed a fuzzy EOQ model in the presence of inspection errors in single sampling plans [8].The proportion of fraction of defectives using fuzzy techniques, α cut intervals and determining the acceptance probability has been taken from the above citations with modifications.

2.1 Statistical Quality Control (SQC): It is a statistical method for determining the extent to which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur are exceeding normal expectations.

2.2 Acceptance Sampling: Acceptance sampling is a branch of statistical science that deals with techniques for deciding whether to accept or reject a lot depending on the findings of sample inspection. An acceptance-sampling scheme is defined as a specific set of procedures that typically consists of acceptance sampling plans in which lot sizes, sample sizes, and acceptance criteria or the amount of 100% inspection and sampling are related.

2.3 Chain Sampling Plan: The Chain Sampling Plan is a sampling procedure in which a lot may be accepted based on the number of defectives in the current sample and the acceptance history of the preceding samples.

2.4 Fuzzy Chain Sampling Plan: This plan is a sampling procedure in which acceptance and rejection of a lot is determined using fuzzy number or fuzzy membership function and the decisions depend on the previous samples of the chain.

2.5 Operating Characteristic Curve (OC curve): It represents the relationship between the probabilities of accepting a lot Pa and the proportion of defectives 'p' in the lot.

3. Methodology

a. **Decide the sampling unit & acceptance rule for the sample**

Sample size is n and acceptance number is c.

For one sample the sample acceptance probability is the Poisson CDF $P(\text{defects} \leq c)$, for small p with $\lambda = np$:

$$P_{\text{sample}}(p) \approx \frac{e^{-\lambda}\lambda^{x}}{x!}, \lambda = np.[3]$$

Fuzzify the Defect Proportion using Statistically principled approach Fuzzify p as a triangular fuzzy number $\tilde{p} = (a1, a2, a3)$.

- a1 is the lowest expected defect rate
- a2 is the actual observed defect rate from proportion
- a3 is the possible upper limit if inspection variation or higher defect tendency occurs.[13] Compute pooled statistics:

Total defects = $d_{current} + d_{previous\,(1)} + d_{previous\,(2)}$

Total inspected = n * 3

p pool = Total defects / n*3 = a2 Standard error:

$$\text{S.E} = \sqrt{\frac{p\ pool\ (1-p\ pool)}{n*3}}$$

95% Confidence Interval for: z = 1.96, Lower CI = Max (0, p pool - z * 1.96) Upper CI = Min(1, p pool + z* 1.96)

Fuzzy fraction proportion of defectives are $\tilde{p} = (a_1, a_2, a_3)$

(a) α-cut decomposition is computed for each $\alpha \in [0,1]$ as α cut bounds

$\tilde{p}[\alpha] = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha]$.
Poisson parameter: is $\lambda_{\text{low}}(\alpha) = n \cdot p_{\text{low}}(\alpha), \lambda_{\text{high}}(\alpha) = n \cdot p_{\text{high}}(\alpha)$.

The fuzzy acceptance probability band is: $P_a(i)[\alpha] = [P_{a,\text{low}}(i)\ [\alpha], P_{a,\text{up}}(i)\ [\alpha]]$ [4]

(b) Decision rule: A defuzzification decision method, is used as follows

Centroid (defuzzified) p: $p_{\text{centroid}} = \frac{a_1+a_2+a_3}{3}, \lambda_{centroid} = n * p_{centroid}$
P_a centroid is computed using the Poisson distribution with (c, $\lambda_{\text{centroid}}$) as the parameters.
If centroid chain acceptance probability ≥ threshold the lot is accepted.

(c) Deciding the chain acceptance probability for defective proportion:

Accept lot only if the current sample and the previous $i$samples each individually show $\leq c$ defectives. Under independence, for eachp in this interval $P_{\text{sample}}(p)$ and then chain acceptance probability $P_{\alpha}^{(\text{chain})}(i; p) = [P_{\text{sample}}(p)]^{i+1}$. is computed.
For the α level an interval,
$[P_{\text{a,min}}(\alpha), P_{\text{,a,max}}(\alpha)] = P_{\alpha}(i)|\alpha| = \lceil P_{\alpha}^{(\text{chain})}(i; \lambda_{\text{high}}(\alpha)), P_{\alpha}^{(\text{chain})}(i; \lambda_{\text{low}}(\alpha)) \rceil$.
is obtained.

(d) Decision rule: A defuzzification/decision method, is used as follows Centroid (defuzzified) p:

$$\text{centroid} = \frac{a_1 + a_2 + a_3}{3} \quad P_{a,\text{centroid}}(i) = (P_{\text{sample}}(n \cdot p_{\text{centroid}}))^{i+1}$$

We require the **centroid** (or modal) chain acceptance probability ≥ threshold T(e.g. 0.4 or 0.9 depending on risk appetite),and the **lower bound at some α** (say α=0.4) ≥ threshold.

Based on Fuzzy Decision Theory

Membership value	**Meaning**
0.0 – 0.4	Rejection zone
0.4 – 0.6	Uncertain / Marginal
≥ 0.6	Acceptance zone

(e) Optimizing the plan:

By varying i and c to meet constraints producer's/consumer's risk, inspection cost we can compute expected number inspected or inspection cost for different (i, c)and pick the best tradeoff.

(f) Fuzzy Chain Sampling Plan to maximize acceptance probability

To determine the highest probability of acceptance when the defect proportion lies roughly between 5% and 8%**.** To maximize the acceptance a larger acceptance number c, and small chain length i is considered (i=0 gives maximum acceptance; i> 0 reduces acceptance as we require several consecutive acceptable samples), binomial cumulative probabilities are used to compute per-sample acceptance:

$P_{\text{sample}}(n,p,c) = \sum nC_x p^x (1-p)^{n-x}$

Chain acceptance: $P_{a,\text{chain}}(i;n,p,c) = (P_{\text{sample}}(n,p,c))^{i+1}$.

$c \approx \lceil n \times 0.08 + 2 \rceil$

n×0.08n times is the **expected number of defects** at the worst-case 8% rate; adding a small margin (+2) makes the plan permissive enough to keep acceptance high across 5% – 8%. And we can use chain length i= 0 for maximum acceptance.

