

GeoGebra and Students' Conceptual Understanding of Mathematical Representations: A Quasi-Experimental Study in Upper Primary Classrooms.

Sher Alam Khan¹, Giorgio Poletti¹, Mazhar Alam², Farooq Nawaz Khan², Janneth Trejo-Quintana³, Daniel Mara⁴, Elena-Lucia Mara⁴

¹Department of Humanistic, University of Ferrara and University of Modena Reggio Emilia, Italy. sher.khan@unimore.it

¹Department of Humanistic, University of Ferrara, Italy. giorgio.poletti@unife.it

²Center for Education & Staff Training, University of Swat, Swat, Pakistan. mazharalam1374@gmail.com

²Center for Education & Staff Training, University of Swat, Swat, Pakistan. farooq@uswat.edu.pk

³Universidad Regional Autonoma De Los Andes: Ambato, Tungurahua, EC. jtrejoquintana@politicas.unam.mx

⁴Faculty of Social and Human Sciences "Lucian Blaga" University of Sibiu, Romania. daniel.mara@ulbsibiu.ro

⁴Faculty of Social and Human Sciences "Lucian Blaga" University of Sibiu, Romania. lucia.mara@ulbsibiu.ro

Corresponding authors: sher.khan@unimore.it, giorgio.poletti@unife.it

Abstract: GeoGebra is a dynamic mathematics software that integrates algebraic and geometric representations through interactive visualization, enabling learners to explore mathematical relationships in multiple forms. By supporting the coordination of symbolic, graphical, and geometric representations, GeoGebra provides opportunities for students to develop deeper conceptual understanding of mathematical ideas. This study investigated the role of GeoGebra-supported instruction in enhancing Grade 6 students' conceptual understanding and achievement in linear equations and coordinate geometry in Pakistan. A quasi-experimental design with nonequivalent control and experimental groups was employed, involving 76 students (38 in each group). The experimental group received GeoGebra-supported instruction, while the control group was taught using traditional lecture-demonstration methods. Grounded in social constructivist theory, the study examined how dynamic visualization and interactive representation influence students' mathematics learning. Data were collected through pretest and posttest assessments and analyzed using independent samples t-tests. The results indicated that the experimental group significantly outperformed the control group in posttest scores, with a large effect size ($t(74) = -4.180, p < 0.001, \text{Cohen's } d = 0.98$), demonstrating the practical impact of GeoGebra-supported instruction. Findings suggest that GeoGebra facilitates students' coordination of algebraic and graphical representations, enabling them to connect abstract mathematical expressions with visual structures. This representational fluency appears to support improved conceptual understanding of linear relationships and coordinate geometry. The study highlights the pedagogical value of integrating dynamic visualization tools into mathematics instruction to support meaningful learning experiences and strengthen students' conceptual foundations in early mathematics education.

Keywords: GeoGebra; Mathematics education; Technology-enhanced learning; Upper primary school mathematics; Quasi experimental design

1. Introduction

Mathematics is often considered a challenging subject for schoolchildren, presenting significant difficulties for teachers. Today's students require more advanced mathematics instruction than previous generations do, which demands that teachers adapt to rapid changes in curricula and teaching approaches. These shifts make teaching



mathematics increasingly complex and highlight the need for effective instructional strategies (Liebendörfer & Schukajlow, 2017; Phuong et al., 2022; Sors et al., 2025).

One of the evolving natures of teaching mathematics is the integration of digital educational technologies at the classroom level. These technologies enable students to explore mathematical concepts and connect abstract ideas with practical applications. However, many teachers still underutilize these tools in their teaching due to numerous reasons, including a lack of training, insufficient skills, and negative attitudes toward the digitalization of education. Research has shown that the integration of digital technologies can enhance students' engagement, motivation, and learning behaviors (Khan et al., 2025; Voogt et al., 2013; Drijvers et al., 2010). For instance, Scippo et al. (2025) reported that digital tools are particularly effective in teaching geometry, significantly improving students' motivation and involvement.

Among the available digital tools, GeoGebra has gained widespread recognition as an effective platform for mathematics education. It provides dynamic geometry functions that allow the creation and manipulation of points, lines, and geometric shapes. GeoGebra is accessible across educational levels, from primary school to university, making it a useful classroom resource. GeoGebra is a free, professionally designed software that is easy to use, accessible, and open-source, offering an interactive and dynamic environment for teaching and learning. This program effectively connects geometry and algebra, making it suitable for different educational environments (GeoGebra, 2019). By facilitating interactive and visual representations, GeoGebra enables teachers to create learning environments that promote exploration, experimentation, and conceptual understanding. Topics such as geometry, algebra, calculus, graphing, and statistics can be explored visually, helping students grasp abstract mathematical concepts more effectively (Chrysanthou, 2008; Diković, 2009; Klllogjeri & Klllogjeri, 2014).

Although GeoGebra is recognized as a powerful tool for mathematical visualization (Kadunz, 2000), its effectiveness in classrooms depends on teachers' instructional decisions and competencies. Studies have highlighted both the advantages and limitations of GeoGebra (Bulut & Neslihan, 2011; Hugener et al., 2009; Reisa, 2010; Summak et al., 2010), indicating that successful integration requires more than mere familiarity with the software. Teachers must develop skills to integrate technology meaningfully into their instruction. Mishra and Koehler (2006) introduced the concept of technological pedagogical and content knowledge (TPACK), which emphasizes the interrelated nature of the technological, pedagogical, and content knowledge necessary for effective teaching. In mathematics education, understanding the relationships among these three components is essential for successful instruction (Hughes, 2005).

Despite the growing evidence supporting the pedagogical potential of GeoGebra, empirical studies examining its effectiveness in teaching fundamental mathematical concepts at the upper primary level remain limited. For instance, Birgin & Yazıcı (2021) found that GeoGebra-supported instruction significantly enhanced eighth-grade students' conceptual understanding and retention of linear equations and slope compared with traditional textbook instruction. Nonetheless, as they pointed out, their results were confined to one specific grade level (eighth grade), a particular mathematical topic (linear equations and slope), and the educational context of Turkey. It remains unclear whether their findings replicate to upper primary students (Grade 6), different but related mathematical topics (linear equations and coordinate geometry), and other national contexts.

The present study addresses this gap by replicating and extending Birgin & Yazıcı's (2021) quasi-experimental design to a new grade level (Grade 6), partially overlapping mathematical topics (linear equations and coordinate geometry), and a new national context (Pakistan), with a larger sample ($n=76$ vs. $n=52$).

This gap is especially noteworthy in Pakistan, where national evaluations continuously highlight issues with mathematics performance. The Annual Status of Education Report (ASER 2023) indicates that only 46% of Grade 5 students can execute basic two-digit division, highlighting a fundamental learning crisis. Additionally, a countrywide study of elementary students revealed that public school students have considerably lower math achievement than private school students (Bhutta et al., 2024). Specifically, few studies, such as Tajik & Maqsood (2019) and Faisal et al. (2024), have utilized quasi-experimental designs to explore the effects of technology-enhanced instruction on students' conceptual understanding and engagement in Pakistan.

Therefore, this study aims to investigate how GeoGebra-supported instruction influences Grade 6 students' conceptual understanding of linear equations and coordinate geometry and examines its impact on mathematics achievement in a Pakistani primary school context. The study is guided by established criteria for evaluating educational technologies (Karolčík et al., 2015).

2. Literature Review

In teaching mathematics, technology plays an important role. When used appropriately, digital technologies can enhance students' engagement and motivation, while also supporting meaningful learning experiences. The effective use of digital tools enables teachers and students to explore and investigate mathematical ideas in more interactive and dynamic ways (Dahal et al., 2019).

Recent studies have therefore focused on the integration of educational technologies in mathematics education. According to the National Council of Teachers of Mathematics (NCTM, 2000), technology has become one of the key principles for enhancing the quality of mathematics learning. As a result, students are required to develop a range of competencies to perform effectively in technology-rich learning environments (Mnguni, 2014).

The integration of technology into mathematics education has created new opportunities for enhancing the learning process. Among various digital tools, GeoGebra has attracted considerable attention as an interactive mathematics software for presenting and exploring mathematical concepts (Doruk et al., 2013; Tamam & Dasari, 2021). GeoGebra combines geometry and algebra in a dynamic environment, allowing students to construct and manipulate points, lines, and other mathematical objects. It also supports the visual representation of mathematical concepts such as functions, coordinates, reflections, and transformations, as well as the exploration of integrals and derivatives (Kllogjeri & Kllogjeri, 2014).

By offering interactive and visually rich representations, GeoGebra has the potential to strengthen teachers' instructional practices and promote more effective student learning (Diković, 2009; Yildiz & Arpacı, 2024). Consistent with this view, Tanvir (2025) reported that students who used GeoGebra demonstrated improvements in their mathematical abilities.