3. Secondary Data Obtained From Inspection Reports for Case Study TABLE1.

Report No.	Description	Inspection Date	Offer & Order Quantity	Inspection Quantity	Final Inspection Plan				Major Defects	Minor Defects	Inspection Result
					Sample Size as per AQL Level II						
					Preferred Lot Size	Sample Size	Major Pass	Minor Pass			
2154	T-Shirt	14.11.24	1969/2003	125	1201-3200	125	7	10	7	9	Pass
1868	T-Shirt	15.11.24	2820/3000	125	1201-3200	125	7	10	7	8	Pass
1879	T-Shirt	19.11.24	1911/2003	125	1201-3200	125	7	10	7	7	Pass
2196	T-Shirt	26.11.24	2469/2505	125	1201-3200	125	7	10	7	8	Pass
1904	T-Shirt	29.11.24	2875/3000	125	1201-3200	125	7	10	7	9	Pass
2160	T-Shirt	15.11.24	1977/2003	125	1201-3200	125	7	10	7	9	Pass
AW25	T-Shirt	19.11.25	1538/1631	125	1201-32030	125	5	5	4	4	Pass

In the above inspection reports sample sizes are n = 80, 125, 200, acceptance numbers for n = 80 are c = 3 and 5(major defect), c= 3 and 7(minor defect), for n= 125 it is taken as 5 and 7(major defect), 5 and 10 (minor defect), for n = 200 it is taken as 3 and 7(major defect), 3 and 7 (minor defect). Chain length is taken as i =2 as per the acceptance quality level in the industry.

3. (a) Evaluation of Sample Inspection data and acceptance probability with reference to the data as in TABLE 1.

In the above inspection reports for sample size n = 125, considering the acceptance values as 7 and 10, and the current lot major defect as 7 and minor defect as 8, for previous lot major defective dprevious(1) is 5 and minor defective is 9 and in the earlier previous lot dprevious(2) majordefective is 9 and minor defective is 11(using average history), according to auditors statement 2%variation around average acceptable rate is allowed. Fuzzy defectives proportion are p: a1 =0.04, a2 = 0.056, a3 = 0.072, Chain length is taken as i=2, decision threshold Pa = 0.5 are the input parameters and single acceptance probabilities are obtained and the fuzzy decision rule is obtained initially. The same procedure is repeated for all the sample sizes with varying acceptance values in the following tables by considering the **major defects**.

TABLE 2.1n = 125, c = 7(major defect), a1 = 0.04, a2 = 0.056, a3 = 0.072

α	pa low	pa high	λa low	λahigh	Pa low	Pa high	Decision
0	0.04	0.072	5	9	0.866628326	0.323896964	Marginal
0.25	0.044	0.068	5.5	8.5	0.809485283	0.385597102	Marginal
0.5	0.048	0.064	6	8	0.74397976	0.452960809	Accept
0.75	0.052	0.06	6.5	7.5	0.67275778	0.598713836	Accept
1	0.056	0.056	7	7	0.598713836	0.598713836	Accept
Centroid	0.056	0.056	7	7	0.598713836	**0.598713836**	

TABLE 2.2n = 125, c = 5(major defect), a1 = 0.024, a2 = 0.04, a3 = 0.056

α	pa low	pa high	λa low	λahigh	Pa low	Pa high	Decision
0	0.024	0.056	3	7	0.916082058	0.300708276	Marginal
0.25	0.028	0.052	3.5	6.5	0.857613553	0.369040684	Marginal
0.5	0.032	0.048	4	6	0.785130387	0.445679641	Accept
0.75	0.036	0.044	4.5	5.5	0.702930435	0.528918687	Accept
1	0.04	0.04	5	5	0.615960655	0.615960655	Accept
Centroid	0.04	0.04			**0.61596065**	**0.61596065**	

TABLE 2.3n = 80, c = 3(major defect), a1 = 0.0125, a2 = 0.037, a3 = 0.0625

α	pa low	pa high	λa low	λa high	Pa low	Pa high	Decision
0	0.0125	0.0625	1	5	0.981011843	0.265025915	Marginal
0.25	0.0186	0.0561	1.49	4.49	0.93560637	0.343985946	Marginal
0.5	0.024	0.0497	1.98	3.98	0.860714237	0.437387192	Accept
0.75	0.0308	0.0433	2.47	3.47	0.763969361	0.543119884	Accept
1	0.037	0.037	2.96	2.96	0.656192759	0.656192759	Accept
Centroid	0.037	0.037			**0.65619275**	**0.65619275**	

TABLE 2.4n = 80, c = 5(major defect),, a1 = 0.0375, a2 = 0.0625, a3 = 0.0875

α	pa low	pa high	λa low	λahigh	Pa low	Pa high	Decision
0	0.0375	0.0875	3	7	0.916082058	0.300708276	Marginal
0.25	0.0437	0.0812	3.5	6.5	0.857613553	0.369040684	Marginal
0.5	0.05	0.075	4	6	0.785130387	0.445679641	Accept
0.75	0.0562	0.0687	4.5	5.5	0.702930435	0.528918687	Accept
1	0.0625	0.0625	5	5	0.615960655	0.615960655	Accept
Centroid	0.0625	0.0625	5	5	**0.61596065**	**0.61596065**	

TABLE 2.5n = 200, c = 7(major defect), a1 = 0.025, a2 = 0.035, a3 = 0.045

α	pa low	pa high	λa low	λahigh	Pa low	Pa high	Decision
0	0.025	0.045	5	9	0.866628326	0.323896964	Marginal
0.25	0.027	0.0425	5.5	8.5	0.809485283	0.385597102	Marginal
0.5	0.03	0.04	6	8	0.74397976	0.452960809	Accept
0.75	0.032	0.0375	6.5	7.5	0.67275778	0.524638526	Accept
1	0.035	0.035	7	7	0.598713836	0.598713836	Accept
Centroid	0.035	0.035	7	7	**0.598713836**	**0.598713836**	

TABLE 2.6n = 200, c = 3(major defect), a1 = 0.005, a2 = 0.015, a3 = 0.025

α	pa low	pa high	λa low	λa high	Pa_low	Pa_high	Decision
0	0.005	0.025	1	5	0.981011843	0.265025915	Marginal
0.25	0.0075	0.0225	1.5	4.5	0.934357546	0.342295956	Marginal
0.5	0.01	0.02	2	4	0.85712346	0.43347012	Accept
0.75	0.0125	0.0175	2.5	3.5	0.757576133	0.536632668	Accept
1	0.015	0.015	3	3	0.647231889	0.647231889	Accept
Centroid	0.015	0.015	3	3	**0.647231889**	**0.647231889**	

3. (a)Interpretation of data from the Tables 2.1 to 2.6:

From the above data at low α values ranging from (0 – 0.5) there is high uncertainty which leads to risky decisions. At high α value ranging from (0.75 – 1) there is low uncertainty and reliable decisions can be taken. Centroid value is around

0.6 and gives the most balanced and stable result.