Accordingly, the integration of GeoGebra into mathematics instruction has been shown to enhance student performance by improving confidence and understanding of complex mathematical concepts. Several studies have reported that the use of GeoGebra contributes to increased student motivation and supports deeper conceptual understanding in mathematics learning (Septian & Monariska, 2020; Zetriuslita et al., 2020). Moreover, the use of GeoGebra across diverse educational settings allows both teachers and students to benefit from the flexibility and accessibility of digital resources, while also engaging in interactive and hands-on learning activities.

Previous research has consistently highlighted the advantages of incorporating GeoGebra into mathematics education. Studies have shown that the use of GeoGebra can enhance students' problem-solving abilities, particularly in areas such as analytical geometry, while also positively influencing the affective, behavioral, and cognitive aspects of learning (Assadi & Cretu, 2023; Frassia & Serpe, 2017; Latifi et al., 2022; Sugandi et al., 2020). In addition, Uwurukundo et al. (2022) reported that GeoGebra supports the development of students' visualization skills, which are essential for understanding and applying mathematical concepts.

Despite the documented benefits of GeoGebra in mathematics education, studies have also highlighted the challenges associated with its effective classroom implementation. These challenges include limited digital competence and teacher training, insufficient pedagogical support, and difficulties in aligning technology with curricular objectives (Batiibwe, 2024). Additionally, the unequal access to digital resources and time constraints within classroom settings may also restrict the effective use of GeoGebra, particularly (Saidu, Rabi'u & Hamzat, 2024). These factors suggest that the effectiveness of GeoGebra depends not only on the software itself, but also on how it is integrated into teaching practice.

3. Theoretical Background

This study is grounded in social constructivist theory, which emphasizes learning as a socially mediated process shaped through interaction, dialog, and the use of cultural and technological tools. From this perspective, learning occurs when learners actively construct knowledge with the support of peers, teachers, and instructional resources. In the present study, GeoGebra applets function as virtual manipulatives and mediating tools that support students' conceptual understanding and enhance motivation in learning mathematics.

Social constructivism is closely associated with Vygotsky's zone of proximal development (ZPD), which describes the distance between what learners can accomplish independently and what they can achieve with guidance. Instruction within the ZPD involves varying levels of support, which are gradually reduced as learners gain competence. In this study, GeoGebra is employed as a scaffolding tool that enables students to visualize, explore, and manipulate mathematical concepts, whereas peer interaction and teacher guidance provide additional social support. Effective scaffolding requires teachers to be aware of students' prior knowledge and cognitive readiness to align instructional support with learners' developmental levels.

To operationalize this approach, the study adopts the four-level instructional framework proposed by Dunphy and Dunphy (2003), informed by the work of Tharp and Gallimore (1991). This framework guided the integration of GeoGebra applets into classroom instruction and structured students' progression from guided learning to independent problem-solving.

Level I involves direct instructional support, where the teacher provides explicit guidance to help students complete specific tasks. Frequent questioning and interaction are used to assess students' understanding of fundamental concepts and to prepare them for subsequent learning stages (Dunphy & Dunphy, 2003). Support may be offered by the teacher or more knowledgeable peers, promoting guided participation and collaborative learning (Tharp & Gallimore, 1991; Siyepu, 2013).

Level II is characterized by increased learner participation and reduced dependence on the teacher. At this stage, students draw upon prior knowledge to engage more independently in mathematical activities, although occasional guidance may still be required (Antonacci, 2000; Siyepu, 2013; Tinungki, 2019). In this study, students actively used GeoGebra applets to explore mathematical concepts, seeking assistance when encountering difficulties.

Level III represents the internalization and automation of learning. Students demonstrate improved proficiency and are able to perform tasks independently with minimal external support, indicating that they have moved beyond the ZPD for specific concepts (Tharp & Gallimore, 1991; Dunphy & Dunphy, 2003).

Level IV emphasizes reflection and self-regulation. At this stage, students revisit and refine their problem-solving strategies, applying previously acquired knowledge to enhance performance. Learning is viewed as a cyclical process, in which students may re-enter the ZPD as they encounter new or more complex tasks, allowing for continuous development through structured support and independent practice (Tharp & Gallimore, 1991).

4. Methodology

4.1 Research Design

This study employed a quasi-experimental research design involving two groups: a control group and an experimental group. The study was conducted at a public upper primary school located in District Shangla, Khyber Pakhtunkhwa (KP), Pakistan. The school was purposively selected because of the availability of a well-equipped computer laboratory and reliable high-speed internet connectivity, which were necessary for implementing GeoGebra-supported instruction.

4.2 Participants

The study was conducted over a period of five weeks. One week was allocated for training teachers and students on the basic functions and educational use of GeoGebra through a student-centered instructional approach. A total of

four mathematics teachers and seventy-six students participated in the study. The students were enrolled in Grade 6th, aged between 12 and 14 years, and came from diverse family backgrounds.

Using a quasi-experimental approach, two nonequivalent groups were formed: a control group ($n = 38$) and an experimental group ($n = 38$).

4.3 Research instrument

A pretest was developed on the basis of the Grade 6th algebra and geometry curriculum to assess students' prior knowledge. The test consisted of 25 multiple-choice questions, with each question receiving one mark. Five items assessed basic familiarity with GeoGebra, whereas the remaining twenty items focused on curriculum-based mathematical concepts. However, the five GeoGebra-related items were used only to assess students' initial familiarity with the software and were not included in the pre and posttest analysis between the control and experimental groups.

To ensure content validity, the instrument was reviewed by experts in technology-enhanced learning and experienced mathematics teachers from other schools. On the basis of their feedback, several items were revised, removed, or added to improve the clarity and validity of the instrument.

4.4 Intervention Procedure

Following the one-week training session and prior to the instructional intervention, the pretest was administered to establish baseline equivalence between the control and experimental groups. An independent samples t-test confirmed that there was no statistically significant difference between the two groups before the intervention.

During the intervention period, the control group was taught via a traditional lecture-based approach, whereas the experimental group received instruction supported by GeoGebra in the computer laboratory. Nineteen computers were available for the experimental group, with students working in pairs to encourage collaborative learning.

Instructional activities included explanations, worked examples, guided demonstrations, worksheets, and student presentations. All activities were carefully designed to align with the different levels of the theoretical framework on the basis of the Zone of Proximal Development. The researcher assumed a facilitative role, monitoring student progress and verifying tasks saved in individual folders on the computer desktops.

Students in the experimental group interacted with GeoGebra applets to explore the relationships between variables and observe dynamic changes in mathematical representations. They were encouraged to communicate their ideas, collaborate with peers, and engage with the content according to their individual learning preferences. The control group completed the same mathematical tasks without using GeoGebra.

4.5 Data analysis

Both descriptive statistics (mean scores, standard deviations, frequencies, and percentages) and inferential statistical techniques were employed. An independent samples t-test was used to examine differences between the control and experimental groups. All hypotheses were tested at the 95% confidence level via a two-tailed test.

4.6 Ethical considerations

Ethical standards for educational research were strictly observed. Approval was obtained from the relevant educational authorities, and official permission was granted by the school principal. The participants were fully informed about the purpose of the study, the voluntary nature of their participation, and their right to withdraw at any time. Written informed consent was obtained from all participants, with consent forms provided in both English and the local language (Urdu). The study was conducted between August 15 and October 15, 2025.

5. Results

5.1 Pretest results

The pre-test analysis revealed that the mean scores and standard deviations of the control group ($M = 8.08$, $SD = 3.15$) and the experimental group ($M = 8.18$, $SD = 3.14$) were nearly identical (Table-1). An independent samples t-test indicated that there was no statistically significant difference between the two groups prior to the intervention ($p > .05$). These results confirm that both groups were comparable in terms of their prerequisite knowledge before the instructional intervention.

Table1. Results of the independent sample *t*-test (pre-test)

Pre-Test	N	Mean	SD	T	df	<i>p</i>
Control	38	8.0789	3.14842	-.146	74	.884
Experimental	38	8.1842	3.14389			

5.2 Posttest Results

Table 2 presents the descriptive statistics for the posttest results. The findings indicate that the experimental group ($M = 12.47$, $SD = 2.90$) achieved higher scores than the control group did ($M = 9.66$, $SD = 2.97$), suggesting improved performance following GeoGebra-supported instruction (Table 2).

Table.2, Group statistics of posttest results

Result	Group	N	Mean	Standard Deviation	Standard Error
Post-Test	Control	38	9.6579	2.97082	.48193
	Experimental	38	12.4737	2.90141	.47067

An independent samples t-test was conducted to examine differences in post-test performance between the two groups (Table 3). Levene's test for equality of variances was not significant ($F = 0.025$, $p = .876$), indicating that the assumption of homogeneity of variance was met. The t-test results revealed a statistically significant difference between the control and experimental groups ($p < .001$), with the experimental group outperforming the control group. We also calculated the effects of the (*r*) value to strengthen our conclusion. Therefore, on the basis of the formula we used, $r = \sqrt{\frac{t^2}{t^2 + df}}$ where *t* is (-4.180) and *df* is (74) as shown in Table 3. Therefore, $r = \sqrt{\frac{(-4.180)^2}{(-4.180)^2 + 74}} = 0.44$, this represents a large effect according to Cohen's guidelines.