The analysis shows that as the confidence parameter α increases, uncertainty in probability and rate parameters decreases, leading to stable acceptance probability. At α = 1 and at the centroid, the acceptance probability converges to around 0.6 in all the cases, which satisfies the acceptance criterion. Hence, the final decision is to accept the lot, indicating a reliable and optimal outcome.

3(b) Graphical Interpretation of Table 2.1 – 2.6

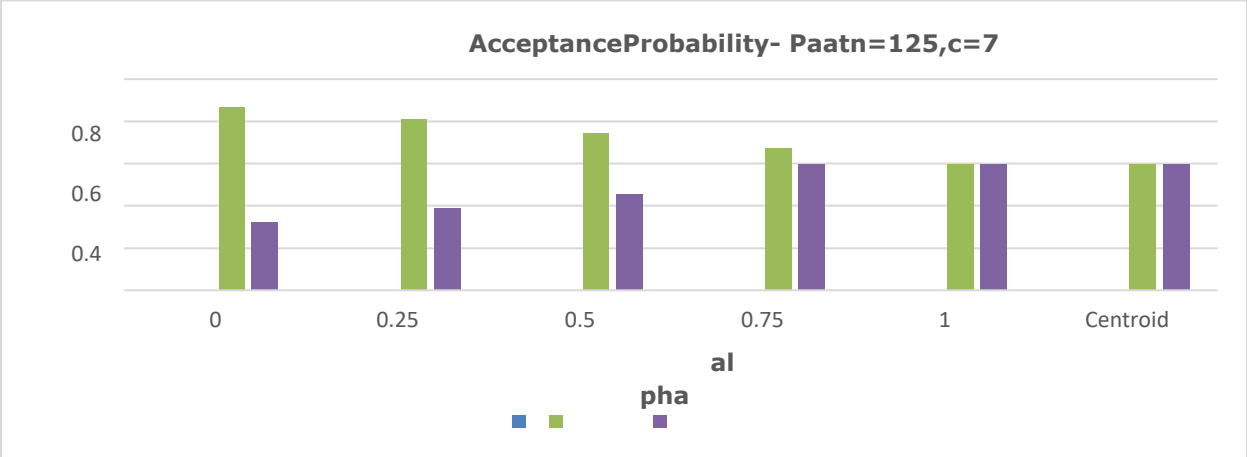


Fig 1.1-Acceptance Probability - Pa at n=125 , c =7

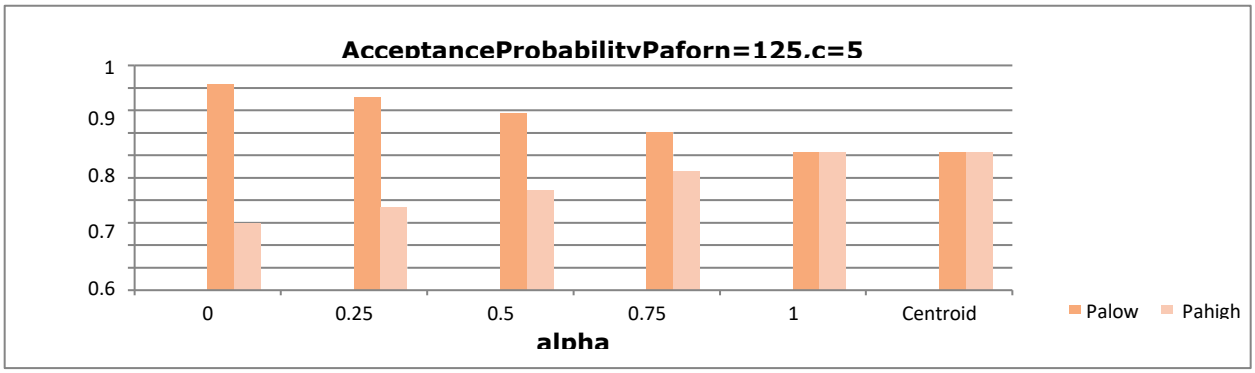


Fig 1.2- Acceptance Probability - Pa at n=125 , c =5

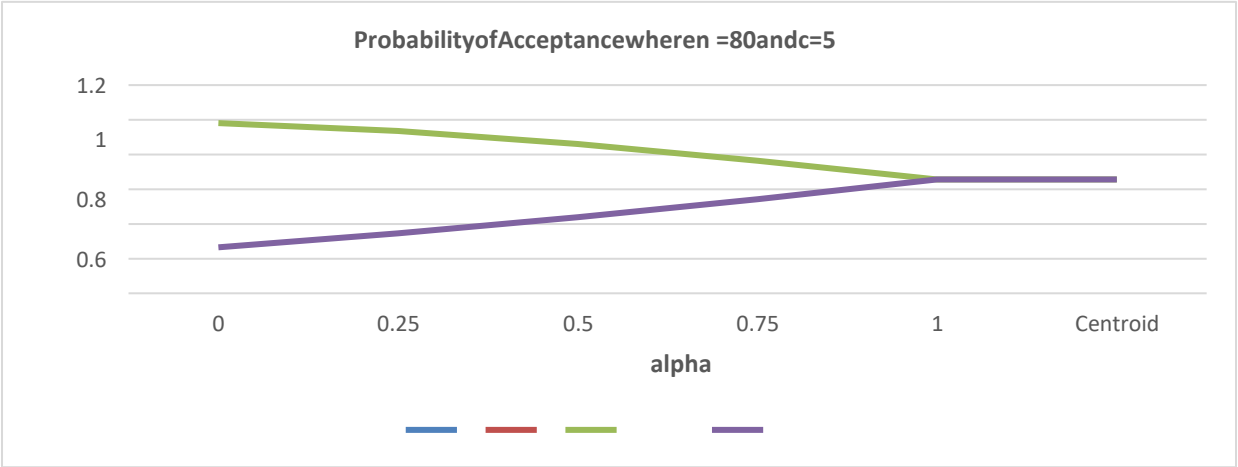


Fig 1.3-Acceptance Probability - Pa at n=80 , c =5

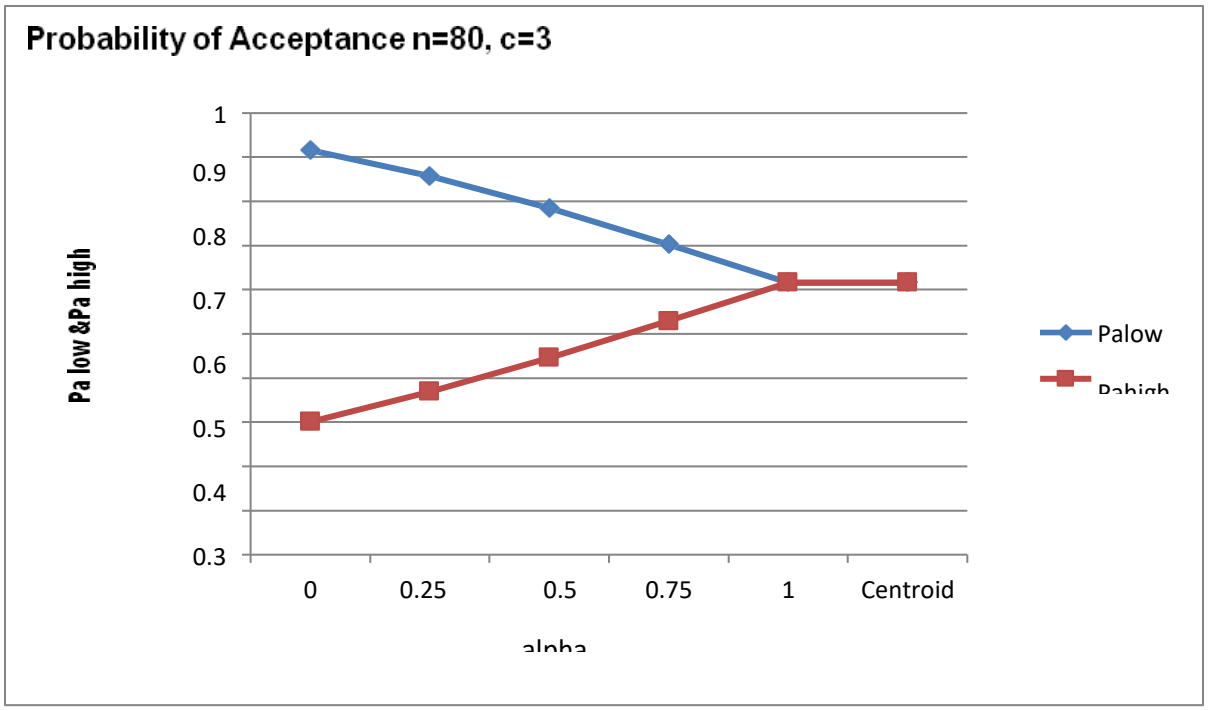


Fig 1.4-Acceptance Probability - Pa at n=80 , c =3

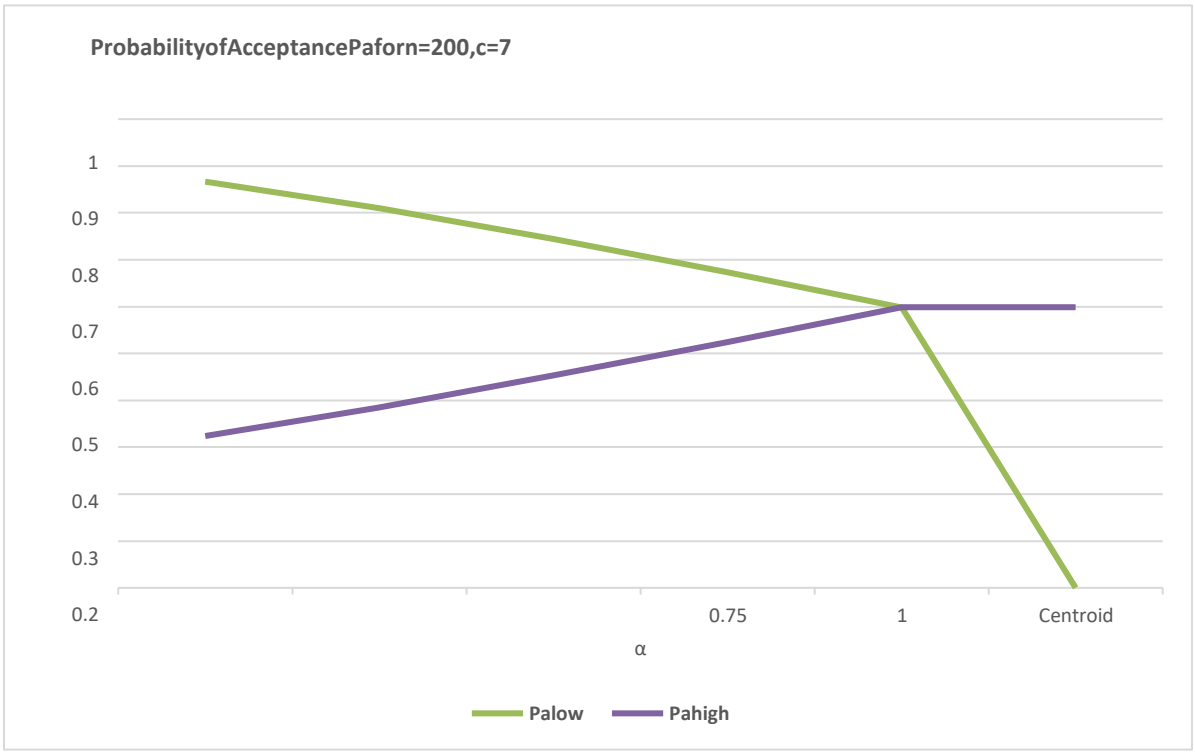


Fig 1.5 - Acceptance Probability - Pa at n=200 , c =7

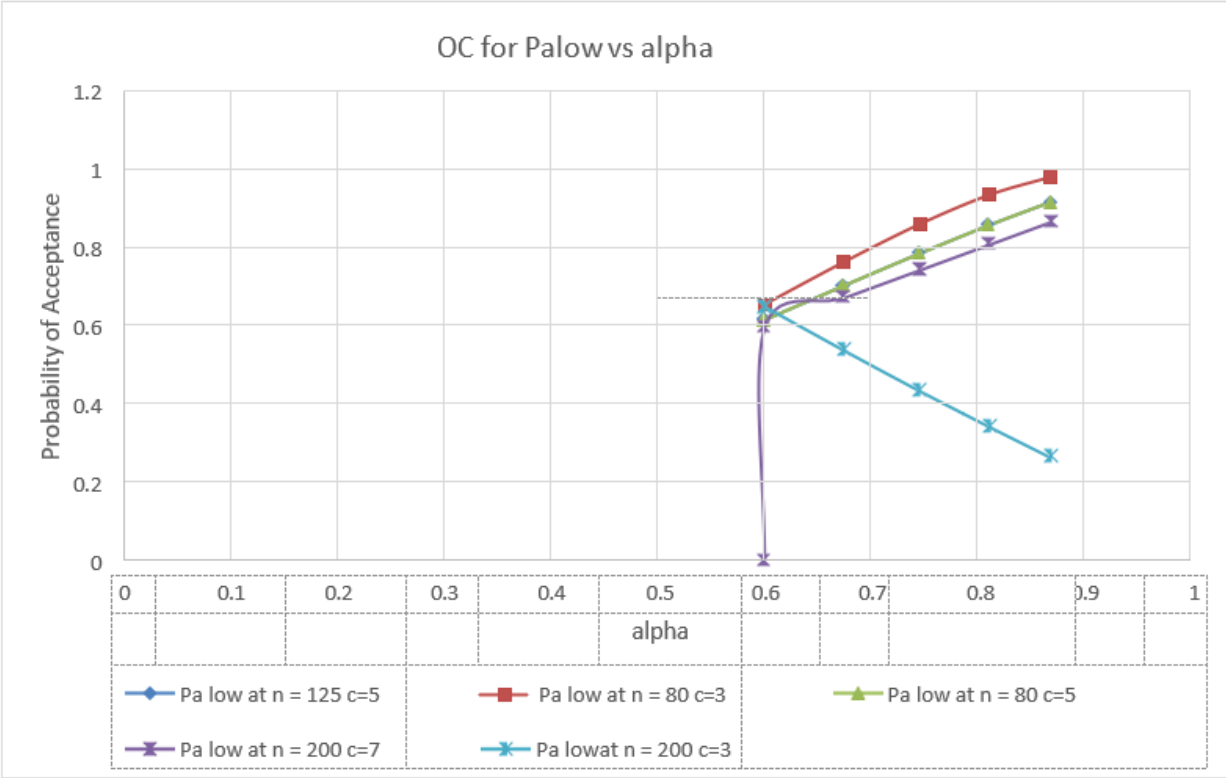


Fig 1.6 - Acceptance Probability - Pa at n=200 , c =3

Fig 1.7 - Acceptance Probability – OC curve for Pa versus alpha 3 (c) Consolidated Graphical interpretations (Fig.1.1 - 1.7)