Table 3. Independent sample t-test for post-test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Post-Test Results	Equal variances assumed	0.025	0.876	-4.18	74	0	-2.81579	0.67364	-4.15804	-1.47354
	Equal variances not assumed			-4.18	73.959	0	-2.81579	0.67364	-4.15806	-1.47352

6. Discussion

This study examined the impact of GeoGebra-supported instruction on Grade 6 students' understanding of linear equations and coordinate geometry in Pakistan. The findings indicate that GeoGebra-integrated instruction significantly improved students' mathematical and conceptual understanding compared to traditional lecture-demonstration methods.

These findings can be explained by GeoGebra's ability to support coordination among multiple representations of mathematical concepts, including algebraic, geometric, and graphical forms. Through dynamic interaction with these representations, students are able to observe relationships between equations and their visual structures, which supports the development of more meaningful conceptual understanding of linear relationships.

Consistent with these results, previous studies (Baye et al., 2021; Birgin & Yazıcı, 2021) have reported similar improvements in students' mathematics learning when GeoGebra is integrated into instruction. The present study extends this evidence by applying GeoGebra in a Grade 6 context in Pakistan and confirming its effectiveness with a relatively larger sample compared to earlier studies.

The post-test results further indicate that students in the experimental group outperformed those in the control group. Although both groups had comparable pre-test scores, the experimental group showed a substantial improvement following the intervention.

From a theoretical perspective, these findings align with social constructivist learning theory, particularly Vygotsky's concept of the zone of proximal development. GeoGebra functioned as a mediating tool that supported students in progressing from guided exploration toward more independent reasoning through interaction with dynamic mathematical representations. Collaborative activities further enhanced this process by encouraging peer discussion and shared problem-solving.

Additionally, GeoGebra's interactive environment allowed students to construct and manipulate mathematical objects such as points, lines, slopes, and geometric transformations in real time. This dynamic visualization of mathematical concepts supported learners in connecting abstract symbolic representations with concrete visual forms, thereby strengthening conceptual understanding.

The effectiveness observed in this study may also be influenced by contextual and instructional factors, including collaborative pair work, increased student engagement, and the novelty of using digital tools in the classroom. These factors highlight that the impact of GeoGebra is closely linked not only to the technology itself but also to how it is integrated into instructional design.

Furthermore, the successful implementation of GeoGebra in this study suggests that effective technology integration in mathematics education depends on structured pedagogical planning, teacher readiness, and alignment with curriculum objectives. In contexts such as Pakistani schools, where technology-enhanced instruction is still developing, these factors play an important role in maximizing learning outcomes. Overall, the findings are consistent with previous research across different contexts, suggesting that GeoGebra-supported instruction has broad relevance in mathematics education.

7. Conclusion

This study investigated the impact of GeoGebra-supported instruction on Grade 6 students' conceptual understanding and achievement in linear equations and coordinate geometry in a Pakistani primary school context. The findings suggest a positive influence of GeoGebra-supported instruction on students' learning outcomes, as evidenced by posttest results.

The results indicate that GeoGebra can support students' conceptual understanding by making abstract mathematical ideas more visible through dynamic visualization and multiple representations. This representational

support enables learners to connect algebraic, graphical, and geometric forms of mathematical concepts, thereby strengthening their understanding of mathematical relationships.

The findings also highlight that technology integration in mathematics classrooms provides opportunities for students to engage more actively in learning activities and develop basic digital skills within a structured instructional environment. However, the study also acknowledges that effective integration of digital tools such as GeoGebra depends on adequate infrastructure, teacher preparedness, and appropriate instructional design.

Overall, the study suggests that GeoGebra-supported instruction, when implemented within well-structured pedagogical practices, can enhance students' conceptual learning experiences in mathematics. The findings emphasize that the effectiveness of GeoGebra is not solely dependent on technology access but also on its meaningful integration into classroom instruction.

Limitations and Future Research

One limitation should be acknowledged. The participating school was selected through convenience sampling because it had access to a fully equipped computer laboratory and reliable internet connectivity. While this enabled the intervention to be conducted effectively, it may not represent typical conditions in many Pakistani public schools. Therefore, findings should be interpreted as reflecting the effectiveness of GeoGebra-supported instruction in relatively well-resourced environments. Future research should examine GeoGebra implementation in schools with varying levels of technological infrastructure to better understand its applicability across diverse educational contexts.