The graphical representation illustrates the variation of acceptance probability with respect to the confidence parameter α. It is observed that the acceptance probability increases monotonically and converges to 0.6063 as α approaches unity for varying values of n and c. This convergence indicates stabilization of the decision process, validating the selection of 0.6 as the acceptance threshold. Hence, higher values of α ensure reliable and consistent decision-making.

Result:

The fuzzy chain sampling analysis confirms that the inspection result, pass aligns it with a quantitative evaluation. Despite a moderate defect rate, the chain history and fuzzy probability interval indicate controlled variability and acceptable process quality. Thus, the lot may be accepted under the Chain Sampling Plan with fuzzy probability reasoning, providing a balanced decision under uncertainty.

4. Deciding the chain probabilities:

Methodology to get chain probabilities

Choose the chain length i (0,1,2,3, ...).

1. For each p (e.g., 0.040, 0.048, 0.056, 0.064, 0.072), we compute Psample(p)
2. Compute chain acceptance: Pa,chain(i; p) =Psample(p) i+1

1TABLE 3:

α	P_a low	p_a high	n p_a low	n p_a high	P a @ p_a low	P a @ p_a high	Pa chain @ p_a low ^ (i+1)	Pa chain @ p_a high ^(i+1)	Pa chain @ $p_a$low^(i+2)	Pa chain @ $p_a$high^(i+2)
0	0.04	0.072	5	9	0.86662	0.32389	0.75104	0.10490	0.65087	0.03397
0.1	0.0416	0.0704	5.2	8.8	0.844921	0.34783	0.71389	0.12098	0.60318	0.04208
0.2	0.0432	0.0688	5.4	8.6	0.821658	0.37277	0.67512	0.13895	0.55472	0.05179