Ethical Approval and Consent to participate: This study was conducted in accordance with the Declaration of Helsinki. Ethical approval was obtained from ethical committee of Elementary and Secondary Education Department District Shangla. The ethics committee has thoroughly evaluated the research protocol and its research objective, methodology, participant recruitment strategy, informed consent process, data confidentiality measures and potential risk and benefits to participants. The committee has determined that the study complies with the ethical guideline. Written informed consent to participate in this study was obtained from Parents Teacher Committee (PTC) from this school.

Consent for publication: All authors have reviewed and approved the final version of the manuscript. Consent for publication has been obtained from all relevant individuals (School Administration, Education Department, Parents Teachers Committee (PTC)).

Competing interests: All authors declare no competing interests.

Funding: This research received no specific grant from any funding agency.

Authors' contributions: Conceptualization and research design were carried out by Sher Alam Khan, Giorgio Poletti, & Farooq Nawaz Khan, Mara Elena Lucia. Methodology development and data collection were conducted by Farooq Nawaz Khan, Mazhar Alam, & Sher Alam Khan. Data analysis and interpretation of findings were performed by Sher Alam Khan, Janneth Trejo-Quintana, and Farooq Nawaz Khan & Daniel Mara. The manuscript was critically reviewed and edited by Daniel Mara & Janneth Trejo-Quintana, Mara Elena Lucia. Overall supervision and support were provided by Giorgio Poletti and Mara Elena Lucia. All authors have read and approved the final version of the manuscript.

Acknowledgements: The author acknowledge the support provided by the co-authors, school administration, teachers and students.

REFERENCES

1. Antonacci, P. (2000). Reading in the Zone of Proximal Development: Mediating Literacy Development in Beginner Readers through Guided Reading. [online] *ScholarWorks at WMU*. Retrieved from https://scholarworks.wmich.edu/reading_horizons/vol41/iss1/2
2. Annual Status of Education Report [ASER]. (2024). *ASER 2023: What did we learn about learning. Idara-e-Taleem-o-Aagahi (ITA)*. <https://aserpakistan.org/blog/ASER-2023-What-did-we-learn-about-learning>