0.3	0.0448	0.0672	5.6	8.4	0.796975	0.39865	0.63516	0.15892	0.50621	0.06335
0.4	0.0464	0.0656	5.8	8.2	0.771025	0.42540	0.59448	0.18097	0.45836	0.07698
0.5	0.048	0.064	6	8	0.743979	0.45296	0.55350	0.20517	0.41179	0.09293
0.6	0.0496	0.0624	6.2	7.8	0.716016	0.48120	0.51267	0.23156	0.36708	0.11142
0.7	0.0512	0.0608	6.4	7.6	0.687321	0.51004	0.47241	0.26014	0.32469	0.13268
0.8	0.0528	0.0592	6.6	7.4	0.658082	0.53933	0.43307	0.29087	0.28499	0.15688
0.9	0.0544	0.0576	6.8	7.2	0.628485	0.56894	0.39499	0.32369	0.24824	0.18416
1	0.056	0.056	7	7	**0.598713**	0.59871	**0.35845**	0.358458	**0.21461**	0.21461

Considering per−sample Pa at p=0.056: Psample≈ 0.598713

For i=0, chain Pa = 0.5987138361^1 = 0.598713836 ≈ 0.59871 For i=1, chain Pa = 0.5987138362^2≈ 0.3584576 ≈ 0.35845 For i=2 chain Pa = 0.5987138363^3≈0.2146139≈0.21461

Interpretation from the table regarding chain values

The analysis shows that as the confidence parameter α increases, uncertainty in probability and rate parameters decreases, at **pa low** leading to stable acceptance probability and at **pa high** as α increases uncertainty increases. At i = 0 highest acceptance is , at centroid p = 5.6%. We can accept 59.9% of lots, at i =1, the centroid value is 0.358458257 a moderate acceptance of the lot is considerable with caution.

Graphical representation of Table 3:

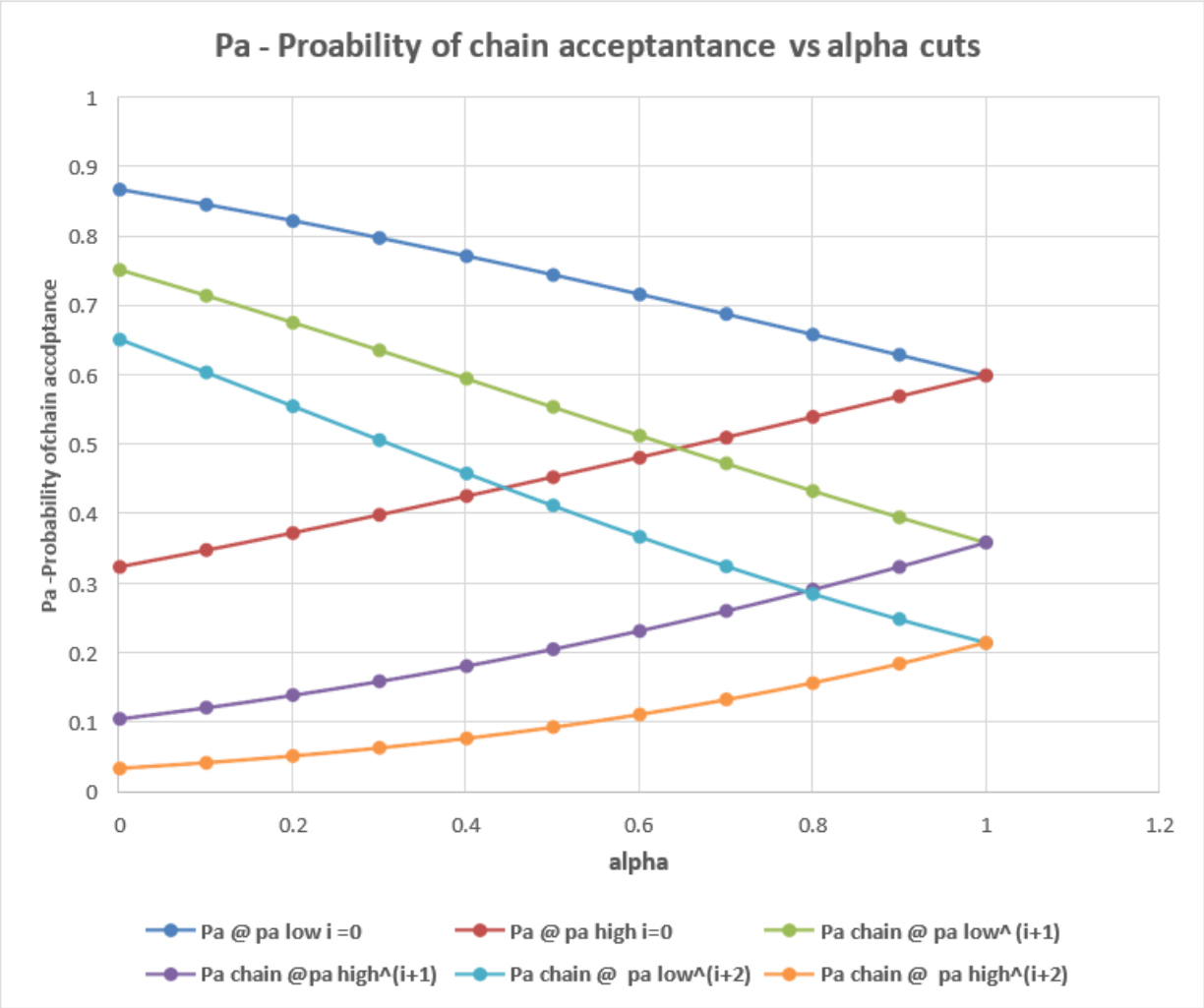


Fig 1.8-Pa - Probability of chain acceptantance vs alpha cuts

5. Application Fuzzy Chain Sampling at different combinations of chain length (i) and acceptance number (c), with the defect proportion modeled as a triangular fuzzy number at alpha equal to 0.5 of the data obtained from the inspection reports as in the following Table 5.

		a_1	a_2	a_3	Centroid p= (a1+a2 +a3)/3			P low = a1 + (a2 - a1)*0.5	P high = a3 - (a3 - a2)*0.5	Pa α 0.5 at plow	Pa α 0.5 at phigh	Pachain centroid	Pachain α0.5 low	Pachain α0.5 high	Meets Criteria

0	5	0.024	0.04	0.056	0.04	5	0.615	0.032	0.048	0.785	0.445	0.615	0.785	0.445	YES
0	6	0.032	0.048	0.064	0.048	6	0.606	0.04	0.056	0.762	0.449	0.606	0.762	0.449	YES
0	7	0.04	0.056	0.072	0.056	7	0.598	0.048	0.064	0.743	0.452	0.598	0.743	0.452	YES
0	8	0.048	0.064	0.08	0.064	8	0.592	0.056	0.072	0.729	0.455	0.592	0.729	0.455	YES
0	9	0.056	0.072	0.088	0.072	9	0.587	0.064	0.08	0.716	0.457	0.587	0.716	0.457	YES
0	10	0.064	0.08	0.096	0.08	10	0.583	0.072	0.088	0.705	0.459	0.583	0.705	0.459	YES
1	5	0.024	0.04	0.056	0.04	5	0.615	0.032	0.048	0.785	0.445	0.379	0.616	0.198	YES
1	6	0.032	0.048	0.064	0.048	6	0.606	0.04	0.056	0.762	0.449	0.367	0.580	0.202	YES
1	7	0.04	0.056	0.072	0.056	7	0.598	0.048	0.064	0.743	0.452	0.358	0.553	0.205	YES
1	8	0.048	0.064	0.08	0.064	8	0.592	0.056	0.072	0.729	0.455	0.351	0.531	0.207	YES
1	9	0.056	0.072	0.088	0.072	9	0.587	0.064	0.08	0.716	0.457	0.345	0.513	0.209	YES
1	10	0.064	0.08	0.096	0.08	10	0.583	0.072	0.088	0.705	0.459	0.339	0.498	0.21	YES
2	5	0.024	0.04	0.056	0.04	5	0.615	0.032	0.048	0.785	0.445	0.233	0.483	0.088	YES
2	6	0.032	0.048	0.064	0.048	6	0.606	0.04	0.056	0.762	0.449	0.222	0.442	0.090	YES
2	7	0.04	0.056	0.072	0.056	7	0.598	0.048	0.064	0.743	0.452	0.214	0.411	0.092	YES
2	8	0.048	0.064	0.08	0.064	8	0.592	0.056	0.072	0.729	0.455	0.208	0.387	0.094	NO
2	9	0.056	0.072	0.088	0.072	9	0.587	0.064	0.08	0.716	0.457	0.202	0.368	0.096	NO
2	10	0.064	0.08	0.096	0.08	10	0.583	0.072	0.088	0.705	0.459	0.198	0.351	0.097	NO
3	5	0.024	0.04	0.056	0.04	5	0.615	0.032	0.048	0.785	0.445	0.143	0.379	0.039	NO
3	6	0.032	0.048	0.064	0.048	6	0.606	0.04	0.056	0.762	0.449	0.135	0.337	0.040	NO
3	7	0.04	0.056	0.072	0.056	7	0.598	0.048	0.064	0.743	0.452	0.128	0.306	0.042	NO
3	8	0.048	0.064	0.08	0.064	8	0.592	0.056	0.072	0.729	0.455	0.123	0.282	0.043	NO
3	9	0.056	0.072	0.088	0.072	9	0.587	0.064	0.08	0.716	0.457	0.119	0.263	0.043	NO
3	10	0.064	0.08	0.096	0.08	10	0.583	0.072	0.088	0.705	0.459	0.115	0.248	0.044	NO
4	5	0.024	0.04	0.056	0.04	5	0.615	0.032	0.048	0.785	0.445	0.088	0.298	0.017	NO
4	6	0.032	0.048	0.064	0.048	6	0.606	0.04	0.056	0.762	0.449	0.081	0.257	0.018	NO
4	7	0.04	0.056	0.072	0.056	7	0.598	0.048	0.064	0.743	0.452	0.076	0.227	0.019	NO
4	8	0.048	0.064	0.08	0.064	8	0.592	0.056	0.072	0.729	0.455	0.073	0.206	0.019	NO
4	9	0.056	0.072	0.088	0.072	9	0.587	0.064	0.08	0.716	0.457	0.069	0.188	0.020	NO
4	10	0.064	0.08	0.096	0.08	10	0.583	0.072	0.088	0.705	0.459	0.067	0.175	0.020	NO