3. Assadi, N., & Cretu, C. (2023). The influence of integrating geogebra software into the educational setting on the affective, behavioral, and cognitive aspects of pre-service mathematics teachers. *Creative Education*, 14(12), 2503–2519. <https://doi.org/10.4236/ce.2023.1412161>
4. Batiibwe, M. S. K. (2024). Integration of GeoGebra in learning mathematics: Benefits and challenges. *East African Journal of Education Studies*, 7(4), 684–697.
5. Baye, M. G., Ayele, M. A., & Wondimuneh, T. E. (2021). Implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students' conceptual understanding of limit in Ethiopian Universities. *Heliyon*, 7(5). <https://doi.org/10.1016/j.heliyon.2021.e07012>
6. Birgin, O., & Uzun Yazıcı, K. (2021). The effect of GeoGebra software–supported mathematics instruction on eighth-grade students' conceptual understanding and retention. *Journal of Computer Assisted Learning*, 37 (4), 925–939. <https://doi.org/10.1111/jcal.12532>
7. Bulut, M.B., Neslihan. (2011). Pre service teachers' usage of dynamic mathematics software. *Turkish Online Journal of Educational Technology*, 10(4), 294–299.
8. Bhutta, S. M., Ahmad, S., & Ansari, A. N. (2024). School system and student performance: Examining Science and Mathematics achievement in Pakistan. *UMT Education Review*, 7 (2), 22–49. <https://doi.org/10.32350/uer.72.02>
9. Chrysanthou I. (2008). The use of ICT in Primary Mathematics in Cyprus: The case of GeoGebra, University of Cambridge, Faculty of Education, *Cambridge*.
10. Dahal, N., Shrestha, D., & Pant, B. P. (2019). Integration of GeoGebra in teaching and learning geometric transformation. *Journal of Mathematics and Statistical Science*, 5(12), 323–332.
11. Diković, L. (2009). Applications GeoGebra into teaching some topics of mathematics at the college level. *Computer Science and Information Systems*, 6(2), 191–203. <https://doi.org/10.2298/csis0902191D>
12. Doruk, B. K., Aktümen, M., & Aytekin, C. (2013). Pre-service elementary mathematics teachers' opinions about using geogebra in mathematics education with reference to 'teaching practices. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 32(3), 140–157. <https://doi.org/10.1093/teamat/hrt009>
13. Drijvers, P., Doorman, M., Boon, P., Reed, H., Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213–234. DOI: <http://dx.doi.org/10.1007/s10649-010-9254-5>
14. Dunphy, B. C., & Dunphy, S. L. (2003). Assisted performance and the zone of proximal development (ZPD); a potential framework for providing surgical education. *Australian Journal of Educational & Developmental Psychology*, 3(2003), 48–58.
15. GeoGebra s (2019). <https://help.GeoGebra.org/topics>
16. Faisal, M. H., Khan, S., Faisal, F., & Makhdom, F. N. (2024). Smart Pathways for Sustainable Education of Teaching and Learning Mathematics at the Elementary Level in Pakistan: The Post-Humanistic Approach. *Journal of Asian Development Studies*, 13 (4), 992–999. <https://doi.org/10.62345/jads.2024.13.4.80>
17. Frassia, M., & Serpe, A. (2017). Learning geometry through mathematical modelling: an example with GeoGebra. *TOJET: The Turkish Online Journal of Educational Technology*, 1(1), 411–418. <https://doi.org/10.37284/eajes.7.4.2454>
18. Hugener, I., Pauli, C., Reusser, K., Lipowsky, F., Rakoczy, K., Klieme, E. (2009). Teaching patterns and learning quality in Swiss and German mathematics lessons. *Learning and Instruction*, 19(1), 66–78. DOI: <http://dx.doi.org/10.1016/j.learninstruc.2008.02.001>
19. Hughes, J. (2005). The role of teacher knowledge and learning experiences in forming technology-integrated pedagogy. *Journal of Technology and Teacher Education*, 13(2), 277–302. <https://www.learntechlib.org/p/26105>.
20. Kadunz, G. (2000). Visualization, image and metaphor: The mediating activity of visualization in learning mathematics. *Journal for Mathematics Didactics*, 21 (3), 280–302.
21. Karolčík, Š., Cipková, E., Hrušický, R., & Veselský, M. (2015). The comprehensive evaluation of electronic learning tools and educational software (CEELTES). *Informatics in Education*, 14(2), 243–264. <https://doi.org/10.15388/infedu.2015.14>
22. Khan, S. A., Poletti, G. & Khan, F. N. (2025). From Chalkboards to Digital Screens: How Classroom Technology Influences Student Academic Behavior, Engagement and Motivation. *Formazione & insegnamento*, 23(2), 104–114. https://doi.org/10.7346/-fei-XXIII-02-25_12
23. Kloggeri, P., Kloggeri, Q. (2014). GeoGebra: A vital bridge linking mathematics with other sciences (available to everyone).
24. Latifi, M., Esegir, A., Elmaroufi, A., Hattaf, K., & Achtaich, N. (2022). Modeling with differential equations and geogebra in high school mathematics education. *Journal of Educational and Social Research*, 12(3), 47–61. <https://doi.org/10.36941/jesr2022-0065>
25. Liebendörfer, M., & Schukajlow, S. (2017). Interest development during the first year at university: Do mathematical beliefs predict interest in mathematics? *ZDM: The International Journal on Mathematics Education*, 49(3), 355–366. <https://doi.org/10.1007/s11858-016-0827-3>
26. Mishra, P., Koehler, M.J. (2006). Technological pedagogical content knowledge: a framework for teacher knowledge. *Teachers College Record*, 108(6), 1017–1054. <https://doi.org/10.1111/j.1467-9620.2006.00684.x>
27. Mnguni, L. E. (2014). The theoretical cognitive process of visualization for science education. *Springer Plus*, 3(1), 1–9. <https://doi.org/10.1186/2193-1801-3-184>