Interpretation of the Fuzzy Chain Sampling Plan Results from the Tabulated data

The table presents acceptance probabilities for a fuzzy chain sampling plan under different combinations of chain length (i) and acceptance number (c), with the defect proportion modeled as a triangular fuzzy number.

For **i = 0** (no chain effect), the acceptance probabilities are the highest across all values of *c*, indicating a more lenient inspection policy. All combinations in this case with threshold value greater than 0.4 satisfy the decision criterion, as reflected by “YES” in the final column.

As the chain length i increases, the chain acceptance probability decreases sharply, since acceptance requires multiple consecutive samples to meet the acceptance condition. This demonstrates the strengthening of consumer protection through the chain mechanism.

6. Fuzzy Chain Sampling Plan to maximize acceptance probability

To determine the highest probability of acceptance when the defect proportion lies roughly between 5% and 8%.To maximize the acceptance a larger acceptance number c, and small chain length i is considered (i=0 gives maximum acceptance; i> 0 reduces acceptance as we require several consecutive acceptable samples), binomial cumulative probabilities are used to compute per-sample acceptance:

$$P_{sample}(n,p,c) = \sum {}^{n}C_{x} p^{x} (1-p)^{n-x}$$

and chain acceptance:

$$P_{a,chain}(i;n,p,c)=(P_{sample}(n,p,c))^{i+1}.$$

where $c \approx \lceil n \times 0.08 + 2 \rceil$ and $n \times 0.08n$ times is the **expected number of defects** at the worst-case 8% rate; adding a small margin (+2) makes the plan permissive enough to keep acceptance high across 5%–8%. And we can use chain length i= 0 for maximum acceptance.$c=\lceil n \cdot 0.08+2 \rceil$ is computed per-sample acceptance and chain acceptance for p=0.5, p=0.065 (mid), and p=0.08.

6(i) Chain length i=0 (single-sample rule — maximum acceptance)

n	c (rule)	Pa @ p=0.05	Pa @ p=0.065	Pa @ p=0.08
80	9	0.993466	0.965604	0.894815
125	12	0.989994	0.936605	0.799634
200	18	0.994176	0.936764	0.749439

Interpretation: using the rule we get very high acceptance at 5% and good acceptance even at 8% (≈75–90% depending on n). This meets the "highest acceptance" objective.

6 (ii) **Chain length i=1(one prior-sample required acceptable — mild protection)**

n	c	Pa_chain @ p=0.05	Pa_chain @ p=0.065	Pa_chain @ p=0.08
80	9	0.986974	0.932391	0.800694
125	12	0.980088	0.877229	0.639415
200	18	0.988387	0.877526	0.561659

Interpretation:

A good acceptance is obtained at 5% but acceptance drops at 8% ,for n=200 acceptance ≈ 56%).

TABLE 6:

Sweep of (i,c) and average acceptance probability across p = 0.05 ,0.06 ,0.07, 0.08								
i	**n**	**c**	**Pa@0.05**	**Pa@0.06**	**Pa@0.07**	**Pa@0.08**	**AvgPa**	**Meets Threshold**

0	20	4	0.99742606	0.994365858	0.989293196	0.981655601	0.990685178	Yes
0	32	5	0.995400254	0.989052625	0.97800674	0.960969987	0.980857401	Yes
0	50	6	0.988213552	0.971075906	0.941686364	0.898128219	0.94977601	Yes
0	80	9	0.993465719	0.978712051	0.947603574	0.894814981	0.953649081	Yes
0	125	12	0.989993867	0.962322067	0.901081136	0.799634145	0.913257804	Yes
0	200	18	0.994176442	0.967200227	0.890709081	0.749438819	0.900381142	Yes
1	20	4	0.994858744	0.988763459	0.978701027	0.963647719	0.981492737	Yes
1	32	5	0.990821666	0.978225094	0.956497183	0.923463316	0.962251815	Yes
1	50	6	0.976566025	0.942988415	0.886773209	0.806634298	0.903240487	Yes
1	80	9	0.986974136	0.957877279	0.897952533	0.800693849	0.910874449	Yes
1	125	12	0.980087856	0.926063761	0.811947213	0.639414766	0.839378399	No
1	200	18	0.988386798	0.935476279	0.793362668	0.561658543	0.819721072	No
2	20	4	0.992298037	0.983192624	0.968222267	0.945970181	0.972420777	Yes
2	32	5	0.986264139	0.967516097	0.935460691	0.88742053	0.944165364	Yes
2	50	6	0.96505578	0.91571333	0.835062239	0.724461025	0.860073093	No
2	80	9	0.98052497	0.937486036	0.850903029	0.716472851	0.871346722	No
2	125	12	0.970280966	0.891171593	0.731630317	0.51129788	0.776095189	No
2	200	18	0.98263087	0.90479287	0.706655333	0.420928715	0.753751947	No
3	20	4	0.989743922	0.977653177	0.9578557	0.928616926	0.963467431	Yes
3	32	5	0.981727575	0.956924335	0.914886861	0.852784495	0.926580816	Yes
3	50	6	0.953681201	0.889227151	0.786366724	0.65065889	0.819983491	No