28. National Council of Teachers of Mathematics (NCTM) (2000). Principles and Standards for School Mathematics: An Overview. *Reston*.
29. Phuong, T. T. T., Danh, N. N., Le, T. T. T., Phuong, T. N., Thanh, T. N. T., & Minh, C. L. (2022). Research on the application of ICT in mathematics education: Bibliometric analysis of scientific bibliography from the scopus database. *Cogent Education*, 9(1), 2084956. <https://doi.org/10.1080/2331186X.2022.2084956>
30. Prince, T. (2025). The impact of GeoGebra on student confidence and perceptions in Calculus III. *International Journal of Mathematical Education in Science and Technology*, 56(4), 634-645. <https://doi.org/10.1080/0020739X.2023.2258883>
31. Reisa, Z. A. (2010). Computer supported mathematics with GeoGebra. *Procedia – Social and Behavioral Sciences*, 9(0), 1449–1455. DOI: <http://dx.doi.org/10.1016/j.sbspro.2010.12.348>
32. Saidu, S., Rabi, A. T., & Hamzat, B. K. (2024). Examining the Challenges of Integrating GeoGebra Software in the Teaching and Learning of Coordinate Geometry in Colleges of Education in the North West Zone of Nigeria. *Faculty of Natural and Applied Sciences Journal of Mathematics, and Science Education*, 6(1), 65–74. Retrieved from <https://fnasjournals.com/index.php/FNAS-JMSE/article/view/575>
33. Scippo, S., Madi, S., & Cuomo, S. (2025). Digital Tessellation for Geometry Learning in Primary School: A Quasi-Experimental Study. *Journal Of Information Technology Education*, 24, 1-31. <https://hdl.handle.net/2158/1417432>
34. Septian, A., Sugianti, R., & Monariska, E. (2020). The application of android-based GeoGebra on quadratic equations material toward mathematical creative thinking ability. *Al-Jabar: Journal Pendidikan Matematika*, 11(2), 261–272. <https://doi.org/10.24042/ajpm.v11i2.6686>
35. Siyepu, S. (2013). The zone of proximal development in the learning of mathematics. *South African Journal of Education*, 33(2), 1-13. <https://hdl.handle.net/10520/EJC134983>
36. Sors, O., Puig, A., & Rodríguez, I. (2025). Analytical and practical study of digital game-based learning under the lens of activity theory: A geometry game case study. *International Journal of Serious Games*, 12(2), 91–113. <https://doi.org/10.17083/ijsg.v12i2.916>
37. Sugandi, A. I., Bernard, M., & Linda, L. (2020). The effectiveness of problem-based online learning assisted by Geogebra on mathematical reasoning skills in the COVID-19 era. *AKSIOMA: Journal Program Studi Pendidikan Matematika*, 9(4), 993–1004. <https://doi.org/10.24127/ajpm.v9i4.3133>
38. Summak, M. S., Bağlıbel, M., Samancıoğlu, M. (2010). Technology readiness of primary school teachers: a case study in Turkey. *Procedia – Social and Behavioral Sciences*, 2(2), 2671–2675. DOI: <http://dx.doi.org/10.1016/j.sbspro.2010.03.393>
39. Tamam, B., & Dasari, D. (2021). The use of geogebra software in teaching mathematics. *Journal of Physics: Conference Series*, 1882(1), 012042. <https://doi.org/10.1088/17426596/1882/1/012042>
40. N. Tajik, M. Maqsood (2019) INTEGRATING ICT IN MATHEMATICS: EVALUATING STUDENTS' ACHIEVEMENT USING GEOGEBRA THROUGH VAN HIELE MODEL, *ICERI2019 Proceedings*, pp. 10725-10732. doi: 10.21125/iceri.2019.2635
41. Tharp, R. G., & Gallimore, R. (1991). Rousing minds to life: Teaching, learning, and schooling in social context. *Cambridge University Press*.
42. Tinungki, G. M. (2019). ZONE PROXIMAL DEVELOPMENT GIVES A NEW MEANING TO THE STUDENTS' INTELLIGENCE IN STATISTICAL METHOD LESSON. *Journal of Honai Math*, 2(2), 129-142. <https://doi.org/10.30862/jhm.v2i2.69>
43. Uwurukundo, M. S., Maniraho, J. F., Tusiime, M., Ndayambaje, I., & Mutarutinya, V. (2024). GeoGebra software in teaching and learning geometry of 3-dimension to improve students' performance and attitude of secondary school teachers and students. *Education and Information Technologies*, 29(8), 10201-10223. <https://doi.org/10.1007/s10639-023-12200-x>
44. Voogt, J., Fisser, P., Pareja Roblin, N., Tondeur, J., van Braak, J. (2013). Technological pedagogical content knowledge – a review of the literature. *Journal of Computer Assisted Learning*, 29(2), 109–121. DOI: <http://dx.doi.org/10.1111/j.1365-2729.2012.00487.x>
45. Yildiz, E., & Arpacı, I. (2024). Understanding pre-service mathematics teachers' intentions to use geogebra: The role of technological pedagogical content knowledge. *Education and Information Technologies*, 29(14), 18817–18838. <https://doi.org/10.1007/s10639-024-12614-1>
46. Zetriuslita, Z., Nofriyandi, N., & Istikomah, E. (2020). The effect of GeoGebra-assisted direct instruction on students' self-efficacy and self-regulation. *Infinity Journal*, 9(1), 41– 48. <https://doi.org/10.22460/infinity.v9i1.p41-48>
47. Zulnadi, H., & Syed Zamri, S. N. A. (2017). The effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. *Eurasia Journal of Mathematics, Science and Technology*. <https://doi.org/10.12973/eurasia.2017.01219>