3	80	9	0.974117944	0.917528882	0.806318751	0.641110641	0.834769054	No
3	125	12	0.960572205	0.85759409	0.659258276	0.408851243	0.721568954	No
3	200	18	0.976908462	0.875115869	0.629424323	0.315460319	0.699227243	No
4	20	4	0.98719638	0.97214494	0.947600127	0.911582007	0.954630863	Yes
4	32	5	0.977211878	0.946448525	0.894765516	0.819500305	0.909481556	Yes
4	50	6	0.942440687	0.863507061	0.74051102821	0.58437511	0.78270842	No
4	80	9	0.967752784	0.897996574	0.76407053	0.573675405	0.800873823	No
4	125	12	0.950960592	0.825281718	0.594045196	0.326931414	0.67430473	No
4	200	18	0.971219379	0.846412267	0.56063396	0.236418209	0.653670954	No

Best Overall Combination

- Sample size (n) = 20
- Acceptance number (c) = 4
- Chain length (i) = 0
- Average Acceptance Probability (Avg Pa) = 0.9907

Interpretation of the Sweep of (i,c) and Average Acceptance Probability

The table examines the effect of chain length (i) and acceptance number (c) on the acceptance probability (Pa) across defect proportions p = 0.05,0.06,0.07, For all combinations of (i,c), the acceptance probability consistently decreases as the defect proportion increases, indicating that the sampling plan is sensitive to deteriorating quality levels.

When i=0 (single-sample plan), the average acceptance probability (Avg Pa) is the highest across all values of c, showing a more lenient inspection policy.

As the chain length i increases from 0 to 4, the Avg Pa steadily decreases, reflecting the strict requirement of accepting multiple consecutive samples.

7. Results and Conclusions

Acceptance probabilities were computed using the Poisson distribution for the given sample size and acceptance number, showing a decrease in acceptance as the defect proportion increased.

Modeling the defect proportion as a triangular fuzzy number captured uncertainty in textile inspection data more effectively than a crisp probability without rejecting the lots and considering the lots with simple constraints.

α-cut decomposition produced fuzzy intervals of acceptance probabilities at different confidence levels, resulting in a banded fuzzy operating characteristic curve instead of a single curve.

Chain acceptance probabilities decreased with increasing chain length, indicating improved consumer protection through stricter acceptance criteria.

Defuzzification using the centroid method provided a representative acceptance probability for decision-making. Conservative decision rules based on lower α-cut bounds further reduced the risk of accepting poor-quality lots.

Optimization of plan parameters showed that suitable combinations of chain length and acceptance number can balance producer's risk, consumer's risk, and inspection cost.

Overall, the fuzzy chain sampling plan demonstrated greater flexibility and realism compared to conventional chain sampling plans in textile quality control.

REFERENCES

1. Ezzatallah Baloui Jamkhaneh, Bahram Sadegpour Gildeh, Gholamhossein Yari,(2009).”Acceptance Single Sampling Plan with Fuzzy Parameter using Poisson Distribution**”.**World Academy of Science, Engineering and Technology 49.
2. Ezzatallah Baloui Jamkhaneh, Bahram SadegpourGildehand Gholamhossein Yari (2011). “Acceptance Single Sampling Plan with Fuzzy Parameter”.Iranian Journal of Fuzzy Systems, Vol. 8, No. 2, (2011) pp. 47-55.
3. Ezzatallah Baloui Jamkhaneh and Bahram Sadeghpour Gildeh (2011).“Acceptance Double Sampling Plan using Fuzzy Poisson Distribution”. World Applied Sciences Journal, 15(12):1692-1702.
4. Ezzatallah Baloui Jamkhaneh and Bahram Sadeghpour Gildeh (2011).“Chain Sampling Plan Using Fuzzy Probability Theory”. Journal of AppliedSciences, 11(24):3830-3838, DOI: 10.3923/jas.2011.3830.3838.
5. Ezzatallah Baloui Jamkhaneh, B. Sadeghpour Gildeh (2012). “Sequential Sampling Plan by Variable with Fuzzy Parameters”. International Journal of Applied Operational Research, Vol. 2, No. 1, pp. 85-95, Spring 2012 Journal homepage: www.ijorlu.ir
6. Gilberto Miller Devo´ s Ganga ,Luiz Cesar Ribeiro Carpinetti,(2012).”A Fuzzy Logic Approach to Supply Chain Performance Management”. International Journal of Applied Operational Research Vol.2,No.1, pp. 85-95, Spring 2012 Journal homepage: www.ijorlu.ir
7. V. Jemmy Joyce & K. Rebbeca Jebaseeli Edna (2020).”Fuzzy Sampling Plan Using ZTPD”.Advances in Mathematics: Scientific Journal 9 (2020), No.4, 1611–1614 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.4.16.
8. Julia T. Thomas and Mahesh Kumar,(2022).“Design of Fuzzy Economic Order Quantity (EOQ) Model in The Presence of Inspection Errors in Single Sampling Plans”. Journal of Reliability and Statistical Studies, Vol. 15, Issue 1 (2022), 211–228. doi: 10.13052/jrss0974-8024.151.
9. Hiroshi Ohta and Akihiro Kanagawa(1989). ”A Note on the Fuzzy for Single Sampling Design Procedure
10. Attribute Plans”. Engineering and Natural Sciences. 1989, 38(1), p.59-67. URL http://doi.org/10.24729/00008481
11. D.Malathi and S.Muthulakshmi (2012). “Special Double Sampling Plan with Fuzzy Parameter”.Indian Journal
12. of Applied Research,Volume :2|Issue : 1 | October 2012 | ISSN - 2249-555X.
13. R. Nandhinini Devi and G.Uma (2018). “Imapact of Fuzzy on Quick Switching System by Attributes Under Poisson Distribution”. International Journal of Pure and Applied Mathematics, Volume 118 No.24.
14. K. Rebecca Jebaseeli Edna and V.Jemmy Joyce (2013). “Chain Sampling Plan for Variable Fraction Non-Conformities’. International Journal of Pure Applied Mathematics, Volume 87 No. 6, 827-833 (on-line version).
15. K. Rebecca Jebaseeli Edna V.Jemmy Joyce (2020).” Fuzzy Modified Chain Sampling Plan for Very Expensive Products”. Advances in Mathematics: Scientific Journal 9, No.4, 1597–1600.
16. Robab Afshari, Bahram SadeghpurGildeh and Adel Ahmadi Nadi (2019).**“**Fuzzy Double Variable Sampling Planunder Uncertainty”. Journal of Quality Engineering and Production Optimization, Vol. 4, No. 2, Summer & Autumn 2019, PP. 83-98 85 III.
17. V . Sangeetha and K. S. Karunya (2022). “Comparison Study on Fuzzy Two Stage Chain Sampling Plan”. International Journal of Statistics and Applied Mathematics, 7(3):01-05.